HYBRID RANS-LES FORMULATIONS FOR WAKE INTERFERENCE PHYSICS IN TANDEM CYLINDERS AND MULTI-COLUMN FLOATERS

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ABSTRACT

Accurate prediction of hydrodynamic forces on tandem bluff bodies at high Reynolds numbers is of interest in many fields of offshore engineering. The most commonly used turbulence modeling strategy for studying these flows is unsteady Reynolds-averaged Navier-Stokes methods (URANS) due to its speed. However, the accuracy of URANS results are problem dependent and usually poor for bluff bodies flow separation predictions. To overcome this deficiency, two different modeling methods have been considered: (i) large eddy simulation (LES) and (ii) non-linear URANS. LES are accurate and computationally feasible for low to moderate Reynolds number flows. However, the cost of LES makes it infeasible at high Reynolds numbers. On the other hand, non-linear URANS methods are fast like URANS, and its accuracy is comparable to LES for certain flows. It is usually not known in advance if the simulations using non-linear methods are accurate. Hybrid models have been proposed in the literature as an alternative to existing methods. They employ a URANS model in the near-body region and LES in the near and far wake regions. Simulations performed using hybrid models are computationally cheaper than LES and more accurate than URANS. Most hybrid models developed in the literature employ linear URANS models. The use of non-linear URANS models in the hybrid context has not received significant attention. In this study, we propose the use of a hybrid model based on a non-linear URANS model. Flow past tandem cylinders, with different spacing ratio, at sub-critical Reynolds number regime, is chosen as the test case. Simulations are also performed using URANS and linear hybrid models for comparison. It is shown that the non-linear hybrid models provides the best agreement to measurement data in the literature. Non-linear URANS models will be shown to provide acceptable prediction of hydrodynamic forces. The models are finally used to predict the current load on a generic multi-column floater.

1 Introduction

When two bodies are placed in close proximity to each other, the flow physics and induced forces are strongly dependent on the body shapes and spacing between the bodies. Canonical flow representing this setup is the flow past tandem cylinders. Three flow interference categories can be identified for this setup [1]: proximity interference, wake interference and no interference. In the proximity regime, drag is negative on the rear cylinder and vortex shedding from front cylinder is suppressed. Tandem cylinders behave like a single body and vortex shedding occurs behind the rear cylinder. In the wake interference regime, flow phenomena such as shear layer reattachment, occasional shedding, can be observed as the separation distance is gradually increased. Beyond a certain L/D ratio, vortex shedding occurs from both the cylinders and there is no interference effect. The three regimes are observed in both laminar and turbulent flows. Accurately simulating the flow physics in all the flow regimes

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for high Reynolds number turbulent flows is challenging due to computational cost. Unsteady Reynolds-averaged Navier-Stokes (URANS) models are incapable of modeling all the three regimes accurately over a wide range of Reynolds numbers of interest.

Laminar flow past cylinders in tandem arrangement has been investigated by several researchers due to its practical relevance and interesting flow physics. Comprehensive review on this topic can be found in Ref. [1]. Turbulent flow past tandem circular cylinders at moderate to high Reynolds numbers has received less attention due to the cost of LES and inadequacies of URANS models even for isolated cylinder flows [2]. Recent success of hybrid methods for single cylinder flows has motivated tandem cylinder simulations.

URANS methods are cheap ($N \sim \log$ $Re$, $Re$ is the Reynolds number), fast and simulations can be mostly performed on high-end workstations. However, there are two major issues in the use of these methods. First, most models are calibrated using canonical attached flows. The predictions of separated flows are usually less successful. Second, only mean flow field data is available from simulations. Instantaneous flow data is essential for many applications. As an alternative to URANS, large eddy simulation (LES) has been receiving a lot of interest in the recent years. LES has been successful in the simulation of wall-boundary flows at moderate Reynolds numbers. LES results are usually accurate within $5-10\%$ of direct numerical simulation or experimental results. However, the cost of LES scales as $N \sim Re^{1.76}$ for wall-bounded flows. This makes it impossible to perform bluff body LES at high Reynolds numbers.

High cost of LES modeling and the limitations of URANS, has led to the development of hybrid models. Hybrid models take advantage of the fact that LES is cheaper away from the wall ($N \sim Re^{0.4}$ for off-body) and that URANS methods need to be employed only in the near-wall region. It was expected that such a coupling should produce much better results than URANS. Consequently, hybrid methods overcome the disadvantages of LES (cost) and URANS (inaccurate) and the computations are relatively cheaper. The cost of hybrid methods is reduced by a factor of $0.07 Re^{0.46}$ compared to LES [3], making it possible to simulate high Reynolds number flows. Hybrid models have been successfully applied for the simulation of complex flows at high Reynolds numbers with varying degree of success [4]. However, unlike LES models, for many flow problems, the choice of the hybrid model did have some effect on the obtained results. A variety of hybrid models are currently in use in the literature [4] and the optimal model among the existing ones is still unclear. A detailed review of the hybrid models can be found in Refs. [4,5]. Comparison of commonly used detached eddy simulation (DES) to two new linear and non-linear hybrid models is the goal of the current study. Tandem cylinder setup at three different spacing to diameter ratio is chosen as the test case. The best model from this analysis will be used for prediction of the hydrodynamic forces on a multi-column floater.

The rest of the study is organized as follows. The turbulence model is presented in Section 2 and the equations are included in the appendix. Section 3 discusses the details of grid generation, the numerical code and the computational setup. Results and discussions are presented in Section 4. Section 5 contains the conclusions of the current study.

2 Governing Equation

The filtered conservation of mass and momentum equations for incompressible flow are given by

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial (\tilde{u}_i \tilde{u}_j)}{\partial x_j} = - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_i \partial x_j} - \frac{\partial D_{ij}}{\partial x_j} \quad (2)$$

where $D_{ij} = \tilde{u}_i \tilde{u}_j - \tilde{u}_j \tilde{u}_i$ is the hybrid stress tensor. It should be noted that there are some assumptions in the above equations. First, filtered and averaged variables are assumed to be equivalent in the near-wall URANS region. Second, the near-wall URANS region is different from traditional URANS. The kinetic energy is both resolved and modeled in the URANS region. The closure of the above set of equations, requires the specification of the hybrid stress tensor, and the transport equation for the turbulent quantities. Four different hybrid models have been considered in the current study. Two of the models employ the K-ω SST model [6] as the baseline URANS model while the other two use a non-linear URANS model (denoted by NSST) as the baseline model. To see the difference between the different hybrid models, let us consider the viscosity and dissipation term in turbulent kinetic energy equation

$$\nu_t^{\text{rans}} = \frac{a_1 k}{\text{max}(a_1 \omega, SF_2)} \quad (3)$$

$$\epsilon^{\text{rans}} = \beta^+ k \omega \quad (4)$$

For a generic hybrid model, the equations can rewritten as follows

$$\nu_t = \min \left( \nu_t^{\text{rans}}, (1 - F_2) C_{\Delta} \Delta \sqrt{k} \right) \quad (5)$$

$$\epsilon = \max \left( \epsilon^{\text{rans}}, (1 - F_2) C_{\omega} k^{3/2} / \Delta \right) \quad (6)$$

With these definitions, the hybrid stress tensor for a linear model is given by

$$D_{ij} = \frac{2}{3} k \delta_{ij} - 2 \nu_t \tilde{S}_{ij} \quad (7)$$
where $S_{ij}$ is the filtered strain rate and $S = \sqrt{2S_{ij}S_{ij}}$ is the strain magnitude. The blending function $F_2$ from the SST model is included to provide shielding effect and avoid grid-induced separation [5]. This function ensures that the model does not switch to LES until the edge of the boundary layer is reached ($F_2 \sim 0$). It is not necessary to use both Eqs. 5 and 6 for the hybrid models. The difference in the models arise from which of these variables are modified from URANS. Four different hybrid URANS-LES models considered in this study are

1. Blended SST (BSST): $\varepsilon^{\text{trans}}$ is not modified and Eq. 4 is used. Turbulent viscosity is calculated as: $\nu_t = f_\mu \nu^{\text{trans}}_t$, $f_\mu$ is a blending damping function which depends on the LES filter width $\Delta$, Kolmogorov length scale and characteristic length scale $L = k^{1/3}/\varepsilon$ [4]. $f_\mu$ has a value of 0 for DNS and 1 for URANS.
2. Unified SST (USST): Turbulent viscosity and dissipation are calculated using Eqs. 5 and 6, respectively.
3. Non-linear blended SST (NBSST): Similar to the blended SST. However, the linear model is replaced with a non-linear model. In this study, the model of Abe et. al [7] is employed.
4. Non-linear unified SST (NUSST): Similar to the unified SST. However, the linear model is replaced with a non-linear model.

3 Numerical Details

First, the grid generation methods are summarized. The grid requirements are different for URANS and hybrid models. For URANS, sufficient number of grid points in the boundary layer is required (without wall-functions). Twenty grid points are used in the boundary layer with a maximum stretching ratio of 1.1. Care is required in generating grids for the hybrid models to avoid modeled stress depletion and grid induced separation. The discussion of these issues is addressed in detail in Refs. [5, 8]. The grids are generated in the present study based on Ref. [8].

3.1 Grid Generation

The grids for the tandem cylinders were generated using GMSH [9]. The grid setup used for the simulations is shown in Fig. 1. The details of the different regions of the grid are marked in Fig. 1(b). The nomenclature follows Ref. [8]. In the near-wall region, stretched hexahedral grids were created to ensure that $y^+ \leq 1$ and at least twenty points are present in the boundary layer. The grid is made up of isotropic cells of size 0.02D (D being the diameter of the cylinder) in the gap between the cylinders, and immediately outside the boundary layer. This was done to avoid abrupt URANS-LES transition away from the cylinders and have isotropic cells in the gap LES region. Outside these regions, the grid was stretched in all the directions. The grids were generated in two-dimension first and extruded in the spanwise ($z$) direction. The total number of grid points was approximately 10 million. URANS simulations were performed in two and three dimensions (~one million grid points) for one of the problems. However, no significant difference was observed between the simulation results. Hence, all URANS simulations were performed in 2D for tandem cylinders.

Multi-column floater grids were generated following Ref. [8]. However, to keep the computational cost reasonable, wall-function approach was employed and $y^+ \sim 50$ at the first grid point.

3.2 Numerical Solver

Tandem cylinder simulations were done in OpenFOAM [10]. Floater simulations were performed using OpenFOAM and STAR-CCM+. The convection term in the momentum equation was discretized using a blended second-order central difference scheme. The convection term in the transport equation for the turbulent variables were discretized using bounded second-order schemes. PISO algorithm is used for pressure-velocity coupling in OpenFOAM. Coupled solvers were employed in STAR-CCM+.

The sketch of the computational domain is shown in Fig. 2. Fixed velocity and outflow boundary conditions were used at the inflow and outflow, respectively. Periodic and slip boundary conditions were employed on the boundaries marked as periodic and slip. Wall boundary conditions were used on the cylinders and floaters. The boundary conditions are the default boundary conditions commonly used in the simulations and no changes were made [6, 11]. For the multi-column floater simulations, periodic boundary conditions were replaced with slip boundary conditions and $D$ was taken to be the length of the floater.

4 Results and Discussion

4.1 Tandem Cylinder

All tandem cylinder simulations are performed at the same Reynolds number as the experiments [14]. Trip wires were used only on the front cylinder for most of the experiments. As the Reynolds number (0.166 million) is not in the super-critical regime laminar separation can be expected in the rear cylinder. This has to be taken into account when comparing the results since URANS models cannot predict laminar separation unless transition based models are employed.

Detailed analysis of results is performed for spacing ratio $L/D = 3.7$. This test case has received maximum attention in the literature. This also allows us to compare with other hybrid simulation results. Based on this analysis, the optimal model is chosen. Two more simulations, at different spacing ratios, are performed using this model. The obtained results are compared with experiments and URANS simulations.

4.1.1 $L/D = 3.7$: The plots of the second-invariant of velocity gradient tensor ($Q$) is shown in Fig. 3. $Q$ contours quantifies the resolved flow structures and identifies differences in the flow field obtained using the different models. The plots of $Q$ show
FIGURE 1: Grid setup used in the simulations. The grids were constructed based on the guidelines of Ref. [8]. (a) Setup of the problem. The grids are shown for \( L/D = 3.7 \) and (b) Different regions of the grid.

FIGURE 2: Problem setup for the simulations: (a) Boundary conditions and spanwise grid size and (b) Dimensions of the domain, in terms of the cylinder diameter D. The upstream and downstream distances were different for the floater simulations and are given in Sect. 4.

FIGURE 3: Second-invariant of the velocity gradient tensor (Q) for the different models: (a) BSST, (b) USST, and (c) NBSST. The contours are colored by the instantaneous streamwise velocity and plotted for \( Q = 1.5U_\infty^2/D \).
that all the models provide resolved flow structures. The roll-up of the shed vortices and breakdown of the vortices usually results in the formation of the Kelvin-Helmholtz instability. This is clearly visible for the USST, and the non-linear models (NBSST and NUSST show similar behavior). For the BSST model, the formation of the Kelvin-Helmholtz instability is not seen. It is
also observed that linear models have a much higher diffusion of the vortices.

The plots of the instantaneous coefficients of drag, lift and the frequency spectrum of the instantaneous lift forces are shown in Fig. 4 for the front and back cylinders. The average $C_D$ and $C_L^{rms}$ for the four models are compared to experiments and other hybrid results available in the literature in Table 1. For the front cylinder, the drag coefficient predicted by all the models show agreement to each other and to the experimental results. Surprisingly, Spalart-Allmaras based hybrid models [12,13] (DDES, and IDDES) show a lower drag force on the front cylinder. The results are also different between IDDES simulations performed using different codes. This was not surprising as the dissipation in LES region comes from the model and numerical scheme. The discrepancy in the prediction of the rms of the lift coefficient on both the cylinders by different models is much higher compared to $C_D$. As these values were not measured in the experiments, it is not possible to comment on the performance of the different models.

The drag on the back cylinder are poorly predicted by all the models. This was expected since no tripping wires were used in the experiments on the rear cylinder and laminar separation can be expected from the rear cylinder due to the spacing ratio and the breakdown of front cylinder vortices in the gap between the cylinders. Laminar separation prediction is still a challenge even for hybrid models [2]. Among the models tested in the literature, the non-linear blended hybrid model EASM [13] provides the best agreement with the measurements while NUSST model performs better than the other three models considered in the current study.

The vortex shedding frequency measured in the experiment corresponds to a Strouhal number ($St$) of 0.24. Most models are accurately able to predict the Strouhal number. It is also seen from the frequency spectrum plot of the lift forces on the back cylinder (Fig. 4) that all the hybrid models exhibit the -5/3 slope suggesting that the wake region is indeed well-resolved LES. On the other hand, if we look at the tabulated values of the Strouhal number from different studies, certain issues are visible. First, the Strouhal number predicted by the IDDES model using two different codes show distinct values. The $St$ obtained in Ref. [12] is comparable to the measured value while the results of Ref. [13] are poor. This reaffirms the importance of the numerical scheme, grid generation, initial conditions and the implementation of the hybrid models in different codes. NUSST provides the best prediction among the four models considered. All further hybrid simulations for the other spacing ratios are performed using the NUSST model.

4.1.2 Comparison of URANS and Hybrid Models: The average $C_D$ predicted by NUSST is compared with the URANS and experimental results in Table 2 for three different spacing to diameter ratios. For the smallest gap ratio of $L/D = 1.4$, the SST is unable to accurately predict the drag, on both cylinders. On the other hand, the non-linear URANS model NSST, improves the prediction of the drag forces. The predictions are comparable to the hybrid results on front cylinder while the NSST model gives reasonable prediction on the back cylinder. NUSST model gives good agreement of the forces on both cylinders. It should be noted that for this small gap ratio, the experimental results are very sensitive to the angle of attack on the front cylinder. The angle of attack on the front cylinder is exactly zero in the simulations. However, it is impossible to obtain a zero degree angle of attack in the measurements. This should be kept in mind while examining the differences in the CFD and experimental results.

As the spacing is increased further to $L/D = 3.0$ and $3.7$, the results from the URANS models are surprisingly better than hybrid predictions. This result does not mean that the URANS models predict the correct flow physics. No trip wires on rear cylinder do not affect the small gap ratio predictions. However, they have a significant effect as the spacing ratio is increased. In fact, when trip wires were used on the rear cylinder for larger gap ratios, it was observed that the measured forces are comparable to the hybrid results [13]. This suggests that the hybrid models provide the best agreement with the measurements. However, the surprising result from this analysis was the performance of the non-linear URANS model. Further investigation and optimization of this model may result in a valuable tool for bluff body force predictions.

4.2 Multi-Column Floater

Three different flow incidence angles are considered (0, 45 and 90 degrees, with respect to the pontoons) for the multi-column floater simulations. Both tandem and side-by-side flow interference effects can be observed between the components. For the zero degrees case, the column-column tandem and side-by-side flow interference effect is the major flow interference phenomena taking place. The ninety degree incidence has three major interference features: pontoon-pontoon interference, column-column tandem interference and column-column side-by-side interference. The pontoon-pontoon interference is the most important effect at this flow incidence angle and the rest are usually negligible. When the flow incidence angle lies between 0 and 90 degrees, partial interference between the columns and pontoons occurs and this results in significant amount of local secondary flows. It can be expected that the URANS models might be able to predict the 0 and 90 degrees case accurately. However, the 45 degrees case is more challenging at least for the linear model. The comparison of the flow field obtained using SST, NSST, and the hybrid NUSST for the forty-five degree case, at the column and pontoon level, are shown in Figs. 5 and 6, respectively. Simulations were run at sub-critical Re to match the experimental conditions. The SST model is unable to properly predict the diffusion of the wake from the front column. On the
TABLE 1: Coefficients of drag and lift and Strouhal number on both the cylinders, predicted using the different turbulence models, for $L/D = 3.7$. Experimental results were taken from Ref. [14]. In the experiments, tripping wires were used only on the front cylinder. Hence, discrepancy is expected on the numerical predictions from the back cylinder as none of the models include transition treatment.

<table>
<thead>
<tr>
<th>Model</th>
<th>$C_D$ Front</th>
<th>% Difference</th>
<th>Back</th>
<th>% Difference</th>
<th>Front</th>
<th>Back</th>
<th>Strouhal Number</th>
</tr>
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<td>0.31</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.24</td>
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<tr>
<td>BSST</td>
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<td>-3.95</td>
<td>0.44</td>
<td>42</td>
<td>0.11</td>
<td>0.65</td>
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<td>38.7</td>
<td>0.106</td>
<td>0.66</td>
<td>0.233</td>
</tr>
<tr>
<td>NUSST</td>
<td>0.62</td>
<td>-3.12</td>
<td>0.41</td>
<td>33</td>
<td>0.13</td>
<td>0.68</td>
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<td>27</td>
<td>0.148</td>
<td>0.695</td>
<td>0.234</td>
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FIGURE 5: Instantaneous flow field at the column and connector level: (a) SST, (b) NSST, and (c) NUSST.

FIGURE 6: Instantaneous flow field at the pontoon level: (a) SST, (b) NSST, and (c) NUSST.
TABLE 2: Coefficient of drag on both the cylinders for different spacing ratios.

<table>
<thead>
<tr>
<th>Model</th>
<th>Front</th>
<th>% Difference</th>
<th>Back</th>
<th>% Difference</th>
</tr>
</thead>
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<td>Experiments</td>
<td>0.64</td>
<td>-</td>
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<td>-</td>
<td>0.34</td>
<td>-</td>
</tr>
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<td>-5.88</td>
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<td>L/D = 1.4</td>
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other hand, the non-linear NSST URANS models provides a reasonable prediction of the wake in the gap between the columns. This was due to the presence of the non-linear term, which can capture the secondary flows with certain degree of success. The comparison of the predicted hydrodynamic coefficients are given in Table. 3. The values are the non-dimensinal forces in the streamwise and spanwise direction and not along the local axis aligned with the model as it is rotated. The experimental results were provided by DNW German-Dutch Wind Tunnels. USST simulation results are also included for comparison. The simulations using the SST, NSST and NUSST models were performed in OpenFOAM while the USST simulation was performed using STAR-CCM+. All the simulations employed wall-function and did not resolve the boundary layer.

For the 0 and 90 degree case, the linear URANS SST model provides a reasonable prediction. However, when the angle of incidence changes to 45 degrees, the performance of the linear model deteriorates. On the other hand, the non-linear model provides a good agreement with the hybrid results. This model has scope for further improvement as the model constants can be further optimized based on new wind tunnel results. This can lead to much better predictions using the non-linear URANS model.

5 Conclusions and Future Outlook

This study presented a comparative study of linear and non-linear URANS models with four newly proposed hybrid models. The non-linear models were constructed based on the extension of the interface definitions which were available in the literature for linear models. Flow past tandem circular cylinders at three different spacing to diameter ratio’s ($L/D = 1.4, 3.0, 3.7$) and a Reynolds number of 0.166 million was chosen as the test case for the evaluation. The results showed that both linear and non-linear hybrid models give accurate predictions of the coefficient of drag with the non-linear model providing slightly better performance. The predictions of the linear and non-linear URANS models showed a dependence on the choice of the constitutive relation for the turbulent stress tensor. The NSST model showed a consistently better performance than the linear SST model. NSST model has scope for further improvement and can lead to much better predictions if additional calibration is performed.

Three main conclusions can be drawn from the current study for tandem bluff body flow simulations:(i) Non-linear URANS models have the capability of predicting the coefficient of drag for proximity interference and no interference regimes. Unsteady bi-stable mode is usually observed around $L/D = 3.0$ and URANS models are not capable of capturing this
<table>
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<th>( C_D/\bar{C}_D )</th>
<th>% Difference</th>
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<td>0</td>
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<td>-5.8</td>
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<td>-5.6</td>
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<tr>
<td>Experiment</td>
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**TABLE 3**: Coefficient of drag for multi-column floater. For each flow incidences, the values were normalized by the respective lift and drag coefficient, measured in the experiment. A dash indicates that the value was within experimental uncertainty. This was expected for 0 and 90 degrees as there is negligible force in the direction perpendicular to the flow.

phenomenon. (ii) Hybrid models are necessary for the accurate prediction of the drag forces for the wake interference regime. These models provided good agreement with the experiments for all the regimes, albeit with slightly higher computational cost. However, the hybrid simulation cost is falling rapidly and it will not be long before it becomes a standard in the offshore industry. (iii) Non-linear hybrid models did not show any significant advantage compared to their linear counterparts. This was not surprising as linear sub-grid models have been found to be sufficient for most LES studies. Simulation results from OpenFOAM and STAR-CCM+ are comparable for the hybrid models. This further affirms the confirmation that linear hybrid models (NUSST not available in STAR-CCM+) are usually sufficient for most flow simulations. Further studies are currently being performed to evaluate the advantages of non-linear hybrid models.

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forward reflect the views of the authors alone, and not necessarily those of the institutions within the Corporate Laboratory.

6 Appendix

The linear model equations can be found in Ref. [6]. The only difference in the non-linear model is the additional non-linear term \( N_{ij} \), which is added to right-hand side of Eq. 7. The non-linear term is given by [7]

\[
N_{ij} = \frac{f_{NL}}{k} \left[ f_S \left( \frac{2 \bar{S}_{ik} \bar{S}_{kj} - \frac{2}{3} \bar{S}_{ik} \bar{S}_{kj} \delta_{ij}}{3} \right) - \bar{S}_{ik} \Omega_{kj} - \bar{S}_{ik} \Omega_{ji} \right] + 2kd_{ij}^{w}
\]

\[ (8) \]

\[
f_{NL} = \frac{4}{3} C_D C_B (1 - f_w(26))
\]

\[ (9) \]

\[
C_B = \frac{1}{1 + 22/3(C_D V_l/k) \Omega^2 + 2/3(C_D V_l/k)^2(\Omega^2 - S^2) f_B}
\]

\[ (10) \]

\[
f_w(\eta) = \exp \left( - \left( \frac{y^+}{\eta} \right)^2 \right)
\]

\[ (11) \]

\[
f_s = 1 - \frac{S^2 (\Omega^2 - S^2)}{(\Omega^2 + S^2)^2} \left\{ 1 + C_{22} C_D (\Omega - S) \frac{V_l}{k} \right\}
\]

\[ (12) \]

where \( \Omega = \sqrt{\bar{\Omega}_{ij} \bar{\Omega}_{ij}} \) is the rotation magnitude, \( S = \sqrt{\bar{S}_{ij} \bar{S}_{ij}} \), \( f_B = 1 + C_\eta C_D V_l/(k (\Omega - S)) \), \( y^+ = u_t \delta_{ij} / \nu \) is a dimensionless wall distance, \( \eta \) is a parameter, and \( C_D = 0.8 \), \( C_\eta = 100 \), and \( C_{22} = 7 \) are model constants. Finally, the expression for the term \( d_{ij}^{\eta} \) can be written as

\[
d_{ij}^{\eta} = -\alpha_w f_w(26) \frac{1}{2} (d_i d_j - \frac{\delta_{ij}}{3} d_k d_k)
\]

\[
+ f_w(26) (1 - f_{r1}) T_d^2 \left\{ - \frac{\beta_w C_w}{1 + C_w T_d^2} \sqrt{S^2 \Omega^2} \left( \bar{S}_{ik} \Omega_{kj} - \Omega_{ik} \bar{S}_{kj} \right) \right\}
\]

\[
+ f_w(26) (1 - f_{r2}) T_d^2 \left\{ \gamma_w C_w \left( \frac{1}{1 + C_w T_d^2} \sqrt{S^2 \Omega^2} \left( \bar{S}_{ik} \Omega_{kj} - \Omega_{ik} \bar{S}_{kj} \right) \right) \right\}
\]

\[ (13) \]

where \( d_i = \partial N_i / \partial x_j \), \( N_i \) is the unit-normal and \( f_{r1} = (\Omega^2 - S^2)/(\Omega^2 + S^2) \). \( T_d \) is the blended time-scale and is given by

\[
T_d = \left\{ 1 - f_w(15) \right\} k / \varepsilon + f_w(15) \delta_w \sqrt{\varepsilon / \varepsilon}
\]

\[ (14) \]

Here \( \varepsilon = \beta^+ \omega_k \) is the turbulence dissipation term. The model constants for the term \( d_{ij}^{\eta} \) are given by

\[
\alpha_w = 1, \quad \beta_w = \frac{1}{4}, \quad C_w = 0.5, \quad \gamma_w = 1.5, \quad \text{and} \quad \delta_w = 1.0
\]

\[ (15) \]