

# HKUST-KAIST-NUS

## Joint Workshop in Mathematics: Algebra, Algebraic Geometry and Lie Theory

**Date:** Thursday - Saturday, 7-9 Nov 2019

### Time & Venue:

7 Nov 1.30pm to 5.00pm

8 Nov 9.30am to 4.30pm

*(both days @ IMS Auditorium)*

9 Nov 9.30am to 12 noon

*(@ NUS Math, S17-04-06)*

### Organisers:

HKUST, KAIST and NUS

### SPEAKERS

#### HKUST:

Huai-Liang Chang  
Jing-Song Huang  
Ivan Ip

#### KAIST:

Sanghoon Baek  
Wansu Kim  
Sijong Kwak

#### NUS:

Huanchen Bao  
Lei Zhang  
De-Qi Zhang



Department of Mathematics  
Faculty of Science

# Programme

Day 1, Thursday 7 Nov @ IMS Auditorium	
<b>12.00pm to 1.30pm</b>	Lunch at the Institute for Mathematical Sciences (IMS)
<b>1.30pm to 2.30pm</b>	Huai-Liang Chang Feynman rule of Gromov Witten theory
<b>2.30pm to 3.00pm</b>	Tea Break
<b>3.00pm to 4.00pm</b>	Sijong Kwak Degree and the sectional genus of higher secant varieties
<b>4.00pm to 5.00pm</b>	De-Qi Zhang Equivariant Minimal Model Program with a View Towards Algebraic and Arithmetic Dynamics
	Dinner

<b>Day 2, Friday 8 Nov @ IMS Auditorium</b>	
<b>9.30am to 10.30am</b>	Wansu Kim Equivariant BSD conjecture over global function fields
<b>10.30am to 11.00am</b>	Tea Break
<b>11.00am to 12.00pm</b>	Lei Zhang Fourier Transforms and Standard L-function of Symplectic Groups
<b>12.00pm to 2.00pm</b>	Lunch
<b>2.00pm to 3.00pm</b>	Jing-Song Huang Generalized Fourier transforms arising from $sl(2)$ -triples and model nilpotent orbits
<b>3.00pm to 3.30pm</b>	Tea Break
<b>3.30pm to 4.30pm</b>	Sanghoon Baek Cohomological invariants and their applications
	Dinner

<b>Day 3, Saturday 9 Nov @ Department of Mathematics S17-04-06</b>	
<b>9.30am to 10.30am</b>	Huanchen Bao  Quantum symmetric pairs and Kazhdan-Lusztig theory
<b>10.30am to 11.00am</b>	Tea Break
<b>11.00am to 12.00pm</b>	Ivan Ip  Positive Peter-Weyl Theorem
	Lunch

# Abstract

## **Feynman rule of Gromov Witten theory**

Huai-Liang Chang

For compact Calabi Yau threefold, BCOV (1993) predicted Feynman rule that determine higher genus Gromov Witten invariants. Recently the conjecture is proved by the discovery of mixed spin  $p$  (msp) fields, using large  $N$  method. A consequence is that  $Fg$  is analytic. I shall introduce the setup and ideas.

## **Degree and the sectional genus of higher secant varieties**

Sijong Kwak

In this talk, we introduce basic information on higher secant varieties to nonexperts and consider degree lower bound and the sectional genus upper bound under some conditions. We would like to define minimal degree varieties and del Pezzo varieties in the category of  $\$q\$-secant varieties. Further, interesting examples and further questions will be raised from the classical viewpoints in projective algebraic geometry.$

## **Equivariant Minimal Model Program with a View Towards Algebraic and Arithmetic Dynamics**

De-Qi Zhang

We will elaborate the notion of 'int-amplified' endomorphism  $f$  of a normal projective variety  $X$ , a property weaker than 'polarized' yet preserved by products. We will show that the existence of such a single  $f$  guarantees that every Minimal Model Program (MMP) is equivariant w.r.t. a finite-index submonoid of the whole monoid  $S\text{End}(X)$  of all surjective endomorphisms of  $X$ . Applications of the equivariant MMP are discussed: Kawaguchi-Silverman conjecture on the equivalence of arithmetic and dynamic degrees of an endomorphism, and characterization of a subvariety with Zariski dense periodic points. Some parts are based on joint work with Meng.

## **Equivariant BSD conjecture over global function fields**

Wansu Kim

Under a certain finiteness assumption of Tate-Shafarevich groups, Kato and Trihan showed the BSD conjecture for abelian varieties over global function fields of positive characteristic. We explain how to generalise this to semi-stable abelian varieties "twisted by Artin character" over global function field (under some additional technical assumptions). This is a joint work with David Burns and Mahesh Kakde. If time permits, I'd like to discuss further speculations for generalisations.

## **Fourier Transforms and Standard L-function of Symplectic Groups**

Lei Zhang

In Tate's thesis, he shows that the theory of the Hecke L-functions can be established through the Fourier transforms over number fields. Following this idea, Godement and Jacquet establish the basic analytic theory of the standard L-function of  $GL(n)$  by using the

Fourier transforms over all square matrices. In the line of this idea, inspired by Braverman-Kazhdan and Ngo's proposal, we use the doubling method of Piatetski-Shapiro and Rallis to construct a Fourier transform on the symplectic groups as an analogy the classical Fourier transform. More precisely, it produces the zeta integral and the standard L-function of symplectic groups as in the doubling method. This is a joint work with Dihua Jiang and Zhilin Luo at University of Minnesota.

### **Generalized Fourier transforms arising from $\mathfrak{sl}(2)$ -triples and model nilpotent orbits**

Jing-Song Huang

The classical Fourier transform on the  $n$ -dimensional Euclidean space can be realized as a unitary operator in the oscillator representation of the symplectic group. We obtained a family of generalized Fourier transforms having properties similar to the classical Fourier transform. Further generalization to real reductive groups lead us to consider the model nilpotent orbits.

### **Cohomological invariants and their applications**

Sanghoon Baek

The notion of cohomological invariants was introduced by Serre in 90's. This notion corresponding the characteristic classes of topology is useful for understanding the structure of torsors under a linear algebraic group by correlating computable cohomology. In this talk, we shall discuss some applications of this invariant to the following three different problems: (1) computation of the numerical invariant measuring the complexity of torsors, (2) rationality problem for classifying spaces, and (3) computation of the Chow groups of flag varieties.

### **Quantum symmetric pairs and Kazhdan-Lusztig theory**

Huanchen Bao

A quantum symmetric pair is a quantization of the symmetric pair of universal enveloping algebras. We initiated the theory of canonical bases arising from quantum symmetric pairs, generalizing Lusztig's canonical bases on quantum groups. The new theory of canonical bases were subsequently applied to establish the (super) Kazhdan-Lusztig theory for ortho-symplectic Lie superalgebras. This is based joint work with Weiqiang Wang.

### **Positive Peter-Weyl Theorem**

Ivan Ip

For a compact Lie group  $G$ , the classical Peter-Weyl Theorem states that the regular representation of  $G$  on  $L^2(G)$  decomposes as the direct sum of its irreducible unitary representations. Similar results are being generalized to the case of real reductive groups by Harish-Chandra, as well as compact quantum groups by Woronowicz, but the case for non-compact quantum groups is pretty much unclear.

In this talk, I will explain the Peter-Weyl Theorem for split real quantum groups of type  $A_n$ . I will talk about the necessary ingredients needed to state and proof the theorem, including the GNS representation of  $C^*$ -algebra, quantum parallel transports, and cluster realization of positive representations. The talk is based on joint work with G. Schrader and A. Shapiro.