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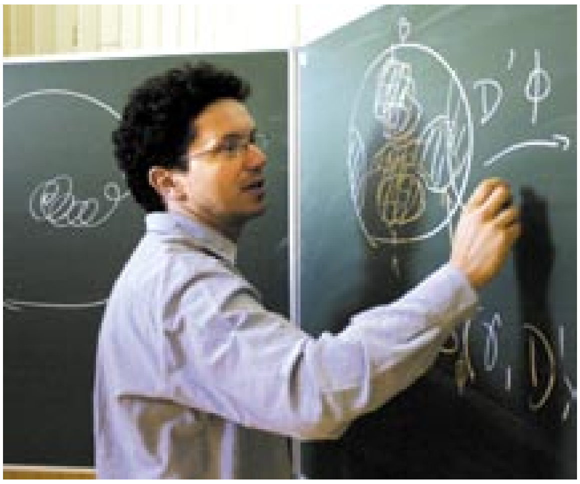
A Probability-Rich ICM and Wendelin Werner's Work >>>



Wendelin Werner at work © J. F. Dars / CNRS Photothèque

[Editor's Note: This article appeared in the March 2007 issue of the IMS Bulletin published by the Institute of Mathematical Statistics under the title "A Probability-Rich ICM Reviewed". *Imprints* is grateful to the editors of the IMS Bulletin for permission to reproduce it here.]

The 2006 International Congress of Mathematicians in Madrid was exceptionally rich in probability theory. Not only was the Fields Medal awarded for the first time to a probabilist, namely Wendelin Werner, it was also awarded to Andrei Okounkov whose work bridges probability with other branches of mathematics. Both Okounkov and Werner had been invited to give a 45-minute lecture each in the probability and statistics section before their Fields Medal awards were announced.

The newly created Gauss Prize (in full, the Carl Friedrich Gauss Prize) for applications of mathematics was awarded to Kiyosi Itô, another probabilist whom we all know. The objective of the Gauss Prize is to honor scientists whose mathematical research has had an impact outside mathematics, such as in technology, in business, or simply in people's everyday lives. A presentation of Itô's work was made by Hans Föllmer in a plenary address to the audience of the congress, in the presence of Itô's daughter, who received the prize and gave a speech on behalf of her 90-year-old father who was prevented by ill health from attending.

The Nevanlinna Prize was awarded to Jon Kleinberg who uses probability in his work. Much of his lecture was about small worlds for which probability was used to formulate the model.

Among the plenary lectures, apart from those delivered by probabilist Oded Schramm and statistician Iain Johnstone (on the use of random matrices in statistics), Percy Deift's lecture on "Universality for mathematical and physical systems" was about random matrices and Avi Wigderson's lecture "P, NP and mathematics" was in part about probabilistic algorithms. Richard Stanley's plenary lecture elaborated on the famous Baik-Deift-Johansson result on the longest increasing sequence in a random permutation (which incidentally has been connected by Andrei Okounkov to another famous result about the largest eigenvalue of random matrices). Even the plenary lecture of Terence Tao, another Fields Medalist at this same congress, was entitled "The dichotomy between structure and randomness" and contained several examples from probability. Finally, in the logic session Rod Downey's 45-minute talk was about algorithmic randomness and computability. These were just a sample of lectures we attended and there could be more talks that reflected the growing importance of probability theory in science and mathematics.

Although the Fields Medal was awarded to a probabilist for the first time, it was not surprising that Wendelin Werner was the one. Werner was born in Germany in 1968, but

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Random Graphs and Large-Scale Real-World Networks >>>

[Editor's Note: In May and June of 2006, the Institute hosted a program on "Random Graphs and Large-Scale Real-World Networks", of which Béla Bollobás was the chair of the Organizing Committee. In this article, he gives his perspectives on the field of random graphs as well as the program organized at IMS.]

The classical theory of random graphs was founded by Erdõs and Rényi almost fifty years ago. Erdõs and Rényi studied random graphs as fascinating and intricate objects in pure mathematics, and used their theory to show the existence of graphs with paradoxical properties. Since then, this theory has gone from strength to strength, with thousands of papers written on the topic.

Not surprisingly, although the theory of random graphs is an area of pure mathematics, possible applications have never been far away — after all, many large-scale graphs occur in real life. For example, the World Wide Web can be viewed as a graph, and so can metabolic and protein networks, food webs, the system of telephone calls, the network in the brain, traffic flows, acquaintances in a society, economic networks, and so on. These graphs resemble the classical binomial random graphs in the sense that they do not seem to have clear-cut structures that are easy to describe — at a glance, the connections seem to be 'random'. However, they are unlike any of the random graphs of the classical models: most networks occurring in the world are far from homogeneous; in particular, their degree distributions are rather different from those in the classical models. Also, real-world networks do not tend to be chosen from reasonably well-defined distributions, but arise as the result of dynamical processes that add and remove vertices and edges from the network.

Although a fair amount of empirical work had been done on real-world graphs for many decades, in particular, on acquaintance and citation networks, mathematical work on them started only in the last decade. For instance, Watts and Strogatz drew attention to the 'small-world phenomenon', and Barabási and Albert noted the 'scale-free' nature of many of the networks concerned, evidenced by, for example, power-law degree distributions. They suggested that such distributions arise in a graph growing by acquiring more and more vertices and edges if the newly arrived vertices get joined to old ones according to some preferential attachment rule. Thus, for example, the World Wide Web seems to be essentially scale free: viewed as a directed graph, the distributions of the in-degrees and out-degrees are well approximated by power law distributions.

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his parents settled in France when he was one year old, and he acquired the French nationality a few years later. After studying at the Ecole Normale Supérieure de Paris, he defended his PhD thesis in Paris in 1993, shortly after getting a permanent research position at the CNRS. He became a Professor at University Paris-Sud Orsay in 1997. Before winning the Fields Medal, he had received many other awards, including the 2000 Prize of the European Mathematical Society, the 2001 Fermat Prize, the 2005 Loève Prize and the 2006 Pólya Prize.

Wendelin Werner's work lies at the interface between probability theory and statistical physics. The fact that the models in consideration enjoy asymptotic conformal invariance properties also leads to using sophisticated tools from complex analysis. Werner's most famous results come from his collaboration with Greg Lawler and Oded Schramm on applications of the so-called SLE (stochastic Loewner evolution) processes. SLE processes are obtained by introducing in Loewner's equation of complex analysis a random driving function which is just a scaled linear Brownian motion. The work of Werner and his co-authors has produced extraordinary applications of SLE processes to long-standing open problems, such as the rigorous calculation of the non-intersection exponents for random walk or Brownian motion. Such exponents govern, for instance, the asymptotic behavior of the probability that two independent planar random walk paths up to time n will have no intersection point.

Another remarkable application was the proof that the Hausdorff dimension of the exterior frontier of a planar Brownian path is equal to 4/3. This fact, which had been conjectured by Mandelbrot more than 20 years ago, was one of the most fascinating open problems of probability theory. SLE processes have many other spectacular applications to different models of statistical physics, such as percolation, self-avoiding random walks or spanning trees on the lattice. The development of these applications, by Wendelin Werner and his co-authors, represents a giant step in the mathematical understanding of these models.

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