

(1) (a) Let  $X = \xi_1 + \cdots + \xi_n$ , where  $\xi_i$  are independent indicator random variables which equal 1 with probability  $1/2$ . What is  $\mathbf{P}(X = k)$ . Compare  $\mathbf{P}(X = k)$  and  $\mathbf{P}(X = m)$  for all pairs of numbers  $(k, m)$ . Which value of  $k$  maximizes  $\mathbf{P}(X = k)$ ? Compute the value of  $\mathbf{P}(|X - n| \leq 2\sqrt{n})$  approximately (assuming that  $n$  is large).

(b) What are the answers if we replace  $1/2$  by a constant  $0 < p < 1$ ? (again assuming that  $n$  is very large and does not depend on  $p$ ).

Hint: Recall the central limit theorem.

(2) Prove that the function  $\binom{y}{n}$  is convex. Read the proof in page 7, Chapter 1.

(3) Let  $S$  be a set of  $n$  elements ( $n$  even). Choose a random subset  $A$  of  $S$  by selecting each element with probability  $1/2$ . Repeat the process to choose another random subset  $B$ . What is the expectation and variance of  $|A \cap B|$ ? What happens if each element of  $B$  is chosen with probability  $1/3$ ? What are the expectation and variance of  $|A \cup B|$  in each case?

(4) Problem 1, page 10.

(5) Problem 2, page 10.

(6) Problem 4, page 11.