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Highlights

- Early onset of co-shedding for gap ratio=0.5 for tandem cylinders with near-wall

- Parallel double-vortex row arrangement in co-shedding regime at \( e/D = 0.5 \)

- Inviscid Krmn stability analysis of parallel vortex double-rows

- Gradual recovery of tandem freestream dynamics from gap ratio = 3 to 5.0

- Enhanced unsteady fluctuations in force signals of downstream cylinder
Dynamics of Tandem Cylinders in the Vicinity of a Plane Moving Wall

James Evon D’Souza, Rajeev Kumar Jaiman* and Chan Keet Mak

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Abstract

We present dynamics of the flow around two cylinders in a tandem configuration along a moving plane wall. A spectral element method is employed to perform the simulations with high accuracy at Reynolds number $Re = 200$. A moving wall with no-slip boundary is considered rather than a stationary wall to avoid the confusing interaction of wall boundary layer and thus focus completely on the influence of wall proximity effects on the force and wake dynamics. The influence of the moving wall with gap ratio $e/D = 0.2$ to $5.0$ and longitudinal center-to-center separation $L/D = 1.5$ to $8.0$ on the unsteady force dynamics is examined for the two cylinder configuration. Through detailed analysis of the flow field dynamics, we observe early transition from reattachment to co-shedding behavior. At co-shedding separation distances, the combined wake interference and wall proximity effects lead to a parallel double-row of vortices for the tandem cylinders at $Re = 200$ for $e/D = 0.5$. For a longitudinal separation of $4D$, the ratio of the street width $h$ to distance between two adjacent vortices in the same row $l$ is in good agreement with that obtained from inviscid theory. Finally, we provide detailed flow visualizations, Strouhal number and force coefficient trends and investigate recovery of freestream behavior.
as the tandem cylinder configuration of varying $L/D$ is gradually distanced further from the moving plane wall.

Key words: tandem cylinders, combined wake-wall interference, stability analysis, vortex suppression, parallel double-vortex configuration, force dynamics.

1 INTRODUCTION

Flow past an isolated cylinder in a free-stream beyond a critical Reynolds number manifests itself as alternate vortex shedding in the cylinder’s wake. This leads to the cylinder being subject to fluctuating lift and drag components. When such a cylinder is placed in proximity to a plane wall, the symmetry of the flow domain is broken as the wall interferes with the symmetric vortex shedding. Such a scenario is encountered in multiple engineering applications. An isolated pipeline in the free oceanic stream, for example, will experience vortex-induced vibrations if such alternate vortex shedding exists. However, if the pipeline is installed on the seabed itself, the interaction with the oceanic currents will be very much different. On occasion, the erosion of the seabed underneath the pipe structure, the installation method or the general unevenness of the terrain may lead to a gap between the sea bed and the pipeline. The above scenario can be modeled as the flow past a cylindrical structure in proximity to a plane wall. It is common practice to use isolated cylinder hydrodynamic coefficients (drag, lift, and inertia) to calculate the pipeline stability even if the pipeline is trenched or lying on the seabed. However, such an approach may lead to severe inconsistencies between predicted and actual

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flow induced structural loading. Hence, the need to characterize the nature of
the flow and induced forces when the cylinder is in proximity to a plane wall.
In particular, the wall-induced lift force is due to two competing mechanisms.
First, the presence of a nearby wall breaks the symmetry of the wake vortic-
ity distribution which results in an effective lift force that tends to move the
cylinder away from the wall. Second, from inviscid theory one can argue that
the flow relative to the cylinder will accelerate faster in the gap between the
cylinder and the wall. The resulting low pressure in the gap will induce a lift
force directed toward the wall.

The proximity effects of a cylinder to a plane wall at a given Reynolds number
are characterized first by defining the gap ratio $e/D$ as the ratio of the spacing
between the cylinder and the moving plane wall and the diameter of the cylin-
der. Taneda’s [1] experiments in a water tunnel by towing a circular cylinder
parallel to a wall at low Reynolds numbers ($Re = 170$) showed a single vortex
street in the cylinder’s wake at a gap ratio $e/D = 0.1$. These vortices be-
came unstable and broke down after a few wavelengths of propagation. It was
reported that a regular double row of vortices was re-established as the gap
ratio was increased to 0.6. Thereafter, several experiments for the flow around
a cylinder in the presence of a single plane boundary have been reported at
moderately high but sub-critical Reynolds number, $Re = 2 \times 10^4$ to $10^5$ [2-5].
The approaching boundary layer implemented in most of the aforementioned
studies was turbulent. The similarity among the results reported was that the
hydrodynamic forces on the cylinder were modified with a slight variation of
the shedding frequency.

Further, the suppression of vortex shedding was observed when the body was
closer than a critical distance from the wall. In most studies, this critical
spacing was found to be approximately at $e/D = 0.3$ to $0.4$ [6,2–4] and decreases with increasing Reynolds number. For smaller gaps, the wake was almost steady and the periodic shedding was strongly inhibited with separation bubbles on the wall. Huang et al. [7] detailed a precise method in order to evaluate the critical gap ratio using 2D stability analysis of the mean streamwise gap velocity. Using Rayleigh’s inflection point theorem they identified the critical gap ratio for $Re = 300$, $400$, $500$ and $600$ observing an inverse relation between critical $e/D$ and Reynolds number. They further concluded that the critical gap ratio corresponds to a local minimum of the streamwise maximum mean velocity in the gap. Price et al. [8] have shown experimentally at $Re = 1200$ that for small gap ratios ($e/D \leq 0.125$), the flow through the gap is suppressed or is extremely weak. The separation of the plane wall boundary layer is observed to occur both upstream and downstream of the cylinder. Although there is no regular vortex shedding, there is a periodicity associated with the outer shear layer. They reported that for intermediate gap ratios ($0.125 \leq e/D \leq 0.5$), the flow is very similar to that of very small gaps, except that there is now a pronounced pairing between the inner shear-layer shed from the cylinder and the wall boundary layer. Gap ratios higher than this are characterized by the onset of regular vortex shedding from the cylinder. Rao et al. [9] performed two-dimensional spectral-element simulations for flow past a circular cylinder translating parallel to a wall and linear stability analysis to determine onset of 3D effects in the wake at different gap heights. They observed that a steady two- to three-dimensional transition takes place at $e/D \leq 0.25$. For larger gap ratios the three-dimensional instability manifested itself post initial unsteady transition of the wake. The validity of the stability analysis results were confirmed through three-dimensional direct numerical simulations at $Re = 200$. They observed that the wake transits into a
highly non-linear state after a non-dimensional time of 160. A comprehensive study of the more complex problem introduced by a rolling cylinder in close proximity to a plane wall has been detailed by Stewart et al [10]. A wide range of Reynolds number from 20 to 500 were studied. The linear stability analysis revealed that the critical Reynolds number for three dimensional effects and the dominant mode were highly dependant on the cylinder rotation rate. Similarities were observed to flow past a backward facing step but no clear connection was established between the respective transition mechanisms.

When an additional cylinder is placed downstream of the first, the flow phenomena become further intricate and are now governed by the relative positioning of the two bluff bodies. Thus, in such a tandem configuration of two cylinders in proximity to a plane wall, an additional parameter in the form of the non-dimensional longitudinal spacing between the two cylinders \( L/D \) governs the flow dynamics. In the present study, this parameter has been defined in terms of the distance separating the centers of the two cylinders \( L \) as shown in Fig. 1. Three flow interference regimes [11,12]: proximity interference, an intermediate wake interference (reattachment) and co-shedding, can be identified for the tandem cylinder configuration. The range of spacing to diameter (\( L/D \)) ratio for each of these categories is problem dependent. In the proximity interference regime for \( 1 \leq L/D \leq 1.2 \) to 1.8, negative drag is produced on the downstream cylinder and vortex shedding from the upstream cylinder is suppressed. The tandem bodies behave like a single bluff body and vortex shedding occurs behind the rear cylinder. In the wake interference or reattachment regime for 1.2 to 1.8 \( \leq L/D \leq 3.4 \) to 3.8, a number of different phenomena such as shear layer reattachment, intermittent vortex shedding, etc. can be observed as the separation distance is gradually increased. In the
regime of large spacing $L/D \geq 3.8$, so-called co-shedding regime, vortex shedding occurs from both the cylinders and there is no interference effect.

The critical separation distance for the onset of the co-shedding regime has been indicated by several researchers [13–15] both numerically and experimentally, for a wide range of Reynolds numbers to be between 3.5 to $5D$ in terms of $L/D$. The two-dimensional numerical simulations conducted for isolated tandem cylinders by Mittal et al. [16] and Meneghini et al. [17] showed a sharp increase of the drag coefficient and Strouhal number when this critical spacing was exceeded. This spacing is termed the drag inversion separation, where the drag coefficient of the downstream cylinder changes from negative to positive as the separation distance is increased. The parameter range of $Re = 200$ with $1.5 \leq L/D \leq 4$ chosen by Meneghini et al. enables comparison between an isolated tandem cylinder configuration in a free stream and the near-wall tandem cylinder configuration investigated in the present study.

Recently, Harichandan et al. [18] investigated the flow past square and circular cylinders in single and tandem configuration at $Re = 100$ and 200 in proximity to a plane wall. In their study, no-slip condition was employed at the stationary plane wall and hence the shear layer associated with the wall boundary layer interferes with the proximity effects. Harichandan observed that the wall shear layer leads to early dissipation of vortices shed from the single circular cylinder at lower Reynolds number $Re = 100$. The wall shear in addition effects the upward movement of the vortex pair in the near wake. However, even at this low Reynolds number, a stable separation bubble was observed to form in the cylinders wake and anchors itself to the plane wall. At $Re = 200$ the positive vortices shed from the lower surface of the cylinder de-stabilize the wall boundary layer leading to transient separation from the wall itself in
the vicinity of the vortex shedding region of the cylinder wake. Furthermore, these positive vortices were reported to form a chain next to the wall via an anchoring mechanism. Rao et al. [19] performed comprehensive spectral element simulations in order to shed light on vortex dynamics and 3D stability of immersed tandem cylinders sliding close to a plane wall at a very small gap ratio $e/D = 0.005$. 2D simulations were performed in order to study onset of periodic unsteady flow for longitudinal spacings between $2.1 \leq L/D \leq 12$ for $Re \leq 200$. They observed that for higher longitudinal spacings of the order of $L/D \approx 12$, the transition to an unsteady flow occurs with synchronous shedding from both cylinders at a lower Reynolds number than that corresponding to a single cylinder near a wall. At intermediate separations $8 \leq L/D \leq 11$, two dominant frequencies have been reported in the FFT spectra, which can be attributed to the alternate shedding of strong and weak vortex pairs from the upstream cylinder and their interactions with the downstream cylinder. More complex phenomena introduced by rotation of the cylinders in single and tandem configurations have been documented by Rao et al. [20] where a reverse rotation was even shown to suppress three-dimensionality in the wake of a single cylinder.

In the present study, spectral-element based direct numerical simulations are performed for flow past two cylinders in a tandem configuration in proximity to a moving plane wall at $Re = 200$. Though cylinder vortex shedding is very much a 3D phenomena at such high Reynolds numbers, the 2D approximation is still useful in providing valuable insight into fundamental vortex dynamics and force trends. For completeness, initial simulations of a single cylinder near a plane moving wall at $Re = 200$ are performed to characterize the fundamental proximity effects on wake dynamics. These have been documented in the
appendices. This exercise further enables gaining confidence in the numerical scheme in capturing the subtle yet profound flow phenomena associated with such flows. Building upon the analysis for a single cylinder, the flow past tandem cylinders at $Re = 200$ held close to a plane moving wall is investigated in detail for gap ratios $e/D = 0.2$ to $5.0$ and longitudinal spacings $L/D \in [1.5, 8]$. To investigate the recovery of freestream behavior, the results of wake-wall interference are compared with the freestream tandem cylinder counterpart at the same longitudinal spacings. The investigations aim to contrast the wake vortex dynamics and force coefficients when the tandem cylinder configuration is placed in the vicinity of a plane moving wall. In particular, we aim to highlight early onset of co-shedding at $e/D = 0.5$ and presence of a non-staggered vortex arrangement in the unsteady wake at $L/D = 3$ and $4$ at $e/D = 0.5$.

The main motivation for this study is to examine and better understand the complex bluff-body/wake and body/body interactions in the neighborhood of a moving wall, where it is clear that the vortex dynamics is quite different from that for interacting bodies in a freestream flow. In turn, this paper represents a step towards an improved understanding of subsea pipelines subject to ocean currents with varying gaps and tandem arrangements.

The remainder of this article is organized as follows. The problem under consideration and the numerical formulation are described in Section 2, along with domain size and spatial resolution studies. Section 3 presents new results and discussions for tandem cylinders for varying gap ratios and longitudinal spacings, where the unsteady flow structures, the stability of parallel double-row vortex arrangement, the mean gap velocity profiles and the variations of the forces and shedding frequencies are systematically analyzed. This is followed by conclusions in Section 4.
2 NUMERICAL METHODOLOGY

2.1 Problem Definition

As shown in Fig. 1, we model the problem as two tandem cylinders of equal diameter above a plane wall that translates at the same velocity as the freestream velocity. The diameter of the cylinder is $D$, the freestream velocity is $U_\infty$, and the distance between the lowest extremity of the cylinder and the moving wall is defined as $e$. $L$ represents the distance between the centers of the two cylinders. The fluid is assumed to be Newtonian and incompressible. The continuity and unsteady Navier-Stokes equations in non-dimensional form are

\[ \nabla \cdot \mathbf{u} = 0, \quad (1) \]
\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}, \quad (2) \]

where $\mathbf{u}$ and $p$ denote the velocity vector $\mathbf{u} = (u, v)$ and pressure, respectively.

The key non-dimensional parameters are Reynolds number $Re = U_\infty D/\nu$, the wall proximity $e/D$ and center-to-center longitudinal separation $L/D$ for tandem configuration, where $\nu$ is the kinematic viscosity of the fluid. A Dirichlet boundary condition ($U_\infty = 1, v = 0$) is employed at the inlet and a Neumann boundary condition ($\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0, p = 0$) is applied at the outlet boundary. A symmetric boundary condition ($\frac{\partial u}{\partial y} = 0, v = 0$) is applied at the upper boundary to simulate far-field conditions. The lower boundary is given a velocity equal to the inflow velocity $U_\infty$, which simulates a moving wall. The no-slip boundary condition is applied on the cylinder surface.
Fig. 1. Schematic of the computational domain and co-ordinate system for tandem cylinders.

The drag and lift coefficients are defined as below

\[ C_D = \frac{1}{\frac{1}{2} \rho U_\infty^2 DL_z} \left[ \int_S (-p n + n \cdot \tau) ds \right] \cdot e_x \]  

(3)

\[ C_L = \frac{1}{\frac{1}{2} \rho U_\infty^2 DL_z} \left[ \int_S (-p n + n \cdot \tau) ds \right] \cdot e_y \]  

(4)

where \( S \) denotes the surface of the cylinder, \( n \) is the outward unit normal to the surface of the cylinder, \( L_z \) is the span of the cylinder and \( \tau \) is the dimensionless viscous stress tensor. Here, \( e_x \) and \( e_y \) are the Cartesian components of the unit vector \( n \) that is normal to the cylinder boundary.
2.2 Numerical Scheme

The numerical method is based on a spectral-element formulation to discretize the transient incompressible Navier–Stokes equations in two dimensions. The flow domain consists of a collection of quadrilateral elements with a finer grid resolution in regions of high gradients near the cylinders and in the wake regions. These quadrilateral spectral elements are split into $N \times N$ nodes that correspond to Gauss-Lobatto-Legendre quadrature points (the nodal basis functions). The method exhibits exponential convergence as $N$ is increased, consistent with global spectral methods [21,22]. Time integration for both the linearized and full Navier-Stokes equations is carried out in primitive variables,
with equal-order interpolants for the velocity and pressure, using a velocity-correction scheme \[23,24\]. In all the analysis and simulations described here, second-order-time integration was employed, and at the time-step sizes required to meet the CFL stability criterion, spatial convergence is the dominant consideration. The representative grid is presented in Fig. 2. The grid independence study is conducted to ensure that the solutions obtained are independent of the domain size, the mesh resolution and the polynomial order. It is conducted in three parts. The effect of domain size is first tested. Next, a grid refinement test ($h$-refinement) is performed. As the spectral-element method is used in this study, the effect of polynomial is performed ($p$-refinement).

Table 1

Sizes of domains compared

<table>
<thead>
<tr>
<th>Parameter</th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_y$</td>
<td>$40D$</td>
<td>$20D$</td>
</tr>
<tr>
<td>$L_{xd}$</td>
<td>$8D$</td>
<td>$8D$</td>
</tr>
<tr>
<td>$L_{zd}$</td>
<td>$52D$</td>
<td>$26D$</td>
</tr>
</tbody>
</table>

Table 2

Comparison of $\bar{C}_D$ and $\bar{C}_L$ for different domain sizes for the downstream cylinder at $\epsilon/D = 0.2$ and $L/D = 8$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>D1</th>
<th>D2</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{C}_D$</td>
<td>0.7215</td>
<td>0.7294</td>
<td>1.08</td>
</tr>
<tr>
<td>$\bar{C}_L$</td>
<td>0.3274</td>
<td>0.3282</td>
<td>0.23</td>
</tr>
</tbody>
</table>
2.3 Effect of Domain Size

The domain is defined in terms of the location of the inlet, top and outlet boundaries relative to the cylinders. Two domain sizes with their boundaries placed at different distances from the cylinders are selected for comparison. The gap ratio is fixed at $e/D = 0.2$ and separation $L/D = 8$. The simulations were run for the same time interval and the forces on the cylinders were monitored. Along the wall-normal direction, the computational domain extends from the wall ($y = 0$) to a top boundary placed at $y = L_y$. Along the stream-wise direction, the computational domain extends from $x = -L_{xu}$ upstream of the center of the upstream cylinder to $x = L_{xd}$ downstream from the center of the downstream cylinder. The larger domain is referred to as $D_1$, while the smaller domain is referred to as $D_2$. The domain sizes are documented in Table A.1. The time-averaged drag and lift coefficients of the cylinder were computed from the force histories. The comparison are tabulated in Table A.2. Based on the results, domain size $D_2$ with values of $8D$, $26D$ and $20D$ were chosen for the inlet and outlet distances and the domain height, respectively. With this choice, the mean drag and lift coefficient differed by $1.08\%$ and $0.23\%$ respectively from the values obtained with the larger domain $D_1$.

2.4 Effect of Mesh Resolution: h-Refinement

A mesh resolution study is done to determine a suitable number of spectral elements required such that the solutions obtained are independent of the mesh resolution. A tandem cylinder configuration with the smallest gap ratio $e/D = 0.2$ and largest cylinder-to-cylinder spacing $L/D = 8$ in the present
investigation was chosen for the mesh resolution study. Three different grid resolutions were used: (i) Coarse: 4112 elements (ii) Medium: 4625 elements (iii) Fine: 6425 elements. Simulations were run at $Re = 200$ with a polynomial order of 5. Table 3 compares the relevant metrics among the three grid systems. The percentage differences were obtained relative to the finest resolution used (Fine mesh). Based on the results, the percentage difference for the medium mesh resolution as compared to the finest resolution were all below 1%, and thus is acceptable. For the subsequent simulations, the computational domains are meshed with a medium mesh resolution of 4625 spectral elements. The finest resolution was not selected as it proved to be computationally expensive and the time taken to complete a single simulation was much longer than that of the medium resolution.

Table 3
Comparison of force coefficients for three different grid resolutions for the down-stream cylinder in a tandem configuration at $e/D = 0.2$ and $L/D = 8$.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Coarse</th>
<th>Difference</th>
<th>Medium</th>
<th>Difference</th>
<th>Fine</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4112 elements</td>
<td>%</td>
<td>4625 elements</td>
<td>%</td>
<td>6425 elements</td>
</tr>
<tr>
<td>$\bar{C}_D$</td>
<td>0.7262</td>
<td>0.44</td>
<td>0.7294</td>
<td>0.3</td>
<td>0.7316</td>
</tr>
<tr>
<td>$C_L$</td>
<td>0.3279</td>
<td>0.09</td>
<td>0.3282</td>
<td>0.18</td>
<td>0.3276</td>
</tr>
<tr>
<td>$C'_D$</td>
<td>0.5806</td>
<td>1.06</td>
<td>0.5836</td>
<td>0.55</td>
<td>0.5868</td>
</tr>
<tr>
<td>$C'_L$</td>
<td>0.4262</td>
<td>1.06</td>
<td>0.4281</td>
<td>0.61</td>
<td>0.4307</td>
</tr>
<tr>
<td>$St$</td>
<td>0.1506</td>
<td>0.13</td>
<td>0.1508</td>
<td>0.0</td>
<td>0.1508</td>
</tr>
</tbody>
</table>
Table 4

Comparison of force coefficients for different polynomial orders for downstream cylinder in tandem configuration.

<table>
<thead>
<tr>
<th>Gap \ Spacing</th>
<th>Force Coefficient</th>
<th>4th Order N = 4</th>
<th>Difference %</th>
<th>5th Order N = 5</th>
<th>Difference %</th>
<th>6th Order N = 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e/D) \ L/D</td>
<td>(C_D)</td>
<td>0.7251</td>
<td>0.47</td>
<td>0.7294</td>
<td>0.11</td>
<td>0.7286</td>
</tr>
<tr>
<td>0.2 \ 8.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 \ 8.0</td>
<td>(\bar{C}_D)</td>
<td>0.3278</td>
<td>0.13</td>
<td>0.3282</td>
<td>0.03</td>
<td>0.3281</td>
</tr>
<tr>
<td>0.5 \ 1.5</td>
<td>(C_D)</td>
<td>0.1491</td>
<td>0.1</td>
<td>0.1491</td>
<td>0.1</td>
<td>0.1494</td>
</tr>
<tr>
<td>0.5 \ 1.5</td>
<td>(\bar{C}_L)</td>
<td>0.0822</td>
<td>0.01</td>
<td>0.0823</td>
<td>0</td>
<td>0.0823</td>
</tr>
<tr>
<td>0.5 \ 4</td>
<td>(\bar{C}_D)</td>
<td>0.4563</td>
<td>0.24</td>
<td>0.4559</td>
<td>0.15</td>
<td>0.4552</td>
</tr>
<tr>
<td>0.5 \ 4</td>
<td>(\bar{C}_L)</td>
<td>0.1481</td>
<td>0.13</td>
<td>0.1482</td>
<td>0.07</td>
<td>0.1483</td>
</tr>
</tbody>
</table>

2.5 Effect of Polynomial Order: \(p\)-Refinement

One advantage of spectral element method is the ability to further refine the mesh resolution at runtime by specifying the number \(N\) of internal node points on each edge of each spectral element. The suitable spectral element mesh resolution has been established in section 2.4 to be 4625 for tandem cylinders. In this subsection, a suitable polynomial order is determined. To test the value of \(N\) required to resolve the flow accurately, four different values of \(N\) were considered. The number of node points for each element tested was \(N = 4\), \(N = 5\) and \(N = 6\). For the tandem configuration, the study was limited to \(N^2 = 6^2\) as \(N^2 = 7^2\) resulted in excessive computation time. In addition, there is a strong Courant restriction on the time step. Hence, it was not feasible to use polynomial order 7. The tests were carried out at \(Re = 200\) at gaps and
spacings enlisted in Table 4 where a comparison is made of the mean drag and lift coefficient between the three cases for the downstream cylinder. At $N^2 = 5^2$ the percentage differences of $\bar{C}_D$ and $\bar{C}_L$ were all lower than 1%. Therefore, an inter-element resolution of $N^2 = 5^2$ was chosen for all computations since it provided an acceptable accuracy for a reasonable computational effort relative to polynomial order of 6. For the sake of completeness, the validation and the convergence results of the single cylinder have been summarized in the appendices.

3 RESULTS AND DISCUSSIONS

The complexity of the flow phenomena involved in flow past a single circular cylinder in proximity to a plane wall is enhanced when two cylinders are placed in a tandem arrangement. An additional variable $L$ is introduced in the form of the longitudinal spacing between the two cylinder centers. It has been documented by several authors ([12],[17]) that the vortical dynamics differ significantly with longitudinal separations $L/D$. However, studies on near wall effects using a stationary plane wall with no-slip boundary result in additional involvement of the wall shear layer along with pure wall-proximity effects experienced by the cylinders. The present study characterizes the sole effect of the varying longitudinal spacings on the wake dynamics and hydrodynamic forces on the tandem cylinders by implementing a moving plane wall. The focus area is chosen as longitudinal separations $L/D = 1.5$ to 8 between the cylinder centers at wall spacings $e/D = 0.2$ to 5.0 at $Re = 200$. 
3.1 Flow Structures

It was identified in the single cylinder investigations documented in Appendix B that at $e/D = 0.2$ alternate vortex shedding is suppressed whereas, for $e/D = 0.5$ an unsteady wake with alternate vortex shedding exists. We first focus on the behavior of a tandem cylinder configuration at these two gap ratios with varying longitudinal separation between the two cylinders. At $e/D = 0.2$ the wake manifests itself as elongated bands of positive and negative vorticity upto a separation of $3D$. These bands are observed to extend to a maximum distance of $14D$ downstream of the cylinders beyond which these structures eventually become unstable and die down. The vorticity contours clearly point at the absence of unsteady vortex shedding at these separations as in Fig. 3b. However, when the cylinder centers are four diameters apart, the wake is no longer steady. As seen in Fig. 3d, vortices shed by the upstream cylinder impinge on the downstream cylinder and then advect downstream. Thus, a transition from a steady to an unsteady wake occurs when the separation is increased from $3D$ to $4D$. Existence of such a transition for cylinders in tandem configuration has been reported by Rao et al. [19] at $e/D = 0.005$ for a separation $L/D \leq 5$. They observed that at $e/D = 0.005$, the flow becomes more stable with a decrease in longitudinal separation and Reynolds number. The flow field at larger separation distances such as $L/D = 8$ is consistent with those observed in the studies by Rao et al. [19].

To begin with, we consider a special case of the tandem configuration at a gap ratio $e/D = 0.5$. Figure 4 presents the instantaneous vorticity contours. The case of flow past a single circular cylinder at $e/D = 0.5$ is provided for comparison. Although distinct changes in flow phenomenon can be observed
Fig. 3. Instantaneous vorticity contours in the wake of the cylinders at $Re = 200$, $e/D = 0.2$ at $L/D = 2, 3, 4$ and $8$ at non-dimensional time $tU_\infty/D = 150$ for flow coming from left to right. An unsteady wake is evident at $L/D = 4$ and $8$. 
with increasing longitudinal separations, periodic unsteady behavior is commonly present at all separations. Figure 4b and Figure 4c show the instantaneous wake vortex patterns for longitudinal separations $L/D = 2$ and $L/D = 3$ respectively. Zdravkovich’s [11] investigations place the reattachment regime in the $1.4 \leq L/D \leq 3.4$ range of longitudinal spacings for isolated tandem configurations. This can be confirmed with the numerical work of Meneghini et al. [17] who observed a shift from reattachment to co-shedding behavior when the longitudinal spacing is changed from $L/D = 3$ to 4 at $Re = 200$ for tandem cylinders in a freestream flow. Accordingly, in an unbounded flow over tandem cylinders at longitudinal spacings $L/D = 2$ to 3, the flow is in the reattachment regime, whereby the shear layers emanating from the upstream cylinder reattach on to the surfaces of the downstream cylinder with a separation bubble behind the upstream cylinder. The presence of the moving wall, however, leads to premature transition from reattachment to co-shedding as evidenced by the change in the vorticity contours when the longitudinal spacing is increased to $3D$.

The reattachment behavior exhibits a wake similar to an isolated tandem configuration. The two cylinders behave as a single bluff body with a single wake downstream of the second cylinder. Shear layers from the upstream cylinder distinctly envelope the basal surface of the downstream cylinder and are eventually shed alternately to form the unsteady wake. With reference to Fig. 4b, the positive lower shear layer emerging from the upstream cylinder reattaches to the front face of the downstream cylinder. On the other hand, the negative upper shear layer emerging from the top face of the upstream cylinder coalesces with the separated negative shear layer of the downstream cylinder. This results in an expansion of the region of negative vorticity emerging from
Fig. 4. Instantaneous vorticity contours in the wake of the cylinders at $Re = 200$, $e/D = 0.5$ with varying $L/D$ at non-dimensional time $tU_\infty/D = 150$ for flow from left to right. Non-staggered vortex arrangement can be seen at $L/D = 3$ and $4$. 
the top face of the downstream cylinder. Wall vortices form below each of the cylinders. Negative vorticity is associated with the moving wall below the tandem arrangement. The negative vorticity eventually forms part of the chain of positive-negative vortices seen along the moving wall. The negative wall vorticity below the tandem cylinders arises to cancel the tangential velocity component at the wall in order to ensure no slip. Sengupta et al. [25] studied in detail the evolution and dynamics of wall vorticity from a shielded vortex in close proximity of a stationary wall. They further pointed out the distribution of wall vorticity as an indicator of the strength of the interaction between a vortex and the wall boundary layer. Similar to their observations, in the present study the positive vortex shed by the cylinder destabilizes the moving wall boundary layer. It induces an asymmetric interaction on the negative vorticity created at the wall. The wall vorticity on the other hand affects the trajectory of the vortices in the cylinder wake. A clear tilt is present in the wake vortices. The shed vortices drift upwards away from the horizontal plane wall. Greater separations such as \( L/D = 8.0 \) exhibit mature co-shedding with stretching of the downstream cylinder positive shear layer along the moving wall prior to shedding.

3.2 Parallel Vortex Arrangement and Instability Analysis

For a gap ratio \( e/D = 0.5 \) and separation \( L/D = 3 \) it is seen that co-shedding has initiated in Fig. 4c. The co-shedding behavior is more mature and distinct at \( L/D = 4 \). At this stage it is important to highlight distinctions in vortex shedding patterns in the reattachment and co-shedding regimes. First, the single cylinder in Fig. 5a clearly presents a Kármán vortex street that is
Fig. 5. Wake structures and vortex height to spacing ratio $h/l$ for (a) single cylinder with Kármán vortex street, and tandem cylinders in (b) reattachment and (c) co-shedding regimes with two parallel rows of vortices in wake at $Re = 200$ and $e/D = 0.5$. Flow is from left to right. Solid contour lines represent positive $z$-vorticity while dashed lines represent negative $z$-vorticity.
skewed away from the horizontal. Based on classical theory of Von Kármán, a staggered arrangement of vortices in the wake of a bluff body is unstable except for a definite ratio between vortex street width $h$ and distance between vortices in the same row $l$ as: $h/l = \frac{1}{\pi} \cosh^{-1} \sqrt{2} = 0.28$. For the single cylinder, the width of the street $h$ is $0.975D$. The distance between two negative vortices in the same row $l_n$ is approximately $3.8311D$ and positive vortices $l_p$ is $3.6882D$. The average distance between two vortices in the same row is computed as $l = (l_n + l_p)/2$. This yields an average $h/l$ ratio for the single cylinder vortex street in Fig. 5a of about 0.2545. In the reattachment regime as well, the wake presents itself as a staggered arrangement of positive and negative vortex cores similar to the single cylinder case. Again, the vortices are skewed upwards from the horizontal due to the presence of the moving plane wall. The vorticity associated with the moving wall impedes the natural trajectory of the shed positive vortices. At $L/D = 2$ as in Fig. 5b, the average $h/l$ is 0.2653.

In the co-shedding regime at $L/D = 3$ and 4, the vortices departing the tandem configuration no longer arrange themselves in an alternate staggered arrangement. Figure 6 presents a sequence of vorticity distributions resulting in the parallel double-row configuration for $L/D = 4$ at distinct instants highlighted in the force histories in Figure 7. The upstream cylinder sheds vortices in a conventional Kármán fashion. Figure 6a presents the flow field just prior to shedding of the negative vortex from the upstream cylinder which impinges upon the surface of the downstream cylinder. It impinges and produces a weak positive vorticity on it. This instant corresponds to the minimum drag experienced by the downstream cylinder. The negative vortex has been completely shed in Fig. 6b. It is convected downstream upon the upper surface of the downstream cylinder. The shed negative free vortex coalesces with the
Fig. 6: Development of unsteady wake with non-staggered vortex arrangement for freestream flow from left to right at $Re = 200$, $e/D = 0.5$ and $L/D = 4$ at $tU_\infty/D =$ (a) 100 (b) 101 (c) 102 (d) 103 (e) 104. The shedding cycle time period is $\approx 5.1$ s and the contours represent $z$-vorticity.
negative vorticity of the downstream cylinder. The downstream cylinder experiences maximum drag just post the time instant captured in Fig. 6d which represents the completion in the merging of the shed free vortex and the negative vorticity shear layer of the downstream cylinder. This combined negative vorticity scheme of the downstream cylinder grows progressively to be eventually shed in Fig. 6b. Immediately after this instant, the downstream cylinder experiences maximum lift. The positive shear layer of the upstream cylinder during the same time grows in a similar fashion till it is shed and impinges on the downstream cylinder in Fig. 6c. From Fig. 6c to 6e this free positive vortex is convected in the gap between the downstream cylinder and the plane wall to eventually merge with the positive vorticity shear layer of the downstream cylinder. A reduction in lift is observed during this period to its minimum value at instant corresponding to Fig. 6e.

Thus, due to continuous impingement of von Kármán vortices from the upstream cylinder, the shedding process of vortices from downstream cylinder
happens simultaneously as opposed to alternately. The vortices do not try to occupy the same region immediately behind the cylinder for the co-shedding regime of $L/D = 4$; instead they have a certain separation attributed to the combined effect of moving wall proximity and the wake-cylinder interaction. This simultaneous shedding of vortices from the upper and bottom side of the downstream cylinder leads to a symmetric non-staggered vortex arrangement. This arrangement can be referred to a symmetric-wake drag with counter-rotating pair. A further quantification of separation distance $h$ and the vortex spacing $l$ is given in Fig. 5c which results in an average $h/l$ value of 0.5965. A similar wake behavior behind a pair of bluff bodies is obtained for a gap ratio of 2.5, where the coupled wake consist of three rows of vortices [26,27]. One of the cylinder wake is very narrow such that the counter-rotating vortices are aligned on the same axis and the speed of the narrow wake is higher than the wider wake street. In the present near-wall case, the center of narrow wake axis can be considered as a moving wall with a constant speed streamline.

*Kármán Stability Analysis*

It is well known that two infinite parallel point vortices as the symmetrical double rows are always unstable Lamb [28] in Article 156. However, we observe that parallel double-row form a stable and persistent vortex system behind the downstream cylinder with the wall proximity effects. This contradiction from the theoretical stability analysis can be explained through the method of images [29] where we consider another row of vortices opposite to the moving wall. The wall is therefore a streamline, and the inviscid boundary condition of zero normal velocity across the wall is satisfied. Three parallel semi-infinite rows can lead to a stable system which is consistent with the inviscid point.
Let us consider three infinite rows of vortices in a parallel configuration at time \( t = 0 \). The vortices in the rows have equal strengths, namely \( \kappa \) but opposite rotation between top and middle and the same and imaged vortices. The top rows are at points \( ml + ih/2 \) \((m = 0,1,2,...)\), the bottom row at the points \( nl + ih/2 \) \((n = 0,1,2,...)\), and the shadow row \( rl + ih/2 \) \((r = 0,1,2,...)\). The complex potential \( w \) for this arrangement of vortices at time \( t = 0 \) is

\[
w = i\kappa \ln \left[ \sin \left( \frac{\pi}{l}(z - ih/2) \right) \right] + i(-\kappa) \ln \left[ \sin \left( \frac{\pi}{l}(z + ih/2) \right) \right] + i\kappa \ln \left[ \sin \left( \frac{\pi}{l}(z + 3ih/2) \right) \right]
\]

(5)

where \( i = \sqrt{-1} \) and \( z \) denotes a generalized vortex location [29].

We adopt the Kármán stability analysis [30] and move the vortices slightly with the following displacements

\[
z_m = \gamma \cos(m\phi) \quad z'_n = \gamma' \cos(n\phi) \quad z''_n = \gamma'' \cos(r\phi)
\]

(6)

in which \( z_m, z'_n, \) and \( z''_n \) are the displacements for the top, bottom and shadow vortices, respectively, and \( \gamma, \gamma' \) and \( \gamma'' \) are small complex numbers, and \( \phi \) is \( 0 < \phi < 2\pi \). By superpositions of the velocity from the vortices corresponding to \( m, n, \) and \( r \) rows and the equation of vortex motion, we evaluate \( \gamma, \gamma', \) and \( \gamma'' \) using the mathematical identities as given in [30]. We find that two parallel rows so-called symmetric double row lead the unstable vortex motion, i.e., the displacement of vortices even grow when they are being displaced slightly with a periodic disturbance.

However, for the parallel two vortex rows and the shadow vortex row on the opposite side of the moving wall, i.e. three parallel vortex rows, we obtain the
following relationship through the stability analysis:

$$\frac{\pi^2}{4} - \frac{\pi^2}{\cosh^2(\frac{\pi h}{l})} = 0$$

which yields $h = 0.562l$. As a remark, the motion is always unstable unless the ratio $h/l$ has a value, namely 0.562. This is consistent with our numerical result $(h/l)_{num} = 0.5965$ in Fig. 5(b).

This parallel non-staggered vortex system is a remarkably different behavior than the isolated tandem cylinders at $L/D = 4$ and the single cylinder with wall proximity of $e/D = 0.5$. In both the configurations, shear layer vortices try to occupy the same region in the immediate vicinity of wake hence lead to two rows of alternately rotating vortices in the form of stable staggered von Kármán vortex street.

3.3 Mean Gap Velocity Profiles

We contrast the mean gap velocity profiles obtained in the tandem configuration with the benchmarked single cylinder results in Section B.1. Figure 8 presents the variation of the mean streamwise velocity $u$ for both upstream and downstream cylinders in the gap between the cylinder and the moving wall for various longitudinal spacings $L/D$ at $Re = 200$ and $e/D = 0.2$.

By comparing the velocity profiles with that of the single cylinder at the same Reynolds number and wall proximity, it can be seen that the upstream cylinder behaves dynamically similar to the case of a single cylinder. At all longitudinal spacings, the velocity magnitudes are marginally lower than that of the single cylinder case. The presence of the upstream cylinder, however, reflects more strongly in the gap velocity profile of the downstream cylinder as seen in Fig.
8. In every case, the streamwise velocity and hence, momentum of the fluid stream confined to the gap between the downstream cylinder and the moving wall is significantly lower than that of the isolated cylinder case. The distinct maxima in the profile diminishes. At $e/D = 0.5$, the upstream cylinder depicts

![Diagram](image1)

(a) Upstream Cylinder

![Diagram](image2)

(b) Downstream Cylinder

Fig. 8. Streamwise mean gap velocity for tandem cylinder configuration at $Re = 200$ and $e/D = 0.2$.

![Diagram](image3)

(a) Upstream Cylinder

![Diagram](image4)

(b) Downstream Cylinder

Fig. 9. Streamwise mean gap velocity for tandem cylinder configuration at $Re = 200$ and $e/D = 0.5$. 

30
a mean gap velocity profile similar to that of the single cylinder for the same gap ratio. Alternate vortex shedding is present on the upstream cylinder in the co-shedding regime. The downstream cylinder is impacted by these vortices shed alternately by the upstream cylinder. The gap velocity profiles for the downstream cylinder naturally depart from the nature exhibited by a single cylinder at the same proximity to the wall. The two distinct local maxima observed in case of the single cylinder vanish.

3.4 Lift and Drag Forces

The drag and lift coefficients are evaluated by non-dimensionalizing the integrated components of pressure and shear in the streamwise and normal directions respectively. We first contrast the upstream and downstream cylinder forces at $e/D = 0.5$ where the parallel non-staggered vortex arrangement is seen. With regard to the wake-wall interference effect, we next elaborate the effect of increasing gap ratio $e/D$ on the force dynamics of downstream cylinder and determine the recovery of freestream tandem cylinder behavior. The force trends further highlight the early onset of co-shedding. Figure 10 compares lift and drag coefficient histories between upstream and downstream cylinders at $e/D = 0.5$. For the upstream cylinder at $L/D = 2.5$, the amplitude of both lift and drag coefficients are much lower than corresponding amplitudes witnessed on the downstream cylinder. On increasing the longitudinal separation between the cylinder centers from $2.5D$ to $3D$, a significant increase is observed in the fluctuating component of unsteady lift and drag on upstream and downstream cylinders. The upstream cylinder that previously experienced predominantly positive lift now experiences periodic suction.
Fig. 10. Time histories of lift and drag coefficients of upstream (1) and downstream (2) cylinders at $Re = 200$ and $e/D = 0.5$ with varying $L/D$. 

(a) $L/D = 2.5$

(b) $L/D = 3$

(c) $L/D = 3.5$

(d) $L/D = 4$
towards the moving wall as well. These along with changes in flow field dynamics elicited in section 3.1 clearly indicate the early onset of co-shedding which is characterized by independent vortex shedding from the surfaces of both cylinders. In the co-shedding regime itself between the upstream and downstream cylinders, there is nearly in-phase and out-of-phase couplings of transverse and inline force signals, respectively at $L/D = 3, 3.5$. As the distance increased to $L/D = 4$, we observe nearly out-of-phase behavior of the lift force and the in-phase mode of drag forces between the two cylinders.

Fig. 11. A comparison between upstream and downstream cylinder drag and lift at $Re = 200$ and $e/D = 0.5$ at different longitudinal separations. Dash-dot (−·−) lines denote the single cylinder counterpart at $e/D = 0.5$. 

(a) Mean drag and lift forces

(b) RMS of fluctuating drag and lift forces.
Figure 11 presents the variation in mean and rms values of drag and lift coefficients as a function of $L/D$. Corresponding coefficients obtained for a single cylinder detailed in Appendix B have been reproduced for comparison. The mean drag experienced by the upstream cylinder is much higher than that of its downstream counterpart. A phenomenon reported in freestream tandem configurations with no moving wall is drag inversion wherein the drag on the downstream cylinder changes from negative to positive at a drag inversion spacing. Meneghini et al. [17] report negative drag on the downstream cylinder at $L/D \leq 3$ for the isolated tandem case which transitions to positive values after initiation of co-shedding. However, in the present study the drag on the downstream cylinder is non-zero and positive for all cylinder separations at $e/D = 0.5$. Hence, the presence of the moving wall results in a more subtle drag inversion phenomenon wherein the drag on the downstream cylinder merely increases in magnitudes. Similar to tandem arrangement in freestream flow, a shielding effect of upstream cylinder can be seen in the mean lift and drag coefficients of the downstream cylinder. With an increase in longitudinal distances, the upstream cylinder dynamics tend towards that of a single cylinder at $e/D = 0.5$. Before the shear-layer reattachment $L/D = 2.5$, the upstream cylinder has small rms fluctuations of the forces, which recover to values similar to that of the single cylinder counterpart after $L/D > 2.5$. The downstream cylinder force coefficients at a separation of $8D$ still show departure from single cylinder coefficients at the same gap ratio. For the downstream cylinder, the combined wake interference and moving wall effects promote the rms fluctuations of lift $C'_L$ and drag $C'_D$ force coefficients.
Fig. 12. Drag and lift trends for the longitudinal separation $1.5 \leq L/D \leq 8$ with varying $e/D$ at $Re = 200$. Force coefficients recover freestream behavior at $e/D \geq 5.0$. ($\ast$) $e/D = 0.2$, ($\ast$) $e/D = 0.5$, ($\square$) $e/D = 0.75$, ($\circ$) $e/D = 1.0$, ($\diamond$) $e/D = 1.5$, ($\circ$) $e/D = 3.0$, ($\times$) $e/D = 5.0$, ($\triangle$) $e/D = \infty$

To characterize the upstream wake and moving wall effects, we now detail the influence of increased distances from the wall on the downstream cylinder force dynamics. Figure 12 presents the nonlinear effect of increasing gap ratio $e/D$ on the force coefficients of the downstream cylinder for the separation range $L/D \in [1.5, 8]$. The drag inversion phenomenon is seen for the isolated
tandem cylinders ($e/D = \infty$, blockage $b = 0.05$). The rms of the fluctuating drag distinctly increases to positive values at $L/D = 4$, consistent with the findings of [17]. At $e/D = 0.2$ the cylinders are in close proximity to the moving wall and the effect of the moving wall on the force dynamics is maximum. At $Re = 200$ this represents a gap at which conventional alternate vortex shedding is suppressed as concluded from the single cylinder investigations. The downstream cylinder mean lift and drag increase monotonically with increasing separation $L/D$ between cylinders at this gap. A sharp increase in rms lift and drag is seen around $L/D \approx 4$. And the maximum fluctuating inline drag force is observed for $e/D = 0.2$ and $L/D = 4$. As $e/D$ is increased to the intermediate distances from $e/D = 0.5$ to 1.0, the longitudinal separation at which this sudden increase takes place recedes to lower $L/D$ values. This is analogous to the trend during early onset of the co-shedding regime detailed earlier at $e/D = 0.5$. In particular, the gap ratios $e/D \in [1.0, \infty]$ show the mean drag inversion from negative to positive values at different critical separation $L/D$. The mean drag and rms of fluctuating lift attain maximum values at $e/D = 1.5$. While negative mean lift has been found from $e/D = 0.75$ to 3.0 for a certain range of longitudinal spacing $L/D$, a monotonic increasing positive lift trend can be seen for $e/D = 0.2, 0.5$ and the mean lift nearly recovers to the single cylinder counterpart at $L/D = 8$. Attachment and detachment of the shear layers lead to dips in mean forces such as that observed at $e/D = 0.75$ and $L/D = 3$. Further investigation is required in order to elaborate the precise mechanisms leading to such oscillatory behavior of the mean drag and lift forces. On increasing the gap ratio further beyond $e/D = 1.5$, the force coefficients gradually migrate to their freestream counterparts. By $e/D = 5.0$ we observe acceptable recovery of freestream dynamics for the downstream cylinder.


Fig. 13. Dependence of Strouhal number of the downstream cylinder on gap ratios and longitudinal separations. Tandem cylinder flow regimes have been demarcated based on free-stream configuration $e/D = \infty$ with blockage $b = 0.05$. (⋆) $e/D = 0.2$, (⋆) $e/D = 0.5$, (square) $e/D = 0.75$, (diamond) $e/D = 1.0$, (triangle) $e/D = 1.5$, (diamond) $e/D = 3.0$, (times) $e/D = 5.0$, (triangle) $e/D = \infty$

3.5 Strouhal Number

We now assess the vortex shedding frequency of the downstream cylinder, non-dimensionally represented by the Strouhal number $St = fU_\infty/D$. The force coefficients have been monitored for several hundred time steps in order to compute the shedding frequency. Figure 13 presents the variation of $St$ number of the downstream cylinder with varying gap ratios and longitudinal separations. The parameter space has been demarcated into proximity interference, intermediate and co-shedding regimes based on freestream tandem cylinder behavior ($e/D = \infty$ and $b = 0.05$). For the freestream tandem configuration, the downstream cylinder in the intermediate (wake interference) regime exhibits much lower values of $St$ than in the proximity interference
Fig. 14. Shear layer interaction between upstream and downstream cylinder with moving wall proximity. The presence of the wall is seen to hasten the co-shedding behavior at $L/D = 3$ which leads to an increase in $St$ frequencies. Instantaneous vorticity contours at $L/D = 3$ for $e/D = 3$ (left) and $e/D = 1.5$ (right) at discrete instants of half a shedding cycle, with shedding period $T$. The flow is from left to right.
and co-shedding regimes. These values are consistent with the results of Meneghini et al. [17] and the values show excellent agreement with those reported in their investigations. A shift from wake-interference to co-shedding behavior reflects in Strouhal number where a sudden transition to higher shedding frequencies is seen. A gradual increase takes place with increased separation between $4 \leq L/D \leq 6$. The presence of the moving wall hastens the onset of co-shedding. This clearly manifests as a premature transition to the higher shedding frequencies associated with co-shedding. Figure 14 presents contour plots of $z$-vorticity visualizing the sudden increase in $St$ as $e/D$ is reduced from 3 to 1.5. For $L/D = 3$ and $e/D = 3.0$, the shear layers emanating from the upstream cylinder stretch and coalesce with the downstream cylinder shear layers. The shedding mechanism from the upstream cylinder occurs in close conjugation with the shear layer growth on the downstream cylinder. However, at $e/D = 1.5$, the upstream shear layers are seen to pinch-off and form a free vortex which rolls over the surface of the downstream cylinder. A constructive interaction with the downstream cylinder shear layer results in the accelerated shedding seen at this gap ratio. In the co-shedding regime, at larger distances ($L/D \geq 6$), as the cylinders are brought closer to the wall from the freestream, the shedding frequency increases until it reaches a maximum at $e/D = 0.75$. The shedding frequency is then seen to reduce from $e/D = 0.75$ to 0.2. At $e/D = 0.2$ and $L/D < 4$, the downstream cylinder does not exhibit any dominant frequency on account of the combined effect of wall proximity and the shielding effect of the upstream cylinder.
4 CONCLUSIONS

In the present study, flow over tandem cylinders near a moving plane wall has been analyzed to investigate the wake and wall interference effects at $Re = 200$. Through detailed analysis of the flow field, we observe early transition from intermediate to co-shedding regime for $e/D = 0.5$. At $e/D = 0.5$, the wake presents itself as a staggered arrangement of positive and negative vortex cores typical of conventional Kármán vortex street patterns at $L/D = 2$ and $2.5$. At co-shedding separation distances of $L/D = 3$ and $4$, the combined wake interference and wall proximity effects lead to a parallel non-staggered arrangement of vortices for the tandem cylinders at $Re = 200$ and a gap ratio $e/D = 0.5$. The predicted wake parameter $h/l$ agrees with the inviscid theory of three parallel rows of vortices. The presence of a plane moving wall hastens the onset of co-shedding for a tandem cylinder configuration which reflects as higher $St$ values for the downstream cylinder than seen in the intermediate regime with shear layer reattachment. Similar to a tandem arrangement in freestream flow, the shedding effect of the upstream cylinder can be seen in the mean lift and drag coefficients of the downstream cylinder at $e/D = 0.5$. The wake interference and the moving wall promote the fluctuations in the lift and drag forces. It is seen that post $e/D = 1.5$, the force coefficients on the downstream cylinder shift gradually towards their isolated tandem configuration counterparts, to be fully recovered at approximately $e/D \geq 5.0$. Further investigations and in-depth analysis via detailed 3D wake visualizations and gap flow will be considered to understand the physical mechanisms leading to the reported behavior in the force coefficients and shedding frequencies as the tandem cylinders are moved away from the wall.
ACKNOWLEDGMENTS

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Appendix A: Numerical Validation - Single Cylinder

For completeness, single cylinder near-wall simulations have been performed at $Re = 200$ and $0.05 \leq e/D \leq 0.5$. This further enables gaining confidence in the present numerical scheme. The single cylinder force coefficients and wake structures obtained from this study have been used in examining and contrasting the behavior of tandem cylinders in the vicinity of a moving wall. As shown in Fig. A.1, we model the problem as a single cylinder above a plane wall that translates at the same velocity as the freestream velocity. The diameter of the cylinder is $D$, the freestream velocity is $U_\infty$ and the distance between lowest extreme of cylinder and bottom wall is defined as $e$. Figure A.2 shows a representative mesh for single cylinder simulations. The numerical method is based on a spectral-element formulation to discretize the transient incompressible Navier–Stokes equations in two dimensions.
Fig. A.1. Schematic of the computational domain and co-ordinate system for single cylinder study.

(a) Spectral element mesh

(b) Zoomed view of mesh in the vicinity of the cylinder

Fig. A.2. Spectral element mesh used for single cylinder simulations
### A.1 Effect of Domain Size

The domain is defined in terms of the location of the inlet, top and outlet boundaries relative to the cylinders. Two domain sizes with their boundaries placed at different distances from the cylinders are selected for comparison. The gap ratio is fixed at \( e/D = 0.3 \). The simulations were run for the same time interval and the forces on the cylinders were monitored. Along the wall-normal direction, the computational domain extends from the wall (\( y = 0 \)) to a top boundary placed at \( y = L_y \). Along the streamwise direction, the computational domain extends from \( x = -L_{xu} \) upstream of the center of the cylinder to \( x = L_{xd} \) downstream from the center of the cylinder. The larger domain is referred to as \( D_1 \), while the smaller domain is referred to as \( D_2 \).

The domain sizes are documented in Table A.1. The time-averaged drag and lift coefficients of the cylinder were computed from the force histories. The comparison are tabulated in Table A.2. Based on the results, domain size \( D_2 \) with values of \( 8D \), \( 26D \) and \( 20D \) were chosen for the inlet and outlet distances and the domain height, respectively. With this choice, the mean drag and lift

---

### Table A.1

Sizes of domains compared

<table>
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<tr>
<th>Parameter</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
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<tr>
<td>( L_y )</td>
<td>( 40D )</td>
<td>( 20D )</td>
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<tr>
<td>( L_{xu} )</td>
<td>( 8D )</td>
<td>( 8D )</td>
</tr>
<tr>
<td>( L_{xd} )</td>
<td>( 52D )</td>
<td>( 26D )</td>
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coefficient differed by 1.15% and 0.2% respectively from the values obtained with the larger domain \( D_1 \).

Table A.2
Comparison of \( \bar{C}_D \) and \( \bar{C}_L \) for different domain sizes

<table>
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<th>Parameter</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>% Difference</th>
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Table A.3
Comparison of \( \bar{C}_D \) for three different grid resolutions for single cylinder

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<th>Medium</th>
<th>Fine</th>
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<td>2675 Elements</td>
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<td>( \bar{C}_D )</td>
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</table>
Table A.4

Comparison of single cylinder $\bar{C}_D$ for different polynomial orders at different distances from the wall.

<table>
<thead>
<tr>
<th>Gap $e/D$</th>
<th>4th Order $N = 4$</th>
<th>Difference $%$</th>
<th>5th Order $N = 5$</th>
<th>Difference $%$</th>
<th>6th Order $N = 6$</th>
<th>Difference $%$</th>
<th>7th Order $N = 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>1.6513</td>
<td>0.51</td>
<td>1.6429</td>
<td>0.01</td>
<td>1.6423</td>
<td>0.04</td>
<td>1.6430</td>
</tr>
<tr>
<td>0.17</td>
<td>1.6623</td>
<td>0.17</td>
<td>1.6649</td>
<td>0.02</td>
<td>1.6661</td>
<td>0.65</td>
<td>1.6652</td>
</tr>
<tr>
<td>0.25</td>
<td>1.7060</td>
<td>0.09</td>
<td>1.7050</td>
<td>0.03</td>
<td>1.7048</td>
<td>0.02</td>
<td>1.7045</td>
</tr>
<tr>
<td>0.35</td>
<td>1.7433</td>
<td>0.48</td>
<td>1.7494</td>
<td>0.02</td>
<td>1.7492</td>
<td>0.69</td>
<td>1.7488</td>
</tr>
<tr>
<td>0.45</td>
<td>1.7499</td>
<td>0.08</td>
<td>1.7401</td>
<td>0.04</td>
<td>1.7406</td>
<td>0.06</td>
<td>1.7395</td>
</tr>
</tbody>
</table>

A.2 Grid independence study: $h$- and $p$-Refinement

For the single cylinder study, three different grid resolutions were implemented: (i) Coarse: 2101 elements (ii) Medium: 2675 elements (iii) Fine: 3175 elements. Simulations were run at $Re = 200$ with a polynomial order of 5. The gap ratio was varied at $0.08 \leq e/D \leq 0.50$. Table A.3 compares the mean drag coefficients among the three grid systems. The percentage differences were obtained relative to the finest resolution used (Fine mesh). Based on the results, the percentage difference for the medium mesh resolution as compared to the finest resolution were all below 1%, and thus is acceptable. For the subsequent simulations, the computational domains for single cylinder are meshed with a medium mesh resolution of 2675 spectral elements. Next, a suitable polynomial order is determined. Four different values of $N$, the number of internal node points on each edge of each spectral element, were considered, $N = 4, 5, 6$ and 7. The tests were carried out at $Re = 200$ at various $e/D$ ratios. Table
A.4 shows a comparison for the mean drag coefficient between the four cases. The resolution of $N^2 = 4^2$ was insufficient to capture the flow characteristics at $e/D = 0.08$. At $N^2 = 5^2$ and $N^2 = 6^2$, the percentage differences of $C_D$ were all lower than 1%. $N^2 = 7^2$ proved to be computationally expensive. Therefore, an inter-element resolution of $N^2 = 5^2$ was chosen for all single cylinder computations.

Table A.5

<table>
<thead>
<tr>
<th>Study</th>
<th>$C_D$</th>
<th>$C'_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Study</td>
<td>1.348</td>
<td>0.451</td>
</tr>
<tr>
<td>Braza et al. (1986)</td>
<td>1.32</td>
<td>0.54</td>
</tr>
<tr>
<td>Roger et al. (1990)</td>
<td>1.29</td>
<td>0.46</td>
</tr>
<tr>
<td>Sa et al. (1991)</td>
<td>1.13</td>
<td>-</td>
</tr>
<tr>
<td>Li et al. (1991)</td>
<td>1.17</td>
<td>0.35</td>
</tr>
<tr>
<td>Farrant et al. (2001)</td>
<td>1.36</td>
<td>0.50</td>
</tr>
<tr>
<td>Meneghini et al. (2001)</td>
<td>1.30</td>
<td>0.47</td>
</tr>
<tr>
<td>Sheard et al. (2003)</td>
<td>1.353</td>
<td>-</td>
</tr>
<tr>
<td>Su (2008)</td>
<td>1.333</td>
<td>0.430</td>
</tr>
</tbody>
</table>
A.3 Validation of Isolated Cylinder

To validate the numerical code, the flow over an isolated cylinder without a wall is simulated in two dimension at $Re = 200$ in the laminar flow regime. A structured spectral mesh with 1900 spectral elements and a polynomial order of 7 was used in the validation. The domain consisted of a collection of quadrilateral spectral elements with a higher element density in regions of high-velocity gradients near the cylinders and in the wake region. The computational domain is a $24D \times 20D$ rectangle surrounding the cylinder. Table A.5 summarizes the comparison of $\bar{C}_D$ and $C'_L$ the present results and those reported in previous literature. The result obtained is in close agreement with those studies that were conducted from the year 2001 onwards. Therefore, the accuracy and validity of the computational method utilized is verified and is proven to be suitable for this study.

Appendix B: Wall Proximity Effects on Single Cylinder

We overview the characteristics of flow, lift and drag coefficients of a single cylinder at $Re = 200$ for a gap ratio varying from 0.05 to 0.5 from a moving plane wall. The critical gap ratio at which cessation of alternate vortex shedding takes place at $Re = 200$ is identified. Figure B.1 presents the variation of lift and drag coefficients with $e/D$ at $Re = 200$. 

Fig. B.1. Dependence of drag and lift coefficients on the gap ratio $e/D$ at $Re = 200$.

The values obtained in the present study show excellent agreement with those presented by Huang et al [7]. They have been used in section 3.4 to assess the behavior of a tandem cylinder arrangement. The rms drag peaks at around $e/D = 0.3$ which is close to the critical gap ratio at which alternate vortex shedding cessation takes place. As the cylinder is moved further away from the wall, the rms of the fluctuating drag decreases in magnitude. The mean lift increases with increasing proximity to the wall. The rms of the fluctuating lift increases with an increase in the gap ratio and asymptotes at $e/D \approx 0.4$.  

\[ (C_D)_{rms} \]
B.1 Mean gap velocity profile and instability analysis

It is of interest to identify the critical gap ratio at which cessation of alternate vortex shedding takes place when a single cylinder is brought close to a moving wall. In this section, Rayleigh's instability criteria has been invoked in order to assess the stability of the shear flow (presence of maximum vorticity) in the gap between the cylinder and the moving wall. Consider the streamwise velocity profile in the gap between the cylinder lower surface and the wall. Let $u_B$ be the base state of the streamwise velocity and $p_B$ of the pressure.

In order to determine the linear instability of the base flow, the momentum and continuity equations are combined to obtain a single equation in perturbation form. Fixing $y$ and wavenumber component $\alpha$ and letting $Re \to \infty$, we obtain the Rayleigh's equation (Drazin and Reid [31])

\[
(u_B(y) - c)(D^2 - \alpha^2)\hat{v} - (D^2 u_B)\hat{v} = 0 \quad (B.1)
\]

or

\[
D^2 \hat{v} - \left(\alpha^2 + \frac{D^2 u_B}{u_B - c}\right)\hat{v} = 0 \quad (B.2)
\]

subject to the following boundary conditions:

\[
\hat{v} = 0 \quad \text{on} \quad y = y_{wall}, y_{cyl} \quad \text{no permeation at each wall} \quad (B.3)
\]

where $D \equiv \frac{d}{dy}$ and $\alpha = \omega/c$ is real with frequency/growth rate $\omega$ and $c$ is an eigenvalue of Rayleigh’s equation such that $c = c_r + ic_i$. Suppose that $u_B$ and $Du_B$ are continuous in $y_{wall} < y < y_{cyl}$. Initially suppose that the flow is unstable that $c_i > 0$. Pre-multiplying equation (B.2) across by $\overline{\hat{v}}$, the complex conjugate of $\hat{v}$ and integrating from $y_{wall}$ to $y_{cyl}$, we obtain,
\[ \int_{y_{wall}}^{y_{cyl}} \tilde{v} D^2 \tilde{v} dy - \int_{y_{wall}}^{y_{cyl}} \left( \alpha^2 + \frac{D^2 u_B}{u_B - c} \right) |\tilde{v}|^2 dy = 0. \]  

(B.4)

with \(|\tilde{v}|^2 = \tilde{v} \bar{v}\). Integrating and simplifying we obtain

\[ - \int_{y_{wall}}^{y_{cyl}} D^2 \tilde{v} |\tilde{v}|^2 dy - \int_{y_{wall}}^{y_{cyl}} \left( \alpha^2 + \frac{D^2 u_B (u_B - \bar{c})}{|u_B - \bar{c}|^2} \right) |\tilde{v}|^2 dy = 0. \]  

(B.5)

The imaginary part of this equation is:

\[ -c_i \int_{y_{wall}}^{y_{cyl}} \frac{D^2 u_B |\tilde{v}|^2}{|u_B - \bar{c}|^2} dy = 0. \]  

(B.6)

Let the integral in equation (B.6) be denoted by I. For (B.5) to be satisfied at least one of \( c_i = 0 \) and \( I = 0 \) must be satisfied. Since \( c_i > 0 \) corresponding to unstable flow, \( I \) must be zero. Since, \( |\tilde{v}|^2 > 0 \) and \( |u_B - \bar{c}|^2 > 0 \), it follows \( D^2 u_B \) must change sign somewhere in the domain \((y_{wall}, y_{cyl})\). Thus, somewhere in the flow, \( u_{yy} = 0 \) must be satisfied. This is Rayleigh’s inflexion point theorem which states that a necessary condition for inviscid instability is the presence of an inflexion point in the background velocity distribution.

The absence of an inflexion point necessarily confers (inviscid) stability. The condition is necessary but not sufficient as earlier stated. Fjortoft’s stronger condition must be additionally satisfied along with Rayleigh’s instability criteria. The condition states that, if the streamwise velocity component \( u(y) \) is a monotonic function with respect to \( y \), a necessary condition for instability is that the product \( u_{yy}(u - u_I) < 0 \) somewhere in the flow where \( u_I \) is the value at the point of inflection.

Figure B.2 shows the mean streamwise velocity profile in the gap between the cylinder lower surface and the moving wall. The \( y \)-coordinate is normalized with respect to the respective gap ratio such that it varies between 0 at the wall and 1 at the cylinder lower surface. The \( x \)-coordinate is the mean streamwise
Fig. B.2. Mean streamwise velocity profiles of single cylinder at $Re = 200$ with varying gap ratio $e/D$.

Fig. B.3. Curvature of mean streamwise $u$ of single cylinder at $Re = 200$ with varying gap ratio $e/D$.

velocity $u$ normalized with respect to the free stream velocity $U_\infty$. In Fig. B.2b, the x–axis is scaled to between 1.3 and 1.62 in order to reveal the dynamic changes in the mean $u$ profile that take place with changes in $e/D$. At the lowest value of $e/D = 0.05$, the velocity profile presents us with a single...
distinct maxima. With an increase in gap ratio, a single maxima persists but its value increases up to $e/D = 0.2$. Between a gap ratio of 0.2 and 0.25, the value of the maxima decreases and the velocity profile increasingly flattens. Between a gap ratio of 0.3 and 0.4 the velocity profile develops two distinct local maxima and these continue to exist at higher gap ratios as evidenced in Fig. B.2b for $e/D = 0.4$ and 0.5.

Figure B.3 presents the curvature of the mean streamwise velocity profile between the cylinder and the plain wall. In Fig. B.3b, the region of interest around $u_{yy} = 0$ has been zoomed into. At extremely small gap ratios such as 0.05 and 0.1, the velocity profile is clearly stable and there is no inflection point. With increasing gap ratios, the curvature profile develops a bulge and it progresses towards the $u_{yy} = 0$. Between $e/D = 0.3$ and 0.4, the $u_{yy}$ profile develops two distinct inflection points. Based on Rayleigh’s inflexion point theorem at a gap ratio of $e/D = 0.4$, the velocity profile is unstable. Huang et al. [7] predicted that the value of the critical gap $e/D$ increases with decreasing Reynolds number. In Huang’s work [7], at $Re = 300$ the critical gap ratio is 0.28. The present study leads us to a critical gap ratio of between 0.3 and 0.35 for $Re = 200$.

References


