Subitizing: What Is It? Why Teach It?

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Three pictures hang in front of a six month-old child. The first shows two dots, the others show one dot and three dots. The infant hears three drumbeats. Her eyes move to the picture with three dots.

Young children spontaneously use the ability to recognize and discriminate small numbers of objects (Klein and Starkey 1988). But some elementary school children cannot immediately name the number of pips showing on the dice. What is this ability? When and how does it develop? Is it a special way of counting? Should we teach it?

Subitizing: A Long History

Subitizing is "instantly seeing how many." From a Latin word meaning suddenly, subitizing is the direct perceptual apprehension of the numerosity of a group. In the first half of the century, researchers believed that counting did not imply a true understanding of number but that subitizing did (e.g., Douglass [1925]). Many saw the role of subitizing as a developmental prerequisite to counting. Freeman (1912) suggested that whereas measurement focused on the whole and counting focused on the unit, only subitizing focused on the whole and the unit; therefore, subitizing underlay number ideas. Carper (1942) agreed that subitizing was a more accurate than counting and more effective in abstract situations.

In the second half of the century, educators developed several models of subitizing and counting. They based some models on the same notion that subitizing was a more "basic" skill than counting (Klahr and Wallace 1976; Schaeffer, Eggleston, and Scott 1974). One reason was that children can subitize directly through interactions with the environment, without social interactions. Supporting this position, Fitzhugh (1978) found that some children could subitize sets of one or two but were not able to count them. None of these very young children, however, was able to count any sets that he or she could not subitize. She concluded that subitizing is a necessary precursor to counting. Certainly, research with infants suggests that young children possess and spontaneously use subitizing to represent the number contained in small sets and that subitizing emerges before counting (Klein and Starkey 1988).

As logical as this position seems, counterarguments exist. In 1924, Beckmann found that younger children used counting rather than subitizing (cited in Solter [19761). Others agreed that children develop subitizing later, as a shortcut to counting (Beckwith and Restle 1966; Brownell 1928; Silverman and Rose 1980). In this view, subitizing is a form of rapid counting (Gelman and Gallistel 1978).

Researchers still dispute the basis for subitizing ability, with patterns and attentional mechanisms the main explanations (Chi and Klahr 1975; Mandler and Shebo 1982; von Glaserfeld 1982). Lower animal species seem to have some perceptual number abilities, but only birds and primates also have shown the ability to connect a subitized number with a written mark or an auditory label (Davis and Perusse 1988).

Two Types of Subitizing

Given that animals are able to subitize, is this procedure a low-level process? Not necessarily. A single mechanism may not underlie all forms of subitizing. We can resolve research contradictions by recognizing that two types of subitizing exist.

Perceptual subitizing

Perceptual subitizing is closest to the original definition of subitizing: recognizing a number without using other mathematical processes. For example, children might "see 3" without using any learned mathematical knowledge. Perceptual subitizing may involve mechanisms similar to those used by animals. Two-year-old children show this ability clearly (Gelman and Gallistel 1978). Perceptual subitizing accounts for some surprising abilities of infants, such as the one described at the beginning of this article.

Perceptual subitizing also plays an even *more* primitive role, one that most of us do not even think about because we take it for granted. This role is making units, or single "things," to count. This ability seems obvious to us. However, "cutting out" pieces of experience, keeping them separate, and coordinating them with number words is no small task for young children. Even when they count their fingers, for example, they have to mentally "cut out" one part of the hand from the next to create units. They then have to connect each of these units with one, and only one, number word.

Conceptual subitizing

But how is it that people see an eight-dot domino and 'lust know" the total number? They are using the second type of subitizing. Conceptual subitizing plays an advanced-organizing role. People who "just know" the domino's number recognize the number pattern as a composite of parts and as a whole. They see each side of the domino as composed of four individual dots and as "one four." They see the domino as composed of two groups of four and as "one eight." These people are capable of viewing number and number patterns as units of units (Steffe and Cobb 1988).

Spatial patterns, such as those on dominoes, are just one kind. Other patterns are temporal and kinesthetic, including finger patterns, rhythmic , and spatial-auditory patterns. Creating and using these patterns through conceptual subitizing help children develop abstract number and arithmetic strategies (Steffe and Cobb 1988). For example, children use temporal patterns when counting on: "I knew there were three more, so I just said, 'Nine . . . ten, eleven, twelve,' rhythmically gesturing three times, one "beat" with each . They use finger patterns to figure out addition problems. Children who cannot subitize conceptually are handicapped in learning such arithmetic processes. Children who can may subitize only small numbers at first. Such actions, however, can be stepping-stones to constructing more sophisticated procedures with larger numbers.

Subitizing and counting

Young children may use perceptual subitizing to make units for counting and to build their initial ideas of cardinality. For example, their first cardinal meanings for number words may be labels for small sets of subitized objects, even if they deter-mined the labels by counting (Fuson 1992; Steffe, Thompson, and Richards 1982). Children use counting and patterning abilities to develop conceptual subitizing. This more advanced ability to group and quantify sets quickly in turn supports their development of number sense and arithmetic abilities. A first grader explains the process for us. Seeing a three-by-three pattern of dots, she immediately says, "Nine." Asked how she reached her answer, she replied as follows:

> When I was about four years old, I was in nursery school. All I had *to* do was count. And so, I just go like 1,2,3,4,5, 6, 7, 8, 9, and I just knew it by heart and I kept on doing it -when I was five too. And then I kept knowing 9, you know. Exactly like this [she pointed to the array]. (Ginsberg 1977, 16)

What Factors Make Conceptual Subitizing Easy or Hard?

The spatial arrangement of sets influences how difficult they are to subitize. Children usually find rectangular arrangements easiest, followed by linear, circular, and scrambled arrangements (Beck-with and Restle 1966; Wang, Resnick, and Boozer 1971). This progression holds true for students from the primary grades to college.

Certain arrangements are easier for specific numbers. Arrangements yielding a better "fit" for a given number are easier (Brownell 1928). Children make fewer errors for ten dots than for eight when dots are in the "domino five" arrangement but make fewer errors for eight dots when using the "domino four" arrangement.

For young children, however, neither of these arrangements is easier for any number of dots. Indeed, two- to four-year-olds show no differences among any arrangements of four or fewer items (Potter and Levy 1968). For larger numbers, linear arrangements are easier than rectangular arrangements. It seems, then, that most preschool children cannot subitize conceptually. Instead, they count one by one. By school age, they can learn to subitize conceptually, although first graders' limits for subitizing scrambled arrangements is about four or five items (Dawson *1953)*.

If the arrangement does not lend itself to grouping, people of any age have more difficulty with larger sets (Brownell 1928). They also take more time with larger sets (Beckwith and Restle 1966).

Some special populations find subitizing particularly difficult. Only a minority (31%) of moderately handicapped children and a slight majority (59%) of mildly handicapped children can success-fully subitize sets of three and four items (Baroody 1986).

Finally, textbooks often present sets that discourage subitizing. Their pictures combine many inhibiting factors, including complex embedding, different units with poor form (e.g., birds that were not simple in design as opposed to squares), lack of symmetry, and irregular arrangements (Carper 1942; Dawson 1953). Such complexity hinders conceptual subitizing, increases errors, and encourages simple one-by-one counting.

Implications for Teaching

Subitizing is an important mathematics skill. But can subitizing be "taught"? Some might argue that it cannot, but that conclusion depends on what is meant by 'teach." *Everybody Counts* says, "In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to *learn* mathematics" (National Research Council 1989, 58). I agree only if we define teaching as mere "telling." But I define teaching more broadly (Clements 1997). Those who stimulate students to learn by setting up experiences, by guiding investigations, and sometimes by telling - are *teaching*. Conceptual subitizing must be learned and there-fore be fostered, or *taught*, in this broad sense. Row might we teach it?

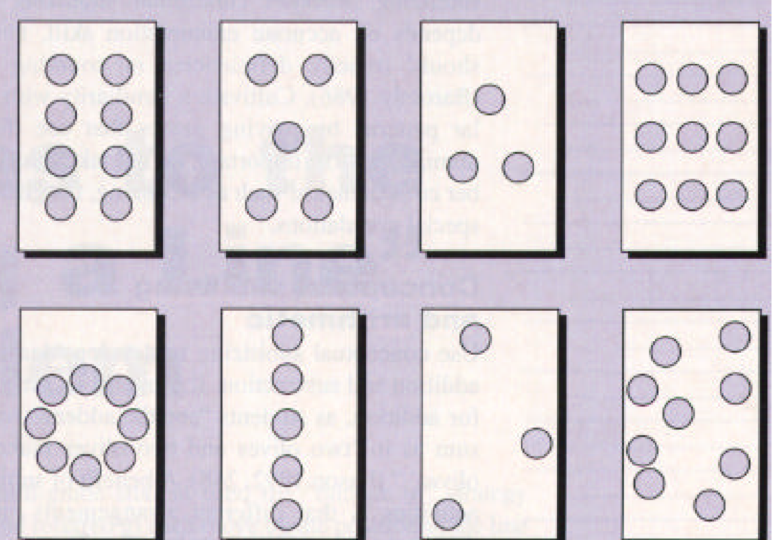
Conceptual subitizing and number

Many number activities can promote conceptual subitizing. One particularly rich activity is "quick images." When I play it with kindergarten students, I have two students stand on opposite sides of an overhead projector. One student holds a pack of cards with holes punched in them (see fig. 1 for some examples). That student places one of the cards on the overhead projector, and the other student takes it off as fast as she or he can. Then the members of the class and I race to announce the number of dots. My students are delighted that they often (and honestly!) beat me to the answer.

We play by using cards like those in the top row of **figure 1,** because research shows that rectangular and dice arrangements are easiest for young children initially. At first I limit the game to small numbers. Only when students are developing conceptual subitizing do we play with more complex patterns like those in the bottom row.

FIGURE 1

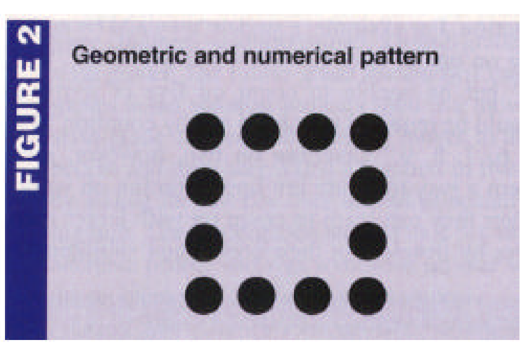
Cards for quick-images activity. Punch holes in three-by-five-inch cards and display them on an overhead projector.



Of the many worthwhile variations of the quick-image activity, some are suitable for students in any elementary grade.

- Have students construct a quick-image arrangement with manipulatives.
- With cards like those in figure 1, play a matching game. Show several cards, all but one of which have the same number. Ask children which card does not belong.
- Play concentration games with cards that have different arrangements for each number. For a version of this game and other helpful activities, see Baratta-Lorton (1976).
- Give each child cards with zero through ten dots in different arrangements. Have students spread the cards in front of them. Then announce a number. Students find the matching card as fast as possible and hold it up. Have them use different sets of cards, with different arrangements, on different days. Later, hold up a written numeral as their cue. Adapt other card games for use with these card sets (see Clements and Callahan [1986]).
- Place various arrangements of dots on a large sheet of poster board. With students gathered around you, point out one of the groups as students say its number as fast as possible. Hold the poster board in a
- different orientation each time you play.
- Challenge students to say the number that is one (later, two) more than the number on the quick image. They might also respond by showing a numeral card or writing the numeral. Alternatively, they can find the arrangement that matches the numeral that you show.
- Encourage students to play any of these games as a free-time or station activity.

The development of imagery is another reason that these activities are valuable. Conceptual subitizing is a component of *visualization* in all its forms (Markovits and Hershkowitz 1997). Children refer to mental images when they discuss their strategies. In addition, we can enhance students' knowledge of both geometry and number by purposely combining the two. For example, play quick images with arrangements like the one in **figure 2** . Older students might say, "A square has four sides, and there were two dots just on each side, and four more on the corners, so I figured twelve."



Also play quick images that involve *estimation.* For example, show students arrangements that are too large to subitize exactly. Encourage them to use subitizing in their estimation strategies. Emphasize that using good strategies and being “close” are the goals, not getting the exact number. Begin with organized geometric patterns, but include scrambled arrangements eventually. Encourage students, especially those in higher grades, to build more sophisticated strategies: from guessing, to counting as much as possible and then guessing, to comparing ("It was more than the previous one"), to grouping (“They are spread about four in each place. I circled groups of four in my head and then counted six groups. So, twenty-four!”). Students do perform better, and use more sophisticated strategies and frames of reference, after engaging in such activities (Markovits and Hershkowitz 1997). For these and for all subitizing activities, stop frequently to allow students to share their perceptions and strategies.

Across many types of activities, from class discussions to textbooks, students should be shown pictures of numbers that encourage conceptual subitizing. Initially, groups to be subitized should follow these guidelines: *(a)* groups should not be embedded in pictorial context; *(b)* simple forms, such as homogeneous groups of circles or squares rather than pictures of animals or mixtures of any shapes, should be used for the units; *(c)* regular arrangements should be emphasized, and most should include symmetry, with linear arrangements for preschoolers and rectangular arrangements for older students being easiest; and *(d)* good figure-ground contrast should be used.

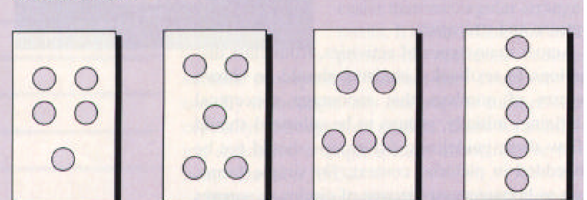
Remember that patterns can also be temporal and kinesthetic, including rhythmic and spatial-auditory patterns. My kindergartners' favorite numeral-writing activities involve auditory rhythms. They scatter around the classroom on the floor with individual chalkboards. I walk around the room, then stop and make a number of sounds, such as ringing a bell three times. They write the numeral 3 on their chalkboards and hold them up. Special populations deserve special attention to subitizing. Because conceptual subitizing often depends on accurate enumeration skill, teachers should remedy deficiencies in counting early (Baroody 1986). Cultivate a familiarity with regular patterns by playing games that use dice or dominoes. Most important, do not take basic number competencies, such as subitizing, for granted in special populations.

Conceptual subitizing and arithmetic

Use conceptual subitizing to develop ideas about addition and subtraction. It provides an early basis for addition, as students "see the addends and the sum as in 'two olives and two olives make four olives' " (Fuson 1992, 248). A benefit of subitizing activities is that different arrangements suggest different views of that number (fig. 3).

FIGURE 3

Different arrangements lead to different decompositions of that number

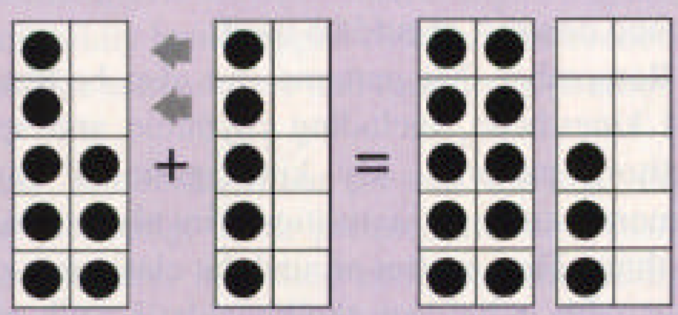


Conceptual subitizing can also help students advance to more sophisticated addition and sub-traction. For example, a student may add by counting on one or two, solving 4 + 2 by saying "4, 5, 6," but be unable to count on five or more, as would be required to solve 4 + 5 by counting "4-5, 6, 7, 8, 9." Counting on two, however, gives them a way to figure out how counting on works. Later they can learn to count on with larger numbers by developing their conceptual subitizing or by learning different ways of "keeping track."

Children can use familiar spatial patterns to develop conceptual subitizing of arithmetic. For example, students can use tens frames to visualize addition combinations (fig. 4). Such pattern recognition can assist students with mental handicaps and learning disabilities as they learn to recognize the five- and ten-frame configuration for each number. "These arrangements ... help a student first to recognize the number and use the model in calculating sums. It is this image of the number that stays with the student and becomes significant" (Flexer 1989). Visual-kinesthetic finger patterns can similarly help, especially with the important number combinations that sum to 10.

FIGURE 4

Using tens frames and subitizing to visualize addition



Eventually, students come to recognize number patterns as both a whole-a unit itself-and a composite of parts-individual units. At this stage, a student is capable of viewing number and number pat-terns as units of units (Steffe and Cobb 1988). For example, students can repeatedly answer what number is "10 more" than another number. "What is 10 more than 23?" "33!" "Ten more?" "43!"

Final Words

"Subitizing is a fundamental skill in the development of students' understanding of number" (Baroody 1987, 115). Students can use pattern recognition to discover essential properties of number, such as conservation and compensation. They can develop such capabilities as unitizing, counting on, and composing and decomposing numbers, as well as their understanding of arithmetic and place value-all valuable components of number sense.

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