RESEARCH STATEMENT

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Contents

1. Overview

My research in recent years dealt with natural questions in the *representation theory of p-adic groups* treated by methods originating in the domain of *quantum algebra*.

Groups that are defined over a p-adic field F, such as the prototypical $GL_n(F)$, together with their infinite dimensional representations, have long been the center of much attention. Aside from being an analogue to the ubiquitous theme of Lie groups and their representations, this subject draws much number-theoretic motivation from its ties to automorphic representations.

I am interested in a *categorical point of view* on the subject. Groups give rise to categories of representations, which can often be approximated by, or shown to be equivalent to, categories which arise through other, often algebraic, means. Multiple descriptions of a category naturally provide more tools for the study of its objects.

My work often concentrates on bringing to a common ground tools from distinct settings, such as representation theory of quantum affine algebras, crystal bases in quantum groups and the geometry of affine Hecke algebras. I am appealed to problems that can be brought, through the use of deep established theories, into a form of explicit *combinatorial questions*.

2. Themes

2.1. **Universality of type A representation categories.**

The nature of Lie-theoretic objects is to take root systems as an essense of their defining data. Thus, a given root system may be used to define a Coxeter group, a Lie group, a p-adic group, an affine Hecke algebra, an affine Lie algebra, a quantum group, etc. The resulting objects are studied through their respective representation categories.

While each of these mathematical objects has its distinct behavior, it is my observation that when the underlying root system is of type A we will often encounter categories with very similar traits.

To be more precise, many of the problems in my work take smooth representations of $GL_n(F)$ as a starting point. The theory of the Bernstein decomposition [Ber84a] for p-adic groups allows us to consider the relevant representation category $\mathcal{C}_n$ as a product of representation categories

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of complex algebras. It was shown [BK99, Hei11] that the algebras in this case are precisely the type A affine Hecke algebras.

The sequence of categories $\{\mathcal{C}_n\}_{n=0}^{\infty}$ may be taken together. By doing so, we see the structure of the Bernstein-Zelevinski parabolic induction functor $\times : \mathcal{C}_{n_1} \times \mathcal{C}_{n_2} \to \mathcal{C}_{n_1+n_2}$.

Now, when decategorifying the resulting sequence of categories, a ring R is obtained (in fact, a bi-algebra). This ring R can be quantized into a lattice inside the quantum group $U_q(\mathfrak{sl}_\infty)^+$. The quantization of the basis of irreducible representations in R takes the form of Lusztig's dual canonical basis [LNT03, Gro99].

Through the Brundan-Kleshchev [BK09] and Rouquier [Rou08] equivalences, this quantization can also take place on a categorical level, by referring to representation categories of KLR-algebras, with their inherent monoidal structure.

From another perspective, the quantum affine Schur-Weyl duality [CP96], identifies the same chain of categories in terms of modules over the quantum affine algebra $U_q(\hat{\mathfrak{sl}}_N)$.

Yet more, one can effectively study $\mathcal{C}_n$ through the Arakawa-Suzuki functors [AS98, Suz98, BC15] which relate it to the category $\mathcal{O}$ of a Lie algebra $\mathfrak{sl}_k$.

In my work I strive to maximize the effect of these bridges, while improving our understanding of the underlying mathematical structure.

Future goals include a transfer of techniques between settings beyond the type A case. For example, it would be immensely useful to produce an analogue of the q-character (of representations of quantum affine algebras) in the setting of affine Hecke algebras. The geometric approach of [GRV94] to the quantum affine Schur-Weyl duality may prove useful for this goal.

2.2. **Prominence of combinatorially accessible classes of representations.** It is often the case in representation theory that general problems are treated first on classes of representations with better accessability. A classical example is that of generic representations. Many theorems are known and conjectures are posed (e.g. in the Langlands program) pertaining to generic irreducible representations of p-adic groups.

My observation is that studies can be expanded beyond the generic case, by carefully picking a class of representations which is amenable to combinatoric techniques. Often such a class is large enough to lead towards a general statement.

For $GL_n(F)$, the class of irreducible ladder representations, introduced to this setting by Lapid-Mínguez [LM14], has proved to be one such example.

A mechanism of multisegments was introduced by Zelevinski [Zel80] to study irreducible representations. The ladder class is defined by certain well-behaved multisegments. An essentially same condition has many manifestations throughout different settings, such as snake modules for quantum affine algebras [MY12], calibrated affine Hecke algebra modules [Ram03], or homogeneous KLR algebra modules [KR10]. The Hecke-algebraic approach of [BC15] suggests the ladder class as an adjusted version of the unitary class.

In the projects described below, generally "wild" questions manage to reach a complete answer when restricted to the ladder case, or its extensions. Curiously, such a restriction does not necessarily constitute a limitation. For example, the ladder class is wide enough to encompass all derivatives of unitarizable represention. Another example is the involvement of the ladder class in the seeds of the cluster algebra structure on the Bernstein-Zelevinski ring, devised by Hernandez-Leclerc [HL10].

3. Projects

3.1. **Local non-generic Gan-Gross-Prasad conjecture.** In [Gur18b], I have made a substantial progress towards the resolution of a conjectural branching rule posed by Gan-Gross-Prasad [GGP]. This rule for general linear groups was the first leap into the non-generic setting of the celebrated GGP problems (see [GGPW12]).

The method of proof for the theorem below involves a transfer of the problem, using the Bernstein decomposition and the quantum affine Schur-Weyl duality, into the realm of quantum affine algebras.

Suppose that π is a given irreducible smooth representation of $GL_n(F)$. We would like to predict the decomposition of the (highly reducible) restriction of π onto the naturally embedded subgroup $GL_{n-1}(F)$.

More precisely, we are seeking for a meaningful description of the collection of pairs (π, σ), $\pi \in \mathrm{Irr}\, GL_n(F)$, $\sigma \in \mathrm{Irr}\, GL_{n-1}(F)$, for which

$$\mathrm{Hom}_{GL_{n-1}(F)}(\pi, \sigma) \neq 0 \ .$$

An effective answer for this general restriction problem is still largely considered unpractical. Yet, Gan-Gross-Prasad stipulated a principle that clear combinatorial rules should describe the pairs inside a subclass of unitarizable representations which is described by *Arthur parameters*.

An Arthur parameter, in this case, can be thought as a completely reducible representation

$$\phi = \oplus_{i=1}^k \psi_i \otimes V_{a_i} \otimes V_{b_i} \ ,$$

of the group $W_F \times SL_2(\mathbb{C}) \times SL_2(\mathbb{C})$, where W_F is the Weil group of the field F. Here $\{\psi_i\}$ are irreducible representations of W_F with bounded image, and V_d, for $d \in \mathbb{Z}_{>0}$, denotes the d-dimensional irreducible representation of $SL_2(\mathbb{C})$.

By the established local Langlands reciprocity, an Arthur parameter ϕ gives rise to an irreducible representation $\pi(\phi)$ of $GL_n(F)$ (representations of Arthur-type).

Theorem 3.1. [Gur18b, Theorem 5.6] *Suppose that*

$$\phi_1 = \oplus_{i=1}^k \psi_i \otimes V_{a_i} \otimes V_{b_i} \ , \quad \phi_2 = \oplus_{i=1}^l \psi'_i \otimes V_{a'_i} \otimes V_{b'_i} \ ,$$

are two Arthur parameters for $GL_n(F), GL_{n-1}(F)$, respectively.

If

$$\mathrm{Hom}_{GL_{n-1}(F)} \left(\pi(\phi_1)|_{GL_{n-1}(F)}, \pi(\phi_2) \right) \neq \{0\}$$

holds, then there are disjoint partitions

$$\{1, \ldots, k\} = I_1 \cup I_2 \cup I_3, \quad \{1, \ldots, l\} = J_1 \cup J_2 \cup J_3 \ ,$$

and bijections $u : I_1 \to J_2$, $d : I_2 \to J_1$, which satisfy

$$\begin{aligned} (a'_{u(i)}, b'_{u(i)}) &= (a_i, b_i + 1),\ \psi'_{u(i)} \cong \psi_i\ \forall i \in I_1, \\ (a'_{d(i)}, b'_{d(i)}) &= (a_i, b_i - 1),\ \psi'_{d(i)} \cong \psi_i\ \forall i \in I_2, \\ b_i &= 1\ \forall i \in I_3, \quad b'_j = 1\ \forall j \in J_3 \ . \end{aligned}$$

The above theorem fully settles one direction of the conjectural branching rule.

One motivation for the formulation of this rule were the works of Clozel [Clo04], Venkatesh [Ven05] and Lapid-Rogawski [LR09] in the setting of *unitary* representations. My theorem is consistent with the results of [Ven05] in a certain sense, albeit in a different setting. Namely, the cases where $I_1 = J_2 = \varnothing$ hold in the statement of Theorem 3.1 produce a refinement of the $SL(2)$-type condition required in [Ven05, Proposition 2(1)].

Future work may attempt to make the analogous refinement back in the unitary setting.

The advantage of posing our problem in a language of quantum groups lies in a recent result of Hernandez [Her17] dealing with cyclic modules for quantum affine algebras. When transferring the notion of a cyclic products back to the p-adic setting, we end up with precisely the notion that was studied by Lapid-Mínguez in [LM16b] for products of pairs of representations of the ladder class.

I have extended Theorem 3.1 into a rule which governs restriction within the full class of unitarizable representations.

Theorem 3.2. [Gur18b, Theorem 5.7] *Let $\pi \in \mathrm{Irr}(GL_n(F))$ and $\sigma \in \mathrm{Irr}(GL_{n-1}(F))$ be unitarizable representation.*

Let

$$\begin{aligned} \pi &\cong \pi_0 \times \pi_1 |\det|^{\alpha_1} \times \pi_1 |\det|^{-\alpha_1} \times \cdots \times \pi_k |\det|^{\alpha_k} \times \pi_k |\det|^{-\alpha_k} \ , \\ \sigma &\cong \sigma_0 \times \sigma_1 |\det|^{\alpha_1} \times \pi_1 |\det|^{-\alpha_1} \times \cdots \times \sigma_k |\det|^{\alpha_k} \times \sigma_k |\det|^{-\alpha_k} \ , \end{aligned}$$

be their decomposition according to the Tadic classification [Tad86]*, that is, π_i, σ_i are irreducible Arthur-type representations.*

If

$$\mathrm{Hom}_{GL_{n-1}(F)}(\pi, \sigma) \neq \{0\}$$

holds, then each pair (π_i, σ_i) *should satisfy the condition outlined in the statement of Theorem 3.1.*

Some case of the converse direction of the branching rule were also established. For example, the following can be viewed as a direct generalization of the classical branching law for generic representations from [JPSS83].

Theorem 3.3. [Gur18b, Theorem 5.10] *Let* (π, σ) *be a pair of Arthur-type representations with* $\pi \in \mathrm{Irr}(GL_n(F))$ *and* $\sigma \in \mathrm{Irr}(GL_{n-1}(F))$*, which satisfies the condition outlined in the statement of Theorem 3.1.*

If at least one of π, σ *is generic, then*

$$\mathrm{Hom}_{GL_{n-1}(F)}(\pi, \sigma) \neq \{0\} \ .$$

One possible future direction of work towards the full resolution of the converse direction is the cohomological study of spaces $\mathrm{Ext}^i(\pi_1, \pi_2)$, for irreducible representations π_1, π_2 of $GL_n(F)$, in the spirit of [Pra18, AP12]. The nature of these spaces is largely unknown, while questions regarding it are amenable to techniques of transfer into other settings, such as representations of KLR-algebras.

3.2. **The p-adic Littlewood-Richardson problem.** Recall again that R is the Bernstein-Zelevinski ring consisting of the sum of Grothendieck groups of smooth finitely generated representations of the groups $\{GL_n(F)\}_{n=0}^{\infty}$, with a product defined in terms of parabolic induction. The multiplicative behavior of irreducible representations $\mathrm{Irr} = \cup_{n\geqslant 0} \mathrm{Irr}(GL_n(F))$ as elements of R remains largely a mystery.

Given two representations $\pi_1, \pi_2 \in \mathrm{Irr}$, my works attempts to describe the decomposition of $\pi_1 \times \pi_2$ into irreducible factors. While a general answer is known in terms of values of Kazhdan-Lusztig polynomials of a corresponding symmetric group, a recent line of research [LM16b, LM16a] together with some classical results [Ber84b] suggests that more transparent descriptions, with significantly reduced computational complexity, are obtainable.

My study [Gur16, Gur17] has produced, among other results, a complete combinatorial rule for the decomposition of $\pi_1 \times \pi_2$ for the case when π_1, π_2 belong to the ladder class. It can be viewed as a generalization of previous decomposition descriptions of Tadic [Tad15], Leclerc [Lec04] and Ram [Ram03].

The rule obviates the need for computation of Kazhdan-Lusztig polynomials in these cases, and settles a conjecture posed by Lapid. In fact, I have shown that this key conjecture, that deals with specific cases in which the length of the representation $\pi_1 \times \pi_2$ is given by *Catalan numbers*, is equivalent to the general rule ([Gur16, Proposition 7.3]).

In order to prove an essential part of the theorem below, the problem had to be lifted into the quantum setting. Namely, as mentioned before, the basis Irr for R is viewed as a specialization at $q = 1$ of the dual canonical basis $\mathcal{B}$ of the quantum group $U_q(\mathfrak{sl}_\infty)^+$.

The irreducible representations which can appear as subquotients in $\pi_1 \times \pi_2$ are naturally parameterized by permutations in a certain symmetric group S_n. For $x \in S_n$, let us write $\Pi(x) \in \mathrm{Irr}$ for that potential candidate.

Theorem 3.4. [Gur16, Theorem 1.2][Gur17, Theorem 1.1] *When* $\pi_1, \pi_2 \in \mathrm{Irr}$ *are ladder representations, we have*

$$[\pi_1 \times \pi_2] = \sum_{x\in S(\pi_1,\pi_2)} [\Pi(x)] \ \in R \ ,$$

for a certain subset $S(\pi_1, \pi_2) \subset S_n$*. Namely, all subquotients appear with multiplicity* 1*.*

The set $S(\pi_1, \pi_2)$ *consists of precisely those permutations* x *which satisfy both:*

(1) A 321 *pattern does not occur in* x*.*

(2) The indicator $\Pi(x)_\otimes$ *appears in the Jacquet module of* $\pi_1 \times \pi_2$*.*

The verification of the last condition is done by a simple computation on any given data of π_1, π_2 and x.

We look at products of two elements of $\mathcal{B}$, which correspond to ladder representations. If we write $b(\pi) \in \mathcal{B}$ for a lifting of a representation $\pi \in \mathrm{Irr}$, then for ladder representations $\pi_1, \pi_2 \in \mathrm{Irr}$ we see an equation of the form

$$b(\pi_1)b(\pi_2) = \sum_{x \in S(\pi_1,\pi_2)} q^{-d(\pi_2,\pi_1;\, \Pi(x))} b(\Pi(x)) \in U_q(\mathfrak{sl}_\infty)^+ \ .$$

The quantum group has a natural embedding into a *quantum shuffle algebra*. This embedding can be seen as a quantization of the character morphism for affine Hecke algebras, which on the level of p-adic groups translates to the Jacquet functor.

We exploited the quantized character map to define a new integer invariant $d_\otimes(\pi_1, \pi_2;\ \sigma)$, for every $\sigma \in \mathrm{Irr}$ whose indicator $\sigma_\otimes$ (when properly defined) appears as a subquotient in the Jacquet module of $\pi_1 \times \pi_2$. The interplay between the invariants d and $d_\otimes$ was the key for the proof of Theorem 3.4.

My techniques in [Gur17] are also suitable for the categorical study of the multiplicative structure of Irr, rather than just its semi-simplified behavior in R. Namely, [Gur17, Conjecture 6.12] gives a fast algorithm for obtaining the unique irreducible sub-representation of the product $\pi_1 \times \pi_2$, when $\pi_1, \pi_2 \in \mathrm{Irr}$ are taken from a wide subclass of ladder representation. The ultimate proof of this conjecture lies in a future write-up of the precise relation between the categories of representation of KLR algebras and affine Hecke algebras. Further comparison with the categorification techniques of [KKKO18] may yield a better insight on these matters.

Future promising directions of research include obtaining a canonical "ladder standard module" (curiously involving the Robinson-Schensted correspondence) in which an arbitrary element of Irr can embedded through a similar process, and the intriguing comparison to the less explicit algorithm for sub-representation outlined in [LM16b].

3.3. **Classification of distinguished representations.** Let G be a reductive group defined over a p-adic field F, and $X = G/H$ be an algebraic G-homogeneous space. The irreducible smooth representations of $G(F)$ that appear as sub-representations of the space of smooth functions $C^\infty(X(F))$ are called H-*distinguished* representations, and are of particular interest in the *relative* harmonic analysis on X. This theme has tight relations with the study Jacquet's relative trace formulae and the systematic conjectural frameworks of Sakellaridis-Venkatesh [SV17] and Prasad [Pra15].

One future goal of research is a satisfactory translation of the notion of distinction into other categories. In particular, a major step would be to understand this notion on the level of affine Hecke algebras. For example, the works of Chan-Savin [CS18, CS17] suggest such a possibility by exhibiting a translation of the notion of generic representations.

My Ph.D. thesis work [Gur15] has established, in the ladder class, the validity of a long-expected conjecture attributed to Jacquet on the nature of $GL_n(F)$-distinguished representations of the group $GL_n(E)$, where E/F is a quadratic extension.

For a representation $\pi \in \mathrm{Irr}(GL_n(E))$, let π^τ denote its Galois twist by the automorphism coming from the extension E/F. It is well-known that each $GL_n(F)$-distinguished π is contragredient to π^τ.

Conversely, I have shown that for ladder representations, such a symmetry is close to being equivalent to distinction. This result takes a combinatorial approach to generalize a known property of discrete series and Speh representations.

Theorem. [Gur15, Theorem A] *Suppose that π is an irreducible ladder representation of $GL_n(E)$. Then π is contragredient to π^τ, if and only if, either π or $\chi_{E/F} \otimes \pi$ are $GL_n(F)$-distinguished.*

Here $\chi_{E/F} = \tilde{\chi}_{E/F} \circ \det$, where $\tilde{\chi}_{E/F}$ is a character of $E^\times$, such that $\tilde{\chi}_{E/F}|_{F^\times}$ is the quadratic character related to E/F. In fact, a precise characterization for the distinction of π and $\chi_{E/F} \otimes \pi$ was given in [Gur15], in terms of the Langlands data of π.

The local Langlands correspondence for $GL_n(E)$ provides more insight on the contragredience condition of π to π^τ. Roughly speaking, it can be expected that the corresponding symmetry on the L-parameter ϕ_π of π, should force ϕ_π to come from an L-parameter of a unitary group for the extension E/F. Langlands functoriality predicts in this case that π should be in the image of a *base change* mapping from the collection of irreducible smooth representations of unitary groups.

In a collaboration with Jia Jun Ma and Arnab Mitra [GMM18], we made the above relation more precise. We were able to pull the results of [Gur15] into a statement which identifies distinguished ladder representations as images of a functorial transfer.

3.4. **Representation-theoretic approach to parabolic Kazhdan-Lusztig polynomials.** Given a Coxeter group W and a standard parabolic subgroup W_J, the parabolic Kazhdan-Lusztig polynomials $\{\hat{P}_{\sigma,\omega}(q)\}$ are a collection of integer polynomials attached to each pair of cosets $\sigma, \omega \in W/W_J$. They were initially defined by Deodhar [Deo87] as a parabolic generalization of ordinary Kazhdan-Lusztig polynomials.

While the ubiquity of Kazhdan-Luzstig polynomials in representation theory is well established, the parabolic analogues attract a growing interest, fueled in part by discoveries on their geometric nature [KT02, LW17]. Yet, explicit values or bounds on coefficients of the polynomials are not known in most cases.

In [LM16a], Lapid-Mínguez worked on reducibility questions in R. They identified conditions under which an irreducible representation $\pi \in \mathrm{Irr}$ has the property that $\pi \times \pi$ is irreducible as well. Such π was called a $\square$-*irreducible* representation, a notion intimately related to that of *real* modules, either for quantum affine algebras or KLR algebras (see [KKKO15]).

They noted that their irreducibility results can be applied to compute values of certain parabolic Kazhdan-Lusztig polynomials. Subsequently, Lapid formulated precise conjectures [Lap17], that relate certain parabolic Kazhdan-Lusztig polynomials to corresponding Kazhdan-Luzstig polynomials of the symmetric group.

In [Gur18a], I suggested a new approach towards the study of parabolic Kazhdan-Lusztig polynomials. This approach provided an explicit monomial formula for some polynomials in the case of $W = S_n$ (the symmetric group) and $W_J = S_m \times \cdots \times S_m$, for a divisor $n = mk$.

Theorem 3.5. [Gur18a, Theorem 4.3] *Suppose that $P_{\sigma_0,\omega}(q) \equiv 1$ (ordinary K-L polynomial), for some permutations $\sigma_0 \leqslant \omega \in S_k$, such that σ_0 is 213-avoiding.*

Then, for every $\sigma_0 \leqslant \sigma \leqslant \omega$,

$$\hat{P}_{r_m(\sigma),r_m(\omega)}(q) = q^{\binom{m}{2}(\ell(\omega)-\ell(\sigma))}$$

holds.

Here $r_m : S_k \to N_W(W_J)/W_J$ is the natural isomorphism.

This proves the basic cases of Lapid's above mentioned conjectures. We show that smoothness properties of Schubert varieties of type A imply simple formulas for certain parabolic Kazhdan-Lusztig polynomials.

Our approach makes use of the basis $\mathcal{B}$ for the quantum group $U_q(\mathfrak{sl}_\infty)^+$. We show that the dual canonical basis approach is the missing link which would push the Lapid-Mínguez irreducibility result into a quantized setting.

When comparing that basis with another PBW-type basis, it is known by results of Lusztig [Lus90] that the values of the transition matrix are given by dimensions of intersection cohomology spaces of certain nilpotent varieties. It is also known that those dimensions can be expressed through coefficients of Kazhdan-Lusztig polynomials. The key observation is that the expressions obtained by inverting this matrix can be thought of as a *double-coset analogue* of parabolic Kazhdan-Lusztig polynomials.

Future work with a similar approach can shed more light on the nature of the double-coset sums involved in the transition matrices and the quantum group meaning of the formulas described by Lapid's conjectures. Exploring the relations to a number of works [Bru06, FKK98, LM02] on similar themes is also an intriguing direction of research.

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