

Evolutionary Music and the Zipf-Mandelbrot Law: Developing Fitness Functions for Pleasant Music

Bill Manaris¹, Dallas Vaughan¹, Christopher Wagner¹,
Juan Romero², and Robert B. Davis³

¹ Computer Science Department, College of Charleston,
Charleston, SC 29424, USA
{manaris, neil.vaughan, wagner}@cs.cofc.edu

² Creative Computer Group - RNASA Lab - Faculty of Computer Science,
University of A Coruña. Spain
jj@udc.es

³ Department of Mathematics and Statistics, Miami University
Hamilton, OH 45011, USA
davisrb@muohio.edu

Abstract. A study on a 220-piece corpus (baroque, classical, romantic, 12-tone, jazz, rock, DNA strings, and random music) reveals that aesthetically pleasing music may be describable under the Zipf-Mandelbrot law. Various Zipf-based metrics have been developed and evaluated. Some focus on music-theoretic attributes such as pitch, pitch and duration, melodic intervals, and harmonic intervals. Others focus on higher-order attributes and fractal aspects of musical balance. Zipf distributions across certain dimensions appear to be a necessary, but not sufficient condition for pleasant music. Statistical analyses suggest that combinations of Zipf-based metrics might be used to identify genre and/or composer. This is supported by a preliminary experiment with a neural network classifier. We describe an evolutionary music framework under development, which utilizes Zipf-based metrics as fitness functions.

1 Introduction

Webster's New World Dictionary (1981) defines beauty as *the quality attributed to whatever pleases or satisfies the senses or mind, as by line, color, form, texture, proportion, rhythmic motion, tone, etc., or by behavior, attitude, etc.* Since computers are ultimately manipulators of quantitative representations, any attempt to model qualitative information is inherently problematic. In the case of beauty, an additional problem is that it is affected by subjective (cultural, educational, physical) biases of an individual – that is, beauty is in the eye (ear, etc.) of the beholder. Or is it?

Musicologists generally agree that music communicates meaning [12]. Some attempt to understand this meaning and its effect on the listener by dissecting the aesthetic experience in terms of separable, discrete sounds. Others attempt to find it in

terms of grouping stimuli into patterns and studying their hierarchical organization [4], [10], [11], [15], [16]. Meyer [13, p. 342] suggests that emotional states in music (sad, angry, happy, etc.) are delineated by statistical parameters such as dynamic level, register, speed, and continuity. Although such state-defining parameters fluctuate locally within a music piece, they remain relatively constant globally.

In his seminal book, Zipf [25] discusses the language of art and the meaning communicated between artists and their audiences. He demonstrates that phenomena generated by complex social or natural systems, such as socially sanctioned art, tend to follow a statistically predictable structure. Specifically, the frequencies of words in a book, such as Homer's Iliad, plotted against their statistical rank on logarithmic scale, produce a straight line with a slope of approximately -1.0 . In other words, the probability of occurrence of words starts high and decreases rapidly. A few words, such as 'a' and 'the', occur very often, whereas most words, such as 'unconditionally', occur rarely. Formally, the frequency of occurrence of the n^{th} ranked word is $1/n^a$, where a is close to 1.

Similar laws have been developed independently by Pareto, Lotka, and Bendford [1], [6], [17]. These laws have inspired and contributed to other fields studying the complexity of nature. In particular, Zipf's law inspired and was extended by Benoit Mandelbrot to account for a wider range of natural phenomena [9]. Such phenomena may generate lines with slopes ranging between 0 (random phenomena) and negative infinity (monotonous phenomena). These distributions are also known as *power-law* distributions [19].

Zipf distributions are exhibited by words in human languages, computer languages, operating system calls, colors in images, city sizes, incomes, music, earthquake magnitudes, thickness of sediment depositions, extinctions of species, traffic jams, and visits of websites, among others.

Research in fractals and chaos theory suggests that the design and assembly of aesthetically pleasing objects – artificial or natural – is guided by hidden rules that impose constraints on how structures are put together [5], [18]. Voss and Clarke [23], [24] have suggested that music might also be viewed as a complex system whose design and assembly is partially guided by rules subconscious to the composer. They have also demonstrated that listeners may be guided by similar rules in their aesthetic response to music.

1.1 Zipf Distribution in Music

Zipf mentions several occurrences of his distribution in musical pieces. His examples were derived manually, since computers were not yet available. His study focused on the length of intervals between repetitions of notes, and the number of melodic intervals; it included Mozart's 'Bassoon Concerto in Bb', Chopin's 'Etude in F minor, Op. 25, No. 2,' Irving Berlin's 'Doing What Comes Naturally,' and Jerome Kern's 'Who' [25, pp. 336-337].

Voss and Clarke [23], [24] conducted a large-scale study of music from classical, jazz, blues, and rock radio stations collected continuously over 24 hours. They measured several fluctuating physical variables, including output voltage of an audio ampli-

fier, loudness fluctuations of music, and pitch fluctuations of music. They discovered that pitch and loudness fluctuations in music follow Zipf's distribution.

Voss and Clark also developed a computer program to generate music using a Zipf-distribution noise source (aka $1/f$ or pink noise). The results were remarkable: *The music obtained by this method was judged by most listeners to be much more pleasing than that obtained using either a white noise source (which produced music that was 'too random') or a $1/f^2$ noise source (which produced music that was 'too correlated'). Indeed the sophistication of this '1/f music' (which was 'just right') extends far beyond what one might expect from such a simple algorithm, suggesting that a '1/f noise' (perhaps that in nerve membranes?) may have an essential role in the creative process.* [23, p. 318]

2 Zipf-Based Metrics for Evolutionary Music

We have developed several Zipf-based metrics that attempt to identify and describe such balance along specific attributes of music [8]. These musical attributes may include the following: pitch, rests, duration, harmonic intervals, melodic intervals, chords, movements, volume, timbre, tempo, dynamics. Some of these can be used independently, e.g., pitch; others can be used only in combinations, e.g., duration. Some attributes are more straightforward to derive metrics from, such as melodic intervals; others are more difficult, such as timbre. These attributes were selected because they (a) have been used in earlier research, (b) have traditionally been used to express musical artistic expression and creativity, and/or (c) have been used in the analysis of composition. They are all studied extensively in music theory and composition. Obviously, this list of metrics is not complete.

We have automated several of these metrics using Visual Basic and C++. This allowed us to quickly test our hypothesis on hundreds of musical pieces encoded in MIDI. The following is a brief definition of selected metrics:

- **Pitch:** The relative balance of pitch of music events, in a given piece of music. (There are 128 possible pitches in MIDI.)
- **Pitch mod 12:** The relative balance of pitch on the 12-note chromatic scale, in a given piece of music.
- **Duration:** The relative balance of durations of music events, independent of pitch, in a given piece of music.
- **Pitch & Duration:** The relative balance of pitch-duration combinations, in a given piece of music.
- **Melodic Intervals:** The relative balance of melodic intervals, in a given piece of music. (This metric was devised by Zipf.)
- **Harmonic Intervals:** The relative balance of harmonic intervals, in a given piece of music. (This metric was devised by Zipf.)
- **Harmonic Bigrams:** The relative balance of specific pairs of harmonic intervals in a piece. (This metric captures the balance of harmonic-triad (chord) structures.)

- **Melodic Bigrams:** The relative balance of specific pairs of melodic intervals in a piece. (This metric captures the balance of melodic structure and arpeggiated chords.)
- **Melodic Trigrams:** The relative balance of specific triplets of melodic intervals in a piece. (This metric also captures the balance of melodic structure.)
- **Higher-Order Melodic Intervals:** Given that melodic intervals capture *the change of pitches* over time, we also capture higher orders of change. This includes the *changes in melodic intervals*, to the *changes of the changes in melodic intervals*, and so on. These higher-order metrics correspond to the notion of derivative in mathematics. Although a human listener may not be able to consciously hear such high-order changes in a piece of music, there may be some subconscious understanding taking place.¹

2.1 Evaluation

We evaluated the effectiveness of our Zipf metrics by testing them on a large corpus of quality MIDI renderings of musical pieces. Additionally, we included a set of DNA-generated pieces and a set of random pieces (white and pink noise) for comparison purposes. Most MIDI renderings of classical pieces came from the Classical Archives [22].

Our corpus consisted of 220 MIDI pieces. Due to space limitations, we summarize them below by genre and composer.

- 1 *Baroque:* Bach, Buxtehude, Corelli, Handel, Purcell, Telemann, and Vivaldi (38 pieces).
- 2 *Classical:* Beethoven, Haydn, and Mozart (18 pieces).
- 3 *Early Romantic:* Hummel, Rossini, and Schubert (14 pieces).
- 4 *Romantic:* Chopin, Mendelssohn, Tarrega, Verdi, and Wagner (29 pieces).
- 5 *Late Romantic:* Mussorgsky, Saint-Saens, and Tchaikovsky (13 pieces).
- 6 *Post Romantic:* Dvorák and Rimsky-Korsakov (13 pieces).
- 7 *Modern Romantic:* Rachmaninov (2 pieces).
- 8 *Impressionist:* Ravel (1 piece).
- 9 *Modern (12 Tone):* Berg, Schönberg, and Webern (15 pieces).
- 10 *Jazz:* Charlie Parker, Chick Corea, Cole Porter, Dizzy Gillespie, Django Reinhardt, Duke Ellington, John Coltrane, Miles Davis, Sonny Rollins, and Thelonius Monk (33 pieces).
- 11 *Rock:* Black Sabbath, Led Zeppelin, and Nirvana (12 pieces).

¹ Surprisingly, our study revealed that, once a Zipfian distribution is encountered in a lower-order metric, the higher-order metrics continue to exhibit such distributions. However, the higher-order slopes progressively move towards zero (high entropy – purely random). This result suggests that balance introduced at a certain level of assembly may influence the perceived structural aspects of an artifact many levels removed from the original. If generalizable, this observation may have significant philosophical implications.

- 12 *Pop*: Beatles, Bee Gees, Madonna, Mamas and Papas, Michael Jackson, and Spice Girls (18 pieces).
- 13 *Punk Rock*: The Ramones (3 pieces).
- 14 *DNA Encoded Music*: actual DNA sequences encoded into MIDI, and simulated DNA sequences (12 pieces).
- 15 *Random (White Noise)*: music consisting of random note pitches, note start times, and note durations “composed” by a uniformly-distributed random number generator (6 pieces).
- 16 *Random (Pink Noise)*: music consisting of random note pitches, note start times, and note durations “composed” by a random number generator exhibiting a Zipf distribution (6 pieces).

2.2 Results

Each metric produces a pair of numbers per piece. The first number, *Slope*, is the slope of the trendline of the data values. Slopes may range from 0 (high entropy – purely random) to negative infinity (low entropy – monotone). Slopes near -1.0 correspond to Zipf distribution. The second number, R^2 , is an indication of how closely the trendline fits the data values – the closer the fit, the more meaningful (reliable) the slope value. R^2 may range from 0.0 (extremely bad fit – data is all over the graph) to 1.0 (perfect fit – data is already in a straight line). We considered R^2 values larger than 0.7 to be a good fit.

Every celebrated piece in our corpus exhibited several Zipf distributions. Random pieces (white noise) and DNA pieces very few (if any) Zipf distributions. Table 1 shows average results for each genre in terms of slope, R^2 , and corresponding standard deviations. The average for all musical pieces (excluding DNA, pink noise, and white noise pieces) across all metrics is -1.2004 , a near-Zipf distribution; the corresponding R^2 (fit) across all metrics is 0.8213.

Additionally, we performed statistical analyses on these results to identify patterns across genres. First we drew side-by-side boxplots of each metric for each genre. In the following results, genres are numbered as 1 - Baroque, 2 - Classical, 3 - Early Romantic, 4 - Romantic, 5 - Late Romantic, 6 - Post Romantic, 7 - Modern Romantic, 9 - Twelve-tone, 10 - Jazz, 11 - Hard Rock, 12 - Pop, 13 - Punk, 14 - DNA, 15 - Pink noise, and 16 - White noise. Although boxplots are not formal inference tools, they are useful in data analysis because any substantial differences in genres should be evident from visual inspection. These side-by-side boxplots revealed several interesting patterns. For instance, genres 14 (DNA) and 16 (random music – uniformly distributed pitch) are easily identifiable by their pitch metric. We also discovered that the first seven genres (baroque, classical, and the five genres with the word “romantic” in them) appear to have significant overlap on all of the metrics. This is not surprising, as these genres are relatively similar – people refer to such pieces as “classical music.” However, further examination suggests that it may be still be possible to identify styles and composers by using combinations of metrics.

We also performed analyses of variance (ANOVAs) on the data to determine whether or not the various metrics had significantly different averages across genres.

Table 1. Average results across metrics for each genre

Genre	Slope	R ²	Slope Std	R ² Std
Baroque	-1.1784	0.8114	0.2688	0.0679
Classical	-1.2639	0.8357	0.1915	0.0526
Early Romantic	-1.3299	0.8215	0.2006	0.0551
Romantic	-1.2107	0.8168	0.2951	0.0609
Late Romantic	-1.1892	0.8443	0.2613	0.0667
Post Romantic	-1.2387	0.8295	0.1577	0.0550
Modern Romantic	-1.3528	0.8594	0.0818	0.0294
Impressionist	-0.9186	0.8372	N/A	N/A
Twelve-Tone	-0.8193	0.7887	0.2461	0.0964
Jazz	-1.0510	0.7864	0.2119	0.0796
Rock	-1.2780	0.8168	0.2967	0.0844
Pop	-1.2689	0.8194	0.2441	0.0645
Punk Rock	-1.5288	0.8356	0.5719	0.0954
DNA	-0.7126	0.7158	0.2657	0.1617
Random (Pink)	-0.8714	0.8264	0.3077	0.0852
Random (White)	-0.4430	0.6297	0.2036	0.1184

We found that all of the p-values are significant, meaning there are differences among genres within our corpus. To better visualize the ANOVA results we generated confidence interval graphs. When displayed using side-by-side boxplots, genre samples may overlap due to simple natural variation and/or some unusual individual pieces within the genres. However, the confidence intervals given in the ANOVA output characterize where the mean slope for each metric within each genre is located. When these intervals do not overlap, there is a statistically significant difference between genres. Figure 1 shows the results for the harmonic interval metric.

Overall, twelve-tone and DNA differ from the other genres significantly. In terms of the pitch metric, Late Romantic appears to differ significantly from Hard Rock and Pop. In terms of pitch mod 12, there are several genres that differ significantly from Jazz. Jazz and Baroque appear to differ in duration. Jazz definitely differs from most other genres in terms of pitch & duration.

Moreover, several other interesting patterns emerge. For instance, in the pitch-mod-12 metric, twelve-tone music exhibits slopes suggesting uniform distribution – average slope is -0.3168 with a standard deviation of 0.1801 . In particular, Schönberg’s pieces averaged -0.2801 with a standard deviation of -0.1549 . This was comparable to the average for random (white noise) pieces, namely -0.1535 . Obviously, this metric is very reliable in identifying twelve-tone music. For comparison purposes, the next closest average slope for musical pieces was exhibited by Jazz (-0.8770), followed by Late Romantic (-1.0741).

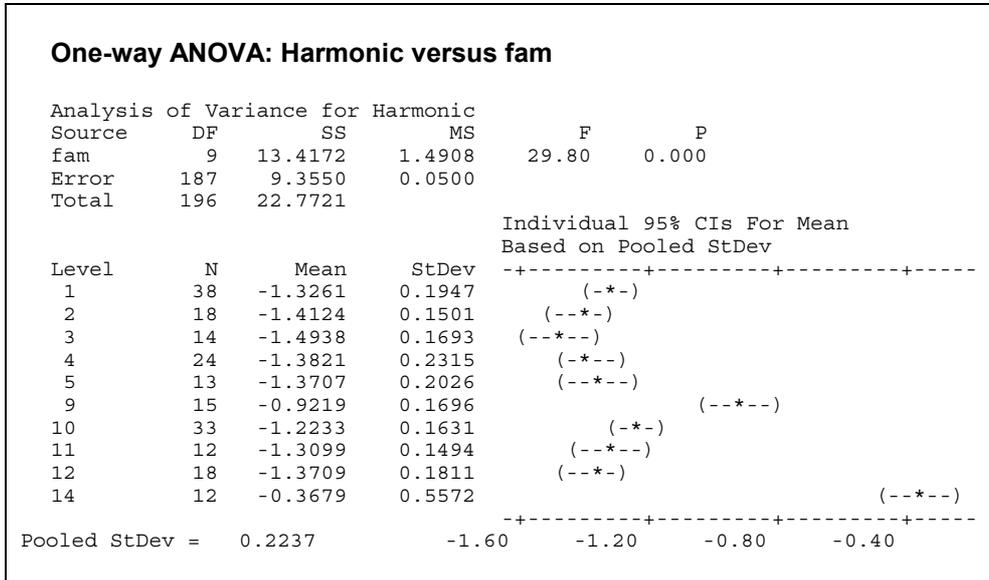


Fig. 1. ANOVA confidence interval graph for the harmonic metric across all genres. Genres 9 (twelve-tone) and 14 (DNA) are identifiable through this metric alone

Overall, these results suggest that we may have discovered certain **necessary but not sufficient conditions** for aesthetically pleasing music. It should be noted that the original inspiration for this project was how the Zipf-Mandelbrot law is used in many other domains to identify “naturally” occurring phenomena – phenomena with a natural “feel” to them [6]. So, it is not surprising to us that we are finding this correlation between pleasant music and instances of Zipf distribution.

3 Composite Metrics

These results suggest that aesthetically pleasing aspects of music may be algorithmically identifiable and classifiable. Specifically, by combining metrics into a weighted composite (consisting of metrics that capture various aspects throughout the possible space of measurable aesthetic attributes) we may be able to perform various classification tasks. We have experimented with composite metrics having (a) various weights assigned to individual metrics and (b) conditional combinations of individual metrics.

Currently, we are exploring various neural network configurations and classification tasks. In one experiment, we developed an artificial neural network to determine if Zipf metrics contained enough information for authorship attribution. For instance, we have used the Stuttgart Neural Network Simulator (SNNS) [20] to build, train and test an artificial neural network. Our corpus consisted of Zipf metrics for two data sets: (a) Bach pieces BWV500 through BWV599, and (b) Beethoven sonatas 1 through 32. From these we extracted training and test sets. The trained neural net-



Fig. 2.a Sample at resolution 1



Fig. 2.b Sample at resolution 2

work was able to identify the composer of a piece it had not seen before with 95% accuracy. We believe that this can easily be improved with a refined training set and/or the fractal metrics discussed in the next section.

Composite metrics, implemented through neural network classifiers, could be used to identify pieces that have similar aesthetic characteristics to a given piece. Composite metrics may also help derive a statistical signature (identifier) for a piece. Such an identifier may be very useful in data retrieval applications, where one searches for different performances of a given piece among volumes of music. For instance, during an earlier study [8], we discovered a mislabeled MIDI piece by noticing that it had identical Pitch-mod-12 slope and R^2 values with another MIDI piece. The two files contained different performances of Bach's Toccata and Fugue in D minor.

3.1 Fractal Metrics

The Zipf metrics presented so far are very promising. However, they have a significant weakness. They measure the global balance of a piece. For instance, consider the sample shown in figure 2.a. The pitch metric of this sample is perfectly Zipfian (slope = -1.0 , $R^2 = 1.0$). However, locally, the sample is extremely monotonous.

This problem can be easily addressed using a fractal method of measurement. The metric is applied recursively at different levels of resolution: We measure the whole sample, then split it into two equal phrases and measure each of these, then split it into four equal phrases, and so on. For instance, at resolution 2 (see sample at figure 2.b), the slope of the left side is negative infinity (monotone). The slope of the right side is -0.585 . By dividing the sample into two parts, the lack of local balance was quickly exposed.

Preliminary tests with music corpora indicate that aesthetically pleasing music exhibits Zipf distributions at various levels of resolution. Depending on the piece of music, this will go on until the resolution reaches a small number of measures. For instance, in Bach's Two-Part Invention No. 13 in A minor (BWV.784), this balance exists until a resolution of three measures per subdivision.

This recursive measuring process may be used to calculate the fractal dimension of a sample relative to a specific metric. This is known as the *box-counting* method. Taylor [21] has used this approach in visual art to authenticate and date paintings by Jackson Pollock using their fractal dimension, D . For instance, he discovered two different fractal dimensions: one attributed to Pollock's dripping process, and the other attributed to his motions around the canvas. Also, he was able to track how Pollock refined his dripping technique – the fractal dimension increased through the years (from $D \approx 1$ in 1943 to $D = 1.72$ in 1952).

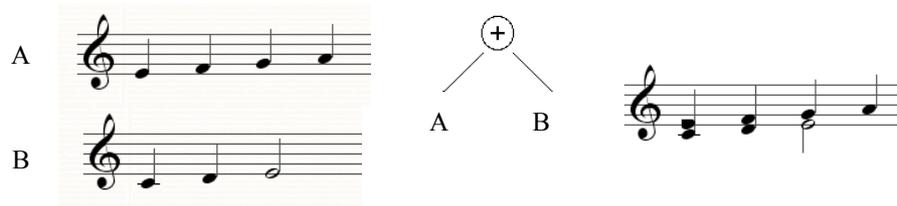


Fig. 3. Example of the addition operation. The tree at the center is the genotype representation. The music scores at left and right are phenotype representations of the two operands and the result of their addition, respectively.

Preliminary results suggest that, as observed for simple Zipf metrics, aesthetically pleasing music exhibits several fractal dimensions near 1, as opposed to aesthetically non-pleasing music or non-music. For instance, the pitch fractal dimension for Bach’s 2-Part Invention in A minor is 0.9678. The results of these experiments are only preliminary. We believe that these fractal metrics will prove much more powerful than simple metrics for ANN-based classification purposes.

4 Evolutionary Music Framework

We have shown that Zipf-based metrics are capable of capturing aspects of the economy exhibited by socially sanctioned music. This capability should be very useful in computer-assisted composition systems. Such systems are developed using various AI frameworks including formal grammars, probabilistic automata, chaos and fractals, neural networks, and genetic algorithms [2], [3], [14].

We are currently evaluating the promise of Zipf-based metrics for guiding evolutionary experiments. Based on our results, we believe that Zipf-based fitness functions should produce musical samples that resemble socially sanctioned (aesthetically pleasing) music. If nothing else, since Zipf distributions appear to be a necessary, but not sufficient condition for aesthetically pleasing music, such fitness functions could minimally serve as an automatic filtering mechanism to prune unpromising musical samples.

4.1 Genotype Operations

Our system is based loosely on Machado’s NEvAR system [7] – a powerful system for evolutionary composition of visual art. In our adaptation of the NEvAR framework, a phenotype is a music score. A genotype is represented as a tree. Leaf nodes are music phrases. Non-leaf nodes are operators that, when interpreted, generate a phenotype (see figure 3).

Genotype operators, such as +, –, and * are related, but not completely analogous, to their mathematical definitions. The following is an overview of low-level genotype operators. We use the word “element” to refer to an arbitrary genotype sub-tree.

- + (addition) takes two elements, A and B, and returns the union of the two preserving their respective start times, end times, and pitches.
- (subtraction) takes two elements, A and B, and returns the set of notes in which B is NOT enveloped by A.
- & (concatenation) takes two elements, A and B, and appends B to the end of A.
- * (multiplication) takes two elements, A and B, and replaces each instance of B with a complete repetition of A, but transposed from A’s starting note to B. Each repetition is appended to the last.

These low-level operations are used to evolve themes. Once a theme has been evolved, higher-level operations are applied to evolve other aspects of the notes, phrases, and piece as a whole. These operations include common compositional devices such as retrograde, diminution, augmentation, inversion, imitation, harmonization, temporal quantization, harmonic quantization, and transposition.

Finally, we include genetic operators for evolving sub-trees such as mutate (sub-tree mutation), and fit (sub-tree evaluation). These allow for introducing improvised phrases within larger compositions.

We maintain control over the probabilities and complexity restrictions of when these operations take place. For instance, if within a certain genre an operation is found to be more prevalent in the first few levels of the tree than the last few, this fact can be used to weight the corresponding probabilities of that operation taking place among the various levels of the generations.

In the NEvAr system, elements are composed of collections of pixels that form a two-dimensional image. Expressions are evaluated on a pixel-by-pixel basis, and the results can be used as the arguments in the next operation in the expression tree. Any sub-tree can be mutated in one of five ways: 1) swapping arbitrary sub-trees, 2) replacing arbitrary sub-trees with randomly created ones, 3) inserting a randomly created node at a random insertion point, 4) deleting a randomly selected node, and 5) randomly selecting an operator and changing it [7].

These sub-tree mutations could prove to be valuable in the context of music, since, unlike most visual art, music is defined almost exclusively by an abstract (in the sense of layers building upon other layers) composition of notes to phrases, phrases to melody, melody to section, and sections to piece. Since this method is closer to the actual process of composition, the results of these operations should minimally produce something that resembles a “standard” musical piece, at least in structure.

The most important question to be answered is, “to what degree should we have certain operations, and where?” If the answer to this is, “anywhere, anytime,” then there will likely be many more non-standard compositions created than if the answer were based on music theory or probability (dependent on whatever genre of music one is attempting to emulate). Although a fitness test (in the case of NEvAr, a human) would usually decide which generations stayed and which did not, a valid fitness test, in terms of musical beauty, is nearly impossible to formulate, since the goal is so hard to articulate in the first place. A better solution may be somewhere in between these answers, where there are weightings and restrictions applied to the generation process (to both elemental operations and sub-tree operations), and a fitness test that at least

discards the generations which are not minimally “musical.” The generations produced would more likely be of a structured type, but there is still the possibility that a less structured generation would make it, provided it passed the fitness test. Among the possible fitness tests, combined Zipf metrics are worthy candidates since they do not depend solely on musical tastes or rules of theory, but on the more abstract idea of balance begetting beauty.

5 Conclusion

We have shown the promise of using the Zipf-Mandelbrot law to measure the balance, and to a certain degree, pleasantness of musical pieces by applying this law to various musical attributes, such as pitch, duration, and note distances. The results of the ANN experiment suggest that a neural network is capable of distinguishing pieces based on their Zipf metrics, and so can be used in part or whole as a fitness test for each generation. Using a neural network as a fitness test could also be used to constrain the generation process to create certain types of music, like Classical, Jazz, etc., or to create pieces that are similar to particular composers.

We also discussed the generation of musical pieces through an evolutionary framework comprised of genetic operations similar to those of the NEvAr framework. This will allow the structured formulation of music, either with or without human interaction. Although this system may produce music that is statistically similar to socially sanctioned music, it is not clear if the result will be truly aesthetically pleasing music. Therefore, this tool could at least assist a human composer by enforcing minimal conditions for aesthetically pleasing music and, thus, producing rough musical sketches for inspiration and further refinement.

Acknowledgements

The authors would like to acknowledge Penousal Machado for his suggestion to combine Zipf metrics with an ANN for authorship attribution. They also acknowledge José Santiago, Cernadas Vilas, Mónica Miguélez Rico, and Miguel Penin Álvarez for conducting the ANN experiment. Tarsem Purewal, Charles McCormick, Valerie Sessions, and James Wilkinson helped derive Zipf metrics and MIDI corpora. This work has been supported in part by the College of Charleston through an internal R&D grant.

References

1. Adamic, L.A.: “Zipf, Power-laws, and Pareto – a Ranking Tutorial” (1999)
<http://ginger.hpl.hp.com/shl/papers/ranking/>

2. Balaban, M., Ebcioğlu, K., and Laske, O., (eds.): Understanding Music with AI: Perspectives on Music Cognition. Cambridge: AAAI Press/MIT Press (1992)
3. Cope, D.: Virtual Music, MIT Press Cambridge (2001)
4. Howat, R.: Debussy in Proportion. Cambridge University Press (cited in [16]) Cambridge (1983)
5. Knott, R.: "Fibonacci Numbers and Nature". www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html (2002)
6. Li, W.: "Zipf's Law". <http://linkage.rockefeller.edu/wli/zipf/> (2002)
7. Machado, P., Cardoso, A.: "All the truth about NEvAr". Applied Intelligence, Special issue on Creative Systems, Bentley, P., Corne, D. (eds), 16(2), (2002), 101-119
8. Manaris, B., Purewal, T., and McCormick, C.: "Progress Towards Recognizing and Classifying Beautiful Music with Computers". 2002 IEEE SoutheastCon, Columbia, SC (2002) 52-57
9. Mandelbrot, B.B.: The Fractal Geometry of Nature. W.H. Freeman New York (1977)
10. Marillier, C.G.: "Computer Assisted Analysis of Tonal Structure in the Classical Symphony". Haydn Yearbook 14 (1983) 187-199 (cited in [16])
11. May, M.: "Did Mozart Use the Golden Section?". American Scientist 84(2). www.sigmaxi.org/amsci/issues/Sciobs96/Sciobs96-03MM.html (1996)
12. Meyer, L.B.: Emotion and Meaning in Music. University of Chicago Press Chicago (1956)
13. Meyer, L.B.: "Music and Emotion: Distinctions and Uncertainties". In Music and Emotion – Theory and Research, Juslin, P.N., Sloboda, J.A. (eds), Oxford University Press Oxford (2001) 341-360
14. Miranda, E.R.: Composing Music with Computers. Focal Press Oxford (2001)
15. Nettheim, N.: "On the Spectral Analysis of Melody". Interface: Journal of New Music Research 21 (1992) 135-148
16. Nettheim, N.: "A Bibliography of Statistical Applications in Musicology". Musicology Australia 20 (1997) 94-106
17. Pareto, V.: Cours d'Economie Politique, Rouge (Lausanne et Paris) (1897) Cited in [9]
18. Salingaros, N.A., and West, B.J.: "A Universal Rule for the Distribution of Sizes". Environment and Planning B(26) (1999) 909-923
19. Schroeder, M.: Fractals, Chaos, Power Laws. W.H. Freeman New York (1991)
20. Stuttgart Neural Network Simulator (SNNS), <http://www-ra.informatik.uni-tuebingen.de/SNNS/>.
21. Taylor, R.P., Micolich, A.P., and Jonas, D.: "Fractal Analysis Of Pollock's Drip Paintings". Nature, vol. 399, (1999), 422 <http://materialscience.uoregon.edu/taylor/art/Nature1.pdf>
22. The Classical Music Archives. www.classicalarchives.com.
23. Voss, R.F., and Clarke, J.: "1/f Noise in Music and Speech". Nature 258 (1975) 317-318
24. Voss, R.F., and Clarke, J.: "1/f Noise in Music: Music from 1/f Noise". Journal of Acoustical Society of America 63(1) (1978) 258-263

25. Zipf, G.K.: Human Behavior and the Principle of Least Effort. Addison-Wesley
New York (1972) (original publication Hafner Publishing Company, 1949)