Econometrica, Vol. 73, No. 2 (March, 2005), 481–516

JOB MATCHING AND THE WAGE DISTRIBUTION

BY GIUSEPPE MOSCARINI[1]

This paper brings together the microeconomic-labor and the macroeconomic-equilibrium views of matching in labor markets. We nest a job matching model à la Jovanovic (1984) into a Mortensen and Pissarides (1994)-type equilibrium search environment. The resulting framework preserves the implications of job matching theory for worker turnover and wage dynamics, and it also allows for aggregation and general equilibrium analysis. We obtain two new equilibrium implications of job matching and search frictions for wage inequality. First, learning about match quality and worker turnover map Gaussian output noise into an ergodic wage distribution of empirically accurate shape: unimodal, skewed, with a Paretian right tail. Second, high idiosyncratic productivity risk hinders learning and sorting, and reduces wage inequality. The equilibrium solutions for the wage distribution and for the aggregate worker flows—quits to unemployment and to other jobs, displacements, hires—provide the likelihood function of the model in closed form.

KEYWORDS: Wage distribution, wage inequality, job matching, specific human capital, worker flows, unemployment, job search, ergodic analysis.

1. INTRODUCTION

THE SEARCH-AND-MATCHING model is the canonical framework for the analysis of labor markets, both in micro-labor economics and in macroeconomics. Despite their common roots and their many similarities, the micro and macroeconomic approaches to matching in labor markets have evolved in parallel, and have addressed different issues. In this paper, we bring together these two views of the labor market, and we extend their scope. In particular, we show that the matching model can reconcile the observed patterns of worker turnover and individual wage dynamics with stylized facts concerning wage inequality.

The job matching theory originally proposed by Jovanovic (1979) quickly became the benchmark model of worker turnover in labor economics, and the inspiration for a vast body of applied microeconomic research. The worker-firm match is modelled as an experience good, whose characteristics are initially uncertain, and are gradually revealed over time by output performance. Optimal inference and the resulting selection of matches provide a natural explanation for a wide range of robust empirical correlations: positive between worker tenure and wage (Topel (1991)), initially positive and soon negative

[1]This paper develops the theoretical analysis from my earlier work "Skill and Luck in the Theory of Turnover," which is now focussed only on the quantitative evaluation of the model presented here. I thank Boyan Jovanovic, the editor and three anonymous referees for detailed suggestions that greatly helped to improve the substance and the presentation of this paper, the audiences of many seminars for helpful comments, and Daniel le Maire for identifying a major typo in the calculations. The usual disclaimer applies. Financial support to this research was generously provided by Yale University and the Alfred P. Sloan Foundation.

between tenure and the hazard rate of separation (Farber (1994)), negative between tenure or wage and the propensity to search on the job (Pissarides and Wadsworth (1994)).

The macroeconomic-equilibrium approach to matching in labor markets has focused instead, in two separate and influential research programs, on unemployment and on the wage distribution. Equilibrium unemployment theory, originating from the work of Diamond (1982), Mortensen (1982), and Pissarides (1985), has become the standard framework of analysis for aggregate labor markets. A defining feature is the lack of commitment power by firms and workers, so that wages are set by continuous bilateral renegotiation, typically Nash bargaining. Conversely, in the "wage posting" literature firms have all the bargaining power. But, to avoid the Diamond (1971) paradox, either new sequential offers arrive to employed workers in a random matching environment, or firms are able to commit to their offers in a directed search setting. Wage dispersion among identically productive matches may result either from asymmetric equilibrium wage-posting strategies (Burdett and Mortensen (1998)) or from imperfectly assortative matching between ex ante heterogeneous workers and firms, due to a lack of coordination in job applications (Postel-Vinay and Robin (2002a), Shi (2002)).

The current state of the literature shows that micro and macroeconomists have re-tooled the search-and-matching model of the labor market to answer their own, different questions. Only recently has the wage-posting literature started to address (Burdett and Coles (2003)) the observed patterns of on-the-job wage dynamics and quits to unemployment that motivated Jovanovic's (1979) job matching model. Conversely, the implications of the latter for wage inequality and for the magnitude of worker flows are unknown, and may well be counterfactual. A rare exception is the main incarnation of equilibrium unemployment theory, the Mortensen and Pissarides (1994) model, which is rooted in the job matching tradition: evolving idiosyncratic uncertainty in match productivities is the engine of turnover. Given its empirical success at the micro level,[2] Jovanovic's (1979) job matching model was the natural candidate to formalize this type of uncertainty; unfortunately, it lacks the tractable aggregation properties that were required for Mortensen and Pissarides' (1994) general equilibrium analysis. Because of this fact, as well as of their focus on aggregate job flows as opposed to individual worker turnover, Mortensen and Pissarides (1994) and extensions thereof have modelled this uncertainty in reduced forms, which have counterfactual implications for the evolution of wages and separation rates *within* a job, such as a wage falling and a separation rate rising on average with tenure. But the gap appears technical, more than conceptual.

[2]See, for example, Flinn (1986) and Lane and Parkin (1998). This empirical literature typically focusses on younger workers, as suggested by the theory, because older workers are much more likely to have found their good match. This does not imply that job matching is not relevant for older workers: while their separations are less frequent, the consequences are often more dire.

In this paper we introduce a synthesis of the theories of job matching and equilibrium unemployment. We propose a unified explanation for labor market turnover and wage inequality based on ex post sorting and wage determination. In this sense, our "unified ex post selection model" is a clear alternative to wage-posting models with sequential search, where wage dispersion among ex ante identical workers and firms originates only from search frictions, in particular from on-the-job search (Burdett and Mortensen (1998), Postel-Vinay and Robin (2002b), Burdett and Coles (2003)). Therefore, an informative comparison between these two alternative views requires that we take into account on-the-job search also in our analysis. We then build on Jovanovic (1984), who extended Jovanovic (1979) to allow for infrequent and random arrival of offers to the worker both when jobless, in the spirit and with the same objectives of equilibrium unemployment theory, and when employed.

When bridging the remaining gap from Jovanovic (1984) to Mortensen and Pissarides (1994), we are presented with three challenges. The first, as mentioned, is technical in nature, and concerns aggregation. In Jovanovic's (1984) well-known Gaussian setup, posterior beliefs about match productivity are two-dimensional, mean and variance, and evolve according to a classical linear filter. In equilibrium, the mean equals the wage and the variance is inversely proportional to job tenure. These features make the model so elegant and empirically appealing, but also not amenable to aggregation. The cross-sectional belief distribution is bivariate and cannot be characterized analytically. Thus, the univariate wage distribution and the aggregate worker flows implied by the model are unknown. This is the likely reason why job matching theory has failed to make progress beyond partial equilibrium analysis.

We introduce a key simplification of this setup, which makes its aggregation tractable, while preserving its desirable equilibrium implications. We assume that *the unknown match quality may take one of only two values*. In effect, each worker-firm match runs a sequential probability ratio test, based on the data provided by cumulative output, of two simple hypotheses: the match is either successful or not. From an economic viewpoint, this simplification is painless: the trade-offs and insights uncovered by job matching theory are preserved in our model. A two-point support of possible match qualities is sufficient to capture the intuition on the general effects of job matching uncertainty. The payoff from our simplification is a uni-dimensional posterior belief about match quality—the chance that the match is successful, following a simple nonlinear filter—which delivers an analytic solution of the ergodic and stationary distribution of posterior beliefs and (expected) match productivities.

When we map posterior beliefs about match quality into wages, we discover a second, conceptual inconsistency between Jovanovic (1984) and equilibrium search. In Jovanovic (1984), the equilibrium wage is "competitive" and equals expected productivity conditional on output history; hence, active firms make zero expected profits, and idle firms lack the rents required to cover the entry cost of creating a vacancy. Costless vacancies, far from solving the problem, would generate frictionless firm-worker contacts and would fail to explain frictional unemployment. Recruiting costs and free entry require that firms have some bargaining power and earn quasi-rents ex post. At the other extreme, if firms had *all* the bargaining power, then wages would reflect just the value of leisure and would not depend on match success. To reconcile costly job creation, frictional matching, and returns to tenure, we allow for rent-sharing. Following the tradition in equilibrium unemployment theory, we assume that wages are set by generalized Nash bargaining, which subsumes Jovanovic's (1984) competitive wages as a special (but problematic) case.

In turn, rent-sharing and on-the-job search raise a third and final conceptual issue. A worker earns and generates rents both for his employer and for alternative employers with which he comes in contact. Therefore, an outside offer made to an employed worker triggers a trilateral renegotiation problem. Competitive wage-setting in Jovanovic (1984) resolved the issue totally in favor of the worker. We assume instead that the two firms play an English auction to win the worker, and we compare this outcome to other solutions to the poaching game proposed so far in the literature.

In this environment, the equilibrium wage is linear in the expected productivity of a match, just like in Jovanovic (1984). But our simpler framework allows for an analytic characterization of the wage distribution, which reveals two substantive results.

First, we find that the equilibrium wage density generically exhibits the three main features of empirical wage distributions: a unique interior mode, skewness like a log-normal, and a long and "fat" right tail of Pareto functional form. The selection of good matches, through optimal quits to unemployment and to other jobs, redistributes mass of workers from the lower to the upper part of the distribution of beliefs about match quality, which determine wages. This explains the skewness and the fat Pareto tail, in spite of symmetric and Gaussian (thin-tailed) noise in output. Therefore, as an explanation for the typical shape of an empirical wage distribution, our unified job matching model is a plausible alternative both to wage-posting models with sequential search[3] and to the frictionless competitive markets tradition dating back to Roy (1951). The latter is based on ex ante self-selection by workers who know their comparative advantages *before* matching with firms. As shown by Heckman and Sedlacek (1985), this mechanism maps a Gaussian distribution of productivities into a Pareto wage distribution. Strikingly, the same equilibrium mapping arises in our very different environment, where sorting occurs entirely ex post and is impeded both by search and learning frictions.

[3]Mortensen (1998) shows that a typical wage-posting search equilibrium implies an increasing or U-shaped wage distribution. Upfront firm-specific investments may restore unimodality and long, fat upper tail. Postel-Vinay and Robin (2002b) introduce employed search and ex post competition for employed workers, but no ex post productivity risk. The implied equilibrium wage distribution is hump-shaped, but lacks the strong skewness and especially the Pareto tail that we observe in the data.

Our second result is a distinct equilibrium implication of imperfect information about match productivity. We find that the larger the idiosyncratic productivity risk that clouds the underlying match quality, the higher the (Pareto) rate of decay of the upper tail of the wage distribution. Intuitively, the harder the inference problem faced by firms and workers, the less effective the sorting process, the fewer workers have time to identify a good match before being exogenously separated from their jobs. The upper tail of the wage distribution contains precisely those matches that have been almost ascertained to be successful. This implication is generally reversed in search models that assume evolving but *observable* idiosyncratic productivity or demand shocks, such as Mortensen and Pissarides (1994). This is important, because all the predictions of Jovanovic (1979) that have found ample empirical support are common to any stochastic selection model, with or without learning. Our (potentially testable) implication may provide a definite test of the importance of learning in the selection process.

We first establish the two main results in a baseline model without on-the-job search. Then, we introduce on-the-job search and analyze its independent contribution to the aggregate selection process. Under the assumption of an English auction for an employed worker, the wage remains a linear function of expected match productivity even with on-the-job search, and the general equilibrium retains qualitatively similar properties.

In addition to these two results, we contribute to the ongoing effort to render equilibrium models of wage dispersion empirically operational. Recent attempts in this direction rely exclusively on wage-posting models, which allow for wage growth only through outside offers (Christensen et al. (2003)) and treat the distribution of worker and firm productivities as unobservables, to be estimated necessarily by nonparametric methods (Postel-Vinay and Robin (2002a)). The analytic solution of our equilibrium model of job matching and unemployment allows for structural estimation of the model parameters by maximum likelihood. The likelihood is the equilibrium wage distribution, whose empirically accurate shape suggests an excellent fit of the model to the data. An econometrician estimating a Mincerian wage equation without knowing match output histories would dump into a Gaussian (symmetric) "error" term the wage dispersion created by evolving beliefs about match quality. If our structural model is correct, this "error" has a skewed distribution, due to job shopping, and therefore the linear estimates are biased.

The paper is organized as follows. Section 2 illustrates the model, Section 3 illustrates equilibrium wages and separation policies, Section 4 illustrates equilibrium turnover, Section 5 illustrates the stationary and ergodic wage distribution, Section 6 introduces on-the-job search, Section 7 closes the general equilibrium of the model with a matching function, Section 8 concludes, and an Appendix collects proofs.

2. THE ECONOMY

A consumption good is produced in continuous time by pairwise firm-worker matches (*jobs*). The average productivity or "quality" of each match, μ, is specific and ex ante uncertain: upon matching, the firm and the worker share a common prior belief on μ, independent of their past histories and concentrated on two points, $p_0 = \Pr(\mu = \mu_H) = 1 - \Pr(\mu = \mu_L) \in (0, 1)$, where μ_L denotes a "bad" match and μ_H $(> \mu_L)$ a "good" match.

The performance of the match is also subject to two additional and orthogonal sources of idiosyncratic noise. First, the cumulative output of a match of duration t is a normal random variable, a Brownian Motion with drift μ and known variance σ^2:

$$X_t = \mu t + \sigma Z_t \sim N(\mu t, \sigma^2 t).$$

Here Z_t is a Wiener process, a continuous additive noise that keeps μ hidden and creates an inference problem. Over time, parties observe output realizations $\langle X_t \rangle$, generating a filtration $\{\mathcal{F}_t^X\}$, and update in a Bayesian fashion their belief from the prior p_0 to the posterior $p_t \equiv \Pr(\mu = \mu_H | \mathcal{F}_t^X)$. The second, more drastic source of idiosyncratic productivity shocks forces jobs out of business at Poisson rate $\delta > 0$. This process captures many important idiosyncratic sources of match dissolution; a few examples are, on the labor demand side, technological obsolescence, natural disasters, changes in specific tax code provisions, idiosyncratic product demand shocks; on the labor supply side, spousal relocation and human capital shocks, such as worker disability, retirement, and death.

The economy is populated by a large mass of ex ante homogeneous firms, ensuring free entry, and by a unit measure of ex ante homogeneous workers. If δ contains a worker attrition component, the population is replenished by new workers. A jobless worker enjoys a flow value of leisure b, while idle firms get zero flow returns. Workers and firms are risk-neutral optimizers and discount future payoffs at rate $r > 0$. Utility is perfectly transferable. We assume $b \in [\mu_L, (1 - p_0)\mu_L + p_0\mu_H]$, so that the matching choice is nontrivial: a new match should always be accepted, because it produces more than the joint value of inactivity b, and should be dissolved if $\mu = \mu_L \leq b$. In practice, parties perform a sequential probability ratio test of simple hypotheses on the viability of the match.

The firm must pay a flow sunk cost κ to keep a vacancy open to applications from unemployed workers. A jobless worker contacts an open vacancy at finite Poisson rate λ according to a matching function described in Section 7. Job search is costless, except for its time-consuming aspect and for discounting. There is no recall of past offers. Every new match restarts from a common prior chance p_0 of success. Unlike in Jovanovic (1984), there is no initial "screening" phase. We defer to Section 6 the introduction of on-the-job search.

Firms and workers cannot commit to a wage contract. Search frictions create rents that the firm and the worker split according to a generalized Nash bargaining rule, assigning a geometric weight β to the worker's surplus.

3. WAGES AND SEPARATIONS

We analyze the steady state equilibrium of this economy. Policy functions and aggregate variables (including the wage distribution) do not change over time, while worker turnover and job churning are continuously driven by purely idiosyncratic uncertainty.

3.1. *Filtering*

A sufficient statistic for output history, which determines the future prospects of a match, thus also the natural state variable of the bargaining game, is the posterior belief p_t that the match was a success ($\mu = \mu_H$). Using the expression for the Gaussian density of total output $y = y_t$, Bayes rule defines the posterior p_t as a $\mathbb{C}^2$ function of the diffusion y, so we can apply Ito's Lemma to obtain the stochastic differential equation for p. This is done by Theorem 9.1 in Liptser and Shyryaev (1977). Conditional on the output process X, the posterior probability of a good match evolves from any prior $p_0 \in (0, 1)$ as a martingale diffusion solving

$$dp_t = p_t(1 - p_t)s\, d\overline{Z}_t, \tag{3.1}$$

where

$$s \equiv \frac{\mu_H - \mu_L}{\sigma}$$

is the *signal/noise ratio* of output, and

$$d\overline{Z}_t \equiv \frac{1}{\sigma}[dX_t - p_t\mu_H\, dt - (1 - p_t)\mu_L\, dt]$$

is the *innovation* process, the normalized difference between realized and unconditionally expected flow output. This is a standard Wiener process with respect to the filtration $\{\mathcal{F}_t^X\}$. Intuitively, beliefs move faster the more uncertain match quality (the term $p(1 - p)$ peaks at $p = 1/2$), and the more informative production, as measured by the signal/noise ratio s.[4]

[4]In this binary structure, unlike in the Gaussian model of Jovanovic (1979, 1984), posterior beliefs' precision $[p_t(1 - p_t)]^{-1}$ does not necessarily increase over time as evidence accumulates. But it does increase on average, because it is a convex function of p_t, which is a martingale. More generally, the qualitative implications of Jovanovic's model depend on the martingale property of beliefs and on optimal selection, *not* on the specific match distribution assumed. In fact, these properties survive essentially intact in this binary framework (see Proposition 3). In contrast, aggregation is tractable in the binary structure, not in the Gaussian model.

3.2. *Equilibrium Wage*

Let $W(p)$ denote the discounted total payoffs that a worker receives in the equilibrium of the bargaining-and-search game, when employed in a match that is successful with current posterior chance p. Similarly, let U denote the worker's value of unemployment, independent of p because of the match-specific nature of μ, $J(p)$ denote the rents of the firms, V denote the value to the firm of holding an open vacancy, and $S(p) = W(p) + J(p) - U - V$ denote the total surplus of this match. By free entry in vacancy creation, $V = 0$, so $S(p) = W(p) + J(p) - U$. We seek to construct an equilibrium where S, W, and J are strictly increasing in p.

The worker's values of being (respectively) unemployed and matched well with probability p solve the Hamilton–Jacobi–Bellman (HJB) equations:[5]

$$rU = b + \lambda[W(p_0) - U],$$

$$rW(p) = w(p) + \Sigma(p)W''(p) - \delta[W(p) - U], \tag{3.2}$$

where

$$\Sigma(p) \equiv \frac{1}{2}s^2 p^2 (1-p)^2$$

is (half) the variance of the change in posterior beliefs. Roughly speaking, this term measures the "speed of learning" about match quality: if posterior beliefs are not expected to change in the next instant, the variance is zero and nothing is learned. The opportunity cost of unemployment, rU, equals its flow benefit b plus the capital gain $W(p_0) - U$ from a new match, which has prior belief p_0 of being successful, accruing at rate λ. Similarly, the opportunity cost $rW(p)$ of working in a job that is successful with posterior chance p equals the flow wage $w(p)$, plus a diffusion-learning term $\Sigma(p)W''(p)$, minus the capital loss following exogenous separation at rate δ. The learning speed $\Sigma(p)$ is converted into payoff units by the convexity of the Bellman value $W''(p)$, because information (here in the form of output) spreads posterior beliefs and empowers more informed decisions by the worker. In fact, if W was a locally affine function, then the martingale property of beliefs $\mathbb{E}[p_{t+dt}] = \mathbb{E}[p_t]$ would imply $\mathbb{E}[W(p_{t+dt})] = \mathbb{E}[W(p_t)]$, namely, an expected value equal to the current value, so nothing valuable would be learned. The worker optimally quits to unemployment at every belief $\underline{p}_W \in [0, 1]$ such that $W(\underline{p}_W) = U$ (*value matching*) and $W'(\underline{p}_W) = 0$ (*smooth pasting*).

The problem of the firm is similar. The free entry condition $V = 0$ will be used later to close the general equilibrium. The value to the employer $J(p)$

[5]Unless otherwise noted, Karlin and Taylor (1981) is the main reference for the standard technical results in diffusion theory exploited in this paper.

of an active match that is successful with posterior chance p solves the HJB equation

$$rJ(p) = \bar{\mu}(p) - w(p) + \Sigma(p)J''(p) - \delta J(p). \tag{3.3}$$

The opportunity cost of production $rJ(p)$ equals expected flow output

$$\bar{\mu}(p) \equiv p\mu_H + (1-p)\mu_L$$

minus the wage $w(p)$, plus the return from learning the quality of the match $\Sigma(p)J''(p)$, minus the expected capital loss due to exogenous separation $(\delta J(p))$. The firm optimally fires the worker at every $\underline{p}_J \in [0,1]$ such that $J(\underline{p}_J) = 0$ and $J'(\underline{p}_J) = 0$.

The generalized Nash bargaining solution selects a wage

$$w(p) \in \arg\max_w [W(p) - U]^\beta [J(p)]^{1-\beta} \tag{3.4}$$

for some $\beta \in (0,1)$ exogenously given. As repeatedly shown in the job search literature, this maximization yields as a necessary and sufficient first-order condition

$$\beta J(p) = (1-\beta)[W(p) - U], \tag{3.5}$$

that is, the worker receives a fraction β of total match surplus: $W(p) - U = \beta S(p)$, $J(p) = (1-\beta)S(p)$. This linear sharing rule aligns the interests of the worker and the match, as a consequence of the (private) efficiency of the Nash solution.

By (3.5), worker and firm agree to separate and turn idle when the posterior belief hits the same threshold(s) $\underline{p}_W = \underline{p}_J \equiv \underline{p}$. Observe that (3.5) implies $\beta J''(p) = (1-\beta)W''(p)$. We prove in the Appendix that using these facts and (3.5) in the HJB equations (3.2) and (3.3) and some algebra yield a simple and intuitive expression for the equilibrium wage.

PROPOSITION 1: *The Nash bargaining maximization* (3.4) *is equivalent to* (3.5) *and both are solved by the wage function*

$$w(p) = (1-\beta)b + \beta[\bar{\mu}(p) + \lambda J(p_0)]. \tag{3.6}$$

The worker receives a wage that weighs with the bargaining share his flow outside option, the opportunity cost of time b, and his flow inside option, expected output $\bar{\mu}(p)$ plus the continuation value of unemployed job search $\beta\lambda J(p_0) = (1-\beta)\lambda[W(p_0) - U]$. The wage is affine and increasing in the posterior belief.

Now consider two extreme cases for the bargaining share. First, observe that $S(p) \leq \mu_H/(r+\delta) < \infty$ for every p, as no match can produce more than this amount in present discounted value. If $\beta = 1$, we deduce $J(p) = (1-\beta)S(p) = 0$ for every p. Then, the wage equation (3.6) reduces to $w(p_t) = \bar{\mu}(p_t) = \mathbb{E}[\mu|\mathcal{F}_t^X]$, namely Jovanovic's (1984) "competitive" wage, equal to expected match productivity. In this case the firm lacks the positive rents $J(p_0) > 0$ that are necessary to cover the cost κ of posting a vacancy and to enter the market. Conversely, if $\beta = 0$, then again from (3.5) the worker gets paid his opportunity cost of time $w(p) = b$, independently of past performance, tenure, etc. In either extreme case, the wage is disconnected from tenure and job finding rate λ.

3.3. *Value Functions and Equilibrium Separation Policy*

Replacing the wage function (3.6) in the worker's and the firm's HJB equations transforms their bargaining-separation game into two separate optimal stopping problems. Using (3.5), (3.6), and boundaries, turns the firm's HJB equation (3.3) into a differential equation in rents $J(p)$ only:

$$(3.7) \qquad (r+\delta)J(p) = (1-\beta)[\bar{\mu}(p) - b] + \Sigma(p)J''(p) - \beta\lambda J(p_0)$$

subject to value matching and smooth pasting at $\underline{p}$. An additional boundary condition is $J(p) \le S(p) < \infty$. We can solve for the value function, which equals the sum of the present discounted value of flow returns and of the option value of separating should things go wrong. We also verify our initial guess that J, thus by (3.5) W and S, are increasing functions of the belief p of a good match.

PROPOSITION 2 (Bargaining and Separation Equilibrium): *When a firm and a worker match, they share a common prior belief* p_0 *that their match is highly productive, continuously observe output in* $[0, t]$, *update the posterior belief* p_t *according to* (3.1), *renegotiate the wage* $w(p)$ *according to* (3.6), *and separate when the posterior declines to a low cutoff* $\underline{p} \in (0, p_0)$ (*when the wage falls to a reservation value* $w(\underline{p})$). *The value function of the firm is the increasing and convex function of beliefs* $p \in [\underline{p}, 1]$:

$$(3.8) \qquad J(p) = c_J p^{1/2-\sqrt{1/4+2(r+\delta)/s^2}}(1-p)^{1/2+\sqrt{1/4+2(r+\delta)/s^2}} + \frac{(1-\beta)[\bar{\mu}(p) - b] - \beta\lambda J(p_0)}{r+\delta},$$

where c_J *and the optimal stopping point* $\underline{p} \in (0, p_0)$ *uniquely solve* $J(\underline{p}) = 0$, $J'(\underline{p}+) = 0$.

4. TURNOVER

The equilibrium play described in Proposition 2 implies a stochastic process for the worker's employment status and, conditional on employment, for the posterior belief of a good match p_t. This starts from p_0 and evolves according to (3.1) following output realizations. It is "killed" at rate δ by exogenous separations and it is stopped when it falls to $\underline{p}$, where parties separate endogenously to restart searching on their own: either way, the worker is absorbed into unemployment for a random duration of mean $1/\lambda$.

Before illustrating our new results, we verify that our model preserves the qualitative correlations between tenure, wages, and separation rates that are observed in the data and that are central to extant theories of worker turnover, as summarized in the Introduction. We also find an analytic expression for the expected residual duration $\tau(p)$ of a p-match. In the absence of endogenous separations at $\underline{p}$, this should equal $1/\delta$. Allowing also for endogenous separations to unemployment, $\tau(p)$ solves

$$\Sigma(p)\tau''(p) - \delta\tau(p) = -1.$$

LEMMA 1 (Expected Tenure): *The expected future duration of a match is the increasing and concave function of the current belief that the match is productive*:

$$\tau(p) = \frac{1}{\delta}\left\{1 - \left(\frac{p}{\underline{p}}\right)^{1/2-\sqrt{1/4+2\delta/s^2}}\left(\frac{1-p}{1-\underline{p}}\right)^{1/2+\sqrt{1/4+2\delta/s^2}}\right\}.$$

Standard in Bayesian learning, in expectation with respect to current beliefs p_t, posterior beliefs $p_{t+\Delta t}$ are a martingale: $\mathbb{E}[p_{t+\Delta t}|0 \le p_{t+\Delta t} \le 1, p_t] = p_t$ for all $\Delta t \ge 0$. But, if we condition on match continuation from t to $t+\Delta t > t$, the belief is a strict submartingale, because it is bounded below by $\underline{p} > 0$ and reflects only good output outcomes. In fact,

$$\begin{aligned}\mathbb{E}\big[p_{t+\Delta t}|\text{ producing in } [t, t+\Delta t], p_t\big] &> \mathbb{E}[p_{t+\Delta t}|\underline{p} \le p_{t+\Delta t} \le 1, p_t] \\ &> \mathbb{E}[p_{t+\Delta t}|0 \le p_{t+\Delta t} \le 1, p_t] = p_t\end{aligned}$$

where the first inequality holds because endogenous separations occur only for low beliefs, and the chance of exogenous match dissolution is independent of p_s, so match continuation is more likely for high beliefs in $[t, t+\Delta t]$; the second inequality follows from $\underline{p} > 0$ and the full support of $p_{t+\Delta t}$ in $(0, 1)$ if not stopped; and the equality is the martingale property. Hence the confidence in a good match rises with tenure on average, although not with probability one as in Jovanovic (1984): here, reversals of confidence in the success of the match are possible. Also standard in Bayesian learning, the value function is convex in beliefs p; hence a worker's value is a submartingale too. The flow wage (3.6) is affine in beliefs, due to the combined assumptions of expected utility and linear sharing rule; hence it is also a submartingale for continuing matches.

Unconditionally on match quality, starting from a current belief p_t, the probability of separating endogenously at some future date $T > t$ ($p_T = \underline{p}$) before finding out that the match is good for sure ($p_T = 1$) equals $(1 - p_t)/(1 - \underline{p})$; therefore, the probability of endogenous separation to unemployment is decreasing in p_t. The hazard rate of exogenous separation, δ, is independent of p_t. Overall, separation is less likely the larger the expected productivity of the match, and thus (on average, by Lemma 1) the longer the worker's tenure. The only exception occurs at the beginning of a match. The continuous sample paths of the diffusion $\langle p_t \rangle$ cannot jump from p_0 down to $\underline{p}$. Therefore, an endogenous separation to unemployment cannot be instantaneous, but "kicks in" only after some time. Thus, on average, the hazard rate of separation initially increases with tenure in the early, "discovery" phase of an employment relationship, and eventually decreases when selection comes to dominate. We cannot prove in general that this relationship is single-peaked, as predicted by Jovanovic's (1979) model and corroborated by empirical evidence in Farber (1994). We summarize these findings in the following proposition.

PROPOSITION 3 (Tenure, Wages, and Job Search Behavior): *Unconditionally on true match quality, but conditional on match continuation, the human wealth of the employed worker $W(\cdot)$, his flow wage $w(\cdot)$, and the rents of his employer $J(\cdot)$ rise on average with tenure. The hazard rate of match separation rate initially increases and eventually decreases with tenure. Expected future tenure is increasing in the current wage.*

5. THE ERGODIC WAGE DISTRIBUTION

The stochastic process describing the equilibrium evolution of the posterior belief of a good match is clearly Markovian and strongly recurrent. Therefore, the stationary density is also ergodic: from any prior belief $p_0 \in (0, 1)$ of a good match draw, the posterior belief converges a.s. to a random variable p_∞, whose unit probability mass is split between total employment on the support $[\underline{p}, 1]$ and an atom of unemployment. If p_∞ has a density on $[\underline{p}, 1]$, say f, then in a large population of workers f can be interpreted also as the ergodic and stationary cross-sectional distribution of employed workers (matches, posterior beliefs). For the following results we refer to Feller's (1954) classic treatment of diffusions on an interval and to his physical interpretation of the dynamic equations.

The Fokker–Planck (Kolmogorov forward) equation of the process describes the dynamics of the transition density. Imposing stationarity, we obtain a differential equation solved by the stationary and ergodic density f of the belief process for $p \neq p_0$:

$$0 = \frac{df(p)}{dt} = \frac{d^2}{dp^2}[\Sigma(p)f(p)] - \delta f(p). \tag{5.1}$$

Intuitively, the distribution at p is reduced by learning in those matches that start at p and move away from there, and is increased by learning in matches that start from other beliefs $p' \neq p$ and end up at p, following either a poor performance of matches with $p' > p$ or a surprisingly good outcome in matches with $p' < p$. The second order term in (5.1) nets out these three learning flows. At the same time, the distribution at p loses mass at rate δ, because of exogenous job destruction. In steady state, all of these flows balance exactly at every belief p in the support $[\underline{p}, 1]$. The equation does not hold at p_0, where the inflow from unemployment and from other jobs (quits) produces a kink in the density, as we will see shortly.

The forward equation is subject to the following three boundary conditions. First, once the match appears sufficiently unpromising and the posterior belief reaches the equilibrium separation point $\underline{p}$, no time is spent pondering the next step, which is separation to unemployment. Since p spends no time at $\underline{p}$, we require $\Sigma(\underline{p})f(\underline{p}+) = 0$, the standard condition for "attainable" boundaries, namely those that can be hit in finite time with positive probability and are either absorbing or reflecting. But $\underline{p} \in (0, 1)$ implies $\Sigma(\underline{p}) > 0$ and therefore

$$f(\underline{p}+) = 0. \tag{5.2}$$

Second, total flows (respectively) in and out of employment must balance:

$$\Sigma(p_0)[f'(p_0-) - f'(p_0+)] = \delta \int_{\underline{p}}^{1} f(p)\,dp + \Sigma(\underline{p})f'(\underline{p}+). \tag{5.3}$$

The left-hand side is the total inflow into employment. The density f has a kink, corresponding to the inflow of workers, at p_0. Intuitively, the rate of change in the c.d.f. as p crosses p_0 from below changes direction at p_0, because new matches start at p_0 and thus add to the cumulative distribution only at or below p_0. The right-hand side of (5.3) sums the total flows out of employment, due (respectively) to exogenous job destructions hitting at rate δ the entire mass of employment $\int_{\underline{p}}^{1} f(p)\,dp$ and to quits to unemployment at $\underline{p}$. To gain intuition on this last term $\Sigma(\underline{p})f'(\underline{p}+)$, notice that for $p = \underline{p} + \varepsilon$ and $\varepsilon > 0$ small, from a Taylor expansion and from (5.2) we get $f(p) \simeq f'(\underline{p}+)\varepsilon$. It follows that, if the ergodic density is initially very flat ($f'(\underline{p}+)$ is small), then $f(p)$ is initially small and so is its c.d.f. $F(p)$. In words, few workers are close to $\underline{p}$ on average, and by continuity of the sample paths of p the outflow from employment through $\underline{p}$ is small. Once again, $\Sigma(\underline{p})$ measures the speed at which learning pushes beliefs over the separation threshold $\underline{p}$.

Third, total flows (respectively) out of and into unemployment must balance:

$$\lambda\left[1-\int_{\underline{p}}^{1} f(p)\,dp\right]=\delta\int_{\underline{p}}^{1} f(p)\,dp+\Sigma(\underline{p})f'(\underline{p}+). \tag{5.4}$$

This is a standard restriction in search models, which gives rise to a Beveridge curve. The left-hand side, the outflow, equals the exit rate λ times the stock of unemployment, given by the unit mass of workers minus the mass of employed. The right-hand side is the inflow into unemployment, which is the outflow from employment in (5.3).

We can solve for (5.1) subject to the relevant boundary conditions.

PROPOSITION 4 (The Ergodic Distribution of Posterior Beliefs about Match Quality): *For $p \in [\underline{p}, 1]$, the ergodic and stationary density of posterior beliefs in equilibrium is*

$$\begin{aligned} f(p) = \Bigg\{ & c_{0f}\left[\left(\frac{1-\underline{p}}{\underline{p}}\frac{p}{1-p}\right)^{\sqrt{1+8\delta/s^2}}-1\right] \\ & \times \mathbb{I}\{\underline{p}\le p< p_0\}+c_{1f}\mathbb{I}\{p_0\le p\le 1\}\Bigg\} \\ & \times p^{-1/2-\sqrt{1/4+2\delta/s^2}}(1-p)^{-3/2+\sqrt{1/4+2\delta/s^2}}, \end{aligned} \tag{5.5}$$

where the coefficients c_{0f} and c_{1f} are the unique and positive solution of a system of two linear equations derived from the boundary conditions (5.3) *and* (5.4) (*see the Appendix*). *f is globally continuous, with a kink at p_0. In $[\underline{p}, p_0]$, f is always increasing*; *in $[p_0, 1]$, f is decreasing if the rate of attrition exceeds the squared signal/noise ratio of output $\delta \ge s^2$, is U-shaped if*

$$\min\{3p_0-1,1\}<\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{2\delta}{s^2}}<2,$$

and is increasing if

$$1<\frac{1}{2}+\sqrt{\frac{1}{4}+\frac{2\delta}{s^2}}\le\min\{3p_0-1,1\}.$$

Rational (Bayesian) learning and optimal match selection map Gaussian output X_t into a piecewise Lévy-stable distribution f of posterior beliefs, which belongs to the Lévy–Pareto *type*. The interpretation of f is empirically more meaningful in wage space. Without loss in generality, we can normalize the scale of output so that $\beta\sigma s = \beta(\mu_H - \mu_L) = 1$. Then, the equilibrium wage function (3.6) becomes $w(p) = p + \omega_0$ for $\omega_0 \equiv (1-\beta)b + \beta[\mu_L + \lambda J(p_0)]$, a location transformation, and the wage density is

$$\phi(w) = f(w - \omega_0) \tag{5.6}$$

also of the Pareto type. In fact, both f and ϕ have a *fat right tail, which is decaying generically* (*for* $\delta \geq s^2$) *but always at slower rate than a Gaussian*.

Proposition 4 has two important implications, the two new results of our analysis. First, the theoretical equilibrium wage distribution $\phi(w)$ may potentially replicate the typical shape of an empirical wage distribution, including its well-known Paretian right tail. Quits to other jobs and to unemployment weed out disproportionately bad matches, censor the left tail, and skew the distribution. The distribution has in fact a globally declining right tail, which gives it overall an empirically accurate shape, if $\delta \geq s^2$. Moscarini (2003) presents a detailed quantitative evaluation of a discrete time version of this model at a monthly frequency, and shows that the restriction $\delta \simeq s^2 = 0.011$ is required to accurately replicate average worker flows and stocks, measures of wage dynamics and inequality, and separation rates in the US economy over the last two decades. To the best of our knowledge, this is the first explanation of the typical shape of a wage distribution based on self-selection in terms of ex post productivity heterogeneity.[6]

The second result is that the *right tail of the wage distribution* $\phi(w)$ *decays faster the larger the ratio* δ/s^2 between the exogenous match dissolution rate δ and the (squared) informativeness of output s^2. The values of δ and s^2 also affect the scale of $\phi(w)$ through the constants of integration c_{0f}, c_{1f}, but the rate of decay of $\phi(w)$ as w rises depends on δ and s^2 only directly, through their ratio. Intuitively, when jobs are at high risk of exogenous destruction (δ is large), or when the output process is very noisy and uninformative, so beliefs move slowly (σ is large and, given the earlier normalization $\mu_H - \mu_L = \beta^{-1}$, the signal/noise ratio s is low), the learning-selection process has no time to produce its effects.

This prediction is unique to an incomplete information environment. A "noisy" economy is "sclerotic": high idiosyncratic output uncertainty unrelated to firm and worker characteristics (high δ and σ) clouds the intrinsic inequality in productivities (μ) and prevents it from being reflected by equilibrium prices. Wages remain concentrated around their starting value $w(p_0)$;

[6]Roy (1951) explains the skewness of the empirical wage distribution as the result of the self-selection of workers in terms of their ex ante known comparative advantages to work in different sectors. Under the assumption of log-normal bivariate skills, Roy produced a wage distribution that lacks the Pareto-like tail and fails to fit the US wage distribution. Heckman and Sedlacek (1985) amend this shortcoming of the Roy model by introducing unobserved worker heterogeneity.

income inequality tends to be dampened, rather than exacerbated, by high idiosyncratic output risk. In contrast, this prediction is reversed in search models with perfectly observable match-specific productivity, such as Mortensen and Pissarides (1994). In that kind of environment, a larger variance of idiosyncratic output shocks raises the incentives to maintain the job active, in order to save on new search costs; in turn, this standard option value effect leads to a reduction in the optimal destruction cutoff, a widening of the range of wages and thus of their inequality.[7]

Indeed, the implications of job matching exploited by the empirical turnover literature, for example, Flinn (1986) and Nagypal (2000), depend only on the existence of a stochastic state variable describing the viability of the match and affecting the wage. But this is consistent also with *observable* and idiosyncratic ex post randomness in either productivity or opportunity cost of working. We could always redefine the belief process in Jovanovic (1979) to be an observable, exogenous productivity process, and the empirical methodologies applied to test the job matching model would not detect this difference. In other words, the empirical literature has tested the *selection*, not really the *learning* implications of the job-matching model. Our prediction, instead, crucially depends on imperfect information. It is interesting to consider how it may be tested empirically, to conclusively accept or reject learning about match quality as a relevant source of wage dynamics and turnover. For example, we may compare industries, occupations, or economies that differ in their technologies or institutions for individual performance evaluation. This is a challenging project, which we leave for future research.

From a methodological viewpoint, our equilibrium solution (5.6) lends itself to a simple econometric implementation. The matching rate λ, endogenous to the model as illustrated later, can be estimated directly from unemployment duration. Then, after choosing μ_L and μ_H to normalize scale and location of output, the remaining parameters of the model $(b, \sigma, \beta, p_0, \delta)$ can be estimated from wage data, by maximizing the likelihood function $\phi(w)$. The analytic solution of the equilibrium allocation makes the structural estimation of the model parametric, thus simple and transparent, particularly in the accuracy of the estimates, a nontrivial issue in the nonparametric case.

6. ON-THE-JOB SEARCH

We now extend our analysis to allow for on-the-job search. Three considerations make this extension compelling. First, on-the-job search is the natural way for mismatched workers to upgrade their employment situation. This

[7] A recent working paper by Prat (2003) analyzes precisely the Mortensen and Pissarides (1994) model with Brownian Motion idiosyncratic job productivity, as in this paper, but without learning and on-the-job search. Wages and separation rates comove with tenure in the empirically accurate directions, and the equilibrium job destruction cutoff declines in the variance of output innovation.

additional margin can potentially alter the aggregate selection outcome, and change our previous conclusions regarding turnover and wage inequality. Second, this upgrading is quantitatively very relevant. The available empirical evidence shows that the bulk of new hires come from other jobs. On average, job-to-job transitions in the US in recent history have been roughly twice as large as flows from nonemployment to employment (see Moscarini (2003) for an overview of the evidence). Finally, as mentioned in the Introduction, our explanation for wage inequality in frictional labor markets, based on ex post selection and wage bargaining, is the natural alternative to wage-posting models, where on-the-job search plays a central role (Burdett and Mortensen (1998)).

We maintain our previous assumptions from Section 2. In addition, we assume that a worker may contact open vacancies at Poisson rate $\psi\lambda$ when searching on the job. Here ψ is the chance at every point in time that an employed worker who wants a new job has the opportunity to actively search for one. Job search is costless also from employment, except for its time-consuming aspect and for discounting. Every new match, even when the worker joins it from another job, restarts from a common prior chance p_0 of success. Now, workers who are pessimistic about the quality of the match with their current employer can try to upgrade, without having to quit to unemployment.

On-the-job search effort is not observable by the firm, so wages cannot be conditioned on it. Wages are set by the same generalized Nash bargaining rule. When an employed worker engages in on-the-job search, any new firm he comes in contact with perfectly observes the wage and the expected productivity (the posterior belief p) of his current match. The incumbent firm knows that the competitor's match with its employee would be successful with chance p_0. The two firms then play an English first-price auction: the poaching firm makes an offer, and then the two firms take turns bidding at time intervals of length $\Delta > 0$, while the existing match remains active. The auction ends after a firm fails to raise the last bid. The worker then receives the highest bid, in exchange for giving up the contact with the losing firm and restarting Nash bargaining with the winner. If indifferent between final bids, the worker stays with the old employer. Since firms cannot commit to wage profiles, in the auction they can only bid a lump-sum transfer, followed by ex post bilateral bargaining once alone with the worker. The worker cannot commit to kick back to his employer any part of his gains from quitting to another firm. All other firms in the economy not involved in the auction observe neither expected match values nor bids.

The analysis of on-the-job search revolves around the wage-setting issue. Our main goal is to show that, under our new assumptions, the equilibrium wage remains an affine function of the expected productivity of a match. Then the previous results concerning equilibrium turnover and wage inequality are modified quantitatively but not qualitatively.

6.1. *The "Poaching Auction"*

Once again, we seek to construct an equilibrium where S, W, and J are strictly increasing in p. We need to characterize equilibrium behavior when an employed worker receives an outside offer. The surplus $S(p)$ of a p-match is clearly the maximum valuation that the worker and the firm can hold for it. When the employed worker successfully contacts a new firm, the two firms immediately play the ascending auction. Their valuations for the worker, respectively $S(p)$ and $S(p_0)$, are common knowledge. Let $\Pi(p|p')$ denote the final bid of firm p against firm p'. Necessarily, $\Pi(p|p') \in [W(p), S(p)]$ for the bid to be acceptable both to the bidding firm and to the worker. The bid $\Pi(p|p')$ is the sum of a lump-sum transfer $\pi(p|p') \equiv \Pi(p|p') - W(p)$ and of the promise (worth $W(p)$ to the worker) to match, produce output, and bargain bilaterally from then on. The lump-sum transfer $\pi(p|p_0)$ offered by a p-employer to its worker can be interpreted as a retention bonus, the transfer $\pi(p_0|p)$ offered by the new firm to poach the worker as a sign-up bonus.

The highest bidder pays the lump-sum transfer to the worker and (re-)starts with him production and bilateral bargaining. Which firm wins, and what is the winning bid? The auction is a symmetric information game in extensive form. It is easy to see that the following strategies are a subgame perfect equilibrium of this game: each firm bids just the bargaining value $\Pi(p|p') = W(p)$ and no lump-sum transfer $\pi(p|p') = 0$. This "noncompetitive" equilibrium is supported by the threat of the more productive of the two firms to outbid the competitor at the next round. For example, if $p < p_0$, the new firm wins. Since $S(\cdot)$ is strictly increasing, $S(p) < S(p_0)$, so the incumbent employer gives up fighting right away, knowing that its highest possible bid $\Pi(p|p_0) = S(p)$ can always be beaten by the new firm with a bid in $(S(p), S(p_0))$. Similarly, if $p \geq p_0$, the new firm does not even try to poach the worker, knowing that the current employer could always retain him. Given our conjecture that the value of a viable relationship is strictly increasing in beliefs to both parties involved, a worker quits to a new firm if and only if $p < p_0$.

Although this auction has other equilibria, and not bidding is a weakly dominated strategy for the less productive firm (the loser), we assume that the two firms play this particular equilibrium, for three reasons. First, if firms faced an arbitrarily small cost of bidding, ours would be the unique subgame perfect equilibrium of the auction, as the winner is known in advance and competing would only be wasteful for the losing firm. Therefore, ours is the unique subgame perfect equilibrium of the costless-bidding auction that is robust to this natural payoff-perturbation refinement. Second, if the less productive firm "trembles" and bids a positive lump-sum transfer with probability ε, the opponent can always successfully respond to the bid. As ε vanishes, all equilibrium payoffs of this perturbed game converge to those of our no-bidding equilibrium, which then survives this perfection refinement too. Third, although competition has no allocative implications, because the more productive firm wins anyway, each employer strictly prefers ex ante not to have to compete ex post.

Ex post competition by firms raises the returns to on-the-job search to the worker, thus his propensity to search on the job, and finally the rate at which employers lose valuable workers.[8]

Notice that our equilibrium outcome is the opposite of one-shot Bertrand competition, where the more productive firm, constrained to bid only once and not to respond to hostile bids, must pay the valuation of the competitor to win the auction (Postel-Vinay and Robin (2002b)). This extreme competition does not change turnover outcomes, but transfers rents to the worker and raises his ex ante incentives to search on the job. This well-known and unpleasant Bertrand paradox with asymmetric but commonly known valuations is here resolved by allowing players to bid repeatedly. Notice that when a worker quits, his old employer loses positive profits $J(p) > 0$, but there is nothing it can do to avoid it, because the surplus of the new match is larger than that of the existing one.[9]

We summarize our conclusions in the following proposition.

PROPOSITION 5 (Equilibrium of the Poaching Auction): *Assume that the equilibrium values of the firm $J(p)$, worker $W(p)$, and match $S(p)$ in the bargaining game are all strictly increasing in p. When a worker matched with a firm at posterior belief p contacts another firm to rematch at belief p_0, the two firms play the following subgame perfect equilibrium of the poaching auction. Each firm bids the worker's bargaining value in its own match and a zero lump-sum transfer; if the less productive firm $p' = \min\{p, p_0\}$ bids more than $W(p')$, the competitor responds accordingly and bids more. Therefore, if $W(p) < W(p_0)$, namely $p < p_0$, the worker quits and restarts bilateral renegotiation with the new firm, earning rents*

[8]Burdett, Imai, and Wright (2004) analyze a two-sided matching model with search frictions, nontransferable utility, and both parties allowed to search while matched. They show that the externality caused by the propensity to search for alternative partners while matched can be so powerful to generate a *continuum* of steady state equilibria.

[9]This reasoning points to asymmetric information as the driving force behind competition for employed workers. With asymmetric information, a standard (either sealed-bid or ascending) private value second price auction yields the same outcome as one-shot Bertrand competition, as a firm always hopes to win. In Burdett and Mortensen (1998), a poaching firm makes a unilateral offer to an employed applicant, independently of the wage he is earning; as emphasized by Postel-Vinay and Robin (2002b), this implies that the poaching firm ignores the current wage, and that the worker's current employer does not respond to the outside offer *by assumption*. In our ascending auction under symmetric information, "no response to outside offers" is a sequential equilibrium strategy for the less productive firm. In Pissarides (1994), employed search takes place as long as it creates a positive surplus for the current match, to be abandoned, while the costs and returns of the new employer do not play any role. Felli and Harris (1996) characterize the unique sequential equilibrium of the repeated poaching game without any commitment but with perfect recall, or equivalently without search frictions, as in Jovanovic (1979). Dey and Flinn (2002) assume that the new offer by the poacher is the result of Nash bargaining, with the total surplus from the old match $S(p)$ as the outside option of the worker. This solution further raises the returns to the worker from searching on the job, but seems to require some recall, that we rule out, and does not survive our backward induction logic.

$W(p_0)$ and a capital gain $W(p_0) - W(p)$; otherwise the worker stays at the old wage and the contact with the new firm is irrelevant.

6.2. *Nash Bargaining During Employment*

In order to solve for equilibrium values and wages, we apply the bargaining solution (3.5). The equivalence between the linear sharing rule (3.5) and the Nash bargaining solution (3.4) must be qualified in the presence of on-the-job search and match heterogeneity. As pointed out by Shimer (2004), the employer may offer to the worker an "efficiency" wage, higher than the one solving (3.5), to make the worker indifferent between searching on the job or not, that is, to set his current value equal to the alternative $W(p_0)$ that he can obtain in a new job. If $p < p_0$, the worker strictly gains. If p is also close enough to p_0, the required "efficiency-wage raise" may be small, and the reduction in firm's profits that it entails may be more than offset by the discrete reduction in the worker's quit rate, making also the firm better off. Therefore, the efficiency wage would be strictly Pareto improving for the match, at the expense of possible future employers of the worker, and would solve (3.4), while the solution to the linear sharing rule (3.5) would violate Nash efficiency axiom.[10] With on-the-job search, the wage not only subtracts from the firm's profits directly, but also affects them through the worker's incentives to on-the-job search, which cannot be contracted upon.

This argument, however, does not apply when firms can make lump-sum transfers in the poaching auction *and* on-the-job search is costless to the worker, as we assume here (unlike in Shimer (2004)). Suppose in fact that an employer with $p < p_0$ did offer such an efficiency wage—an employer with $p \geq p_0$ would have no reason to do so, because its worker would not quit anyway in equilibrium, as per Proposition 5. The worker would then accept the higher efficiency wage, and nonetheless he would keep searching on the job, knowing that any new firm with belief $p_0 > p$ (thus $S(p_0) > S(p)$) would be able to observe the efficiency wage and to outbid it with a lump-sum sign-up bonus, followed again by bilateral Nash bargaining. We conclude that lump-sum transfers with costless on-the-job search neutralize efficiency wages, just like they offset severance payments (Lazear (1990)). Therefore (3.4)$\Leftrightarrow$(3.5), and we proceed to solve for the wage from (3.5).[11] Notice that, if on-the-job search were costly, then the sign-up bonus by the new firm could never cover

[10] I thank a referee for first pointing out this possibility to me.

[11] The assumption that the firm can credibly promise to pay the worker a lump-sum payment, but not a stream of future wages, can be justified as follows: the firm can commit to payments to the worker for a length of time $\Delta t > 0$ after starting production, and we take $\Delta t \to 0$. In contrast, Burdett and Mortensen (1998) and Burdett and Coles (2003) study the full commitment case $\Delta t = \infty$, while a model with no commitment and no lump-sum payments such as Shimer (2004) corresponds to $\Delta t = 0$.

the effort costs sunk by the worker in past on-the-job search, due to the familiar Diamond (1971) effect. In this case, there would be room for an efficiency wage, to an extent that depends continuously on the cost of on-the-job search. Henceforth, our model is a valid approximation to the case of small on-the-job search costs.

The linear sharing rule aligns the interests of the worker and the match. The worker quits if and only if $W(p) < W(p_0) \Leftrightarrow S(p) < S(p_0)$. Ignoring issues of matching congestion, quits are always socially efficient: they occur just when they should to raise aggregate productivity. But they are not privately efficient, as the incumbent employer always strictly loses.

6.3. *Bellman Equations and Equilibrium Wage Function*

Given the continuation values guaranteed by subgame perfect equilibrium play in the poaching auction (see Proposition 5), the value of unemployment U solves the same HJB equation as in the previous model, while the worker's value of being matched well with probability p now solves

$$\begin{aligned} rW(p) = w(p) + \Sigma(p)W''(p) - \delta[W(p) - U] \\ + \psi\lambda \max\langle W(p_0) - W(p), 0\rangle. \end{aligned}$$

Relative to (3.2), the only novelty is the last term, the capital gain following a profitable quit to another job, which resets the prior to p_0 (see Proposition 5). The worker keeps searching on the job whenever $W(p)$ falls short of the value $W(p_0)$ that he can obtain from a fresh start at a new firm. In this case he gains exactly $W(p_0) - W(p)$. The optimality of stopping on-the-job search at p_0 follows from the conjectured monotonicity of W, and implies a smooth pasting condition $W'(p_0-) = W'(p_0+)$.

The value to the employer of an active match that is successful with posterior chance p solves the HJB equation

$$\begin{aligned} rJ(p) = \bar{\mu}(p) - w(p) + \Sigma(p)J''(p) - \delta J(p) \\ - \psi\lambda J(p)\mathbb{I}\{W(p) < W(p_0)\} \end{aligned}$$

with $\mathbb{I}\{\cdot\}$ an indicator function, so $\mathbb{I}\{W(p) < W(p_0)\} = 1$ if and only if the worker seeks outside offers.

When the worker quits to another job, he forfeits positive rents $W(p) - U > 0$ for even larger ones $W(p_0) - U$ in the new match, while his employer suffers an unrecoverable loss $J(p) \propto W(p) - U > 0$. Observe that the linear sharing rule (3.5) implies $\mathbb{I}\{W(p) < W(p_0)\} = \mathbb{I}\{J(p) < J(p_0)\}$ and $\beta J''(p) = (1-\beta)W''(p)$. We prove in the Appendix that, using these facts and (3.5) in the HJB equations (3.2) and (3.3), combined with some algebra, yield a simple and intuitive expression for the equilibrium wage. The Nash bargaining maximization (3.4) is equivalent to (3.5) and both are solved by the wage function

$$w(p) = (1-\beta)b + \beta\big[\bar{\mu}(p) + \lambda J(p_0)\big(1 - \psi\mathbb{I}\{J(p) < J(p_0)\}\big)\big]. \tag{6.1}$$

Now the endogenous outside option from unemployed job search $\lambda J(p_0)$ is reduced by a fraction ψ when the match looks unpromising and the worker searches on the job, $W(p) < W(p_0)$ or $J(p) < J(p_0)$, in order to compensate the firm for the potential loss of a valuable employee. The wage is affine and increasing in the posterior belief, and jumps up at p_0 as the worker ceases on-the-job search and the firm no longer faces the potential quit of its employee. Employed search improves the worker's outside option, at the expense of joint match surplus. Notice that the discontinuity in the wage w at p_0, an obvious implication of the simplifying assumption of all-or-nothing on-the-job search, guarantees continuity in the value functions W and J, so there is no incentive to renegotiate further on lump-sum transfers when $p_t = p_0$.

Finally, it is easy to verify that our conclusions in the two extreme cases of bargaining shares ($\beta = 0$ and $\beta = 1$) are still valid.

6.4. *Value Functions, Separations, and Turnover*

We can now proceed, as in the previous section, to extend Proposition 2 as follows. The worker searches on the job for a new match at prior p_0 (again) if and only if $p_t < p_0$, and always accepts outside offers, to which his employer never responds. The value function of the firm is the increasing and convex function of beliefs $p \in [\underline{p}, 1]$:

$$\begin{aligned} J(p) = {} & \big[c_{0J} p^{1-\alpha_0}(1-p)^{\alpha_0} + k_{0J} p^{\alpha_0}(1-p)^{1-\alpha_0}\big]\mathbb{I}\{\underline{p} \le p < p_0\} \\ & + c_{1J} p^{1-\alpha_1}(1-p)^{\alpha_1}\mathbb{I}\{p_0 \le p \le 1\} \\ & + \frac{(1-\beta)[\bar{\mu}(p) - b] - \beta\lambda J(p_0)(1 - \psi\mathbb{I}\{\underline{p} \le p < p_0\})}{r + \delta + \psi\lambda\mathbb{I}\{\underline{p} \le p < p_0\}}, \end{aligned} \tag{6.2}$$

where

$$\alpha_0 \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(r+\delta+\psi\lambda)}{s^2}}, \quad \alpha_1 \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(r+\delta)}{s^2}},$$

the coefficients c_{0J}, k_{0J}, c_{1J} and the optimal stopping point $\underline{p} \in (0, p_0)$ uniquely solve the system of four algebraic equations:

$$J(\underline{p}) = 0, \quad J'(\underline{p}+) = 0, \quad J(p_0-) = J(p_0+), \quad J'(p_0-) = J'(p_0+). \tag{6.3}$$

The expected residual duration of a p-match should now equal $1/\delta$ for $p > p_0$ when outside offers are rejected, and $1/(\delta + \psi\lambda)$ for $p < p_0$ when they are accepted. Lemma 1 extends as follows. The expected future duration of a match is the increasing and concave function of the current belief that the match is productive:

$$\tau(p) = \frac{1 + c_{0\tau} p^{1-\gamma_0}(1-p)^{\gamma_0} + k_{0\tau} p^{\gamma_0}(1-p)^{1-\gamma_0}}{\delta + \psi\lambda} \mathbb{I}\{\underline{p} \leq p < p_0\} + \frac{1 + c_{1\tau} p^{1-\gamma_1}(1-p)^{\gamma_1}}{\delta} \mathbb{I}\{p_0 \leq p \leq 1\}, \tag{6.4}$$

where

$$\gamma_0 \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2(\delta + \psi\lambda)}{s^2}}, \quad \gamma_1 \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2\delta}{s^2}},$$

and the coefficients $\{c_{0\tau}, k_{0\tau}, c_{1\tau}\}$ solve $\tau(\underline{p}) = 0$, $\tau(p_0-) = \tau(p_0+)$, $\tau'(p_0-) = \tau'(p_0+)$.

Since quits to other jobs occur only at low beliefs below the prior p_0, we can also reiterate our discussion regarding the martingale properties of beliefs and wages conditional on the continuation of the match. The flow wage (6.1) is almost everywhere affine in beliefs (except for the jump at p_0). The hazard rate of a quit $\psi\lambda\mathbb{I}\{\underline{p} \leq p_t < p_0\}$ is also decreasing in p_t. Proposition 3 still holds, and can be supplemented as follows: On-the-job search is more common among low-tenured workers.

6.5. *The Ergodic Wage Distribution*

The piecewise affine map (6.1) from beliefs p to wages in the case of on-the-job search allows to easily extend also our results concerning the wage distribution. The ergodic distribution of beliefs now solves

$$0 = \frac{d^2}{dp^2}[\Sigma(p)f(p)] - (\delta + \psi\lambda\mathbb{I}\{\underline{p} \leq p < p_0\})f(p).$$

The distribution at p loses mass also at rate $\psi\lambda$ when the worker is unhappy and searching on the job ($\mathbb{I}\{\underline{p} \leq p < p_0\} = 1$). Only the second boundary condition, the balance of total flows (respectively) in and out of employment, is modified by the introduction of on-the-job search, as follows:

$$\Sigma(p_0)[f'(p_0-) - f'(p_0+)] = \delta \int_{\underline{p}}^{1} f(p)\,dp + \Sigma(\underline{p})f'(\underline{p}+) + \psi\lambda \int_{\underline{p}}^{p_0} f(p)\,dp.$$

The total flows out of employment on the right-hand side now include quits to other jobs, the quit rate $\psi\lambda$ times the mass $\int_{\underline{p}}^{p_0} f(p)\,dp$ of unhappy workers searching on the job.

We can solve for the ergodic distribution of posterior beliefs about match quality:

$$f(p) = c_{0f} p^{-1-\gamma_0}(1-p)^{\gamma_0-2}\left[\left(\frac{1-\underline{p}}{\underline{p}}\frac{p}{1-p}\right)^{2\gamma_0-1} - 1\right]\mathbb{I}\{\underline{p} \le p < p_0\} + c_{1f} p^{-1-\gamma_1}(1-p)^{\gamma_1-2}\mathbb{I}\{p_0 \le p \le 1\}, \tag{6.5}$$

where the coefficients c_{0f} and c_{1f} are the unique and positive solution of a system of two linear equations, derived from the stationarity conditions for employment and unemployment. f is globally continuous, with a kink at p_0. The density f has the same qualitative properties as those illustrated in Proposition 4. In particular, on-the-job search operates in unpromising matches, so it adds to selection at the lower end of the wage distribution without changing its rate of decay at high wages, where workers do not engage in on-the-job search anyway.

After our normalization $\beta(\mu_H - \mu_L) = 1$, the equilibrium wage function (6.1) becomes a pure location transformation $w(p) = \omega_{\mathbb{I}\{\underline{p}\le p<p_0\}} + p$, where

$$\omega_{\mathbb{I}\{\underline{p}\le p<p_0\}} \equiv b + \beta\big[\mu_L - b + \lambda J(p_0)(1 - \psi\mathbb{I}\{\underline{p} \le p < p_0\})\big].$$

For $w \ge \underline{w} \equiv \omega_1 + \underline{p}$, and denoting $w_0 \equiv \omega_0 + p_0$, the wage density is now

$$\phi(w) = f(w - \omega_{\mathbb{I}\{w<w_0\}}).$$

Therefore, ϕ also belongs to the Pareto type, and the same conclusions that we obtained earlier still hold. The properties of the right tail of the wage distribution do not depend on on-the-job search, because workers rationally choose not to search on the job when the match is very promising.

7. THE MATCHING FUNCTION AND GENERAL EQUILIBRIUM

The description of the economy is completed by a frictional matching process, and the equilibrium is closed by a free entry condition that determines the job-finding rate λ, so far taken as given. An increasing, concave, and linearly homogeneous matching function $m(a, v)$, satisfying Inada conditions, yields the flow of new matches as a function of the stocks of open vacancies v and of *job applicants* a, both unemployed and employed:

$$a = 1 - \int_{\underline{p}}^{1} f(p)\,dp + \psi\int_{\underline{p}}^{p_0} f(p)\,dp. \tag{7.1}$$

For concreteness, albeit this is inessential to the main results, let $m(a, v) = a^\eta v^{1-\eta}$ for $\eta \in (0, 1)$. As all workers are ex ante identical, we assume random matching. Due to the linear homogeneity of the matching function, only *labor market tightness* $\theta \equiv v/a$ matters for equilibrium,

$$\lambda = \frac{m(a,v)}{a} = m\left(1, \frac{v}{a}\right) = \theta^{1-\eta}. \tag{7.2}$$

As is standard, the free entry condition equates the cost of the vacancy to the expected rents from filling the job:

$$\kappa = \frac{m(a,v)}{v} J(p_0) = \theta^{-\eta} J(p_0). \tag{7.3}$$

A *Stationary General Equilibrium* is a vector of scalars $\{\lambda^*, \theta^*, \underline{p}^*, a^*, v^*\}$, and a triple of functions $\{J^*, w^*, f^*\}$ defined on the unit interval, which satisfy (6.1)–(6.3), (6.5), (7.1)–(7.3), and $v^* = a^*\theta^*$. Without on-the-job search ($\psi = 0$), the first four conditions reduce (respectively) to: (3.6), (3.8), the first two in (6.3), and (5.5).

By the linear homogeneity of the matching function, the firm's vacancy-filling rate depends only on θ, thus (by 7.2) on λ. The free-entry condition (7.3) can then be rewritten as $J(p_0) = \kappa\lambda^{\eta/(1-\eta)}$, which describes a continuous increasing relationship between λ and the starting rents $J(p_0)$, going from 0 to ∞. This relationship is termed the "job creation curve."

Proposition 2 allows unique solution, *given a value of* λ, for the value function $J(\cdot|\lambda)$, thus for the starting value $J(p_0|\lambda)$, another continuous relationship between λ and $J(p_0|\lambda)$ that we dub the "profit curve."

A Stationary General Equilibrium is an intersection between the job creation and the profit curves: formally, we seek λ^* such that $J(p_0|\lambda^*) \times (\lambda^*)^{-\eta/(1-\eta)} = \kappa$. The values of the other variables in equilibrium are found recursively from the unique λ^*. The proof of the final proposition shows that an increasing job-finding rate λ reduces the initial rents of a firm $J(p_0|\lambda)$ from a positive value $J(p_0|0) > 0$ to $J(p_0|\infty) = 0$. Intuitively, a higher λ strengthens the worker's bargaining power at the expenses of the firm's profits. Therefore $J(p_0|\lambda)\lambda^{-\eta/(1-\eta)}$ decreases from ∞ to 0 as λ rises from 0 to ∞, and we obtain the following conclusion.

PROPOSITION 6: *There exists a unique Stationary General Equilibrium*, *which features positive employment*.

8. CONCLUSIONS

We have introduced a tractable analytical framework to reconcile the microeconomic-labor and macroeconomic-equilibrium views of matching in labor markets. The model inherits, from the former, empirically accurate correlations between wages, tenure, and turnover, and from the latter an equilibrium structure that is able to account for involuntary unemployment, worker flows between jobs and in/out of jobs, and job creation. Our new contribution is the ability to account also for some empirically robust features of wage inequality, and to link the wage distribution to aggregate worker flows.

The model provides a natural explanation for additional stylized facts, such as the sizable and persistent wage loss caused by exogenous displacement. The tractability of the model makes it also an open-ended, flexible tool, which can be extended in several directions; for example, it easily accommodates ex ante heterogeneity in worker skills. In this sense, the empirical wage distribution that our model should and does replicate must be interpreted as conditional on observable worker and firm characteristics. The emerging empirical literature exploiting matched employer-employee data (e.g. Abowd, Kramarz, and Margolis (1999), Postel-Vinay and Robin (2002a)) has shown that such residual wage dispersion is pervasive and sizable. Moscarini (2003) builds on this model to explain, both qualitatively and quantitatively, a host of additional facts concerning cross-skill inequality in labor markets, such as the strong inequality across worker groups in entry rates into unemployment and in within-group unexplained wage dispersion.

Finally, we briefly discuss some possible further extensions of this model and the robustness of our conclusions. Both a continuous search effort choice and a screening phase—where the firm and worker draw an informative signal of their match quality, before starting production—would add some smoothness to the model, and potentially eliminate the gap in the support of the wage distribution corresponding to the prior belief p_0. For example, it is easy to accommodate a finite-valued screening signal, which would just expand the number of starting "prior beliefs" and the corresponding number of continuity conditions required to solve for the Bellman value J and the distribution of beliefs f. However, these extensions do not promise to add any important insights. More importantly, the search effort-cost function and the distribution of the initial screening signal would represent unobservable features of the environment affecting the distribution of wages. We find it more parsimonious and instructive to isolate the effects of learning and selection with a single source of model uncertainty.

More interesting are the alternative solutions of the poaching game, such as Bertrand competition with or without ensuing renegotiation. Bertrand competition for an employed worker makes most sense as an auction under asymmetric information about match values. Analyzing this case in the present environment is part of this research agenda. Equally interesting are the effects of costly on-the-job search and/or the impossibility of lump-sum transfers, which create a moral hazard problem and the scope for an "efficiency wage" element in the Nash bargaining solution. Our solution is a good approximation only for situations of small costs of on-the-job search and of short-lived commitment to wage contracts, thus of modest incentive effects. Under either modification, we would obtain a new separation cutoff $\underline{p}$, but the results illustrated in Section 5 relating to the belief distribution would survive qualitatively intact. The linearity of the wage function in beliefs, however, would not survive either modification, due to outside option effects from Bertrand competition or to efficiency-wage incentive effects. These "strategic" nonlinearities may substantially alter the shape of the wage distribution, and confound the pure effects of selection.

Another important direction of future research concerns the empirical implementation of the model. The model parameters can be estimated from labor market price information, by maximizing the equilibrium wage likelihood function, and then input into a calibration to predict labor market quantities. The derivation of the wage density also produces explicit formulae for the flows of workers between employment states, which enter the boundary conditions of the belief distribution: voluntary quits to unemployment, quits to other jobs, exogenous displacements, and hires from unemployment. These flows have been carefully measured in the equilibrium unemployment literature. The cross-restrictions provided by price and quantity data allow to substantially raise the standards in testing the empirical accuracy of the model, and consequently to enhance its reliability for policy analysis.

Dept. of Economics and Cowles Foundation for Research in Economics, Yale University, P.O. Box 208268, New Haven, CT 06520-8268, U.S.A.; giuseppe.moscarini@yale.edu; www.econ.yale.edu/faculty1/moscarini.htm.

Manuscript received February, 2003; final revision received October, 2004.

APPENDIX

Solving for the Wage Function (6.1)

We cover the more general case of on-the-job search introduced in Section 6. The special case of no on-the-job search can be obtained by setting $\psi = 0$, and proves Proposition 1.

To simplify notation let $\mathbb{I}_S \equiv \mathbb{I}\{W(p) < W(p_0)\}$ denote the indicator function of on-the-job search. Subtract the two HJB equations of the worker and multiply through by $(1-\beta)$:

$$\begin{aligned}
&(1-\beta)r[W(p)-U] \\
&\quad = (1-\beta)\big\{w(p) - b + \Sigma(p)W''(p) - \delta[W(p)-U] \\
&\qquad\qquad + \psi\lambda\mathbb{I}_S[W(p_0)-W(p)] - \lambda[W(p_0)-U]\big\}.
\end{aligned}$$

Multiply both sides of the firm's HJB equation by β:

$$\beta r J(p) = \beta\{\bar{\mu}(p) - w(p) + \Sigma(p)J''(p) - \delta J(p) - \psi\lambda J(p)\mathbb{I}_S\}.$$

Subtract the two equations hereby obtained:

$$\begin{aligned} & r\{(1-\beta)[W(p)-U]-\beta J(p)\} \\ & \quad = (1-\beta)\big\{w(p)-b+\Sigma(p)W''(p)-\delta[W(p)-U] \\ & \qquad\qquad + \psi\lambda\mathbb{I}_S[W(p_0)-W(p)]-\lambda[W(p_0)-U]\big\} \\ & \qquad - \beta\{\bar{\mu}(p)-w(p)+\Sigma(p)J''(p)-\delta J(p)-\psi\lambda\mathbb{I}_S J(p)\}, \end{aligned}$$

using (3.5) and its implication $\beta J''(p)=(1-\beta)W''(p)$ to simplify terms,

$$\begin{aligned} 0 &= w(p)+(1-\beta)\big\{-b-\lambda[W(p_0)-U]+\psi\lambda\mathbb{I}_S[W(p_0)-W(p)]\big\} \\ & \quad - \beta\{\bar{\mu}(p)-\psi\lambda\mathbb{I}_S J(p)\} \\ &= w(p)-b(1-\beta)-\bar{\mu}(p)-\lambda(1-\beta)[W(p_0)-U] \\ & \quad + \psi\lambda\mathbb{I}_S\big\{(1-\beta)[W(p_0)-W(p)]+\beta J(p)\big\}. \end{aligned}$$

Finally, using

$$\begin{aligned} & (1-\beta)[W(p_0)-W(p)]+\beta J(p) \\ & \quad = (1-\beta)[W(p_0)-U]-(1-\beta)[W(p)+U]+\beta J(p) \\ & \quad = (1-\beta)[W(p_0)-U]+0=\beta J(p_0), \end{aligned}$$

we obtain the desired expression, equivalent to (6.1):

$$0=w(p)-b(1-\beta)-\bar{\mu}(p)-\lambda\beta J(p_0)+\psi\lambda\mathbb{I}_S\beta J(p_0).$$

Solving for the Value Function of the Firm and the Separation Cutoff (6.2)–(6.3)

We cover the more general case of on-the-job search. The special case of no on-the-job search can be obtained by setting $\psi=0$, and proves Proposition 2.

J is the value function of an optimal Bayesian experimentation problem with flow returns $\bar{\mu}(p)$ that are linear in beliefs p by the expected utility hypothesis: ergo J is convex in beliefs by a standard improvement argument. It follows that $\underline{p}$ is unique and J is everywhere continuous and almost everywhere *twice* differentiable. So standard HJB Verification Theorems for optimal stopping problems apply, including value matching and smooth pasting at $\underline{p}$ (Shyryaev (1978, 3.8)). Since J is convex, nonnegative (a firm could always separate to obtain zero), and flat at the lower bound $\underline{p}$ where it is also zero, it must be globally increasing where strictly positive, so $\mathbb{I}\{J(p)<J(p_0)\}=\mathbb{I}\{p<p_0\}$ and it is optimal to stop on-the-job search at and only at p_0. Continuity of J and J' at p_0 are value matching and smooth pasting conditions for this stopping choice.

By direct verification and using $\mathbb{I}\{\underline{p} \leq p < p_0\} = \mathbb{I}\{J(p) < J(p_0)\}$, the general solution to the firm's HJB equation is

$$J(p) = \frac{(1-\beta)[\bar{\mu}(p) - b] - \beta\lambda J(p_0)(1 - \psi\mathbb{I}\{\underline{p} \leq p < p_0\})}{r + \delta + \psi\lambda\mathbb{I}\{\underline{p} \leq p < p_0\}}$$
$$+ c_{iJ} p^{1-\alpha_i}(1-p)^{\alpha_i} + k_{iJ} p^{\alpha_i}(1-p)^{1-\alpha_i}$$

for $i = \mathbb{I}\{p_0 \leq p \leq 1\}$ and constants of integration c_{iJ}, k_{iJ}. Imposing the boundary condition $J(1) \leq S(1) < \mu_H/(r+\delta) < \infty$ yields $k_{1J} = 0$, else the term $k_{1J} p^{\alpha_1}(1-p)^{1-\alpha_1}$ would explode to $\pm\infty$ as $p \uparrow 1$ by $\alpha_1 > 1$, violating continuity and monotonicity of J, which imply $J(p) \in [0, J(1)]$.

We have left five unknowns, the three remaining constants $\{c_{0J}, k_{0J}, c_{1J}\}$, $J(p_0)$ and the separation point $\underline{p}$. To find them we have five equations. The simplest algorithm is as follows. Fix an arbitrary positive value $J(p_0) = \bar{J}_0$ and consider the linear system of three equations in $\{c_{0J}, k_{0J}, c_{1J}\}$:

(i) continuity from the left $J(p_0-) = \bar{J}_0$:

$$\begin{aligned}
&[r + \delta + \beta\lambda + \psi\lambda(1-\beta)]\bar{J}_0 \\
&\quad = (1-\beta)[\bar{\mu}(p_0) - b] \\
&\qquad + (r + \delta + \psi\lambda)\big[p_0^{1-\alpha_0}(1-p_0)^{\alpha_0} c_{0J} + p_0^{\alpha_0}(1-p_0)^{1-\alpha_0} k_{0J}\big];
\end{aligned}$$

(ii) continuity from the right, $\bar{J}_0 = J(p_0+)$, which implies value matching for stopping on-the-job search at p_0:

$$(r + \delta + \beta\lambda)\bar{J}_0 = (1-\beta)[\bar{\mu}(p_0) - b] + (r+\delta) p_0^{1-\alpha_1}(1-p_0)^{\alpha_1} c_{1J};$$

(iii) smooth pasting for stopping on-the-job search at p_0, $J'(p_0+) = J'(p_0-)$:

$$\begin{aligned}
&p_0^{\alpha_0 - 1}(1-p_0)^{-\alpha_0}(\alpha_0 - p_0) k_{0J} \\
&\qquad + p_0^{-\alpha_0}(1-p_0)^{\alpha_0 - 1}(1 - \alpha_0 - p_0) c_{0J} + \frac{(1-\beta)(\mu_H - \mu_L)}{r + \delta + \psi\lambda} \\
&\quad = p_0^{-\alpha_1}(1-p_0)^{\alpha_1 - 1}(1 - \alpha_1 - p_0) c_{1J} + \frac{1-\beta}{r+\delta}(\mu_H - \mu_L).
\end{aligned}$$

Solve this system for $\{c_{0J}, k_{0J}, c_{1J}\}$ given the guess $\bar{J}_0$, and plug both into value matching at separation, $J(\underline{p}) = 0$:

$$\begin{aligned}
&-\beta\lambda(1-\psi)\bar{J}_0 + c_{0J}\underline{p}^{1-\alpha_0}(1-\underline{p})^{\alpha_0}(r + \delta + \psi\lambda) \\
&\qquad + k_{0J}\underline{p}^{\alpha_0}(1-\underline{p})^{1-\alpha_0}(r + \delta + \psi\lambda) \\
&\quad = -(1-\beta)[\bar{\mu}(\underline{p}) - b]
\end{aligned}$$

and smooth pasting $J'(\underline{p}) = 0$:

$$\frac{(1-\beta)(\mu_H - \mu_L)}{r+\delta+\psi\lambda} + c_{0J}\underline{p}^{-\alpha_0}(1-\underline{p})^{\alpha_0-1}(1-\alpha_0-\underline{p}) + k_{0J}\underline{p}^{\alpha_0-1}(1-\underline{p})^{-\alpha_0}(\alpha_0-\underline{p}) = 0.$$

Finally iterate over values of $\bar{J}_0$ and corresponding vector $\{c_{0J}, k_{0J}, c_{1J}\}$ until the last two equations yield the same value of $\underline{p}$.

Solving for Expected Match Duration (6.4)

We cover the more general case of on-the-job search. The special case of no on-the-job search can be obtained by setting $\psi = 0$, and proves Lemma 1.

For the ODE solved by $\tau(p)$ see Karlin and Taylor (1981, Chapter 15). The general solution in the claim can be verified directly. The boundary conditions are those stated in the claim plus $\tau(1) = 1/\delta$, because a job that is good for sure can be destroyed only exogenously ($p = 1$ is an absorbing belief as $\Sigma(1) = 0$). By the same reasoning as in the previous proof this implies $k_{1\tau} = 0$. To see why τ is increasing, notice that $\tau(\cdot) \leq 1/\delta$ because job destruction is always a risk, with strict inequality somewhere, so from the general solution we get $c_{1\tau} < 0$. Next:

$$\tau'(p) = \frac{1}{\delta+\psi\lambda}\big[c_{0\tau}p^{-\gamma_0}(1-p)^{\gamma_0-1}(1-\gamma_0-p) + k_{0\tau}p^{\gamma_0-1}(1-p)^{-\gamma_0}(\gamma_0-p)\big]\mathbb{I}\{\underline{p} \leq p < p_0\} + \frac{1}{\delta}c_{1\tau}p^{-\gamma_1}(1-p)^{\gamma_1-1}(1-\gamma_1-p)\mathbb{I}\{p_0 \leq p \leq 1\}$$

so $\tau'(p) > 0$ for $p > p_0$ by $1-\gamma_1-p < 1-\gamma_1 < 0$. By contradiction, suppose $0 \geq \tau'(p)$ for some $p \in (\underline{p}, p_0)$. Since $\tau'(\underline{p}) \geq 0$ and $\tau'(p_0-) = \tau'(p_0+) > 0$, by continuity of τ' in $[\underline{p}, \overline{p_0}]$ and the Mean Value Theorem either there is only one such p, with $\tau'(p) = 0$, equivalent to

$$c_{0\tau} = -k_{0\tau}\left(\frac{p}{1-p}\right)^{2\gamma_0-1}\frac{\gamma_0-p}{\gamma_0+p-1}, \tag{8.1}$$

in which case the claim obtains, or there are two roots of (8.1). But tedious algebra shows that the right-hand side of (8.1) is either globally increasing or decreasing in p, according to the sign of $k_{0\tau}$, hence (8.1) may have at most one root.

Solving for the Ergodic Distribution (6.5)

We cover the more general case of on-the-job search. The special case of no on-the-job search can be obtained by setting $\psi = 0$, and proves Proposition 4.

Let $\zeta(p) = p^2(1-p)^2 f(p)$; we obtain from the forward equation another differential equation:

$$p^2(1-p)^2\zeta''(p) = \frac{2(\delta + \psi\lambda\mathbb{I}\{p < p_0\})}{s^2}\zeta(p).$$

The general solution is

$$\zeta(p) = \zeta_i(p) = C_{if}p^{1-\gamma_i}(1-p)^{\gamma_i} + k_{if}p^{\gamma_i}(1-p)^{1-\gamma_i}$$

for $i = \mathbb{I}\{p \geq p_0\}$. Therefore the ergodic density is

$$\begin{aligned} f_i(p) &= \zeta(p)p^{-2}(1-p)^{-2} \\ &= C_{if}p^{-1-\gamma_i}(1-p)^{\gamma_i-2} + k_{if}p^{\gamma_i-2}(1-p)^{-1-\gamma_i}, \end{aligned}$$

the sum of Inverted-Beta-1 functions, for some constants C_{0f}, C_{1f}, k_{0f}, k_{1f} to be determined.

Integrating $f_1(p)$ between p_0 and 1 we observe that $\int_{p_0}^{p}(1-x)^{-1-\gamma_1}\,dx = (\gamma_1)^{-1}(1-x)^{-\gamma_1}|_{p_0}^{p} \to \infty$ as $p \uparrow 1$ by $\gamma_1 > 0$. Hence we must have $k_{1f} = 0$ to satisfy the requirement that the density has finite mass $0 < \int_{\underline{p}}^{1} f(z)\,dz < 1 < \infty$. In contrast, $\int_{p_0}^{1}(1-x)^{\gamma_1-2}\,dx < \infty$ by $2-\gamma_1 < 1$, or $\gamma_1 > 1$, so C_{1f} can be nonzero.

Next, let

$$\xi_1 \equiv \left(\frac{1-\underline{p}}{\underline{p}}\right)^{2\gamma_0-1}.$$

Recall that the separation point $\underline{p} \in [0, p_0]$ has been determined in the equilibrium of the bargaining game. Therefore, for the purpose of solving for the ergodic density, this is known. The boundary condition "no time spent at the separation boundary" $f_0(\underline{p}+) = 0$ then reads $k_{0f} = -\xi_1 C_{0f}$, which implies $C_{0f}k_{0f} < 0$. Now let $c_{0f} \equiv -C_{0f}$, $c_{1f} \equiv C_{1f}$, so that $c_{0f}k_{0f} > 0$. Using $k_{0f} = -\xi_1 C_{0f}$ we simplify the ergodic density to the expressions in the main body of the paper:

$$\begin{aligned} f_0(p) &= C_{0f}\big[p^{-1-\gamma_0}(1-p)^{\gamma_0-2} - \xi_1 p^{\gamma_0-2}(1-p)^{-1-\gamma_0}\big] \\ &= c_{0f}p^{-1-\gamma_0}(1-p)^{\gamma_0-2}\left[\left(\frac{1-\underline{p}}{\underline{p}}\frac{p}{1-p}\right)^{2\gamma_0-1} - 1\right], \\ f_1(p) &= c_{1f}p^{-1-\gamma_1}(1-p)^{\gamma_1-2}. \end{aligned}$$

It remains to determine the constants of integration c_{0f}, c_{1f} using the other two boundary conditions. We first compute the worker stocks and flows. Changing variable $p' = p/(1-p)$,

$$\begin{aligned}
&\int_{\underline{p}}^{p_0} p^{-1-\gamma_0}(1-p)^{\gamma_0-2}\,dp \\
&\quad = \int_{\underline{p}/(1-\underline{p})}^{p_0/(1-p_0)} \left(\frac{p'}{1+p'}\right)^{-1-\gamma_0}\left(\frac{1}{1+p'}\right)^{\gamma_0-2}\frac{dp'}{(1+p')^2} \\
&\quad = \int_{\underline{p}/(1-\underline{p})}^{p_0/(1-p_0)} (p')^{-1-\gamma_0}(1+p')\,dp' \\
&\quad = \frac{(\frac{\underline{p}}{1-\underline{p}})^{-\gamma_0} - (\frac{p_0}{1-p_0})^{-\gamma_0}}{\gamma_0} + \frac{(\frac{\underline{p}}{1-\underline{p}})^{1-\gamma_0} - (\frac{p_0}{1-p_0})^{1-\gamma_0}}{\gamma_0 - 1}
\end{aligned}$$

which is positive because $p_0 \geq \underline{p}$ and $\gamma_0 > 1$, and similarly

$$\begin{aligned}
&\int_{\underline{p}}^{p_0} p^{\gamma_0-2}(1-p)^{-1-\gamma_0}\,dp \\
&\quad = \frac{(\frac{p_0}{1-p_0})^{\gamma_0-1} - (\frac{\underline{p}}{1-\underline{p}})^{\gamma_0-1}}{\gamma_0 - 1} + \frac{(\frac{p_0}{1-p_0})^{\gamma_0} - (\frac{\underline{p}}{1-\underline{p}})^{\gamma_0}}{\gamma_0}
\end{aligned}$$

is also positive for the same reason. Finally

$$\begin{aligned}
\int_{\underline{p}}^{p_0} f_0(p)\,dp &= -c_{0f}\int_{\underline{p}}^{p_0} p^{-1-\gamma_0}(1-p)^{\gamma_0-2}\,dp \\
&\quad + c_{0f}\xi_1 \int_{\underline{p}}^{p_0} p^{\gamma_0-2}(1-p)^{-1-\gamma_0}\,dp \equiv c_{0f}\xi_2,
\end{aligned}$$

where $\xi_2 \geq 0$ is defined implicitly by the previous two expressions.

Similarly let

$$\begin{aligned}
\xi_3 &\equiv \int_{p_0}^{1} p^{-1-\gamma_1}(1-p)^{\gamma_1-2}\,dp \\
&= \frac{1}{\gamma_1}\left(\frac{p_0}{1-p_0}\right)^{-\gamma_1} + \frac{1}{\gamma_1 - 1}\left(\frac{p_0}{1-p_0}\right)^{1-\gamma_1} > 0
\end{aligned}$$

so that

$$\int_{\underline{p}}^{1} f(p)\,dp = \int_{\underline{p}}^{p_0} f_0(p)\,dp + \int_{p_0}^{1} f_1(p)\,dp = c_{0f}\xi_2 + c_{1f}\xi_3.$$

Finally

$$\Sigma(\underline{p})f_0'(\underline{p}+) = c_{0f}\xi_4,$$
$$\Sigma(p_0)[f_0'(p_0-) - f_0'(p_0+)] = c_{0f}\xi_5 - c_{1f}\xi_6,$$

where

$$\xi_4 \equiv \frac{s^2}{2}\underline{p}^{-\gamma_0}(1-\underline{p})^{\gamma_0-1}(2\gamma_0 - 1) > 0,$$
$$\xi_5 \equiv \frac{s^2}{2}\big[\xi_1 p_0^{\gamma_0-1}(1-p_0)^{-\gamma_0}(3p_0 - 2 + \gamma_0)$$
$$- p_0^{-\gamma_0}(1-p_0)^{\gamma_0-1}(3p_0 - 1 - \gamma_0)\big] > 0,$$
$$\xi_6 \equiv \frac{s^2}{2}p_0^{-\gamma_1}(1-p_0)^{\gamma_1-1}(3p_0 - 1 - \gamma_1).$$

Here the inequality $\xi_5 > 0$ follows from the definition of ξ_1, $\underline{p} \leq p_0$, and $\gamma_0 > 1$, while the sign of ξ_6 (thus, the sign of $f_1'(p_0)$) depends on the parameter conditions stated in Proposition 4. With these definitions, the balance of flows in and out of unemployment is

$$\lambda(1 - c_{0f}\xi_2 - c_{1f}\xi_3) = \delta(c_{0f}\xi_2 + c_{1f}\xi_3) + c_{0f}\xi_4$$

and the balance of flows in and out of employment is

$$c_{0f}\xi_5 - c_{1f}\xi_6 = \psi\lambda c_{0f}\xi_2 + \delta(c_{0f}\xi_2 + c_{1f}\xi_3) + c_{0f}\xi_4.$$

We write these two remaining boundary conditions in matrix form to obtain the expression in claim $\Xi(c_{0f}\ c_{1f})' = (\lambda\ 0)'$, where

$$\Xi \equiv \begin{pmatrix} (\lambda+\delta)\xi_2 + \xi_4 & (\lambda+\delta)\xi_3 \\ -\xi_5 + (\psi\lambda+\delta)\xi_2 + \xi_4 & \delta\xi_3 + \xi_6 \end{pmatrix}.$$

Notice that this matrix is a function of the separation cutoff $\underline{p}$ determined in the bargaining equilibrium. The system can be solved by standard methods.

Finally, a substantial amount of algebra (omitted) shows that the boundary conditions also imply that the density is continuous at p_0: $f(p_0-) = f(p_0+)$, therefore the inflow at p_0 creates a kink but not a jump in the density. This also implies $c_{0f}, c_{1f} > 0$ by a simple contradiction argument using $c_{0f}k_{0f} > 0$ found earlier.

Proof of Proposition 6

Since the worker-firm pair cannot produce more than μ_H flow expected output, $J(p_0|\lambda)$ is bounded uniformly in λ above by $\mu_H(r+\delta)^{-1}$. Then $\lim_{\lambda\to\infty} J(p_0|\lambda)\lambda^{-\eta/(1-\eta)} = 0$.

When $\lambda = 0$, the following facts are true: $\alpha_0 = \alpha_1$, after manipulation of value matching and smooth pasting at p_0 we have $k_{0J} = c_{1J}(\alpha_0 - \alpha_1) = 0$, $c_{0J} = c_{1J}$, and finally for all $p \geq \underline{p}$

$$J(p|0) = \frac{(1-\beta)[\bar{\mu}(p) - b]}{r+\delta} + c_{1J}p^{1-\alpha_1}(1-p)^{\alpha_1}.$$

We show that this implies $J(p_0|0) > 0$ and therefore $\lim_{\lambda\to 0} J(p_0|\lambda) \times \lambda^{-\eta/(1-\eta)} = \infty$. By contradiction suppose

$$J(p_0|0) = 0 = \frac{(1-\beta)[\bar{\mu}(p_0) - b]}{r+\delta} + c_{1J}p_0^{1-\alpha_1}(1-p_0)^{\alpha_1}.$$

The assumption $\bar{\mu}(p_0) \geq b$ then implies $c_{1J} \leq 0$. By value matching this also implies $\underline{p} = p_0$, and then by smooth pasting

$$\begin{aligned} J'(p_0|0) &= 0 \\ &= \frac{(1-\beta)(\mu_H - \mu_L)}{r+\delta} + c_{1J}p_0^{-\alpha_1}(1-p_0)^{\alpha_1 - 1}(1-\alpha_0 - p_0), \end{aligned}$$

which implies $c_{1J} > 0$, the desired contradiction, because $\mu_H - \mu_L > 0$ and $\alpha_0 > 1$.

From these two limits and from continuity, it follows that the required fixed point $\lambda^* > 0$ always exists.

To establish that it is unique, it suffices to show that the profit curve $J(p_0|\lambda)$ is decreasing in λ, because then it cuts exactly once the job creation curve. Consider two values λ_0 and λ_1 with $\lambda_1 > \lambda_0 > 0$. By contradiction, suppose that $J(p_0|\lambda_1) \geq J(p_0|\lambda_0)$. Then $\lambda_1 J(p_0|\lambda_1) > \lambda_0 J(p_0|\lambda_0)$. This implies

$$\begin{aligned} J(1|\lambda_1) &= \frac{(1-\beta)(\mu_H - b) - \beta\lambda_1 J(p_0|\lambda_1)}{r+\delta} \\ &< \frac{(1-\beta)(\mu_H - b) - \beta\lambda_0 J(p_0|\lambda_0)}{r+\delta} = J(1|\lambda_0), \end{aligned}$$

so by continuity there exists $p' \in [p_0, 1)$ such that $J(p'|\lambda_1) = J(p'|\lambda_0)$ and $J'(p'|\lambda_1) \leq J'(p'|\lambda_0)$. Now there are two possibilities.

First, for all $p < p'$, $J'(p|\lambda_1) \leq J'(p|\lambda_0)$, which implies $J(p|\lambda_1) > J(p|\lambda_0)$ and $\underline{p}(\lambda_1) < \underline{p}(\lambda_0)$. From the firm's HJB equation solved by $J(\cdot|\lambda)$

for $\lambda = \lambda_0, \lambda_1$,

$$\Sigma(p)J''(p|\lambda) = [r + \delta + \psi\lambda\mathbb{I}\{p < p_0\}]J(p|\lambda) - (1-\beta)[\bar{\mu}(p) - b] + \beta\lambda J(p_0|\lambda)(1 - \psi\mathbb{I}\{p < p_0\}),$$

we deduce $J''(p|\lambda_1) > J''(p|\lambda_0)$ for all $p > \underline{p}(\lambda_0)$ and then from $J'(\underline{p}(\lambda_0)|\lambda_1) > 0 = J'(\underline{p}(\lambda_0)|\lambda_0)$ we obtain $J'(p|\lambda_1) > J'(p|\lambda_0)$, a contradiction.

Second, there exists $p'' < p'$ such that $J'(p''|\lambda_1) > J'(p''|\lambda_0)$ and $J(p''|\lambda_1) = J(p''|\lambda_0)$. But then again from the ODE above we infer $J''(p|\lambda_1) > J''(p|\lambda_0)$ at $p = p''$ and, by the instability of the ODE, for all larger values of p, implying the same contradiction, $J'(p|\lambda_1) > J'(p|\lambda_0)$ for all values of $p > p''$.

REFERENCES

ABOWD, J., F. KRAMARZ, AND D. MARGOLIS (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.

BURDETT, K., AND M. COLES (2003): "Equilibrium Wage-Tenure Contracts," *Econometrica*, 71, 1377–1404.

BURDETT, K., AND D. MORTENSEN (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39, 257–273.

BURDETT, K., R. IMAI, AND R. WRIGHT (2004): "Unstable Relationships," *Frontiers of Macroeconomics*, 1, Article 1, http://www.bepress.com/bejm/frontiers/vol1/iss1/art1.

CHRISTENSEN, B. J., R. LENTZ, D. MORTENSEN, G. NEUMANN, AND A. WERVATZ (2003): "On the Job Search and the Wage Distribution," *Journal of Labor Economics*, forthcoming.

DEY, M., AND C. FLINN (2002): "An Equilibrium Model of Health Insurance Provision and Health Determination," Mimeo, New York University, Department of Economics.

DIAMOND, P. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3, 156–168.

——— (1982): "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 90, 881–894.

FARBER, H. (1994): "The Analysis of Interfirm Worker Mobility," *Journal of Labor Economics*, 12, 554–593.

FELLER, W. (1954): "Diffusion Processes in One Dimension," *Transactions of the American Mathematical Society*, 77, 1–31.

FELLI, L., AND C. HARRIS (1996): "Learning, Wage Dynamics, and Firm-Specific Human Capital," *Journal of Political Economy*, 104, 838–868.

FLINN, C. (1986): "Wages and Job Mobility of Young Workers," *Journal of Political Economy*, 94, S88–S110.

HECKMAN, J., AND G. SEDLACEK (1985): "Heterogeneity, Aggregation and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market," *Journal of Political Economy*, 93, 1077–1125.

JOVANOVIC, B. (1979): "Job Matching and the Theory of Turnover," *Journal of Political Economy*, 87, 972–990.

——— (1984): "Matching, Turnover and Unemployment," *Journal of Political Economy*, 92, 108–122.

KARLIN, S., AND H. TAYLOR (1981): *A Second Course in Stochastic Processes*. New York: Academic Press.

LANE, J., AND M. PARKIN (1998): "Turnover in an Accounting Firm," *Journal of Labor Economics*, 16, 702–717.

LAZEAR, E. (1990): "Job Security Provisions and Employment," *Quarterly Journal of Economics*, 105, 699–726.

LIPTSER, R., AND A. SHYRYAEV (1977): *Statistics of Random Processes*, Vol. 2. Berlin: Springer-Verlag.

MORTENSEN, D. (1982): "Efficiency of Mating, Racing, and Related Games," *American Economic Review*, 72, 968–979.

——— (1998): "Equilibrium Unemployment with Wage Posting: Burdett–Mortensen Meet Pissarides," Mimeo, Northwestern University, Department of Economics.

MORTENSEN, D., AND C. PISSARIDES (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61, 397–415.

MOSCARINI, G. (2003): "Skill and Luck in the Theory of Turnover," Mimeo, Yale University, Department of Economics.

NAGYPAL, E. (2000): "Learning-By-Doing versus Learning about Match Quality: Can We Tell Them Apart?" Mimeo, Stanford University, Department of Economics.

PISSARIDES, C. (1985): "Short Run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages," *American Economic Review*, 75, 676–690.

——— (1994): "Search Unemployment with On-the-Job Search," *Review of Economic Studies*, 61, 457–475.

PISSARIDES, C., AND J. WADSWORTH (1994): "On-the-Job Search: Some Empirical Evidence from Britain," *European Economic Review*, 38, 385–401.

POSTEL-VINAY, F., AND J.-M. ROBIN (2002a): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, 2295–2350.

——— (2002b): "The Distribution of Earnings in an Equilibrium Search Model with State-Dependent Offers and Counteroffers," *International Economic Review*, 43, 989–1016.

PRAT, J. (2003): "A Matching Model of Creative Destruction with Idiosyncratic Productivity Shocks," Mimeo, European University Institute, Department of Economics.

ROY, A. (1951): "Some Thoughts on the Distribution of Earnings," *Oxford Economic Papers* (New Series), 3, 135–146.

SHI, S. (2002): "A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Based Technology," *Review of Economic Studies*, 69, 467–491.

SHIMER, R. (2004): "On-the-Job Search, Bargaining, and Wage Dispersion," Mimeo, University of Chicago, Department of Economics.

SHYRYAEV, A. (1978): *Optimal Stopping Rules*. Berlin: Springer-Verlag.

TOPEL, R. (1991): "Specific Capital, Mobility and Wages: Wages Rise with Job Seniority," *Journal of Political Economy*, 99, 145–176.