1. Let \( a_n \) be the number of ways to fill a \( 2 \times n \) checkboard using any combination of \( 1 \times 1 \) and \( 1 \times 2 \) dominoes.
   
   (a) Find \( a_1, a_2, \) and \( a_3 \).
   
   (b) Find a recurrence relation for \( a_n \).
   
   (c) Find \( a_{10} \).

2. You are planning the first round of a 1v1 basketball tournament. Let \( a_n \) be the number of ways to pair \( 2n \) people up for their first match.
   
   (a) What are \( a_1, a_2, a_3 \)?
   
   (b) Find a recurrence relation for \( a_n \).
   
   (c) Try to solve this recurrence.
   
   (d) Try to explain why this solution makes sense via a different argument.

3. How many ways are there to climb a flight of \( n \) stairs if we can take jumps of size either 1, 2, or 3 steps at once? (Find a recurrence and try to solve it [with the help of a calculator of some kind!])

4. Use the formula for the Fibonacci numbers to finish the following sentence (and try to prove it): “If \( n > 5 \), then \( F_n \) is whatever integer is closest to \( Ax^n \)" (Find values \( A \) and \( x \) that make this true!)

5. Take a piece of paper and draw lines on it so that each pair of lines intersect, but no three lines intersect in a single point. Let \( a_n \) be the number of different regions of the paper left after drawing \( n \) lines.
   
   (a) Draw pictures for \( n = 1, 2, 3 \).
   
   (b) Compute \( a_1, a_2, a_3 \).
   
   (c) Find a recurrence relation for \( a_n \).
   
   (d) Solve it.

6. Here’s a “double recurrence!” (i.e., we have two sequences that are related to each other!) For this, let \( a_1 = b_1 = 1 \). Then to find the next numbers, let \( \frac{a_n}{b_n} = \frac{1}{1 + \frac{a_{n-1}}{b_{n-1}}} \). So \( \frac{a_2}{b_2} = \frac{1}{1 + \frac{1}{1}} = \frac{1}{2} \), which implies \( a_2 = 1 \) and \( b_2 = 2 \).
   
   (a) Find a recurrence relations for \( a_n \) and \( b_n \) in terms of \( a_{n-1} \) and \( b_{n-1} \).
   
   (b) Use this to find \( a_{10} \).

7. You have three types of tiles: \( 1 \times 1 \) tiles, \( 2 \times 1 \) tiles, and L shaped tiles of area 3 (these are \( 2 \times 2 \) tiles missing a square). Let \( T(n) \) denote the number of ways you can use these tiles to tile a \( 2 \times n \) checkerboard.
   
   (a) Find \( T(1), T(2), T(3) \).
   
   (b) Find a recurrence relation for \( T(n) \).
   
   (c) Find a formula for \( T(n) \).

8. Let \( B(n) \) denote the size of the biggest entry in the \( n \)th row of Pascal’s triangle. So for instance \( B(0) = B(1) = 1, B(2) = 2, \) and \( B(4) = 6 \).
   
   (a) Argue that \( B(n) \leq B(n+1) \)
   
   (b) Argue that \( 2B(2n+1) \leq B(2n + 2) \)
   
   (c) Argue that \( 2^{(n-1)/2} \leq B(n) \)
   
   (d) On the other hand, use a different argument to show \( B(n) \leq 2^n \)
   
   (e) Try to improve or generalize these results.
A triangulation of a convex \( n \)-gon is a way to draw \( n - 2 \) non-overlapping cords in order to cut it up into triangles. For example, the following picture shows all 42 triangulations a convex 7-gon. Let \( T(n) \) denote the number of triangulations of a convex \( n \)-gon. (Note, order and such really matters for triangulations. So \( T(7) = 42 \))

(a) Find \( T(3), T(4), \) and \( T(5) \) by drawing out all the options.
(b) Try to find a recurrence for \( T(n) \) [it’s a bit tricky-looking, and hard to solve!]
(c) Use it to find \( T(10) \).

Figure 1: All 42 triangulations of the 7-gon