

# Full tables for “On efficiently solving the subproblems of a level-set method for fused lasso problems”

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We present statistics of the tested biomedical data in Table 1.

Table 1: Statistics of the biomedical data sets.

probname	$m; n$	$\lambda_{\max}(AA^T)$
DLBCLH	160;7399	8.38e+02
DLBCLN	160;7399	8.38e+02
DLBCLS	47;4026	6.23e+02
lungH1	203;12600	6.64e+03
lungH2	149;12533	6.96e+03
lungM	96;7129	4.46e+03
lungO	39;2880	2.30e+03
NervousSystem	60;7129	3.34e+03
ovarianP	253;15153	1.30e+04
ovarianS	216;373401	2.90e+05

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Table 2 reports the detailed numerical results for SSNAL, SLEP, ADMM, iADMM and LADMM in solving all the instances in the biomedical datasets.

Table 2: The performance of various algorithms on fused lasso regularized least squares problems on high-dimensional biomedical datasets (accuracy  $\eta \leq 10^{-6}$ ).  $m$  is the sample size and  $n$  is the dimension of features. In the table, “a” = SSNAL, “b” = SLEP, “c” = ADMM, “d” = iADMM, and “e” = LADMM. “nnz” denotes the number of nonzeros in the solution obtained by SSNAL.

probname $m; n$	$\alpha_1; \alpha_2$	nnz( $x$ ) ; nnz( $Bx$ )	$\eta$					time (hours:minutes:seconds)				
			a	b	c	d	e	a	b	c	d	e
DLBCLH 160;7399	$10^{-3}$ ; 10	1966 ; 235	1.3-7	7.8-6	9.9-7	7.6-7	9.9-7	01	16	06	13	10
	$10^{-3}$ ; 2	818 ; 261	3.6-7	1.2-5	9.9-7	9.9-7	9.9-7	01	16	11	22	18
	$10^{-3}$ ; 0.2	211 ; 291	7.0-8	2.9-5	9.9-7	6.6-7	2.4-6	01	16	16	39	19
	$10^{-3}$ ; 0.01	157 ; 306	9.1-8	4.7-5	9.8-7	6.5-7	3.7-6	01	14	17	37	18
	$10^{-4}$ ; 10	1995 ; 240	2.9-7	2.1-5	8.5-7	9.8-7	1.3-6	01	16	14	36	19
	$10^{-4}$ ; 2	848 ; 275	3.6-7	3.4-5	9.1-7	8.7-7	3.4-6	01	16	20	43	20
	$10^{-4}$ ; 0.2	213 ; 292	4.9-9	7.6-5	8.7-7	9.1-7	2.0-5	01	16	26	1:09	20
$10^{-4}$ ; 0.01	158 ; 306	1.5-7	1.0-4	4.3-6	9.9-7	5.8-5	01	14	30	1:24	18	
DLBCLN 160;7399	$10^{-3}$ ; 10	1966 ; 235	1.3-7	7.8-6	9.9-7	7.6-7	9.9-7	01	16	06	13	10
	$10^{-3}$ ; 2	818 ; 261	3.6-7	1.2-5	9.9-7	9.9-7	9.9-7	01	16	10	22	18
	$10^{-3}$ ; 0.2	211 ; 291	7.0-8	2.9-5	9.9-7	6.6-7	2.4-6	00	16	16	39	19
	$10^{-3}$ ; 0.01	157 ; 306	9.1-8	4.7-5	9.8-7	6.5-7	3.7-6	00	14	17	37	18
	$10^{-4}$ ; 10	1995 ; 240	2.9-7	2.1-5	8.5-7	9.8-7	1.3-6	01	16	14	36	19
	$10^{-4}$ ; 2	848 ; 275	3.6-7	3.4-5	9.1-7	8.7-7	3.4-6	01	16	20	43	20
	$10^{-4}$ ; 0.2	213 ; 292	4.9-9	7.6-5	8.7-7	9.1-7	2.0-5	01	16	26	1:09	20
$10^{-4}$ ; 0.01	158 ; 306	1.5-7	1.0-4	4.3-6	9.9-7	5.8-5	01	14	30	1:25	18	
DLBCLS 47;4026	$10^{-3}$ ; 10	685 ; 69	2.2-7	4.6-6	9.9-7	5.3-8	9.9-7	00	09	02	04	03
	$10^{-3}$ ; 2	264 ; 79	8.5-7	2.0-6	8.7-7	7.4-7	9.9-7	00	10	02	04	04
	$10^{-3}$ ; 0.2	76 ; 90	2.6-7	6.1-6	7.1-7	9.9-7	9.9-7	00	09	04	09	08
	$10^{-3}$ ; 0.01	44 ; 84	4.2-7	2.0-5	5.9-7	6.9-7	9.9-7	00	07	06	12	10
	$10^{-4}$ ; 10	676 ; 70	3.3-7	1.6-5	6.9-7	2.6-7	9.9-7	00	08	05	09	09
	$10^{-4}$ ; 2	274 ; 83	5.7-7	1.3-5	4.7-7	9.9-7	9.9-7	00	09	06	11	09
	$10^{-4}$ ; 0.2	76 ; 90	6.6-7	1.8-5	3.2-7	5.1-7	2.4-6	00	08	09	20	11
$10^{-4}$ ; 0.01	44 ; 84	4.5-8	4.2-5	6.2-7	9.7-7	2.5-5	00	07	14	33	10	
lungH1 203;12600	$10^{-3}$ ; 10	1556 ; 244	9.0-8	1.2-4	7.2-7	9.9-7	9.9-7	01	28	07	11	25
	$10^{-3}$ ; 2	514 ; 325	4.9-7	1.1-4	8.7-7	9.3-7	3.9-5	01	28	13	22	33
	$10^{-3}$ ; 0.2	199 ; 357	2.6-7	1.8-4	9.9-7	9.4-7	9.8-2	01	27	18	32	32
	$10^{-3}$ ; 0.01	188 ; 365	2.1-7	5.7-4	9.9-7	8.3-7	2.9-2	01	24	16	38	29
	$10^{-4}$ ; 10	1856 ; 301	4.4-8	4.4-4	9.8-7	9.4-7	9.9-3	01	27	15	35	33
	$10^{-4}$ ; 2	551 ; 344	9.2-8	7.2-4	9.9-7	6.5-7	6.7-2	01	28	28	1:02	34
	$10^{-4}$ ; 0.2	206 ; 373	1.5-8	1.4-3	9.9-7	9.5-7	7.6-2	01	27	45	1:28	34
$10^{-4}$ ; 0.01	195 ; 375	5.6-8	1.7-3	9.9-7	8.5-7	4.2-2	01	25	37	1:34	31	
lungH2 149;12533	$10^{-3}$ ; 10	1293 ; 136	3.5-7	4.2-5	8.5-7	5.2-7	9.9-7	00	25	03	05	07
	$10^{-3}$ ; 2	646 ; 186	6.6-8	3.9-5	9.9-7	8.5-7	9.9-7	00	26	06	10	19
	$10^{-3}$ ; 0.2	261 ; 249	8.8-7	2.2-4	9.9-7	9.8-7	1.4-4	00	25	14	27	31
	$10^{-3}$ ; 0.01	137 ; 268	4.6-7	1.5-4	9.9-7	7.9-7	2.2-5	00	22	24	50	29
	$10^{-4}$ ; 10	1901 ; 202	1.8-7	1.8-4	7.5-7	9.9-7	4.9-3	01	26	07	16	31
	$10^{-4}$ ; 2	775 ; 236	2.6-7	1.3-4	8.2-7	9.9-7	2.4-2	01	27	15	36	34
	$10^{-4}$ ; 0.2	263 ; 266	1.1-7	9.2-4	9.9-7	5.6-7	2.3-3	01	25	42	1:42	32
$10^{-4}$ ; 0.01	146 ; 285	1.2-7	9.9-4	1.4-7	8.3-7	2.0-2	01	23	50	1:46	30	
lungM	$10^{-3}$ ; 10	801 ; 125	5.3-7	2.4-5	9.9-7	8.5-7	9.9-7	00	12	03	04	08
	$10^{-3}$ ; 2	236 ; 144	1.4-7	6.5-5	9.9-7	7.5-7	9.9-7	00	12	04	06	13

Table 2: The performance of various algorithms on fused lasso regularized least squares problems on high-dimensional biomedical datasets (accuracy  $\eta \leq 10^{-6}$ ).  $m$  is the sample size and  $n$  is the dimension of features. In the table, “a” = SSNAL, “b” = SLEP, “c” = ADMM, “d” = iADMM, and “e” = LADMM. “nnz” denotes the number of nonzeros in the solution obtained by SSNAL.

probname $m; n$	$\alpha_1; \alpha_2$	nnz( $x$ ) ; nnz( $Bx$ )	$\eta$					time (hours:minutes:seconds)				
			a	b	c	d	e	a	b	c	d	e
96;7129	$10^{-3}$ ; 0.2	98 ; 179	5.1-8	2.2-4	9.9-7	5.5-7	1.3-5	00	12	12	14	15
	$10^{-3}$ ; 0.01	90 ; 175	3.1-7	2.1-4	7.1-7	9.9-7	3.1-5	00	10	08	18	14
	$10^{-4}$ ; 10	919 ; 148	2.5-7	1.3-4	1.2-6	5.5-7	1.2-5	00	12	25	17	16
	$10^{-4}$ ; 2	270 ; 164	3.5-7	3.1-4	9.6-7	9.6-7	4.4-5	00	12	09	14	17
	$10^{-4}$ ; 0.2	101 ; 182	2.7-7	2.3-4	6.7-7	9.2-7	7.1-5	00	12	18	33	17
	$10^{-4}$ ; 0.01	92 ; 180	4.2-7	6.5-4	4.6-6	4.4-7	1.8-4	00	11	24	41	15
lungO 39;2880	$10^{-3}$ ; 10	383 ; 48	2.1-7	3.3-5	8.7-7	8.8-7	9.5-7	00	05	01	03	05
	$10^{-3}$ ; 2	102 ; 65	2.4-8	1.2-4	8.4-7	6.0-8	8.5-6	00	05	02	05	07
	$10^{-3}$ ; 0.2	40 ; 74	7.8-8	3.5-4	9.5-7	9.8-7	1.3-5	00	05	05	10	07
	$10^{-3}$ ; 0.01	36 ; 72	6.6-8	2.5-4	9.9-7	4.2-7	3.4-5	00	04	03	07	06
	$10^{-4}$ ; 10	422 ; 59	2.3-7	8.0-5	9.9-7	2.2-7	2.6-5	00	05	07	17	07
	$10^{-4}$ ; 2	102 ; 68	1.9-7	1.7-4	8.8-7	5.6-7	4.5-5	00	05	05	11	08
	$10^{-4}$ ; 0.2	40 ; 75	1.3-7	3.0-4	4.1-7	6.9-7	1.0-4	00	05	10	24	08
	$10^{-4}$ ; 0.01	36 ; 71	5.6-8	7.4-4	5.3-7	8.7-7	5.5-5	00	05	07	20	07
NervousSystem 60;7129	$10^{-3}$ ; 10	785 ; 97	1.4-7	8.9-5	8.6-7	8.7-7	3.0-6	00	13	05	09	15
	$10^{-3}$ ; 2	184 ; 100	1.5-7	1.1-4	7.5-7	9.9-7	9.9-7	00	13	05	09	15
	$10^{-3}$ ; 0.2	65 ; 113	2.9-7	3.4-4	9.2-7	3.4-7	1.2-5	00	12	10	19	16
	$10^{-3}$ ; 0.01	57 ; 113	5.9-8	1.1-4	9.9-7	8.4-7	1.5-5	00	11	07	15	15
	$10^{-4}$ ; 10	823 ; 98	1.1-7	1.1-4	9.9-7	9.9-7	9.5-5	00	13	12	21	16
	$10^{-4}$ ; 2	187 ; 99	8.4-8	3.8-4	9.9-7	7.2-7	4.8-5	00	13	12	26	17
	$10^{-4}$ ; 0.2	64 ; 112	3.9-8	5.9-4	4.9-7	8.6-7	1.5-4	00	12	18	42	17
	$10^{-4}$ ; 0.01	59 ; 116	1.4-7	5.7-4	9.8-7	9.9-7	5.3-4	00	11	20	45	15
ovarianP 253;15153	$10^{-3}$ ; 10	2089 ; 102	4.2-8	7.3-5	5.6-7	6.6-7	1.8-5	01	1:02	14	23	1:09
	$10^{-3}$ ; 2	824 ; 144	1.6-7	1.6-4	9.9-7	9.0-7	9.7-4	01	1:01	19	36	1:06
	$10^{-3}$ ; 0.2	331 ; 248	1.6-7	6.8-4	7.6-7	6.5-7	1.9-3	01	57	34	1:35	1:04
	$10^{-3}$ ; 0.01	180 ; 285	1.3-7	6.2-4	9.9-7	9.9-7	2.7-3	01	53	59	2:55	1:00
	$10^{-4}$ ; 10	3906 ; 285	1.3-7	2.1-4	7.9-7	1.3-7	9.2-3	01	58	21	1:02	1:04
	$10^{-4}$ ; 2	1259 ; 350	2.7-7	4.5-4	8.2-7	9.7-7	1.1-2	01	58	45	1:51	1:05
	$10^{-4}$ ; 0.2	468 ; 401	2.4-7	2.2-3	7.5-7	7.4-7	3.1-3	01	57	1:13	3:41	1:05
	$10^{-4}$ ; 0.01	255 ; 412	9.9-8	1.4-3	2.9-5	1.9-5	4.4-3	01	55	1:55	6:11	1:01
ovarianS 216;373401	$10^{-3}$ ; 10	5748 ; 294	5.9-7	1.2-3	7.1-7	2.5-7	2.9-3	13	23:02	12:44	19:25	22:46
	$10^{-3}$ ; 2	1958 ; 352	6.7-7	3.8-3	2.1-6	8.5-7	9.4-3	15	20:17	46:31	57:19	23:23
	$10^{-3}$ ; 0.2	332 ; 396	2.2-7	8.4-3	1.8-4	1.0-4	2.8-2	15	19:24	41:45	1:11:39	22:43
	$10^{-3}$ ; 0.01	205 ; 409	6.3-7	8.5-3	1.6-3	3.6-4	3.8-2	14	18:41	41:12	1:15:03	21:55
	$10^{-4}$ ; 10	5792 ; 358	4.4-7	4.7-3	2.6-5	2.6-5	9.5-2	20	26:23	39:51	1:33:09	29:27
	$10^{-4}$ ; 2	1963 ; 380	2.5-7	7.3-3	1.1-3	1.8-3	9.0-2	20	16:39	45:03	1:17:03	22:59
	$10^{-4}$ ; 0.2	343 ; 410	3.0-7	6.0-3	2.9-4	1.4-3	2.1-1	19	16:36	41:00	1:37:19	23:26
	$10^{-4}$ ; 0.01	212 ; 422	2.5-7	6.6-3	1.2-3	6.8-2	1.8-1	18	18:15	44:24	1:50:34	23:13

We present statistics of the tested UCI datasets in Table 3.

Table 3: Statistics of the UCI test instances.

probname	$m; n$	$\lambda_{\max}(\mathcal{A}\mathcal{A}^*)$
E2006.train	16087;150360	1.91e+05
log1p.E2006.train	16087;4272227	5.86e+07
E2006.test	3308;150358	4.79e+04
log1p.E2006.test	3308;4272226	1.46e+07
pyrim5	74;201376	1.22e+06
triazines4	186;635376	2.07e+07
abalone7	4177;6435	5.21e+05
bodyfat7	252;116280	5.29e+04
housing7	506;77520	3.28e+05
mpg7	392;3432	1.28e+04
space_ga9	3107;5005	4.01e+03

In Table 4, we report the detailed numerical results for SSNAL, SLEP, ADMM, iADMM and LADMM in solving fused lasso problems with large-scale UCI datasets.

Table 4: Same as Table 2 but for large-scale UCI datasets.

probname $m; n$	$\alpha_1; \alpha_2$	$\text{nnz}(x); \text{nnz}(Bx)$	$\eta$					time (hours:minutes:seconds)				
			a	b	c	d	e	a	b	c	d	e
E2006.train 16087;150360	$10^{-6}; 1$	4 ; 5	1.4-8	8.1-4	4.0-7	6.7-7	8.7-6	03	18:44	28:42	4:39	19:31
	$10^{-6}; 0.5$	8 ; 13	2.1-7	2.1-3	2.5-7	3.4-7	3.2-3	03	18:49	36:42	7:34	18:39
	$10^{-6}; 0.2$	8 ; 15	1.8-7	2.6-3	1.7-7	4.4-7	5.6-3	04	22:13	45:19	9:31	23:19
	$10^{-6}; 0.01$	25 ; 47	6.1-8	9.9-4	8.8-8	5.3-7	6.4-3	04	21:12	50:04	9:48	19:48
	$10^{-7}; 1$	450 ; 586	8.1-7	2.1-3	9.1-7	3.5-7	8.1-3	14	20:13	48:00	13:47	19:59
	$10^{-7}; 0.5$	657 ; 1069	9.3-7	4.2-3	2.7-7	4.9-8	8.5-3	19	20:13	42:04	13:29	20:05
	$10^{-7}; 0.2$	1007 ; 1844	4.6-8	4.3-3	1.2-7	5.0-7	8.8-3	40	19:23	44:33	16:04	19:32
	$10^{-7}; 0.01$	1424 ; 2764	1.7-7	4.3-3	4.2-7	8.9-7	9.1-3	1:13	19:41	45:20	18:01	20:14
E2006.test 3308;150358	$10^{-6}; 1$	10 ; 15	1.4-8	3.7-4	2.5-7	5.6-7	2.4-3	02	5:19	2:51	2:02	6:38
	$10^{-6}; 0.5$	14 ; 24	2.6-8	6.8-4	3.6-8	5.7-7	4.2-3	02	5:32	2:55	2:17	6:37
	$10^{-6}; 0.2$	25 ; 46	3.1-8	1.3-3	3.4-8	6.1-7	4.5-3	02	5:33	2:57	2:22	6:51
	$10^{-6}; 0.01$	49 ; 95	1.7-8	4.9-4	9.2-8	3.3-7	4.7-3	02	5:20	2:59	2:22	6:39
	$10^{-7}; 1$	512 ; 802	4.0-8	7.1-4	6.1-7	1.1-7	5.0-3	05	5:32	3:55	3:49	6:54
	$10^{-7}; 0.5$	765 ; 1384	2.8-8	1.2-3	5.4-7	3.9-7	5.1-3	12	5:31	4:10	4:56	6:53
	$10^{-7}; 0.2$	1026 ; 1927	7.3-8	6.6-4	9.4-7	7.8-7	5.1-3	22	5:26	4:44	6:42	6:44
	$10^{-7}; 0.01$	1317 ; 2581	2.9-7	1.1-3	7.1-7	7.8-7	5.1-3	53	5:07	4:23	5:15	6:24
log1p.E2006.train 16087;4272227	$10^{-3}; 1$	12 ; 7	1.7-7	4.4-5	7.4-7	3.2-7	1.9-3	25	2:47:42	35:24	13:11	3:00:01
	$10^{-3}; 0.5$	4 ; 5	4.0-8	2.5-4	3.5-7	2.7-7	5.3-3	24	2:52:08	36:17	14:33	3:00:01
	$10^{-3}; 0.2$	4 ; 5	6.6-7	2.0-4	9.9-7	3.6-7	2.3-3	23	2:50:42	32:13	12:00	3:00:01
	$10^{-3}; 0.01$	5 ; 9	9.6-8	1.8-5	6.5-7	4.7-7	2.4-3	25	2:45:56	43:08	16:21	3:00:01
	$10^{-4}; 1$	222 ; 192	2.4-8	6.8-5	9.4-7	7.2-7	9.9-3	54	2:53:39	39:35	16:31	3:00:01
	$10^{-4}; 0.5$	256 ; 340	1.2-7	1.3-4	9.9-7	8.8-7	1.3-2	53	2:47:20	52:33	32:45	3:00:01
	$10^{-4}; 0.2$	373 ; 620	5.0-7	1.4-4	7.3-7	5.9-7	1.1-2	1:04	2:41:46	50:18	30:04	3:00:01
	$10^{-4}; 0.01$	576 ; 1100	9.8-7	1.6-4	7.8-7	6.7-7	1.4-2	1:09	2:44:02	1:01:16	54:23	3:00:01
log1p.E2006.test 3308;4272226	$10^{-3}; 1$	4 ; 3	5.5-7	1.7-4	4.8-7	4.4-7	4.1-5	17	1:43:15	6:13	6:08	1:56:24
	$10^{-3}; 0.5$	4 ; 5	6.1-7	1.5-4	1.5-8	5.3-7	8.3-4	17	1:44:15	6:20	6:05	1:56:52
	$10^{-3}; 0.2$	5 ; 7	2.8-8	1.9-4	8.4-7	8.5-7	1.0-3	20	1:43:14	6:34	5:56	1:53:09
	$10^{-3}; 0.01$	8 ; 15	5.8-8	1.4-4	2.7-7	2.0-7	5.2-4	21	1:39:06	8:32	8:45	1:52:15

Table 4: Same as Table 2 but for large-scale UCI datasets.

probname $m; n$	$\alpha_1; \alpha_2$	$\text{nnz}(x); \text{nnz}(Bx)$	$\eta$					time (hours:minutes:seconds)				
			a	b	c	d	e	a	b	c	d	e
	$10^{-4}; 1$	413 ; 408	2.6-7	1.7-4	7.8-7	9.2-7	1.0-3	55	1:40:28	11:10	13:16	1:51:49
	$10^{-4}; 0.5$	597 ; 842	7.3-8	2.5-4	5.1-7	6.8-7	1.6-3	58	1:41:13	11:54	14:40	1:54:20
	$10^{-4}; 0.2$	797 ; 1387	1.0-7	1.6-4	5.2-7	7.1-7	2.0-3	54	1:38:03	13:34	15:01	1:51:59
	$10^{-4}; 0.01$	1059 ; 2035	2.0-7	2.2-4	2.5-7	9.8-7	2.7-3	42	1:35:41	12:16	13:37	1:46:12
pyrim5 74;201376	$10^{-3}; 1$	327 ; 97	4.3-7	1.1-3	9.9-7	9.9-7	1.0-4	05	11:14	20:36	49:59	13:07
	$10^{-3}; 0.5$	174 ; 123	8.5-7	5.6-3	9.9-7	9.9-7	3.2-4	04	8:27	12:17	29:20	9:58
	$10^{-3}; 0.2$	128 ; 124	6.1-7	1.4-3	9.1-5	8.2-5	8.6-4	05	8:28	17:18	47:59	9:19
	$10^{-3}; 0.01$	75 ; 145	1.7-7	1.9-3	6.8-5	2.0-4	4.3-4	04	7:33	16:17	59:36	8:31
	$10^{-4}; 1$	350 ; 141	3.1-7	8.3-3	2.2-4	1.2-4	2.2-3	08	9:49	25:49	1:49:37	12:25
	$10^{-4}; 0.5$	233 ; 142	3.0-7	6.8-3	6.2-5	1.4-4	2.5-3	07	8:27	19:55	1:35:54	9:33
	$10^{-4}; 0.2$	165 ; 140	6.1-7	5.1-3	1.5-4	2.1-4	1.3-3	07	8:33	18:22	1:36:41	9:16
	$10^{-4}; 0.01$	91 ; 156	3.1-8	6.2-3	7.1-3	1.7-3	1.6-3	06	7:26	17:16	1:42:19	8:21
triazines4 186;635376	$10^{-3}; 1$	1078 ; 261	8.2-7	3.1-3	1.4-4	1.3-3	3.1-3	23	1:01:56	2:08:07	3:00:01	1:04:04
	$10^{-3}; 0.5$	679 ; 260	2.9-7	3.4-3	2.9-5	5.9-3	3.8-3	25	1:02:49	2:07:21	3:00:02	1:03:59
	$10^{-3}; 0.2$	455 ; 236	3.2-7	3.9-3	8.2-5	4.0-3	4.3-3	24	53:41	1:46:59	3:00:01	1:15:14
	$10^{-3}; 0.01$	217 ; 302	1.4-7	2.8-3	3.0-4	1.4-1	4.8-3	27	54:52	1:56:48	3:00:01	56:11
	$10^{-4}; 1$	1192 ; 326	2.2-7	1.6-2	2.2-3	8.9-1	2.0-2	41	1:02:01	2:11:02	3:00:05	1:04:30
	$10^{-4}; 0.5$	875 ; 334	4.5-7	1.2-2	9.9-3	8.6-1	5.2-2	37	1:00:20	2:13:51	3:00:10	1:02:02
	$10^{-4}; 0.2$	533 ; 365	1.7-7	1.4-2	6.7-2	9.2-1	4.0-2	38	53:35	2:26:04	3:00:06	54:24
	$10^{-4}; 0.01$	223 ; 355	3.3-7	1.3-2	2.6-2	7.8-1	2.3-2	40	1:07:27	2:37:41	3:00:01	1:11:24
abalone7 4177;6435	$10^{-3}; 1$	26 ; 15	5.3-8	2.1-5	6.2-7	9.4-7	7.0-7	01	3:16	51	45	2:19
	$10^{-3}; 0.5$	29 ; 17	8.4-8	3.7-5	8.8-7	6.8-7	9.9-7	01	3:15	53	41	2:18
	$10^{-3}; 0.2$	27 ; 36	1.7-7	4.1-5	5.7-7	2.0-7	8.3-7	01	3:18	1:38	1:33	2:17
	$10^{-3}; 0.01$	25 ; 43	7.0-7	7.6-5	8.6-7	7.0-7	9.9-7	01	3:19	2:15	2:17	2:29
	$10^{-4}; 1$	61 ; 57	6.9-7	5.7-4	9.9-7	9.6-7	9.9-7	02	3:22	1:40	2:23	2:50
	$10^{-4}; 0.5$	50 ; 75	1.7-7	8.8-4	4.4-7	7.4-7	9.9-7	02	3:22	1:52	2:28	2:56
	$10^{-4}; 0.2$	52 ; 95	2.5-7	1.6-3	2.7-7	6.4-7	7.8-5	03	3:21	2:26	3:55	3:27
	$10^{-4}; 0.01$	58 ; 110	5.7-8	3.3-3	9.0-7	8.7-7	1.0-4	04	3:19	4:11	5:31	3:25
bodyfat7 252;116280	$10^{-5}; 1$	37 ; 25	3.2-7	9.2-4	9.9-7	9.1-7	9.9-7	03	7:24	2:38	6:47	7:53
	$10^{-5}; 0.5$	36 ; 29	6.4-7	2.1-4	9.9-7	6.0-7	2.6-6	03	7:14	2:22	7:59	7:39
	$10^{-5}; 0.2$	25 ; 33	3.7-7	2.2-3	9.9-7	7.9-7	9.9-6	04	7:21	2:15	7:02	7:44
	$10^{-5}; 0.01$	25 ; 43	3.1-7	7.8-4	5.8-7	8.4-7	2.8-5	03	6:56	2:19	8:59	7:20
	$10^{-6}; 1$	151 ; 103	1.8-7	2.1-3	6.8-7	9.3-7	8.0-4	05	7:06	2:34	12:47	7:30
	$10^{-6}; 0.5$	142 ; 136	7.9-7	1.4-3	9.2-7	9.4-7	9.1-4	05	7:05	3:03	19:30	7:33
	$10^{-6}; 0.2$	111 ; 169	3.2-7	2.0-3	9.9-7	9.9-7	8.7-4	06	7:06	4:52	32:28	7:33
	$10^{-6}; 0.01$	101 ; 190	9.2-8	1.4-3	9.9-7	9.9-7	8.7-4	06	6:41	9:33	47:33	7:02
housing7 506;77520	$10^{-3}; 1$	131 ; 117	8.1-7	1.4-4	9.3-7	8.0-7	9.9-7	02	7:19	2:09	8:25	5:42
	$10^{-3}; 0.5$	126 ; 149	4.0-7	2.1-4	9.9-7	9.2-7	2.4-6	02	7:21	4:20	17:24	7:33
	$10^{-3}; 0.2$	117 ; 199	4.5-8	1.5-4	9.9-7	8.7-7	3.1-5	02	7:18	3:57	16:10	7:30
	$10^{-3}; 0.01$	151 ; 284	4.8-7	3.6-4	9.9-7	7.6-7	1.0-4	02	7:03	4:26	18:11	7:16
	$10^{-4}; 1$	296 ; 304	1.6-7	2.4-3	9.9-7	6.7-7	2.1-4	03	7:07	3:38	27:04	7:24
	$10^{-4}; 0.5$	253 ; 352	1.6-7	4.4-3	9.9-7	7.6-7	4.0-4	03	7:26	6:44	1:02:18	7:36
	$10^{-4}; 0.2$	268 ; 446	1.9-7	3.3-3	9.1-7	7.6-8	1.3-3	04	7:25	6:29	51:13	7:41
	$10^{-4}; 0.01$	276 ; 543	2.7-7	7.3-3	9.9-7	7.0-7	1.3-3	04	6:59	8:55	1:36:21	7:16
mpg7 392;3432	$10^{-3}; 1$	32 ; 39	1.7-7	2.1-6	3.1-7	1.5-7	9.9-7	00	10	01	01	01
	$10^{-3}; 0.5$	35 ; 46	1.2-7	4.0-6	3.0-7	1.5-7	9.1-7	00	10	01	01	01
	$10^{-3}; 0.2$	43 ; 68	1.0-7	7.5-6	8.9-7	1.2-7	9.4-7	00	10	01	02	01
	$10^{-3}; 0.01$	42 ; 74	4.1-8	8.1-6	3.0-7	5.3-7	9.3-7	00	09	01	02	03

Table 4: Same as Table 2 but for large-scale UCI datasets.

probname $m; n$	$\alpha_1; \alpha_2$	$\text{nnz}(x) ; \text{nnz}(Bx)$	$\eta$					time (hours:minutes:seconds)				
			a	b	c	d	e	a	b	c	d	e
	$10^{-4} ; 1$	149 ; 142	5.6-7	4.1-5	5.3-7	8.7-7	2.0-6	00	10	02	05	12
	$10^{-4} ; 0.5$	131 ; 171	8.6-7	5.0-5	6.0-7	8.8-7	2.0-6	00	10	02	05	12
	$10^{-4} ; 0.2$	137 ; 200	7.1-7	2.3-4	9.9-7	5.9-7	1.7-5	00	09	02	07	12
	$10^{-4} ; 0.01$	128 ; 222	2.2-7	1.7-4	5.8-7	9.1-7	8.8-5	00	09	02	07	11
space9 3107;5005	$10^{-3} ; 1$	14 ; 13	1.2-7	9.0-7	3.2-7	4.4-7	3.7-7	00	04	07	03	06
	$10^{-3} ; 0.5$	16 ; 16	2.6-7	9.0-7	7.3-8	4.2-7	8.4-7	00	08	08	04	07
	$10^{-3} ; 0.2$	15 ; 21	6.9-7	8.3-7	6.5-7	5.0-7	8.1-7	00	26	15	12	09
	$10^{-3} ; 0.01$	14 ; 23	8.3-8	8.5-7	9.9-7	9.7-7	1.4-7	00	55	17	15	09
	$10^{-4} ; 1$	47 ; 38	1.4-7	6.7-6	6.1-7	8.0-7	9.5-7	01	1:55	18	28	10
	$10^{-4} ; 0.5$	34 ; 41	9.8-7	9.9-7	9.3-7	7.7-7	9.9-7	01	1:51	22	33	10
	$10^{-4} ; 0.2$	35 ; 52	1.3-7	1.1-5	9.9-7	8.6-7	9.9-7	01	1:56	24	31	14
$10^{-4} ; 0.01$	37 ; 62	1.1-7	3.4-5	9.9-7	9.5-7	9.9-7	01	1:55	25	34	24	

In Table 5, we report the detailed results for SSNAL-LSM in solving the least squares constrained fused lasso problems for large-scale datasets.

Table 5: The performance of SSNAL-LSM on least squares constrained fused lasso problems with large-scale datasets (accuracy  $\epsilon = 10^{-6}$ ).  $m$  is the sample size and  $n$  is the dimension of features. “nnz” denotes the number of nonzeros in the solution. The computation time is in the format of “hours:minutes:seconds”.

probname $m; n$	$\gamma$	nnz( $x$ ) ; nnz( $Bx$ )	$\mu^*$	iteration	$\eta$	time
E2006.train 16087;150360	1.0-1 1.5-1 2.0-1	840 ; 547 1 ; 1 1 ; 1	1.30 - 2 6.86 + 3 1.11 + 4	40 22 22	1.2-7 1.5-7 2.0-7	3:07 13 12
log1p.E2006.train 16087;4272227	1.0-1 1.5-1 2.0-1	345 ; 177 20 ; 6 20 ; 6	2.38 + 1 2.49 + 3 3.94 + 3	27 25 25	1.3-7 7.0-7 3.3-7	9:17 4:55 4:46
E2006.test 3308;150358	5.0-2 7.5-2 1.0-1	2393 ; 2240 603 ; 680 1 ; 1	1.20 - 3 3.36 - 3 2.56 + 2	43 41 21	5.9-7 5.1-8 5.8-7	4:45 1:08 06
log1p.E2006.test 3308;4272226	5.0-2 7.5-2 1.0-1	3685 ; 2609 1504 ; 1003 20 ; 7	1.40 + 0 3.27 + 0 1.15 + 2	34 31 24	2.3-7 5.8-7 8.5-7	15:40 8:59 4:10
pyrim5 74;201376	1.0-1 2.0-1 3.0-1	254 ; 49 38 ; 9 54 ; 10	3.74 - 1 1.18 + 0 2.45 + 0	24 24 23	4.7-8 5.6-7 7.1-8	40 37 35
triazines4 186;635376	1.0-1 2.0-1 3.0-1	1338 ; 194 782 ; 47 243 ; 18	1.59 - 1 1.91 + 0 9.33 + 0	27 23 19	8.2-7 9.4-7 4.7-7	5:32 3:26 2:35
housing7 506;77520	1.0-1 2.0-1 3.0-1	238 ; 134 34 ; 21 17 ; 12	3.87 + 0 7.06 + 1 1.83 + 2	28 23 21	5.7-7 5.0-7 4.0-7	36 20 17
bodyfat7 252;116280	1.0-4 1.0-3 1.0-2	731 ; 391 322 ; 150 2 ; 3	2.70 - 6 9.87 - 5 1.98 - 1	35 33 25	4.9-8 7.0-7 4.1-7	1:00 43 19
DLBCLH 160;7399	1.5-1 2.0-1 2.5-1	755 ; 233 692 ; 216 653 ; 194	6.21 - 2 8.43 - 2 1.08 - 1	25 22 24	2.9-7 4.5-7 8.6-8	03 02 02
DLBCLN 160;7399	1.5-1 2.0-1 2.5-1	755 ; 233 692 ; 216 653 ; 194	6.21 - 2 8.43 - 2 1.08 - 1	25 22 24	2.9-7 4.5-7 8.6-8	02 02 02
DLBCLS 47;4026	1.5-1 2.0-1 2.5-1	211 ; 58 227 ; 53 220 ; 54	7.13 - 2 1.10 - 1 1.54 - 1	23 24 23	2.8-7 1.7-7 2.6-7	01 01 01
lung1 203;12600	1.5-1 2.0-1 2.5-1	373 ; 226 329 ; 198 248 ; 138	7.38 - 2 1.09 - 1 1.56 - 1	25 26 24	5.7-8 2.4-8 6.2-7	04 04 03
lung2 149;12533	1.5-1 2.0-1 2.5-1	293 ; 42 226 ; 29 184 ; 20	2.39 - 1 3.72 - 1 5.12 - 1	23 23 23	6.2-7 2.3-7 8.1-8	02 02 02
lungM 96;7129	1.5-1 2.0-1 2.5-1	183 ; 95 127 ; 77 79 ; 45	2.02 - 2 2.93 - 2 4.21 - 2	25 23 23	3.1-7 1.5-7 4.0-8	01 01 01
lungO 96;7129	1.5-1	90 ; 59	1.27 - 2	24	1.8-7	01

Table 5: The performance of SSNAL-LSM on least squares constrained fused lasso problems with large-scale datasets (accuracy  $\epsilon = 10^{-6}$ ).  $m$  is the sample size and  $n$  is the dimension of features. “nnz” denotes the number of nonzeros in the solution. The computation time is in the format of “hours:minutes:seconds”.

probname $m; n$	$\gamma$	nnz( $x$ ) ; nnz( $Bx$ )	$\mu^*$	iteration	$\eta$	time
39;2880	2.0-1	79 ; 52	$1.71 - 2$	26	6.8-7	01
	2.5-1	62 ; 42	$2.35 - 2$	26	1.1-7	01
NervousSystem 60;7129	1.5-1	147 ; 71	$5.16 - 2$	26	1.0-7	01
	2.0-1	122 ; 57	$7.39 - 2$	25	3.4-9	01
	2.5-1	129 ; 61	$9.92 - 2$	24	1.3-7	01
ovarianP 253;15153	1.5-1	591 ; 53	$5.66 - 2$	25	1.1-7	06
	2.0-1	686 ; 30	$1.53 - 1$	22	1.1-9	05
	2.5-1	368 ; 22	$2.65 - 1$	23	7.0-7	05
ovarianS 216;373401	1.5-1	1506 ; 218	$6.02 - 2$	26	4.6-7	2:05
	2.0-1	1395 ; 175	$8.40 - 2$	25	4.2-7	1:53
	2.5-1	1123 ; 133	$1.11 - 1$	23	5.5-7	1:43