Do Exclusivity Arrangements Harm Consumers?*

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Abstract

This paper explores the ramifications of exclusivity arrangements, e.g., iPhone’s partnership with wireless carriers, for market competition and consumer welfare. Two firms compete in a primary good market, and a monopolistic firm offers a value-adding good. The primary good can be consumed alone, while the value-adding good must be consumed with the primary good. The monopolistic firm forms an exclusivity partnership with one of the primary good providers. Buyers are able to consume the value-adding good only if they patronize the monopolistic firm’s exclusive partner. This practice allows the monopolistic firm to extract surplus from the primary good market. Surprisingly, consumers benefit from the exclusivity arrangement. However, overall social welfare declines, despite improvements to consumer welfare.

JEL Nos: L1, L2, L4, L5

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1 Introduction

Firms often use exclusivity arrangements to exploit their market power. For instance, public interest was aroused by Apple’s initial marketing strategy for the iPhone, in which Apple has granted only a few wireless carriers the exclusive right to carry the product. In the U.S., Apple locked its iPhone exclusively into AT&T’s network for nearly four years before extending a contract to Verizon in early 2011. Prior to March 2012, China Unicom had been the exclusive carrier in China, Apple’s second-largest market. The suspension of iPhone’s exclusivity practice in the European and Singaporean markets is largely due to an “unfavorable” legal environment, which Apple had not foreseen.¹

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¹Source: “German Court Ruling Triggers Major Review for iPhone Sales across Europe,” Global Insight, November 2007.
In this paper, we provide a stylized analysis of exclusivity arrangements in a context that resembles Apple’s iPhone case. The model includes an assembly between an “upstream” firm (e.g., Apple) that occupies a substantially wide market niche and duopolistic “downstream” firms (e.g., wireless carriers). Each of the downstream firms produces a primary (essential) good, which provides a platform for consumers to use a value-adding (nonessential) complementary good offered by the upstream firm. An exclusivity partnership between the monopolistic firm and one of the duopolistic firms limits the availability of the value-adding good on the other platform. As a result, the buyers of the value-adding good will be forced to patronize the exclusive partner of the monopolistic firm. We provide an equilibrium analysis of firms’ behavior under such an exclusivity arrangement. This allows us to formally evaluate the ramifications of exclusivity practice for consumer welfare and market efficiency.

Exclusivity arrangements exist in many other contexts. For example, Electronic Arts, a major game developer, has launched games that can only be played on Sony’s PlayStation. Many publishers sell electronic versions of their works exclusively on selected platforms, e.g., Amazon’s Kindle. However, the exclusivity arrangement between Apple and its partner wireless carriers has unique characteristics that fundamentally distinguish it from the usual practice, in which the primary good provider (e.g., the Kindle and the iPad), rather than the value-adding good provider (e.g., e-books and video games), typically predominates in an exclusivity partnership. The bargaining power of upstream firms is often largely limited by the nonessential nature, functional dependence, and/or ample supply of close substitutes of the value-adding goods they produce. Their sales largely rely on the extensive consumer base and distribution networks of the primary good (i.e., platform) providers. It is the platform’s strategies, as a market intermediary between consumers and value-adding or complementary good providers, that contribute more to the ultimate market structure (see Rysman, 2009).

The opposite, however, is observed in the iPhone case: Its marketing strategy has been widely regarded as an attempt to “change the existing relationship radically between mobile handset manufacturers and mobile operators.” Apple’s continuing marketing success and unique product image, along with its independently integrated product lines, allow its products (e.g., the iPhone) to substantially differentiate themselves from rival devices and acquire a unique and wide market niche. Its leadership in the smartphone-handset market has given

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2Our current model would require modifications to fit the aforementioned examples where the primary-product providers have more market power. For example, regarding the timing of the game, the duopolistic firms would move first to price the primary product, followed by the monopolistic firm to price the value-adding product. In addition, the exclusive firm would negotiate a more favorable contract to extract the surplus from the exclusivity arrangement. Instead of surrendering all the rents generated from the exclusivity, it could seek an even-split or more, depending on its bargaining position relative to the monopolist’s (Dukes and Gal-Or, 2003).

3Source: “German Court Ruling Triggers Major Review for iPhone Sales across Europe,” Global Insight, November 2007.
Apple the upper hand in its negotiations with wireless carriers. Its exclusivity arrangement has substantially affected the balance of power in the downstream wireless market, and has been viewed by carriers as an effective means of preempting their rivals. For instance, the CEO of China Mobile, the leading wireless provider in the massive Chinese telecommunication market, acknowledged in public that the firm had been under strong pressure to include the iPhone in its lineup in order to please its unhappy customers.\footnote{Source: “Updated: China Mobile wants iPhone and iPad,” iPhonAsia.com, March 2010.} Verizon explicitly demanded, in its negotiations with Apple, that Sprint and T-Mobile be excluded from iPhone’s distribution channels.\footnote{Source: “Verizon may pay Apple for iPhone semi-exclusive,” by Marguerite Reardon, CNET news (http://news.cnet.com/8301-30686_3-20024767-266.html).}

Apple’s distinctive exclusivity practice has caused substantial controversy and aroused strong regulatory concerns. In 2009, four U.S. senators led a petition to persuade the Federal Communications Commission (FCC) to investigate the exclusivity arrangement between Apple and AT&T.\footnote{They argued that “for many consumers, the end result of these exclusionary arrangements is being channeled to purchase wireless service from a carrier that has monopolistic control over the desired handset and having to pay a premium price for the handset because the market is void of any competition for the particular handset” (Source: “Department of Justice launches review of handset arrangements,” by Tom Braithwaite and Richard Waters, Financial Times, July 7, 2009).} The FCC and the Department of Justice (DOJ) then launched an investigation into exclusionary handset arrangements. In Paris, a court rejected Apple’s exclusivity agreement with France Telecom and ordered Apple to unlock the mobile device, a ruling that was widely regarded as a victory for French consumers. Despite the high-profile debates caused by iPhone’s exclusivity, its implications for market efficiency and consumer welfare have yet to be formally analyzed. This paper attempts to fill in the gap.

Our study serves as an early step to uncover various underlying concerns about this popular practice. Yet, this analysis yields some interesting implications. In contrast to the popular view that exclusivity arrangements jeopardize consumer welfare, our analysis demonstrates otherwise. Our results cast doubt, for instance, on the court ruling against Apple in France on the grounds that it threatened consumers’ interests. Even so, the results of welfare analysis should be interpreted with caution. First, the practice redistributes surplus among different consumers, and leads some to gain at the expense of others. Second, social welfare declines despite the gains in consumer welfare. These observations highlight the complexity of evaluating the ramifications of exclusivity arrangements.

We offer a stylized but potentially useful analysis of an economic relationship that resembles the iPhone context, with a particular focus on the strategy of the upstream firm (value-adding good provider) and its impact on competition in the downstream (platform) market and consumer welfare. Aside from the iPhone deal, we observe other similar but less pronounced applications of this setting: for some examples, under a multi-million-dollar...
contract, Hulu has acquired exclusive streaming video on demand (SVOD) rights to CBS’s hit drama series, *CSI*, since April 2015;\(^7\) so has Amazon Prime to PBS’s “phenomenally successful” *Downton Abby* since June 2013.\(^8\) Similarly, artist exclusivity has emerged where top artists including Jay Z and Taylor Swift would leverage their popularity when negotiating exclusive deals with music streaming services.\(^9\) In the motion pictures industry, Disney is the first major studio to have secured a hefty $300 million in exchange for Netflix to offer exclusive streaming of all its new releases from September 2016.\(^10\)

In our model, consumers are uniformly distributed on a unit square. Two duopolistic downstream firms, which produce a primary good, are located at opposite ends of the \(x\)-axis, while a monopolistic upstream firm, which produces a value-adding good, is located at the center of the \(x\)-axis. The primary good can be consumed alone, while the value-adding good must be used together with the primary good in a fixed (one-to-one) proportion. In the exclusivity case, the game proceeds as follows. First, the monopolistic firm announces its exclusivity contract, which specifies the price of its value-adding good. Second, it runs an auction to sell its exclusive partnership, and invites the duopolistic primary good providers to bid for the partnership. Under the exclusive arrangement, the monopolistic firm “locks” its product to the primary good offered by its exclusive partner. Finally, the duopolistic firms simultaneously set their prices, and consumer purchases take place subsequently. In the multi-homing case, when the assumption of exclusivity is relaxed, the monopolistic firm allows both duopolistic firms to sign a contract for selling the value-adding good.

We show that, under certain assumption about consumer valuation of the value-adding good, the monopolistic firm benefits from exclusivity in spite of its loss of market share due to exclusion. This practice allows it to leverage its market power in the (nonessential) value-adding good market so as to extract surplus from the primary good market. Under exclusivity, the monopolistic firm has every incentive to forgo the profit from the value-adding product market, so as to use another vehicle (i.e., an exclusivity fee) to extract the lost consumer surplus. Paradoxically, consumers as a whole also benefit from the exclusivity practice. The reasons are as follows.

Exclusivity fundamentally alters all three firms’ pricing behaviors, leading to an overall

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\(^7\)Source: “Hulu to stream all episodes of *CSI*,” by Daniel Hurwitz, USA Today, February 19, 2015 (http://wwwusatoday.com/story/life/web-to-watch/2015/02/19/hulu-to-stream-all-episodes-of-csi/23663329/).


gain in consumer welfare. First, exclusivity enhances price competition: The excluded firm is forced to undercut its rival to avoid losing its market share even further, which intensifies price competition and, in turn, benefits consumers in the primary good market. This is a “competition-enhancing” effect. Second, under exclusivity, the monopolistic firm strategically “underprices” its value-adding good through a two-part pricing strategy. The lower price boosts demand for the good and increases the appeal of the primary good offered by its partner. This effect amplifies the rent that accrues to its exclusive partner, which allows the monopolistic firm to recoup the foregone sales revenue (from its value-adding good) through the higher revenue from the bidding contest, and consumers of the value-adding product benefit from the low price. Furthermore, the two-part pricing strategy strengthens the competition-enhancing effect: A lower price for the value-adding good would further handicap the excluded firm in the primary good market, thereby compelling it to undercut its rival even more.

The rest of the paper is organized as follows. A review of the relevant literature is provided in Section 2. The model is set up in Section 3 and Section 4 analyzes the exclusivity and multi-homing cases in turn. Section 5 discusses welfare implications and Section 6 concludes.

2 Relevant Literature

Our analysis is related to the extensive literature on tying and bundling. The conventional framework for tying usually involves a multi-product firm that monopolizes at least one good, and focuses on the firm’s incentive to bundle its own products. A tie-in sale has been interpreted as a price-discriminating device (see Adams and Yellen, 1976; McAfee et al., 1989; Bakos and Brynjolfsson, 1999; and Armstrong, 2006a), or as a foreclosure or entry-deterrence strategy (Whinston, 1990; Choi and Stefanadis, 2001; Carlton and Waldman, 2002; and Nalebuff, 2004). In a recent paper, Mialon (2014) demonstrates the anti-competitive effect of a bundling strategy when it motivates merger.

A handful of papers have identified tying sales as an effective means of altering price competition between firms. These studies typically involve one firm monopolizing one good and competing against others in the market for another good. Carbajo et al. (1990) and Chen (1997) demonstrate that a firm may prefer to sell its multiple independent goods in bundles, as that creates product differentiation. In contrast, Carlton et al. (2010) assume that consumers only value a system that consists of two goods, with one (primary good) monopolistically supplied and the other (complementary good) competitively supplied. They show that a firm that monopolizes the primary good may prefer to tie its complementary good. Tying allows the monopolist to alter price competition in the complementary good
market, thereby shifting rent from that market to its own.\footnote{In contrast to most existing studies, Carlton et al. (2010) allow for reverse tying, such that consumers can add a second complementary good to the bundle “system.”} Gans (2011) extends Carlton et al. (2010) to a more general framework in which the primary good offered by the monopolistic firm can be consumed alone. Furthermore, consumers value the complementary good offered by different firms asymmetrically. Both Carlton et al. (2010) and Gans (2011) demonstrate the social cost associated with tying and its ambiguous effects on consumer welfare. More recently, Miao (2010) studies a monopolistic system maker’s (e.g., Microsoft’s) decision to introduce a separate application or an upgraded system that integrates the application (bundle) when the application can be supplied by other firms. Miao (2010) focuses on the intertemporal and compatibility concerns of the monopolist in introducing systems of different generations.

Our paper is related to this set of studies because we also focus on a mechanism that leverages the monopolistic power of one market to alter pricing competition in the other and “squeeze” its rent. However, there are a few fundamental differences. First, unlike these studies, we do not consider multi-product firms. In our context, a monopolistic firm artificially locks its own (nonessential complementary) product to its exclusive partner’s primary good, and rent is shifted through a side payment. Second, the monopolistic firm in our model offers a nonessential good whose consumption relies on a competitively supplied essential good that can be consumed alone. This flavor has rarely been included in the literature. Most existing studies assume either (1) that only the monopolist produces the essential good, or (2) that consumers must consume two goods together in a system.\footnote{One notable exception is provided by Gans and King (2006), who consider the bundling of goods between different firms. In this context, two independent and unrelated goods are each produced by two sellers. They show that two coalitions would endogenously arise in equilibrium, each comprising two firms that produce the two goods. Consumers can purchase a bundle of the two goods at discount from either coalition.}

Our paper is also related to the literature on exclusivity arrangements, which has conventionally focused on exclusivity arrangements in vertical distribution channels (e.g., Hart and Tirole, 1990; O’Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Fumagalli and Motta, 2006; and Jing and Winter, 2014).\footnote{There is a small amount of research on exclusivity in the context of two-sided markets. In a recent working paper, Chowdhury and Martin (2016) investigate the relevant conditions under which a platform (e.g., newspapers) may bundle a critical product (e.g., columns and comic strips).} Our study mainly differs from these works in two respects. First, this literature typically studies strategic wholesale contracting between an upstream manufacturer and downstream retailers, with the former selling its product to one of the latter and relinquishing control over retail prices. Our choice of an “affiliated” market over a standard vertical market structure is driven by the iPhone’s unique sales strategy. The contractual arrangement we consider corresponds to the “affiliation” mode (e.g., Hagiu and Lee, 2011),\footnote{The “affiliation” mode is more typically featured in the literature on two-sided markets, which focuses on} under which an upstream firm (e.g., Apple) retains control over
the price of its own product and sells the product directly to consumers, while “locking” its product to that of its downstream exclusive partner. As revealed by our analysis, the ability of the monopolistic firm to price its good triggers profound strategic interactions. It also significantly affects downstream market structure and welfare distribution.

Hagiu and Lee (2011) bridge the two strands of literature on vertical relation and two-sided markets, and are the first to differentiate explicitly between upright sale and affiliation. They analyze how “multihoming” or exclusivity may endogenously arise when content is matched to platforms, in either contractual mode. In contrast, we focus on the welfare implications of exclusivity arrangements under “exclusive affiliation.” Our modeling approach also differs subtly from that of Hagiu and Lee (2011). First, we assume that the monopolistic firm first commits to an exclusivity plan and the two duopolistic firms then bid to become the exclusive partner. Hagiu and Lee (2011) allow competing platforms to offer contracts that specify payments contingent on ultimate affiliation choices (exclusivity or multihoming). Second, Hagiu and Lee (2011) assume that platform providers set their prices first, while we assume that the nonessential, value-adding good provider, as a forward-looking first mover, leads in pricing its goods, which manipulates behavior in the primary good market. These modeling nuances fit different contexts of interest. In particular, our model is intended to reflect the basic premise that the monopolistic firm dominates in the exclusivity partnership, i.e., with a superior ability to choose and commit to business modes and contractual terms. The two papers thus complement each other. We also demonstrate later in the paper that (1) the monopolistic firm benefits from such practice in the current context, and (2) these modeling features are consistent with stylized facts.

Finally, our analysis relates to the licensing literature as well. To provide an analogy, the monopolist in our model represents a licensor who invites downstream firms to bid for the right to sell its value-adding product. The monopolist in our model directly sells the product to consumers. In contrast, the licensor in the literature typically sells (intangible) property rights or assets to licensees, which are then used as a valuable input to produce a final product.

Early studies on licensing consider three selling strategies, including fixed-fee, royalties, and auctions (e.g., Kamien and Tauman, 1984, on fixed-fee plus royalties; Kamien and Tauman, 1986, on fixed-fee vs. royalties; Katz and Shapiro, 1985b, 1986, both on licensing

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15In a sense, both papers assume that dominant firms move first in pricing their goods.

16Several studies in the licensing literature incorporate spatial competition similar to our model. For example, Poddar and Sinha (2004) compare different selling strategies between an outside and inside innovators and show welfare improvements in all cases in the spatial framework; Kabiraj and Lee (2011) introduce product differentiation from both the horizontal and vertical dimensions and argue that fixed-fee licensing is profitable provided the quality differential is sufficiently large.

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auctions) and find that compared to fixed-fee and royalties, the most profitable method is for the inventor to auction licenses (Kamien, 1992). For example, Katz and Shapiro (1986) examine the impact of the licensor’s ownership to downstream firms, both as an independent entity and a joint venture, on market outcome, and consider optimal licensing strategies under different scenarios. More recently, Sen and Tauman (2007) consider a selling strategy that consists of an entry fee and royalties. This is similar to the two-part tariff the monopolist adopts in our context, in which the monopolist receives its rents through a fixed fee and earns zero profit from the sales of its product. The difference is that two different groups – the winning firm and consumers, respectively – pay for each part of the tariff in our model, while the same party pays for both parts under a typical two-part tariff scheme such as that in Sen and Tauman (2007).

3 Setup

We consider a two-good market as illustrated in Fig. 1. A unit mass of consumers is uniformly distributed within the square. Two competing firms \((i = 1, 2)\) provide a primary good \(X\), while a monopolistic firm sells a value-adding good \(Y\). The primary good \(X\) can be consumed alone. The value-adding good \(Y\), however, must be consumed along with \(X\) in a one-to-one proportion. To provide an analogy, suppose that \(X\) represents voice and data services in the wireless market,\(^{17}\) while \(Y\) represents a premium smartphone handset (e.g., an iPhone). Furthermore, the marginal costs of producing these products are normalized to zero.

As depicted in Fig. 1, the two competing firms are located at the end points of the horizontal axis, 0 and 1. The monopolistic firm is located at the center of the \(x\)-axis, i.e., the point with the coordinate \((\frac{1}{2}, 0)\). Each consumer’s preference is characterized by her position \((x, y)\). If a consumer purchases primary good \(X\) from firm 1, she incurs a travel cost of \(d_X x\), while she incurs a travel cost of \(d_X (1-x)\) if she purchases the good from firm 2. Similarly, if a consumer purchases product \(Y\) from the monopolistic firm, she bears a travel cost of \(d_Y y\), where \(y\) measures the cost of mismatch between consumer \((x, y)\) and \(Y\).\(^{18}\)

Each consumer has inelastic demand of up to one unit for each good. She receives a utility \(u\) if she consumes product \(X\) and \(v\) if she consumes product \(Y\). Further, the utility \(u\) is assumed to be sufficiently large to sustain full market coverage of \(X\). If a consumer purchases both \(X\) and \(Y\) from firm 1, she receives \(U_1 = u - p_X - d_X x\) in the primary good market and \(U_Y = v - p_Y - d_Y y\) in the value-adding good market, respectively. If she purchases

\(^{17}\)It is reasonable to assume that each consumer possesses a basic phone that allows him/her to enjoy non-premium voice services. For simplicity, the cost of the basic phone is normalized to be zero.

\(^{18}\)We thank an anonymous reviewer for these valuable points.
both from firm 2, the consumer receives $U_2 = u - p_2^X - d_X(1 - x)$ and $U_Y = v - p_Y - d_Yy$, respectively. When a consumer chooses not to purchase $Y$, she receives zero utility in the value-adding good market. A consumer’s preference between firms 1 and 2 for the primary good is independent of the travel cost she has to bear when consuming $Y$. To provide an analogy, a consumer’s valuation of a smartphone is ex ante independent of preference between wireless networks.

A few remarks are in order for the model. First, this model can be viewed as a variant of Hotelling’s linear-city framework, in which consumers have multi-dimensional preferences. However, the current model differs from most existing models in a multi-characteristic Hotelling game. For example, Irman and Thisse (1998) allow a good to have $n$ characteristics such that firms can differentiate their products in $n$ dimensions. In contrast, both goods in the current model have only one characteristic. Second, for simplicity, we let the monopolistic firm be positioned at the center of the $x$-axis, or $(\frac{1}{2},0)$, and the two primary good providers at $(0,0)$ and $(1,0)$, respectively. However, our main predictions remain qualitatively the same when these simplifying assumptions are relaxed. Holding other things equal (i.e., without an exclusivity arrangement), only horizontal locations matter for firms 1 and 2, while only the vertical location matters for the monopolistic firm. Hence, a more general setting would allow the two duopolistic firms to be located at $(0, y)$ and $(1, y)$, respectively, for any $y \in [0,1]$, and the monopolistic firm at $(x,0)$, with $x$ deviating from $\frac{1}{2}$.

To focus our attention on the most relevant case, it is further assumed that (marginal) travel costs are in an intermediate range. These assumptions are stated as follows.

**Assumption 1** $v < d_X < \frac{7}{6}v$.

**Assumption 2** $v < d_Y < 2v$.

The first assumption rules out the possibility of full foreclosure in equilibrium. That is, when the monopolistic firm locks $Y$ to the primary good offered by one firm, the other firm will not lose its most loyal consumers (i.e., those who are located in the vicinity of the position of that firm). In the second assumption, the condition $v < d_Y$ implies that the market for $Y$ is never fully covered. Exclusivity does not pay off otherwise, and welfare analysis would be less interesting when the monopolistic firm does not implement the strategy in the first place. However, $d_Y$ is also assumed to not be prohibitively high, i.e., $d_Y < 2v$.\textsuperscript{19} Under this condition, the monopolistic firm still retains sufficient market coverage. This assumption thus allows our analysis to focus on the most relevant case, in which the monopolistic firm serves a nontrivial market niche.

\textsuperscript{19}Or, equivalently, the complementary good is assumed to substantially add to consumers’ utility.
4 Analysis

The equilibrium when the monopolistic firm is allowed to practice exclusivity is first derived. Then, we consider a multi-homing case in which the monopolistic firm offers a non-exclusive contract to both duopolistic firms.

4.1 Exclusivity

The monopolistic firm forms an exclusivity partnership with one of the duopolistic firms.\textsuperscript{20}

We consider a three-stage game. The timing of moves is stated as follows.

1. The monopolistic firm announces its exclusivity plan and runs an auction to sell its exclusive partnership. The exclusivity contract commits to locking the monopolistic firm’s product to the primary good offered by the winning firm. It stipulates a price $p_Y \geq 0$ for the value-adding good,\textsuperscript{21} and demands that only consumers of the partner firm be eligible to purchase $Y$ (at the contractually stipulated price $p_Y$).

2. Upon observing the exclusivity plan, the two competing firms bid for the partnership. We assume that the bidding contest takes the form of a sealed-bid, first-price auction. Firms simultaneously submit their bids $b_i \in \mathbb{R}_+$. The higher bidder wins and enters the exclusive partnership. A tie would be broken randomly. Further, the contest has free entry.

3. The duopolistic firms simultaneously announce their prices for $X$, $p_i^X$ ($i = 1, 2$).

4. Consumers observe $p_Y$ and $p_i^X$ and make their purchases.

A few remarks are in order before we proceed to solve for the equilibrium. First, the bargaining between the monopolistic firm and the two downstream firms is modeled as a bidding contest, in which the duopolistic firms submit their bids of lump-sum “subsidies” for the exclusivity partnership. Second, we assume that the monopolistic firm commits to its price $p_Y$ as part of the terms that it demands for the exclusivity deal prior to the bidding contest. Hence, the duopolistic firms price their products after the exclusivity partnership is formed, which leads to a sequential-pricing game. This modeling nuance closely mirrors Apple’s marketing practice for the iPhone and is consistent with casual observations from

\textsuperscript{20}For the purpose of this paper, we do not consider the option of “outright sale” by the monopolistic firm in the model. An outright sale refers to a vertical relation case in which the monopolistic firm does not control the pricing and/or sales of its value-adding good and relays all these decisions to its downstream firm(s).

\textsuperscript{21}For the sake of analytical convenience and expositional efficiency, it is assumed that the monopolistic firm is unable to price $Y$ below its marginal cost, i.e., $p_Y \in [0, \infty)$. This assumption allows for tractability. It can also be interpreted as a regulatory restriction.
the U.S. wireless market. For instance, Apple announced (e.g., in preorder) the price of the iPhone and its network technology long before the associated wireless plan was revealed. This setting reflects the superior bargaining power of the monopolistic firm and its ability to manipulate the behavior of downstream firms (i.e., their pricing). Indeed, stylized facts evidence that wireless carriers have been “more willing to give in to Apple’s terms.” Furthermore, we demonstrate below (see Proposition 2) that when practicing the exclusivity strategy, it is in the monopolistic firm’s best interest to bundle the price of the value-adding good $p^Y$ into the contract.

4.1.1 Price Competition in the Primary Good Market

Without loss of generality, let firm 1 be the winner. Each consumer faces one of three purchase options: (1) purchasing $X$ from firm 2; (2) purchasing $X$ only from firm 1; or (3) purchasing the “bundle” of both $X$ (from firm 1) and $Y$ (from the monopolist). Fig. 1 illustrates the market segmentation with an exclusivity arrangement.

In this scenario, firm 1 secures a larger market share. As Fig. 1 shows, some consumers who would otherwise patronize firm 2 (i.e., those who are located in the right half of the square) may switch to firm 1 if they highly value product $Y$, i.e., when they are located sufficiently close to the center of the horizon axis.

Let $D_i$ denote a firm $i$’s market share. The consumers of firm 1 can be split into two groups. One group purchases $X$ only, which we denote by $D_{1X}$, while the other group purchases both $X$ and $Y$, which we denote by $D_{1XY}$. The following lemma depicts the equilibrium in the primary good market under an exclusivity plan ($p^Y$).

**Lemma 1** (i) The partner firm (firm 1 by default) charges $p^X_1 = d_X + \frac{(v-p^Y)^2}{6d_Y}$ for the primary good and secures a market share of $D_1 = \frac{1}{2} + \frac{(v-p^Y)^2}{12d_Xd_Y}$. A share of consumers $D_{1XY} = \frac{(v-p^Y)^2}{2d_Y} - \frac{(v-p^Y)^3}{6d_Xd_Y^2} + \frac{(v-p^Y)^2}{4d_Xd_Y} + \frac{(v-p^Y)^2}{12d_Xd_Y}$ purchases both $X$ and $Y$. Firm 1 earns a total profit of $\pi^*_1 = \left(d_X + \frac{(v-p^Y)^2}{6d_Y} \right) \cdot \left( \frac{1}{2} + \frac{(v-p^Y)^2}{12d_Xd_Y} \right)$ from the primary good market.

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23 For example, when Verizon announced its iPhone launch in January 2011, it only discussed the price (which started at $199) for the phone, but “wouldn’t discuss service plans” (Source: “Verizon Unwraps iPhone,” by Shayndi Raice and Yukari Iwatani Kane, *Wall Street Journal*, January 12, 2011).
24 Note that wireless carriers independently design their service plans, which are typically uniform within their own wireless networks and independent of the handset a subscriber uses.
26 For the sake of brevity, we do not consider the case in which a foreclosure (i.e., one firm drives the other out of the market by undercutting price) can arise in the equilibrium. This possibility is precluded by the two regularity assumptions stated in Section 3.
(ii) Firm 2 charges a price \( p_X^2 = d_X - \frac{(v-p_Y)^2}{6d_Y} \), secures a market share of \( D_2 = \frac{1}{2} - \frac{(v-p_Y)^2}{12d_Xd_Y} \), and earns a profit of \( \pi^*_2 = (d_X - \frac{(v-p_Y)^2}{6d_Y}) \cdot \left( \frac{1}{2} - \frac{(v-p_Y)^2}{12d_Xd_Y} \right) \).

(iii) Firm 1 secures a greater market share than firm 2, i.e., \( D_1 > D_2 \).

Proof. See Appendix. ■

Comparing the equilibrium prices for \( X \), it is straightforward to see that \( p_X^1 \) is greater than \( p_X^2 \). That is, to obtain the privilege of purchasing \( Y \), firm 1’s consumers have to pay a higher price for \( X \) than those who remain with firm 2 and forgo the opportunity of buying \( Y \) now enjoy a discount in the primary good market: Firm 2 must undercut to retain its clientele.

4.1.2 Equilibrium at the Bidding Stage

As Lemma 1 indicates, exclusivity allows the winning firm to obtain a competitive edge in the primary good market. For a given \( p^Y \), it earns an extra profit of \( \Delta \pi = \pi_1 - \pi_2 = \frac{(v-p_Y)^2}{3d_Y} \). The bidding subgame boils down to a two-player symmetric complete-information auction. Each firm \( i \) always has an incentive to marginally outbid the rival for any bid \( b_j \in [0, \Delta \pi] \).

As a standard result, each firm bids \( \Delta \pi \) in the unique pure-strategy Nash equilibrium, and one firm (firm 1 by default) is chosen as the exclusive partner.\(^{27}\)

4.1.3 Equilibrium Exclusivity Plan

Now we turn our attention to the equilibrium strategy of the monopolistic firm. The firm collects a profit of \( \pi^Y(p^Y) \) from selling \( Y \). It also receives revenue through the partner firm’s bid \( \Delta \pi \). Hence, the overall profit of the monopolistic firm is given by \( \pi^m = \pi^Y(p^Y) + \Delta \pi \). At the beginning of the game, the monopolistic firm chooses \( p^Y \in [0, \infty) \) to maximize \( \pi^m \), internalizing its effect on primary good market competition. Standard technique yields the solution to the subgame perfect equilibrium of the game.

Proposition 1 (a) In the unique subgame perfect equilibrium of the game, the monopolistic firm charges \( p_Y = 0 \) for the value-adding good and receives a profit (from the winning bid) of \( \pi^{m*} = \frac{v^2}{3d_Y} \).

(b) The partner firm (firm 1 by default) secures a market share \( D_1^* = \frac{1}{2} + \frac{v^2}{12d_Xd_Y} \) and earns \( \pi_1 = \left( d_X + \frac{v^2}{6d_Y} \right) \cdot \left( \frac{1}{2} + \frac{v^2}{12d_Xd_Y} \right) \) from the primary good market, while the losing firm (firm 2 by default) secures a market share \( D_2^* = \frac{1}{2} - \frac{v^2}{12d_Xd_Y} \), and earns \( \pi_2 = \left( d_X - \frac{v^2}{6d_Y} \right) \cdot \left( \frac{1}{2} - \frac{v^2}{12d_Xd_Y} \right) \).

\(^{27}\)One would obtain the same equilibrium outcome if the contest takes the form of an English auction with alternate bidding rules.
Proof. See Appendix. ■

In equilibrium, the monopolistic firm simply charges the marginal cost \( p^Y = 0 \) to consumers and earns zero profit from retailing the product. The foregone revenue from selling \( Y \) is compensated for by a higher bid at the auction. Under this two-part pricing strategy, the monopolistic firm charges the value-adding product at its marginal cost and does not profit from retailing its own product. The low price, however, allows its exclusive partner to acquire a greater advantage in the downstream primary good market. The extra profits that the exclusive partner receives under the contract eventually find their way back to the monopolistic firm through a higher bid, which compensates for the sales revenue that the monopolistic firm has sacrificed.

This prediction of the monopolistic firm’s underpricing is largely consistent with the stylized facts in the iPhone case. For instance, the price of an iPhone 4 ranges from $199 to $299, depending on model specifications. Apple pays manufacturers an average of $244 for each iPhone, according to Apple’s financial filings.\(^{28,29}\) Although the details of Apple’s exclusivity contracts have not been disclosed, it was estimated that AT&T paid Apple an average of approximately $550 for each iPhone under its exclusivity contract.\(^{30}\)

4.1.4 Discussion

In setting up the model, we have assumed that the monopolistic firm moves first in pricing its product and that it bundles its price in the exclusivity contract. This assumption arguably reflects the monopolistic firm’s superior ability to dominate its negotiations with the downstream duopolistic firms.

We now relax this assumption and allow the monopolistic firm to not commit to \( p^Y \) during the bargaining process, but to announce it after the bidding. The underlying question is whether it pays for the monopolistic firm to include \( p^Y \) in the exclusivity contract. Next, we consider a case in which the three firms are allowed to set their prices after an exclusivity contract has been awarded to the winning bidder. The following result is obtained.

**Proposition 2** The monopolistic firm strictly prefers to bundle its price \( p^Y \) into the exclus-

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\(^{29}\)When the iPhone was first introduced to the market, it had a price tag of $499. The high price, as a typical marketing tool to sell “hot” new products, contained a premium that early adopters were willing to pay. The price of an iPhone quickly declined and stabilized. The current price schedules should be considered as a more appropriate benchmark, as they are set to target mainstream consumers instead of a small group of early adopters. The price dynamics of the iPhone are consistent with those of many other popular electronics products when newer models are released and technology advances. Our model, however, is not designed to capture the dynamic features of Apple’s pricing strategy.

Proof. See Appendix.

If the monopolistic firm sets the price after settling the amount of transfer in exchange for the exclusivity partnership, it would then be tempted to charge a higher price for its value-adding good. The higher price leads to a loss of market share for the monopolistic firm, which reduces the rent its exclusive partner could collect ex post from the primary good market. Anticipating that, the duopolistic firms would bid less for the partnership, which jeopardizes the overall profit of the monopolistic firm. We conclude that the monopolistic firm prefers to commit to its price and include it in the exclusivity contract.

In the current setting, we consider Apple as a monopolistic firm in the value-adding good market (i.e., the iPhone market). However, in the more broadly defined smartphone market, multiple producers, including Samsung and Nokia, coexist. In the presence of competition, depending on whether or not Apple practices exclusivity, rivals would certainly react differently. The exclusivity deal increases firm asymmetry/product differentiation in the primary good market, which would in turn soften competition. Formal modeling of this situation, however, is beyond the scope of this paper.

4.2 Multi-Homing

We now relax the assumption that the monopolistic firm commits to contract for exclusivity. The timing of moves is as follows.

1. The monopolistic firm charges a fee $\lambda$ for a contract, but does not restrict the number of signing parties. It stipulates a price $p_Y \geq 0$ for the value-adding good, and consumers of a signing firm may be eligible to purchase $Y$.

2. Upon observing the contract, the duopolistic firms simultaneously make their signing decisions.

3. The duopolistic firms simultaneously announce their prices for $X$, $p_i^X (i = 1, 2)$.

4. Consumers observe $p_Y$ and $p_i^X$ and make their purchases.

The competition in the primary good market is analogous to that in a conventional Hotelling model. The demand for firm 1 is then determined by the equation $u - p_1^X - d_X x = \ldots$ for the sake of analytical convenience and expositional efficiency, it is assumed that the monopolistic firm is unable to price $Y$ below its marginal cost, i.e., $p_Y \in [0, \infty)$. This assumption allows for tractability. It can also be interpreted as a regulatory restriction.
\( u - p_2^X - d_X(1 - x) \). We then have

\[
D_i = \frac{(p_j^X - p_i^X) + d_X}{2d_X}.
\]  

(1)

Hence, a firm \( i \)'s profit function \( \pi_i(p) = p_i^X (p_j^X - p_i^X + d_X) \). A unique equilibrium exists, in which firms each charge \( p_1^{X*} = p_2^{X*} = d_X \) and earn (net of any contract fee) \( \pi_1^{X*} = \pi_2^{X*} = \frac{d_X}{2} \).

In Fig. 1, all consumers on the left half of the square purchase \( X \) from firm 1 and those on the right half from firm 2.

At the signing stage, in equilibrium, both duopolistic firms sign the contract for selling product \( Y \), under a fee structure outlined in the following lemma.

**Lemma 2** The duopolistic firms are willing to pay a contract fee up to \( \lambda^* = \frac{(v-p^Y)^2}{12d_Xd_Y} \) for the right to sell product \( Y \).

**Proof.** See Appendix. ■

The signing decisions resemble a game of prisoners’ dilemma. The duopolistic firms both sign the contract in equilibrium, although they would be better off if neither signs. However, the latter is not an equilibrium, because either would have incentive to be the exclusive partner.

In the market for product \( Y \), the marginal consumer is determined by setting \( v - p^Y - d_Y y = 0 \), which leads to a demand of \( \frac{(v-p^Y)}{d_Y} \). The monopolistic firm thus collects profits from selling \( Y \) and contract fees, or \( \pi_Y = \frac{(v-p^Y)p^Y}{d_Y} + 2\lambda^* \). Hence, at the beginning of the game, the monopolistic firm chooses \( p^Y \in [0, \infty) \) to maximize \( \pi_Y \), internalizing its effect on primary good market competition. Standard technique yields the equilibrium outcome of the game.

**Proposition 3** (a) Under multi-homing, in equilibrium, the monopolistic firm charges \( p^{Y*} = w - \frac{4d_Xd_Y}{w} \) where \( w = \sqrt[3]{64d_X^3d_Y^3v^3 + \frac{81}{4}d_X^2d_Y^2v^2 + \frac{9}{2}d_Xd_Yv} \) for the value-adding good and receives a profit (inclusive of contract fees) of \( \pi_Y^{*} = \frac{(v-w+\frac{4a}{b})^2}{3b} (2 + \frac{(v-w+\frac{4a}{b})^2}{4ab}) \).

(b) The duopolistic firms split the primary product market and each earns a profit of \( \frac{d_X}{2} - \frac{(v-w+\frac{4d_Xd_Y}{6d_Y})^2}{12d_Xd_Y} \).

(c) There exists \( p^{Y*} \in (0.26v, 0.29v) \) that maximizes the equilibrium profit under multi-homing, \( \pi_Y^{*} \).

**Proof.** See Appendix. ■

Proposition 3 indicates that consumers pay a higher price for the value-adding product under multi-homing than exclusivity. With multi-homing, the intensive competition in the
primary product market, due to symmetry, reduces the extractable rent through contract fees for the monopolistic firm, who now finds it optimal to charge a higher price for $Y$ than in the exclusivity case. Graphically, everyone located in the area where $y < y_0$ in Fig.1 purchases $Y$ from the monopolistic firm.

5 Who Benefits from an Exclusivity Arrangement?

The above equilibrium result allows us to explore the welfare implications of exclusivity arrangements.

5.1 Comparison

For the computation to be tractable, we normalize $v$ to be one when comparing the equilibrium outcomes between exclusivity and multi-homing.

**Proposition 4** When $v = 1$, the monopolistic firm earns a higher profit under exclusivity $(\frac{v^2}{3d_Y})$ than it would receive in the multi-homing case $(\pi_Y^*)$.

**Proof.** See Appendix. ■

Proposition 4 claims that the monopolistic firm prefers exclusivity to multi-homing in the value-adding product market, at least for several values of $v$.\(^{32}\) Under exclusivity, the monopolistic firm strategically “underprices” its value-adding good through a two-part pricing strategy. The lower price boosts demand for the good, which in turn increases the appeal of the primary good offered by its partner (a “product differentiation” effect). This effect amplifies the rent that accrues to its exclusive partner, which allows the monopolistic firm to recoup the foregone sales revenue (from its value-adding good) through the higher revenue from the bidding contest. In comparison, under multi-homing, the symmetry in the primary product market intensifies price competition, which lowers the rent extracted by the monopolistic firm through contract fees. The logic for this observation will be further explained in Section 5.3.

By making the value-adding product exclusively available, the monopolistic firm inevitably sacrifices market share.\(^{33}\) In the context of the iPhone, as of 2009, the iPhone

\(^{32}\)In addition, one might assume there exists some cost, $C$, of contracting with each provider. These might also represent the costs to wireless companies of making their networks compatible with the iPhone, for example, which would reduce extractable rents from the multi-homing equilibrium, and for sufficiently high costs, it would further induce the monopolistic firm to opt for exclusivity.

\(^{33}\)It can be shown that the monopolistic firm faces a smaller market share for $Y$ under exclusivity $(\frac{v}{2d_Y} - \frac{v^3}{6d_Xd_Y} + \frac{v}{4d_Xd_Y})$ than multi-homing $(\frac{v-p_Y^*}{d_Y}$ where $p_Y^* \in (0.26v, 0.29v)$). Under Assumptions 1&2, $\frac{v}{2d_Y} - \frac{v^3}{6d_Xd_Y} + \frac{v}{4d_Xd_Y} < \frac{v-0.20v}{d_Y} < \frac{v-p_Y^*}{d_Y}$, which holds since $\frac{v}{d_X}(\frac{1}{2} - \frac{v}{3d_Y}) < 0.42$. 

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only accounted for 2.5% of the headset market worldwide. Surprisingly, however, its operating profit surpassed those of the industry leaders.\textsuperscript{34} Moreover, offering a single version of the iPhone during the initial phase was a risk-averse and \textit{efficient} strategy. Apple had chosen AT&T to be the exclusive carrier not just because it was then the largest wireless provider in the U.S., but more importantly, for a practical reason: AT&T operates on a GSM network, which perfectly fits Apple’s ambition of marketing the iPhone globally, while Verizon and Sprint are both on CDMA networks.\textsuperscript{35}

5.2 Consumer Welfare

Primarily, we investigate the effect of exclusivity on consumer welfare. Consumers derive utility from consuming \( X \) and \( Y \). They make payments to these firms in exchange for the products, and also bear the travel costs. Let \( W_1 \) and \( W_2 \) denote consumer welfare in the exclusivity case and in the multi-homing case (without exclusivity), respectively. Our analysis allows us to conclude the following:

Proposition 5 \textit{Consumers as a whole benefit from exclusivity arrangements, i.e.,} \( W_1 > W_2 \).

\textbf{Proof.} See Appendix.

Overall, consumer surplus increases when exclusivity is implemented. In contrast to the prevailing view, our analysis suggests that banning exclusivity arrangements may, paradoxically, hurt consumers as a whole.

A number of effects loom large in the presence of exclusivity arrangements. On the one hand, exclusivity generates two positive effects on consumer welfare. First, the monopolistic firm strategically underprices \( Y \), which effectively internalizes the externality between \( Y \) and firm 1’s \( X \) as a pair of complements. Under exclusivity, customers (in the red area in Fig. 1) who patronize the exclusive partner (firm 1) receive not only \( X \), but also the privilege of enjoying \( Y \) (at an additional price \( p_Y \)). Hence, a lower \( p_Y \) makes firm 1’s \( X \) more appealing than its rival’s, which amplifies the rent from the exclusivity arrangement, thereby inflating firms’ bids. By underpricing \( Y \), the monopolistic firm sacrifices its retailing revenue but is “subsidized” by firm 1 through its bid. This practice allows a subset of consumers (\( D_{1XY} \)) to enjoy \( Y \) at a lower price. The mechanism is referred to as a two-part pricing strategy.

Second, this practice intensifies price competition in the primary good market. In equilibrium, firm 2 charges a lower price than it does in the multi-homing case. Handicapped


by firm 1’s exclusivity partnership, firm 2 undercuts its rival to protect its clientele. Firm 1, as the exclusive partner, demands a premium price for its $X$. However, its product also entitles a consumer to buy $Y$ at a low price, which could also improve consumer welfare. In summary, exclusivity intensifies price competition in the primary good market, thereby allowing (a subset of) consumers (in the white area to the right of $x = \frac{1}{2}$ and above $y = y_0$ in Fig. 1) to pay less for their consumption of $X$. This mechanism is referred to as a “competition-enhancing” effect.

Furthermore, the competition-enhancing effect has a positive interaction with the two-part pricing strategy. Note by Lemma 1 that $p_2^X$ and $p^Y$ are strategic complements, i.e., $\frac{\partial p_2^X}{\partial p^Y} > 0$. The competition-enhancing effect is magnified when the price of $Y$ is lower. A lower $p^Y$ exacerbates firm 2’s disadvantage, which adds downward pressure to its pricing of $X$.

On the other hand, this practice distorts market competition and consumer behavior, thereby resulting in disutility to consumers as well. Exclusivity inflicts welfare loss on four subsets of consumers. First, a subset of firm 1’s “loyal customers” (those who are located sufficiently close to zero along the $x$-axis) would purchase $X$ only (in the blue area in Fig. 1). They end up paying more for $X$, because firm 1 charges a premium price. Second, a subset of firm 2’s initial customers (in the red triangle area to the right of $x = \frac{1}{2}$ in Fig. 1) in the multi-homing case, who highly value $Y$, would switch to firm 1. The benefit these consumers receive from a lower $p^Y$ can be offset by the higher travel costs required for consuming $X$. Third, a subset of firm 1’s initial customers (in the white area to the left of $x = \frac{1}{2}$ in Fig. 1), who purchase $X$ only, would switch to firm 2 because of its lower price for $X$. These consumers bear a higher travel cost to take advantage of paying less for $X$. Finally, a subset of firm 2’s “loyal customers” (in the white area to the right of $x = \frac{1}{2}$ and below $y = y_0$ in Fig. 1) are excluded from consuming $Y$. Taken together, the positive effects unambiguously dominate the negative ones.

Consumers, as a whole, benefit from the practice. Additional analysis reveals the intricate redistribution of consumer surplus: while consumers, as a whole, realize a gain from the market of $Y$, they suffer a loss in the market of $X$ due to the distortion.\footnote{See the detailed proof for Proposition 5 in the Appendix.} In summary, the overall gain in the value-adding good market dominates the loss in the primary good market, leading to an overall rise in welfare under exclusivity. This result adds new insight into the iPhone’s widely debated exclusivity practices. However, the result must be interpreted with caution, as exclusivity triggers welfare redistribution among consumers: Some gain at the expense of others. Furthermore, our welfare analysis is based on assumptions of inelastic demand and a covered market for the primary product.
5.3 Primary Good Producers

We now explore the effects of exclusivity on the profitability of the primary good industry and obtain the following.

**Proposition 6** *In the primary good market, both firms are worse off when the monopolistic firm is allowed to practice exclusivity. Moreover, overall profit from both product markets declines under exclusivity.*

**Proof.** See Appendix.

With a higher product price and an expanded market share, firm 1 receives higher profits from its sales in the primary good market under exclusivity. However, the two duopolistic firms are left in a fierce bidding war. By practicing exclusivity, the monopolistic firm is able to leverage its market power to extract surplus from the primary good market, as the winning firm surrenders its rent in the primary good market through its bid. Meanwhile, the losing firm responds by aggressively undercutting the price, which in turn worsens the outside option for the winning firm and allows the monopolistic firm to charge an even higher fee. The monopolistic firm’s ability to practice exclusivity jeopardizes both firms’ profitability in the primary product market.

Another interesting observation is that overall profit declines under exclusivity, which implies that the monopolistic firm benefits from the exclusivity arrangement at the expense of the competing duopolistic firms. In theory, exclusivity may be profitable for the monopolistic firm – even if both duopolistic firms (before submitting the bidding fee) earn less than in the multi-homing case – as long as the loss to the losing firm sufficiently exceeds that to the winning firm.

Our results are consistent with the stylized facts: It has been reported that AT&T suffered a loss from its exclusive iPhone deal with Apple. Even though it had successfully lured subscribers away from its competitors, AT&T ended up with a dip in its profits: In the second quarter of 2009, profits fell by $0.27 billion compared to the same quarter a year earlier.\(^{37}\) AT&T’s loss was mainly due to the heavy subsidy it paid for the iPhone. As previously mentioned, on average, AT&T was estimated to have subsidized $550 of the price of each iPhone, which is $200 - $300 more than the estimated cost of other smartphones.\(^{38}\) AT&T’s major competitors’ profits also declined during the same period.


5.4 Social Welfare

In sum, both the monopolistic firm and consumers (as a whole) benefit from exclusivity arrangements, while the duopolistic firms in the primary good market lose. Despite the gain in consumer surplus, the overall social surplus declines under exclusivity, which is stated as follows.

**Proposition 7** Social welfare declines as the result of an exclusivity arrangement.

**Proof.** See Appendix.

Under the assumption of full coverage and perfectly inelastic demand, we show that the cost of this practice in the primary product market (for the two duopolistic firms) more than offsets the gains that accrue to consumers and the monopolistic firm. Our analysis thus indicates the complexity of evaluating the ramifications of exclusivity arrangements: Social welfare falls in our model under exclusivity when the two competing firms’ profits in the primary good market are taken into consideration.

5.5 An Auxiliary Case

We now introduce an alternative case to multi-homing as a benchmark to compare with exclusivity. Such a case may arise when the monopolistic firm is prohibited from charging a lump-sum fee absent an exclusive contract. Specifically, all firms price and sell their products independently. Consumers who purchase $Y$ can purchase $X$ from either duopolistic firm. The timing of the game is as follows:

1. The monopolistic firm announces its price $p^Y$.
2. The duopolistic firms simultaneously announce their prices for $X$, $p^X_i$ ($i = 1, 2$).
3. Consumers observe $p^Y$ and $p^X_i$ and make their purchases.

As in the multi-homing case, the duopolistic firms are again symmetric in the market for $X$. A unique equilibrium exists, in which firms each secure a profit of $\frac{dX}{2}$ by selling $X$ at $dX$. In the market for product $Y$, the monopolistic firm faces a demand of $\frac{(v - p_Y)}{dy}$, and its profit is given by $\pi(p_Y) = \frac{p^Y(v - p^Y)}{dy}$. In equilibrium, it charges $\frac{v}{2}$ and earns a profit of $\frac{v^2}{4dy}$.

All results in this paper would carry over when we compare exclusivity with this auxiliary case.\[39\] However, the monopolistic firm’s motive for exclusivity would be different. In essence, the monopolistic firm successfully eliminates market inefficiency through the employment of exclusivity arrangement; that is, it collects profit via a lump-sum payment from the

\[39\] These results and related proofs are available from the authors upon request.
exclusive partner, rather than under the “inefficient” monopoly pricing. By removing the monopoly marginalization, the exclusivity arrangement becomes welfare-enhancing. In that case, exclusivity may be deemed to reflect monopolistic price flexibility.\footnote{We thank an anonymous reviewer for this insightful point.}

6 Conclusion

In this paper, we construct a stylized model to investigate the welfare implications of exclusivity arrangements, using the iPhone as a motivating example, given the wide influence and unique characteristics of its marketing practice. We demonstrate that this practice distorts competition and leads to market inefficiency. Consumers as a whole, however, benefit from it. The practice leads to redistribution among consumers, as some gain at the expense of others. Meanwhile, the monopolistic firm extracts additional surplus from the primary good market, which renders the two firms in that market strictly worse off. Overall, social welfare declines.

Our analysis offers one perspective on the controversial practice of exclusivity arrangements and yields useful implications. Caution must be exercised, however, when interpreting the findings, as the analysis is built on a stylized model that glosses over many nuances, such as strategic interactions among firms that practice exclusivity arrangements across multiple markets. Our study takes an early step in understanding the ramifications of this popular practice and related welfare implications. Hence, we would caution readers to view our study as a positive exploration that attempts to uncover various underlying concerns, rather than a normative judgment.

A more conclusive appraisal of an exclusivity arrangement’s effects on welfare and competition, which would more constructively addresses antitrust concerns, demands a more comprehensive model to formally include many other relevant factors. A large room for extensions is left open. First, our model assumes inelastic demand and a fully covered market for the primary good; additional qualifications would result if we do not require full coverage. In that case, competition in the primary good market would be less intense, which attenuates the “competition-enhancing” effect that contributes to the gain in consumer welfare – namely, intensified competition brought about by exclusivity. More importantly, this would also reduce the monopolistic firm’s ability to extract rent from the primary good market, which would cast doubt on the profitability of an exclusivity practice in the first place. Second, we do not explicitly consider the monopolistic firm’s incentive to develop innovative products. Our analysis shows that, under certain assumption about consumer valuation of the value-adding good, exclusivity allows the monopolistic firm to reap higher returns, and implies that banning exclusivity adversely affects the firm’s incentive to innovate. The over-
all effect on social welfare needs to be re-examined to address this concern, which deserves a formal analysis in a more general setup. Third, we focus on the niche market of a given product, but abstract away the potential entry of competing substitutes. How exclusivity initiated by market leadership affects subsequent market entry remains an intriguing question. Alternatively, the monopolistic firm’s opportunistic concerns could also be included in the model. Another reasonable setting would allow duopolistic firms to collude in bidding for the exclusivity partnership. In that case, substantially more extensive strategic interactions could be expected, although the modeling framework would be subtle and technically sophisticated. Such extensions are beyond the scope of the current study, but will remain a priority for the authors in future research.

References


Appendix

Proof of Lemma 1

Proof. The mass of consumers who purchase $X$ alone from firm 1 is $D_Y^X = (1 - \bar{y})x = \left(1 - \frac{(y - p_Y)}{d_Y}\right) \left(\frac{p_X^Y - p_Y^X}{2d_X} + dx\right)$, and the mass of consumers who purchase both $X$ and $Y$ is $D_{1}^{XY} =$
Proof. In the value-adding product $Y$ market, we first search for the optimal price \( p_Y \) of the monopolistic firm. We have

\[
\frac{d\pi^m}{dp_Y} = \frac{d\Delta \pi}{dp_Y} + D_1^{XY} + p_Y D_1^{XY} \quad \text{(3)}
\]

\[
= - \frac{2(v - p_Y)}{3d_Y} + (v - p_Y) \left( \frac{1}{2d_Y} - \frac{(v - p_Y)^2}{6d_X d_Y} + \frac{(v - p_Y)}{4d_X d_Y} \right)
\]

\[- p_Y \left( \frac{1}{2d_Y} - \frac{(v - p_Y)^2}{2d_X d_Y} + \frac{(v - p_Y)}{2d_X d_Y} \right) \]

When \( p_Y = 0 \), the last equation becomes

\[
\frac{d\pi^m}{dp_Y} = - v \left[ \frac{1}{6d_Y} + \frac{v^2}{6d_X d_Y} - \frac{v}{4d_X d_Y} \right] \quad \text{(4)}
\]

We now establish the following: Whenever the condition \( \frac{1}{6d_Y} + \frac{v^2}{6d_X d_Y} - \frac{v}{4d_X d_Y} > 0 \) is met,
\( \frac{d\pi_m}{dp^y} \) must be negative for all \( p^y \in [0,v] \). We rewrite (3) as

\[
\frac{d\pi_m}{dp^y} = -v\left[ \frac{1}{6d_Y} + \frac{(v-p^y)^2}{6d_Xd_Y^2} - \frac{(v-p^y)}{4d_Xd_Y} + \frac{p^y}{6d_Y} \right] - p^Y\left( \frac{1}{2d_Y} - \frac{(v-p^y)^2}{6d_Xd_Y^2} + \frac{(v-p^y)}{4d_Xd_Y} \right) + \frac{(v-p^y)}{d_Y} - 1\left( \frac{v-p^y}{2d_Xd_Y} \right) p^Y
\]

(5)

The assumption \( v < \frac{d_Y}{2} \) leads to \( 0 < \frac{(v-p^y)^2}{4d_Xd_Y} - \frac{(v-p^y)}{6d_Xd_Y^2} < \frac{v}{4d_Xd_Y} - \frac{v^2}{6d_Xd_Y^2} \) for all \( p^y \in [0,v] \). Hence, we must have \( \frac{1}{6d_Y} + \frac{(v-p^y)^2}{6d_Xd_Y^2} - \frac{(v-p^y)}{4d_Xd_Y} > 0 \). We now claim that \( -\frac{1}{3d_Y} + \frac{2(v-p^y)^2}{3d_Xd_Y^2} - \frac{3(v-p^y)}{4d_Xd_Y} < \frac{1}{6d_Y} + \frac{(v-p^y)^2}{6d_Xd_Y^2} - \frac{(v-p^y)}{4d_Xd_Y} \). We establish it by verifying \( 3(v-p^y) - \frac{2(v-p^y)^2}{4d_Xd_Y} > \frac{(v-p^y)}{4d_Xd_Y} - \frac{(v-p^y)^2}{d_Y} \), which is equivalent to \( \frac{(v-p^y)^2}{2d_Xd_Y^2} > \frac{(v-p^y)^2}{2d_Xd_Y^2} \). The last inequality holds iff \( \frac{(v-p^y)^2}{d_Y} < 1 \) (by Assumption 1).

Hence, we must have the expression in equation (5) \( -(v-p^y)\left[ \frac{1}{6d_Y} + \frac{(v-p^y)^2}{6d_Xd_Y^2} - \frac{(v-p^y)}{4d_Xd_Y} \right] < 0 \). We then need to verify the condition \( \frac{1}{6d_Y} + \frac{v^2}{6d_Xd_Y^2} - \frac{v}{4d_Xd_Y} > 0 \). It boils down to \( \frac{1}{4} - \frac{v}{d_X}(\frac{1}{2} - \frac{v}{3d_Y}) > 0 \), which follows under Assumptions 1&2. \( \blacksquare \)

**Proof of Proposition 2**

**Proof.** We now allow the monopolistic firm to set \( p^y \) after striking the exclusivity deal. In this case, the monopolistic firm sets \( p^y \) to maximize its sale revenue \( p^Y D_1^{XY} \), instead of \( \Delta \pi + p^Y D_1^{XY} \). It should be noted that for a given \( p^y, D_1^{XY} \) is still the same as that in the basic setting, with \( D_1^{XY} = \frac{(v-p^y)}{2d_Y} - \frac{(v-p^y)^2}{6d_Xd_Y} + \frac{(v-p^y)^2}{4d_Xd_Y} \). In this case, the monopolistic firm must set \( p^y > 0 \). For a given \( p^y \), the monopolistic firm still receives a total profit of \( \pi^m(p^y) = \Delta \pi + p^Y D_1^{XY} = \frac{(v-p^y)^2}{3d_Y} + p^Y\left[ \frac{(v-p^y)^2}{2d_Y} - \frac{(v-p^y)^3}{6d_Xd_Y} + \frac{(v-p^y)^2}{4d_Xd_Y} \right] \). As we have established in the proof of Proposition 1, any \( p^y \) must be strictly suboptimal, because \( \frac{d\pi^m(p^y)}{dp^y} \) strictly decreases with \( p^y \) when \( p^y \in [0,v] \). We then conclude that the monopolistic firm prefers to include \( p^Y \) in its contract. \( \blacksquare \)

**Proof of Lemma 2**

**Proof.** In the signing stage, the duopolistic firms face two options (sign vs. not sign), and receive the payoff summarized in the matrix below:
Proof of Proposition 3

Proof. Under multi-homing, the monopolistic firm would charge $\lambda^*$ to each signing firm, and receives a profit of

$$\pi_Y = \frac{(v - p^Y)}{d_Y} p^Y + \frac{(v - p^Y)^2}{3d_Y} [1 - \frac{(v - p^Y)^2}{12d_X d_Y}]. \quad (6)$$

Evaluating $d_Y \pi_Y$ with respect to $p^Y$ yields

$$\frac{d(d_Y \pi_Y)}{dp^Y} = (v - 2p_Y) - \frac{2(v - p^Y)}{3} + \frac{(v - p^Y)^3}{9d_X d_Y}$$

$$= \frac{(v - p^Y)}{3} [1 + \frac{(v - p^Y)^2}{3d_X d_Y}] - p^Y. \quad (7)$$

Solving the first-order condition of the previous expression, we have

$$p^{Y*} = w - \frac{4d_X d_Y}{w} \quad (8)$$

where

$$w = 3 \sqrt[3]{64d_X^3 d_Y^3 v^3 + 81d_X^2 d_Y^2 v^2 + \frac{9}{2} d_X d_Y v}.$$ Then, the monopolistic firm receives a profit of

$$\pi^*_Y = \frac{(v - w + \frac{4ab}{w})^2}{3b} \left( 2 + \frac{(v - w + \frac{4ab}{w})^2}{4ab} \right). \quad (9)$$

Given equation (7), we have $\frac{d(d_Y \pi_Y)}{dp^Y} \bigg|_{p^Y = 0} > 0$, and $\frac{d(d_Y \pi_Y)}{dp^Y} \bigg|_{p^Y = v} < 0$. Furthermore, $\frac{d^2(d_Y \pi_Y)}{dp^Y} = -\frac{1}{3} [1 + \frac{(v - p^Y)^2}{3d_X d_Y}] - \frac{2(v - p^Y)^2}{9d_X d_Y} - 1 < 0$. Hence, there must exist a unique $p^{Y*} \in (0, v)$.
that maximizes $\pi_Y$. It satisfies $p^*_Y = \frac{(v-p^*_Y)}{3d_Y}[1 + \frac{(v-p^*_Y)}{3d_Xd_Y}]$. We can rewrite (6) as

$$\pi_Y^* = \frac{(v - p^*_Y)}{d_Y} \cdot \frac{(v - p^*_Y)}{3d_Y}[1 + \frac{(v - p^*_Y)}{3d_Xd_Y}] + \frac{(v - p^*_Y)^2}{3d_Y}[1 - \frac{(v - p^*_Y)^2}{12d_Xd_Y}]$$

$$= \frac{(v - p^*_Y)^2}{3d_Y} - \frac{2}{4d_Xd_Y} \cdot \frac{(v - p^*_Y)^2}{4d_Xd_Y}.$$ 

Thus, we can further pin down the optimal price $p^*_Y$ to solve (7). Given that $v^2 < d_Xd_Y < 2.33v^2$, when $p^Y = 0.26v$, we have $\frac{d(d_Y \pi_Y)}{dp^Y}\bigg|_{p^Y=0.26v} = \frac{0.74v}{3}(1 + \frac{0.74v^2}{3d_Xd_Y}) - 0.26v \approx \frac{0.045v^3}{d_Xd_Y} - 0.013v > 0$; When $p^Y = 0.29v$, $\frac{d(d_Y \pi_Y)}{dp^Y}\bigg|_{p^Y=0.29v} = \frac{0.71v}{3}(1 + \frac{0.71v^2}{3d_Xd_Y}) - 0.29v \approx \frac{0.0398v^3}{d_Xd_Y} - 0.053v < 0$. Thus, we conclude that $p^*_Y \in (0.26v, 0.29v)$.

**Proof of Proposition 4**

**Proof.** To compare the profits under exclusivity and multi-homing, we have

$$D = \pi^E_Y - \pi^M_Y$$

$$= \frac{v^2}{3d_Y} - \frac{(v - w + \frac{4d_Xd_Y}{w})^2}{3d_Y} \left(2 + \frac{(v - w + \frac{4d_Xd_Y}{w})^2}{4d_Xd_Y}\right)$$

where \( w = \sqrt[3]{\sqrt[3]{\frac{64d_X^3d_Y^3}{3} + \frac{81}{4}d_X^2d_Y^2v^2} + \frac{9}{2}d_Xd_Yv}. \) For exclusivity to be optimal ($D > 0$), we need $\frac{(v - w + \frac{4d_Xd_Y}{w})^2}{4d_Xd_Y} + 2(v - w + \frac{4d_Xd_Y}{w})^2 - v^2 < 0$, or

$$\left(v - w + \frac{4d_Xd_Y}{w}\right)^2 < 2d_Xd_Y \sqrt{4 + \frac{v^2}{d_Xd_Y}} - 4d_Xd_Y.$$  \(11\)

We can rewrite (11) as \( (v - 3\sqrt{yv\left(\sqrt{y + \frac{81}{64}} + \frac{9}{8}\right)} + \frac{y}{3\sqrt{yv\left(\sqrt{y + \frac{81}{64}} + \frac{9}{8}\right)}}) < \sqrt{y\left(y + v^2\right) - y} \) where \( y = 4d_Xd_Y \). Under Assumptions 1&2, we have \( y \in (4v^2, 9.33v^2) \).

To make the calculation tractable, we normalize \( v \) to be 1. It is then easy to show that $f(y) = (1 - 3\sqrt{y\left(\sqrt{y + \frac{81}{64}} + \frac{9}{8}\right)} + \frac{y}{3\sqrt{yv\left(\sqrt{y + \frac{81}{64}} + \frac{9}{8}\right)}})^2 - \sqrt{y\left(y + 1\right)} + y < 0$, because $f'(y) \neq 0$ when $y \in (4, 9.33)$ (see Fig. 2). Thus, we have shown that $D > 0$ when $v = 1$. In addition, one may infer from Fig. 2 that when $v$ sufficiently close to 1, the same conclusion still holds. Finally, when plotting equation (10), Fig. 3 again illustrates that when $v = 1$, exclusivity is more profitable than multi-homing for the monopolistic firm ($D > 0$).
Proof of Proposition 5

Proof. The equilibrium results allow us to compute the values of \( \bar{x} \), \( \bar{y} \) and \( y \). Hence, \( \Delta x = \bar{x} - x = \frac{v^2}{2d_X} \). The position of \( y \) is given by \( \frac{x}{dy} \). We have, respectively, \( \bar{x} = (p_Y - p_X^*) + dx + v = \frac{1}{2} + \frac{v}{2d_X} - \frac{v^2}{6d_Xd_Y} \), and \( x = \frac{1}{2} - \frac{v^2}{6d_Xd_Y} \). Hence, \( \Delta x = \bar{x} - x = \frac{v}{2d_X} \). The position of \( y \) is given by \( \frac{x}{dy} < 1 \).

In the multi-homing case, consumers pay in total \( d_X \) to the two firms that provide product \( X \). The consumption of \( X \) entails traveling cost \( \frac{dX}{dx} \). The consumption of product \( Y \) yields a surplus \( (v - p_Y^*) \bar{y} \) by \( \frac{dX}{dy} \). Hence, consumers’ welfare can be written as \( W_2 = u - d_X - \frac{dX}{4} + \frac{(v - p_Y^*)^2}{2dy} \). Under exclusivity, consumer welfare is given by \( W_1 = u - M_X - T_X + D_1^{XY} v - T_Y \).

Let \( M_X \) denote their payments to Firms 1 and 2, and \( T_X \) and \( T_Y \) give their travel costs for \( X \) and \( Y \), respectively. We now compute each component separately. First consider the total payment in the base product market, \( M_X \). We have \( M_X = (d_X + \frac{v^2}{6d_Y})(\frac{1}{2} + \frac{v^2}{12d_Xd_Y}) + (d_X - \frac{v^2}{6d_Y})(\frac{1}{2} - \frac{v^2}{12d_Xd_Y}) = d_X + \frac{v^4}{36d_Xd_Y^2} \). Apparently, consumers pay more to firms 1 and 2 under an exclusivity arrangement, since \( M_X > d_X \).

Next, the overall travel cost for the consumption of product \( X \) is given by \( T_X = \left[ \frac{v^2}{2} + \frac{(1-x^2)}{2}\right] d_X + T_i - T'_i \), where \( T_i - T'_i \) is given by \( T_i - T'_i = d_X \int_0^\bar{x} (2x - 1) \int_0^{\bar{y} - \frac{x}{dy}(x-x)} 1dydx = d_X \int_0^\bar{x} (2x - 1)[y - \frac{x}{dy}(x-x)]dx = d_X \bar{y}[(\Delta x + x + \frac{1}{2})(x + x) - \frac{2}{3}(x^2 + \bar{x}x + x^2) - (\Delta x + x)] = \frac{d_X}{3} \left[ (\bar{x}^2 - 2\bar{x} + 3\frac{\Delta x}{2}) \right]. \)

Hence,

\[ T_X = d_X[(\bar{x}^2 - \bar{x} + \frac{1}{2}) + \frac{\bar{y}}{3}(\bar{y}^2 - 2\bar{y}^2 + \bar{y}^3 - 3\frac{\Delta x}{2})]. \tag{12} \]

We can rewrite equation (12) as \( \frac{T}{d_X} = \left[ \frac{v^2}{2} + \frac{(1-x^2)}{2} \right] + \frac{v^2}{3}(\bar{y}^2 - 2\bar{y}^2 + \bar{y}^3 - 3\frac{\Delta x}{2}) \) and \( \frac{T}{d_Y} = \left[ \frac{v^2}{2} + \frac{(1-x^2)}{2} \right] + \frac{v^2}{3}(\bar{y}^2 - 2\bar{y}^2 + \bar{y}^3 - 3\frac{\Delta x}{2}) \) \( + \frac{d_X}{4} \left[ \frac{v^4}{12d_Xd_Y} - \frac{v^4}{18d_Xd_Y^2} \right] \) given \( d_Y \) is chosen to make \( T_X \) exceed \( \frac{2}{3}v \). Thus, compared to the multi-homing case, consumers incur higher transportation costs in the primary product market. Overall, consumers in the \( X \) market are worse off.

To purchase the value-adding good \( Y \), consumers incur travel cost

\[ T_Y = \frac{xy^2}{2} d_Y + d_Y \int_0^y \int_{-\bar{y}}^{\bar{y}} 1dx d\bar{y} = \frac{dy \cdot x \bar{y}^2}{2} + \frac{dy \Delta x \bar{y}^2}{6}. \]

Finally, we examine the consumer surplus for consuming \( Y \), which is given by \( W_Y = D_1^{XY} v - T_Y = [\frac{v}{2d_Y} - \frac{v^3}{6d_Yd_Y^2} + \frac{v^2}{4d_Xd_Y}]v - [\frac{\bar{y}^2}{2} d_Y + \frac{dy \Delta x \bar{y}^2}{6}] = \frac{v^2}{4d_Y} - \frac{v^4}{12d_Xd_Y} + \frac{v^4}{6d_Xd_Y^2}. \)

Compared to the multi-homing case, consumer surplus from consuming the value-adding
product $Y$ is higher under an exclusivity arrangement. It can be easily shown that $\frac{v^2}{4d_Y} - \frac{v^4}{12d_Yd_Y} + \frac{v^3}{6d_Yd_Y} > \frac{(v-p)^Y}{2d_Y}$. Since $p^Y \in (0.26v, 0.29v)$, we have $\frac{v^2}{4d_Y} - \frac{v^4}{12d_Yd_Y} + \frac{v^3}{6d_Yd_Y} > \frac{0.2736v^2}{d_Y}$, or $\frac{v}{6d_X} \left( 1 - \frac{v}{2d_Y} \right) > 0.0238$ under Assumptions 1&2.

Now we are at the position to compare the overall consumer welfare under both cases. Now we can rewrite equations for $W_1$ and $W_2$ as $W_1 = u - M_X - T_X + W_Y = u - \left( d_X + \frac{v^4}{36d_Xd_Y} + \frac{d_X}{4} + \frac{v^3}{12d_Xd_Y} - \frac{v^4}{18d_Xd_Y^2} \right) = u - \left( \frac{5d_X}{4} + \frac{v^4}{18d_Xd_Y} - \frac{v^3}{12d_Xd_Y} - \frac{v^2}{4d_Y} \right)$ and $W_2 = u - \frac{5d_X}{4} + \frac{(v-p^Y)^2}{2d_Y}$. Comparing them yields $W_1 - W_2 = \frac{v^2}{2d_Y} = \frac{v^4}{18d_Xd_Y} + \frac{v^3}{12d_Xd_Y} - \frac{v^2}{4d_Y} > \frac{v^2}{18d_Xd_Y} + \frac{v^3}{12d_Xd_Y} - \frac{0.2736v^2}{d_Y} > 0$, given $p^Y \in (0.26v, 0.29v)$. The last inequality holds because $\frac{v}{6d_X} \left( \frac{1}{2} - \frac{v}{3d_Y} \right) > 0.0238$ under Assumptions 1&2. Thus, $W_1 - W_2 > 0$. ■

**Proof of Proposition 6**

**Proof.** Recall that in equilibrium, both firms bid $\Delta \pi$. The duopolistic firms end up with the same overall profit, although firm 1 earns more from the primary good market. Compare $\pi_2^*$ with the equilibrium profit of $\frac{d_X}{2}$ in the multi-homing case. The claim is straightforward, given that $\frac{d_X}{2} - (d_X - \frac{v^2}{6d_Y})(\frac{1}{2} - \frac{v^2}{12d_Xd_Y}) = \frac{v^2}{6d_Y} - \frac{v^4}{72d_Xd_Y} > 0$, under Assumptions 1&2.

Recall from Proposition 1 that the monopolistic firm receives $\frac{v^2}{3d_Y}$, while after surrendering the bidding fee, both firms 1 and 2 earn the same profit, $(d_X - \frac{v^2}{6d_Y})(\frac{1}{2} - \frac{v^2}{12d_Xd_Y})$. Hence, the overall profit under exclusivity amounts to $d_X + \frac{v^4}{36d_Xd_Y^2}$, while profit in the multi-homing case is $d_X + \frac{(v-p^Y)p^Y}{d_Y}$. It is then straightforward to show that, under Assumptions 1&2, $d_X + \frac{v^4}{36d_Xd_Y^2} < d_X + \frac{0.1924v^2}{d_Y} < d_X + \frac{(v-p^Y)p^Y}{d_Y}$ given $p^Y \in (0.26v, 0.29v)$. ■

**Proof of Proposition 7**

**Proof.** We now consider the overall change in social welfare. We have $\Delta W = -\frac{v^2}{18d_Xd_Y^2} + \frac{v^3}{12d_Xd_Y} - \frac{(v-p^Y)^2}{2d_Y} + d_X + \frac{v^4}{36d_Xd_Y^2} - (d_X + \frac{(v-p^Y)p^Y}{d_Y}) = \frac{v^2}{4d_Y} + \frac{v^3}{12d_Xd_Y} - \frac{v^4}{36d_Xd_Y^2} - \frac{(v-p^Y)^2}{2d_Y} - \frac{(v-p^Y)p^Y}{d_Y}$. Given that $p^Y \in (0.26v, 0.29v)$, we must have $(v-p^Y)(v+p^Y) \in \left( \frac{0.45795v^2}{d_Y}, \frac{0.4662v^2}{d_Y} \right)$. Thus, $\Delta W < \frac{v^2}{4d_Y} + \frac{v^3}{12d_Xd_Y} - \frac{v^4}{36d_Xd_Y^2} - \frac{0.45795v^2}{d_Y} < \frac{v^2}{4d_Y} \left( \frac{v}{3d_X} - \frac{v^2}{9d_Xd_Y} - \frac{3}{5} \right) < 0$, under Assumptions 1&2. ■