Job-to-Job Quits and Corporate Culture

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February 2008
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Abstract

This paper presents an analysis of moral hazard in On-the-Job Search (OJS) in an equilibrium setting. In a frictional labor market, when an employee receives an outside offer, her employer is naturally tempted to compete against it to save the cost of hiring a replacement. Casual observation in the labor market, however, suggests that this type of *ex post* competition is rare, presumably because it would raise the worker's ex ante returns to OJS, if only for pure rent-seeking purposes, i.e. just to get a raise. Firms may credibly commit to ignore outside offers to their employees, let them go without a counteroffer, and suffer the loss, in order to keep in line the other employees' incentives to not search on the job. This commitment perpetuates a coordination failure among co-workers: if they all started searching on the job at a level that would be optimal should the firm indeed compete *ex post* against poachers, the firm would indeed be helpless and would have to compete. Therefore, a transition to a "competitive corporate culture", where firms do compete *ex post* and worker search intensively on the job, appears irreversible. I study a version of Burdett and Mortensen (1998)'s OJS model where workers choose the intensity of OJS covertly, thus creating a moral hazard problem, and firms cannot commit in any way not to compete against outside offers. I investigate the conditions for wage posting and no matching of outside offers to be a sequential equilibrium strategy, supported by the coordination failure among co-workers. I find that the incentives to deviate from this equilibrium are strongest for low-productivity, low-wage firms−which are small, thus have few employees to discipline−and for their employees−who have the most to gain from active OJS.

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1. Introduction

Academic economists and professional athletes actively pursue and expect to receive outside job and wage offers while employed. The *ex post* wage competition that arises between the current and the perspective employer appears to be a major source of wage growth for these workers. While there exists no systematic evidence on such *ex post* competition, anecdotal evidence suggests that this phenomenon is quite rare outside such specialty labor markets. Firms dislike employees that spend time and effort to generate outside offers, because turnover is costly and outside offers force employers either to raise wages or to let valuable employees go.[1] A common theme in the personnel business literature is that threatening the firm to quit to another job to obtain a wage raise amounts to a "breach of loyalty", which probably means something that the firm profoundly dislikes, and oftentimes sanctions (see Welch 2001).

Firms have a compelling reason to resist the temptation to compete *ex post* against poachers: namely, to minimize the incentives of their other employees to seek rents through on-the-job search (OJS). But the firm's commitment to let employees go without a fight is hard to sustain, for two types of reasons: hiring and training costs have to be incurred to replace the quitter, and the promise of future competition allows the firm to cut the current wage without compromising recruitment. That is, wage raises following outside offers may serve to "backload" wages, as required to retain workers by the optimal contracts of Stevens (2004) and Burdett and Coles (2003). Efficiency wages have been long advocated as means to reduce turnover, but they build on the assumption of no *ex post* competition. It appears difficult to prevent firms from competing *ex post* to retain their workers, and yet firms seem to crave this commitment power.

In this paper I investigate a reputational foundation for the firm's commitment not to respond to outside offers. The basic idea is that renouncing *ex post* competition serves to *perpetuate a coordination failure among employees at the same firm*. Responding to one outside offer may provide a coordination device to all employees, to raise the intensity of their OJS to the level that would be individually optimal should indeed the firm match all outside offers from then on; once this coordination takes place, the firm is helpless and must indeed fight. So the firm may prefer to sacrifice individual workers and let them go without a fight, in order to keep the incentives of the other workers in line. This wage-setting practice, along with the resulting workers' OJS behavior, defines the "corporate culture" at the firm.

The existence and the importance of moral hazard in OJS are hardly questionable. OJS effort is neither fully observable nor contractible .It is necessary to explain observed wage dispersion by means of search frictions (Christensen *et alii* 2005). Workers search for alternative work not only to quit, but also for rent-seeking purposes. Letting workers quit is costly to their employer. We claim that moral hazard explains why firms do not match

[1] A Google search of the sentence "ten reasons not to accept a counteroffer" returns 2,850 hits. For example, the recruiting firm Feggestad&Hill, Inc. publishes, on its website, a page on "Counteroffers - Career Suicide" (http://www.fhjobs.com/counter.htm). This lists as facts: "When a company makes a counteroffer, you will be considered a risk and treated different in the future. [...]", "Successful and respected companies don't usually make counteroffers. An open door policy should help create an environment that people want to stay in. If you started looking, there is something better out there."

outside offers and labor markets remain relatively non-competitive, in spite of high job-to-job turnover. The key is the search behavior of the worker *before* receiving an outside offer. After the outside offer arrives, the firm can always match it. Wage policies are used for retention primarily through their effect on OJS effort spent by workers.

A natural objection to our story is that reputation can support various equilibrium outcomes, and thus typically lacks strong predictive power. We believe, however, that our particular foundation built on moral hazard and a coordinated response by co-workers is empirically plausible and compelling. We argue that firm membership and boundaries are truly defined by superior information, relative to the outside world, regarding the wage-setting practice of the entrepreneur: the first reaction that he must face when changing his wage-setting practice is by his own workers, because they are the first to know. In fact, we posit an informational definition of the boundary of a firm: a collection of workers who can observe the wage-setting behavior of their common employer. This definition was proposed by Fang and Moscarini (2003), who also discuss the reasons why it is empirically plausible and interesting from a theoretical viewpoint.

In addition, we let firms free to poach and to compete *ex post* for *other* firms' workers, and these other workers to search optimally on the job without any constraint. So these outside actors have no additional margins to threaten a firm and to support particular equilibria. The only real threat can come from existing employees, through the intensity of their OJS. Mortensen (1978) already remarked that counteroffers may lead workers to search too much for outside offers.

Once all workers raise their OJS effort in response to one matched offer, and then it is more profitable for the firm to compete, then we enter a subgame "with no return", where the firm now matches all offers for ever. The threat of the firm to stop matching offers is not credible, because workers know that, by sticking to the current coordinated OJS, they can force the firm to keep matching, and this makes them better off at the current wage. That is, once all workers spend high OJS, one individual worker who deviates and stops searching actively on the job just loses opportunities for internal raises, without changing the firm's behavior. The firm is forced to match by all workers' behavior and by the *ex post* temptation to retain the deviant worker.

We adopt the most popular wage-posting model of OJS and firm size, namely Burdett and Mortensen (1998) [BM]. In the BM model, a firm has a wage policy, which entails four types of commitment: to an offer, to a constant wage, to unconditional wage offers, that do not depend on the applicant's employment status, and to no *ex post* competition against poachers. Postel-Vinay and Robin (2002) allow for no commitment and *ex post* competition, but not for moral hazard in OJS, and find that *ex post* competition relieves the monopsonistic downward pressure on wages that still survives in BM98, albeit not as extreme as in the Diamond paradox. Coles (1999) allows for discounting and shows that *ex post* commitment to offered wages is an equilibrium, but still he does not allow firms to fight back when workers receive outside offers. Stevens (2004) and Burdett and Coles (2003) explore more general wage-tenure contracts, which still require some exogenous commitment power, because a firm has an incentive to renege on them *ex post.* They, too, rule out moral hazard in OJS effort.[2] We relax the commitment not to compete *ex post*. The firm's offers can be thought of as a bargaining solution, where the firm has all the bargaining power, but is constrained by the OJS moral hazard problem. In this sense, we aim to reconcile wage-posting models with the spirit of the Nash bargaining tradition in equilibrium search, based on continuous renegotiation and no commitment to offers of any kind.

To the best of our knowledge, Postel-Vinay and Robin (2004) [PVR04] is the only equilibrium dynamic analysis of moral hazard in OJS. They assume that firms have an exogenous ability to commit to a no-matching strategy at the moment of hire. Under constant returns to scale, firm size plays no role and each job is a firm. In our view, firm size in terms of employment is a key determinant of the firm's incentives to commit, so it must be properly modelled endogenously in the incentive constraints. In fact, Brown and Medoff (1996) conclude that large firms are less likely to match outside offers. In our interpretation, this is because in large firms there are more co-workers whose incentives have to be kept in line. PVR04 find that low-wage firms are more likely to commit not to match outside offers, and that the labor market is segmented, if at all, into low-wage, small firms that offer no "career prospects", and high-wage, large firms that compete *ex post*. Our preliminary findings, that large firms are less likely to match outside offers, based on the effects of firm size, go in the opposite direction.

2. Setup

We study a continuous time search model of the labor market without recall. A unit measure of risk-neutral workers and a measure m of firms maximize payoffs, discounted at rate $r > 0$ by workers and undiscounted by firms. Each firm has CRS technology with parameter p, distributed with a c.d.f. $J(p)$ over a support $[\underline{p}, \overline{p}] \subseteq \mathbb{R}$, and can hire as many workers as desired and as allowed by the frictional search process. An unemployed worker receives a payoff b and contacts a firm at rate λs_0, at no cost. An employed worker is paid a wage w and contacts a firm randomly at rate $\lambda (s_1 + s)$, where s_1 is fixed, $s \in \{0, 1\}$ is an OJS effort choice, with cost $c(0) = 0 < c(1) = c$.[3] To simplify the algebra, we assume $s_0 = s_1$. Matches separate exogenously at rate δ, endogenously when a worker quits to another job. Firms make unilateral and unconditional wage offers and can commit not to cut them,

[2] An earlier literature in the tradition of "personnel Economics" (e.g., Waldmann 1984) analyzed the strategic situation that arises with offers and counteroffers in simple two-period models, that feature no market equilibrium analysis. More generally, the ideas discussed in this paper are reminiscent of a large literature on efficiency wages and turnover. The current search literature revisits these questions in a frictional equilibrium setting, rather than in the vacuum of the individual worker-firm relationships.

[3] If no OJS effort produces no outside offers ($s_1 = 0 < s_0$) and we make λ endogenous through a standard matching function, then the Diamond paradox can be restored, and BM's resolution fails. In fact, if all firms offer the reservation wage b, a worker gains nothing from spending costly OJS effort, because there is no better wage to be found in the market. Given that no employed worker generates job applications, no firm would find it profitable to deviate and to post a wage above b, because an unemployed job applicant would still accept b, while an employed job applicant would have no chance to locate that firm. In BM, $s_1 > 0$, so workers would come in contact for free with a deviant firm, which would be able to hire someone, even if every single worker would expect to be hired there with zero probability. We follow BM and assume $s_1 > 0$, so that OJS breaks the Diamond paradox even when discretionary OJS effort is zero.

but cannot commit not to respond to outside offers received by its employees. Co-workers employed by the same firm and any other firm directly competing with it for a worker can observe a firm's wage-setting behavior; all other workers and firms cannot.

3. The BM Equilibrium Allocation

We describe the allocation that we aim to support in equilibrium in the wage-posting game. Workers do not search actively on the job ($s = 0$), but still receive outside offers for free by $s_1 > 0$, and rationally accept an offer only if it makes them better off. Firms commit to wage offers that are unconditional on the employment status of the job applicant, equal for all of their workers, and not reneged *ex post*, and do not respond to outside offers.

This allocation is identical to the equilibrium of BM's wage-posting game when firms are heterogeneous. In BM, firms are not allowed to counter outside offers to their employees, and workers have no control over their OJS intensity. In this paper we study how the BM allocation can still be an equilibrium when firms can match outside offers and workers can control the intensity of their OJS, creating a moral hazard problem. Because of these two extensions, we have to derive some novel implications of the BM equilibrium.

Let $k \equiv \lambda s_0/\delta = \lambda s_1/\delta$, F denote the (normalized) c.d.f. of offered wages, and $l(w)$ the steady state size of a firm offering wage w. BM show that the reservation wage for a worker is b, each firm of type p offers a unique profit-maximizing wage

$$\omega(p) = \arg\max_{w \geq b} l\left(w\right)\left(p - w\right) \tag{3.1}$$

and earns total profits

$$\pi_0(p) = l(\omega(p))[p - \omega(p)].$$

Therefore, the wage-offer distribution coincides with the firm type distribution

$$F\left(w\right) = J(\omega^{-1}(w)). \tag{3.2}$$

The cdf of wages paid to employed workers satisfies

$$G\left(w\right) = \frac{F\left(w\right)}{1 + k\left[1 - F\left(w\right)\right]}.$$

From this expression, BM derive the ratio of wage densities, which is the share of total employment that works in a firm offering wage w (ratio between the share of workers in the economy being paid w and the share of firms in the economy offering the wage w):

$$\frac{g\left(w\right)}{f\left(w\right)} = \frac{G\left(w\right)}{F\left(w\right)}\frac{1+k}{1 + k\left[1 - F\left(w\right)\right]} = \frac{1+k}{\left\{1 + k\left[1 - F\left(w\right)\right]\right\}^2}.$$

Stationary unemployment solves $\lambda s_0 u = \delta\left(1 - u\right)$, or $u = \left(1+k\right)^{-1}$. Putting all pieces together, steady state firm size equals

$$l\left(w\right) = \frac{g\left(w\right)}{f\left(w\right)}\left(1 - u\right) = \frac{k}{\left\{1 + k\left[1 - F\left(w\right)\right]\right\}^2}$$

which closes the model. We may now solve explicitly the firm's optimization problem (3.1). This has a NFOC

$$p - \omega(p) = \frac{l(\omega(p))}{l'(\omega(p))} = \frac{1 + k\left[1 - F(\omega(p))\right]}{2kf(\omega(p))}. \tag{3.3}$$

This can be easily inverted in closed form

$$p = \omega^{-1}(w) = w + \frac{1 + k\left[1 - F(w)\right]}{2kf(w)} \tag{3.4}$$

so that (3.2)

$$F(w) = J\left(w + \frac{1 + k\left[1 - F(w)\right]}{2kf(w)}\right)$$

yields is a nonlinear ODE in F for any given assumption about J.

We now describe worker's behavior both on and off the BM equilibrium path. In equilibrium, workers do not search actively on the job. To check that deviations are unprofitable, it suffices to check a one-step deviation: raising OJS effort to $s = 1$ on the current job and reverting to $s = 0$ from next job on is dominated by always choosing $s = 0$.

The value of being employed by a $p-$firm at wage $w = \omega(p)$ while searching on the job at intensity $s \in \{0, 1\}$ is $W_s(w|p)$, and the value of unemployment when expecting to search on the job at intensity $s = 0$ once employed is U. A firm of productivity p offers a wage $w = \omega(p)$ in the candidate equilibrium, and does not match outside offers. As remarked by BM, this implies that p does not enter explicitly the value of the worker, the wage is a sufficient statistic for the match productivity, and we can write $W_s(w)$. Therefore, $W_0(w)$ is the value in the BM equilibrium, of being employed and never, now or in the future, searching actively on the job ($s = 0$); $W_1(w)$ is the value of being employed and searching actively on the job only in the current employment spell, namely, the value of a one-step deviation from the BM equilibrium where no worker ever spends OJS effort and firms offer wages according to $F(w)$. Under either strategy, the worker has a reservation wage R_s such that $W_s(R_s) = U$.

Under the easily verified conjecture that $W_s(w)$ is increasing in w, the Bellman equations read:

$$\begin{aligned} rW_s(w) &= w + \delta[U - W_s(w)] + \lambda(s + s_0)\int_w^\infty \left[W_s(x) - W_s(w)\right] dF(x) - cs \\ rU &= b + \lambda s_0 \int_{R_s}^\infty [W_s(x) - U] dF(x). \end{aligned}$$

Taking a derivative and rearranging

$$W_s'(w) = \frac{1}{r + \delta + (k\delta + \lambda s)\left[1 - F(w)\right]}. \tag{3.5}$$

Integrating by parts and replacing for $W_s'(w)$ from (3.5)

$$\int_w^\infty \left[W_s(x) - W_s(w)\right] dF(x) = \int_w^\infty \frac{1 - F(x)}{r + \delta + (k\delta + \lambda s)\left[1 - F(x)\right]} dx \equiv q_s(w)$$

which is a known magnitude, given the wage offer distribution F.

To derive the reservation wage R_s, use the definition $W_s(R_s) = U$ in the Bellman equations

$$\begin{aligned} rU &= R + \lambda(s + s_0) \int_{R_s}^{\infty} [W_s(x) - U]\, dF(x) - cs \\ rU &= b + \lambda s_0 \int_{R_0}^{\infty} [W_0(x) - U] dF(x) \end{aligned}$$

subtract these equations, and use $\lambda s_0 = k\delta$. Tthe reservation wage solves

$$R_s = b + cs + k\delta_0 q_0(R_0) - (\lambda s + k\delta) q_s(R_s).$$

As argued earlier, when $s = 0$ as in the BM equilibrium, we get immediately,

$$R_0 = b + 0 + k\delta_0 q_0(R_0) - k\delta_0 q_0(R_0) = b.$$

Otherwise, R_1 solves implicitly

$$R_1 - b - c - k\delta q_0(b) + (\lambda + k\delta)\, q_1(R_1) = 0.$$

and we look for a root R_1 of this equation. The LHS has slope with respect to R_1

$$1 - (\lambda + k\delta)\, q_1'(R_1) = \frac{r + \delta}{r + \delta + (\lambda + k\delta)\,[1 - F(R_1)]} \in (0, 1)\,,$$

so there exists exactly one such value of R_1, possibly negative.

Finally we have explicit expressions for the worker's values:

$$U = b + k\delta q_0(b)$$

$$W_s(w) = \frac{w - cs + \delta b + k\delta^2 q_0(b) + (k\delta + \lambda s) q_s(w)}{r + \delta}. \tag{3.6}$$

4. The Returns to OJS when Firms Compete for Employed Workers

On-the-job search effort is typically difficult to observe and to contract upon. At the same time, it is an important source of job separations. Hence, on-the-job search creates a classic moral hazard problem: the worker can spend costly effort in an activity that affects the firm's payoffs, and the firm can only affect this choice through appropriate incentives. The firm has essentially two instruments: the wage it offers to the worker when he is first hired, and the counteroffers to the worker when he receives outside offers. The first key piece of information is, therefore, the incentive effect of either instrument. Before specifying the extensive form of the ex post bargaining game, we can write the resulting payoffs of the worker in a general form.

Let $W_s^M(w|p)$ denote the value to a worker of working at wage w and searching at intensity s with a $p-$firm that competes against outside offers. This value depends on productivity p

independently of the wage w, because p determines the ability of the firm to compete against future competitors.

Renegotiation is triggered only when an outside offer w' arrives from one of the other firms, playing the BM equilibrium strategies, such that

$$W_0(w') > W_s^M(w|p).$$

Any other offer can be safely ignored by the current employer. As $W_0(\cdot)$ is increasing, this is equivalent to

$$w' > \xi_s(w|p)$$

where the wage $\xi_s(w|p)$ uniquely solves

$$W_s^M(w|p) = W_0(\xi_s(w|p)). \tag{4.1}$$

Therefore, conditional on an outside offer, the worker obtains a gain with probability $1 - F(\xi_s(w|p))$, while with complementary chance his condition is unchanged, as the new offer is not competitive enough to lead the employer to respond.

Notice that a firm, competing or not, cannot pay more than a flow wage p. When this happens, the firm has no more room to compete ex post, therefore the worker anticipates that competition is irrelevant: for $s = 0, 1$,

$$W_s^M(p|p) = W_s(p) \tag{4.2}$$

where $W_s(p)$ can be obtained from (3.6). This also implies $\xi_s(p|p) = p$.

We denote by $Q_s^M(w|p)$ the expected worker's payoff from ex post competition between firms, where the expectation is taken before the outside offer is known. This payoff depends on the extensive form of the poaching game, to be specified later. In this notation:

$$W_s^M(w|p) = \frac{w - cs + \delta b + k\delta^2 q_0(b) + (k\delta + \lambda s)\left[1 - F(\xi_s(w|p))\right] Q_s^M(w|p)}{r + \delta + (k\delta + \lambda s)\left[1 - F(\xi_s(w|p))\right]}. \tag{4.3}$$

Replacing this expression and (3.6) in (4.1) yields an implicit equation in $\xi_s(w|p)$.

We consider three different extensive forms of the poaching subgame. First, the current employer makes a counteroffer, and the poacher does not respond further. Second, the current employer and the poacher play Bertrand competition. Third, they play an ascending English auction with repeated, infinitely quick bids. We analyze the robustness of the BM equilibrium allocation in each of these environments. That is, we assume that all firms follow the BM wage policy, making wage offers $\omega(p)$ that are independent of the applicant's employment status, and committing not to cut them or raise them for any reason. In this environment, we study the incentives of an individual firms to deviate and to compete ex post, while maintaining the ability to commit not to cut accepted wage offers. If such incentives are not strong enough, then the BM allocation remains an equilibrium under such less restrictive assumptions.

5. The Poaching Game: One Counteroffer

Suppose that the current employer responds to the outside offer $w' \sim F$, and then the worker chooses between the new offer w' and his employer's counteroffer w''. Then, either the response is unnecessary, $w'' = w$, because the outside offer w' is not competitive, $w' \leq \xi_s(w|p)$, and the worker stays anyway at the current wage; or, the response is irrelevant, because the outside offer is too strong and "out of reach": $w' > \xi_s(p|p) = p$, and the worker quits anyway to grab the new offer w'; or, finally, $\xi_s(w|p) < w' \leq p$, and then the response w'' is relevant, because it induces the worker to stay where he is, rather than quit as in the BM equilibrium. In the first two cases, the returns to the worker from OJS are the same whether the employer competes or not against outside offers. In the third case, the employer raises the wage to make the worker indifferent between staying or not, i.e. it responds to w' with a new wage w'' such that $W_0(w') = W_s^M(w''|p)$, namely, $w'' = \xi_s^{-1}(w'|p)$. But then, again, the worker obtains from his employer's counteroffer the same value that he would obtain without a counteroffer in the BM allocation. The only difference is that the worker stays with his current employer at upgraded value $W_s^M(\xi_s^{-1}(w'|p)|p) = W_0(w')$, rather than quitting to obtain $W_0(w')$ from the poacher as in the BM equilibrium. The worker cares about payoffs, not about the identity of the employer per se. We conclude that in this case the worker's continuation value is the same as in the BM allocation:

$$Q_s^M(w|p) = W_s(w).$$

Based on this result, it is intuitive, and easy to show, that the worker's payoffs *before* receiving outside offers are just the same as in a BM equilibrium with OJS intensity s:

$$W_s^M(w|p) = W_s(w). \tag{5.1}$$

The employer's counteroffer has no incentive effects and does not change the worker's returns to OJS, therefore the returns to work in general.

In this case, the BM allocation cannot be an equilibrium, independently of the heterogeneity in firm's productivities. The BM allocation requires that all workers search at the same intensity on the job, as the offer arrival rate is independent of the wage. Clearly, it cannot be optimal for the highest-paid workers to search actively on the job, as the potential gains are negligible relative to the cost c.[4] Therefore, in the BM allocation all workers choose $s = 0$, namely, it must be the case that

$$W_0(w) \geq W_1(w)$$

for all wages w in the support of the wage distribution G. From (5.1), this implies

$$W_0^M(w|p) \geq W_1^M(w|p).$$

[4]The reader may wonder whether this is an artifice of a 0-1 OJS choice. But, if we made OJS effort s a continuous choice with a convex cost function $c(s)$, $c(0) = 0$, then in equilibrium OJS effort and then the arrival rate of outside offers would depend on the current wage. This would also invalidate the BM equilibrium, with or without firm heterogeneity, as the wage distribution would also reflect these additional moral hazard effects. See Christensen *et alii* (2004) for a partial analysis of this case.

This inequality implies that a firm can compete ex post without raising its workers' incentives to search on the job actively. But then, ex post competition can only help a firm, because it retains workers and does not impact on the frequency of outside offers. Hence, no firm will find it optimal to ignore all outside offers without trying to counteroffer, as exogenously assumed in BM. The BM allocation cannot be an equilibrium when an employer can respond to outside offers but the poacher cannot respond further.

6. The Poaching Game: English Auction

Next suppose that, by responding to an outside offer, a firm triggers an ascending auction against the poaching firm. After the outside offer arrives, the two firms (beginning with the current employer) take turns, at time intervals of length Δ, in bidding a new binding wage offer. When a bid fails to strictly raise the value to the worker of accepting it, the auction ends. The winning wage might not be the higher of the two, because the winning firm may also promise to compete against poachers in the future, and the worker cares about overall value, not just flow wage.

This game has multiple subgame perfect equilibria. One has the same outcome as Bertrand competition: if the outside offer $\omega(p')$ made in the BM allocation by a p' firm makes the worker quit his p employer, the more productive of the two firms bids the opponent's maximum valuation for the worker, and wins in one round.

This equilibrium, however, has the same undesirable features that we found in Bertrand competition. As argued by Moscarini (2005), there is another, more natural equilibrium of the poaching ascending auction: the incumbent employer responds to an outside offer if and only if this is threatening, but is made by a less productive competitor; after that, or in all other circumstances, no further bids are made. That is, if a worker earning w at a $p-$firm receives an outside offer w' such that $W_0(w') \leq W_s^M(w|p)$, then the incumbent firm does nothing; if $W_s^M(w|p) < W_0(w')$, namely $w' > \xi_s(w|p)$, but the poacher is not as productive as the incumbent, namely $p' = \omega^{-1}(w') < p$, then the incumbent offers a raise to $w'' = \xi_s^{-1}(w'|p)$ such that $W_s^M(w''|p) = W_0(w')$, and the auction ends there; otherwise, the worker accepts the outside offer w' and quits. The worker obtains from an outside offer and from the consequent auction an expected continuation value, when the outside offer $w' > \xi_s(w|p)$ is competitive:

$$Q_s^M(w|p) = \int_{\xi_s(w|p)}^{\infty} W_0(w') \frac{dF(w')}{1 - F(\xi_s(w|p))}.$$

The implied wage raise over $W_s^M(w|p)$ comes from the current employer when $\xi_s(w|p) < w' \leq p$, and from the poacher otherwise.

These "non-competitive" bidding strategies are supported by the threat of the more productive firm to outbid the rival. These strategies could not be an equilibrium when firms are constrained to just *one* round of bids, as in the Bertrand game, because the losing firm could deviate and outbid the more productive rival, who would have no opportunity to respond. But, when firms *can* wait, and then respond to hostile bids as many times as necessary, then any deviation by the losing firm would be punished and neutralized. This equilibrium of the ascending auction survives any trembles, as it is subgame perfect. Unlike the Bertrand-type equilibrium, it also survives the introduction of an infinitesimal cost of bidding. Simply put: when everybody knows who will win, there is no point in competing, especially if competition entails some arbitrarily small costs.

The outcome of this equilibrium of the ascending auction appears identical to the one in Section 5, where only the incumbent employer is entitled to a counteroffer. This similarity, however, is deceiving. In Section 5, a firm p offering the BM wage $\omega(p)$ can match all outside offers that are threatening but within reach, $w' = \omega(p') \in (\omega(p), p]$, knowing that, by assumption, even more productive poachers would not respond further. Thus, the worker would not alter his OJS behavior relative to the BM allocation $s = 0$. In that case, we concluded that sticking to the BM wage offer and matching the threatening but not unbeatable outside offers is a profitable deviation for a firm, because this deviation maintains worker inflow and their wages, but reduces the outflow. In the case of an ascending auction that we are now studying, a p firm offering the BM wage $\omega(p)$ would have to respond in equilibrium only to outside offers w' by more productive competitors, as $\omega(\cdot)$ is increasing, so $w' > \omega(p)$ if and only $p' = \omega^{-1}(w') > \omega^{-1}(\omega(p)) = p$. Hence, sticking to the BM offer $\omega(p)$ and then playing the noncompetitive equilibrium of the ascending auction is no longer a profitable deviation: in this case no competition ever takes place, just as assumed in BM, but for endogenous reasons.

Unfortunately, this difference between the cases of one counteroffer (Section 5) and ascending auction does not imply that the latter can support the BM equilibrium. The reason is that, when ascending auctions for employed workers take place, a p firm has a *further* dimension to deviate from the BM wage policy $\{\omega(p), \text{no match}\}$: it can cut (some of) its wage offers below $\omega(p)$, and then bid against outside offers that beat this lower wage but come from less productive competitors. In particular, the following is *one possible* profitable deviation by firm p: maintain the BM offer $\omega(p)$ to all workers who apply from another job, and not compete for them, but cut the offer to unemployed workers to a lower wage w that gives them a value $W_0(\underline{p}) = W_S^M(w|p) < W_0(\omega(p))$, where $\underline{p}$ is the lowest productivity firm, and compete for them in auctions. We claim that this deviation is sequential and does not reduce the inflow or increase the outflow of workers, thus it preserves firm size. Yet, it reduces some wages, thus it raises average profits above the BM equilibrium level.

First, the deviation is sequential, i.e. a best response in the subgame it triggers. When an outside offer $\omega(p')$ arrives to workers who are paid $\omega(p)$, this is threatening to their current employer if and only if $\omega(p') > \omega(p)$, i.e. $p' > p$. But then the equilibrium of the auction prescribes exactly not competition, as in BM. When the outside offer arrives to the low-paid workers, it is ex post optimal to compete, when profitable.

To see why this strategy does not reduce worker inflow below that in the BM allocation, consider the two types of job applicants. Unemployed job applicants accept the offer $W_0(\underline{p})$, because they do so in the BM equilibrium from $\underline{p}$ firms, so $W_0(\underline{p}) \geq U$. Employed job applicants also accept $\omega(p)$ from this deviating firm whenever they do accept it from other non-deviating p firms, because the promise to compete ex post against poachers added to the same wage cannot hurt the value of employment.

To see why this strategy does not increase worker outflow above that of the BM allocation, notice that the employed job applicants who are hired at wage $\omega(p)$ by the deviating p firm subsequently receive relevant (threatening to the employer) outside offers $\omega(p') > \omega(p)$ only from more productive poachers $p' > p$, so they know that the p employer will not respond, but let them go, just like in the BM equilibrium. Hence, these workers have the same incentives to search on the job, thus they choose $s = 0$, as in BM, and quit at the same rate. The unemployed job applicants who are hired by this deviating firm at value $W_0(\underline{p})$ behave like the workers who, in the BM allocation, are hired by a firm $\underline{p}$ and choose $s = 0$. In fact, the value of employment is the same by construction, $W_0(\underline{p})$, the lowest value the worker can earn when employed, so it is always beaten by (almost) any outside offer. But, the p firm matches these outside offers exactly in equivalent worker's value, except when they are too high. Either way, just like in Section 5 where only the incumbent could respond to an outside offer, the worker faces the same continuation value of OJS as in the BM allocation, only sometimes he obtains the raise from his employer rather than by competitors. Thus, faced with the same returns to OJS, the worker chooses the same OJS intensity, $s = 0$. But the p firm, unlike the $\underline{p}$ firm, does occasionally fight, and loses workers only to more productive competitors, just like in the BM allocation. Hence, the quitting rate of new hires from unemployment at the deviating p firm, the product of the arrival rate and of the conditional probability that the poacher wins the auction, is the same as in BM, and so it the total worker outflow.

This concludes the argument. It is too tempting for firms to cut their unemployed hires' wages and then just match outside offers ex post in the auction, when possible, to retain them. Firms may do even better than this deviation, by reducing the wage of employed hires. We now move to analyze Bertrand competition. Why does this deviation not work in that case? There, the firm cannot help competing even against outside offers of the kind $\omega(p') \in (\omega(p), p)$, although it knows it will lose in the end. This temptation of the firm to compete generates extra returns to OJS for the workers who earn $\omega(p)$, relative to the BM allocation, and possibly induces them to raise their OJS effort. So we cannot rule out an increase in outflow when the firm follows the deviation illustrated in this section.

7. The Poaching Game: Bertrand Competition

Next, suppose that the poacher can respond to the counteroffer and, if it does so, then the poacher has the opportunity of responding one last time. Then, if the current employer responds, the two firms effectively play Bertrand competition under symmetric information. The unique equilibrium of this game prescribes that the more productive of the two firms bids the opponent's valuation for the worker, and wins. In this case, the heterogeneity in firms' productivities plays an important role.

7.1. Homogeneous Firms

First, suppose all firms are identical, and this is common knowledge. Then Bertrand competition effectively transfers all rents to the worker, the familiar Bertrand paradox. After the two counteroffers are made, the worker obtains all output and optimally stops any active

OJS. The resulting value is

$$Q_s^M(w|p) = W_0(p).$$

This is a powerful capital gain for the worker, who may then be induced to search on the job when working for a firm that competes against poachers. But, it is easy to see that a firm has no ex post incentive, and a strong ex ante incentive, not to trigger this Bertrand game. In fact, by responding to an outside offer, a firm may retain the worker, but loses all rents anyway: ex post, it makes no difference to the firm's profits whether a worker quits or stays and extracts all rents. Ex ante, active OJS raises the workers' incentives to active OJS, $s = 1$, thus the frequency of outside offers and the occurrence of necessary and painful ex post competition for the employer. The average effect of competing is simply that the firm effectively loses rents. The commitment not to compete ex post is easily supported as an equilibrium strategy. The BM allocation is an equilibrium when all firms are identical and a response to an outside offer triggers Bertrand competition, namely, one final counter-counteroffer by the poacher.

7.2. Heterogeneous Firms

If firms differ by their productivity, there exist instances where ex post Bertrand competition both helps the incumbent employer and changes the worker's incentives to search on the job. An employer p may retain some workers who would otherwise quit, without giving up all rents, when the outside offer is $w' > \xi_s(w|p)$, thus threatening, but also $w' \leq \omega(p)$, thus beatable by a profitable counteroffer $p' = \omega^{-1}(w') < \xi_s(w|p) = p$. The employer triggers Bertrand competition and wins by promising the worker the maximum value $W_0(p')$ that the poacher could ever offer in the BM allocation. Because $\xi_s(w|p) < w' = \omega(p')$, the winning bid gives the worker a value $W_0(p')$ that is higher than the one that he is currently enjoying. In practice, an employer p paying a wage w responds to an outside offer $w' \in (\xi_s(w|p), \omega(p))$ with a wage raise from w to $w'' = \xi_s^{-1}(\omega^{-1}(w')|p)$, that is, to a new wage w'' such that $p' = \omega^{-1}(w') = \xi_s(w''|p)$.

If the outside offer is threatening $w' > \xi_s(w|p)$, but also $w' \in (\omega(p), p]$, thus beatable in principle but not in equilibrium by a profitable counteroffer, Bertrand competition follows and the poacher wins (as $w' = \omega(p') > \omega(p)$ implies $p' > p$) by raising its outside offer to the maximum wage p that the current employer can afford. In this case, the worker still quits as in the BM allocations, but gains some more thanks to the aggressive response of his employer. If, instead, the outside offer is $w' \leq \xi_s(w|p)$, then the worker will not accept it anyway, and his employer can just ignore it. Finally, if the outside offer is $w' > p$, out of the reach of the incumbent employer, then in the equilibrium of the Bertrand game the worker will walk away with a value upgraded to $W_0(w')$.

In light of the possible benefits from responding to counteroffers, the question is, then, why wouldn't an incumbent employer do it, as BM assume. The only possible drawback of competition is the moral hazard cost, the reaction of the employees who anticipate that ex post competition will make the cost of OJS worth its while, and raise their OJS effort to $s = 1$, forcing the firm to more frequent competition or attrition. Since the additional returns to higher OJS, relative to the BM allocation, accrue to the worker only in those circumstances where his employer is forced to respond but in the end wins, the only reason for a worker to raise OJS is rent-seeking, to force the current employer to raise the wage. Conditional on outside offers, the worker remains with his employer with the same probability as in the BM allocation; only, he often gets a raise by his employer.

In light of the possible events following an outside offer that we spelled out, we can write the worker's expected continuation value upon receiving a competitive outside offer, but before knowing how high it, equals

$$Q_s^M(w|p) = \int_p^\infty W_0(w')dF\,(w') + [F(p) - F(\omega(p))]W_0(p) + \int_{\xi_s(w|p)}^{\omega(p)} W_0(\omega^{-1}(w'))dF(w'). \quad (7.1)$$

The first capital gain occurs when the new outside offer is immediately too strong to fight it, the second when the employer can respond to it but eventually loses the Bertrand game, the third when the employer can successfully respond. When the outside offer is too weak, no competition ensues and the workers gains nothing.

Integrating by parts

$$\int_p^\infty W_0(w')dF(w') = W_0(p)[1 - F(p)] + q_0(p)$$

$$\begin{aligned}\int_{\xi_s(w|p))}^{\omega(p)} W_0(\omega^{-1}(w'))dF(w') &= -W_0(p)[1 - F(\omega(p))] + W_0(\omega^{-1}(\xi_s(w|p))))[1 - F(\xi_s(w|p)))] \\ &\quad + \int_{\xi_s(w|p)}^{\omega(p)} W_0'(\omega^{-1}(w'))\frac{d\omega^{-1}(w')}{dw'}[1 - F(w')]dw'.\end{aligned}$$

Let

$$\chi(x|p) \equiv \int_x^{\omega(p)} W_0'(\omega^{-1}(w'))\frac{d\omega^{-1}(w')}{dw'}[1 - F(w')]dw' + W_0(\omega^{-1}(x))[1 - F(x)]$$

changing variable from w' to $p' = \omega^{-1}(w')$, so $dp' = \frac{d\omega^{-1}(w')}{dw'}dw'$ and using (3.5) to replace for $W_0'(p')$

$$\chi(x|p) = \int_{\omega^{-1}(x)}^{p} \frac{1 - F(\omega(p'))}{r + \delta + k\delta[1 - F(p')]}dp' + W_0(\omega^{-1}(x))[1 - F(x)]$$

a function of parameters and the BM wage offer function $\omega(p)$ and distribution F. Notice the slope

$$\begin{aligned}\chi'(x|p) &= -\frac{1 - F(x)}{r + \delta + k\delta[1 - F(\omega^{-1}(x))]}\frac{d\omega^{-1}(x))}{dx} + W_0'(\omega^{-1}(x))\frac{d\omega^{-1}(x)}{dx}[1 - F(x)] - f(x)W_0(\omega^{-1}(x)) \\ &= [1 - F(x)]\frac{d\omega^{-1}(x))}{dx}\left\{-\frac{1}{r + \delta + k\delta[1 - F(\omega^{-1}(x))]} + W_0'(\omega^{-1}(x))\right\} - f(x)W_0(\omega^{-1}(x)) \\ &= -f(x)W_0(\omega^{-1}(x)) < 0.\end{aligned}$$

Collecting all terms into 4.3, the value to the worker of spending OJS effort $s \in \{0, 1\}$ while working for a Bertrand-competing employer is in this case

$$W_s^M(w|p) = \frac{w - cs + \delta U + (k\delta + \lambda s)\{q_0(p) + \chi(\xi_s(w|p)|p)\}}{r + \delta + (k\delta + \lambda s)[1 - F(\xi_s(w|p))]}. \tag{7.2}$$

To solve for the cutoff wage $\xi_s(w|p)$, replace this expressions for $W_s^M(w|p)$ and $W_0(\xi_s(w|p))$ from (3.6) into (4.1)

$$\frac{w - cs + \delta b + k\delta^2 q_0(b) + (k\delta + \lambda s)\{q_0(p) + \chi(\xi_s(w|p)|p)\}}{r + \delta + (k\delta + \lambda s)[1 - F(\xi_s(w|p))]} = \frac{w - cs + \delta b + k\delta^2 q_0(b) + (k\delta + \lambda s)q_s(w)}{r + \delta}$$

and simplify to obtain:

$$\frac{(k\delta + \lambda s)\{q_0(p) + \chi(\xi_s(w|p)|p)\}}{r + \delta + (k\delta + \lambda s)[1 - F(\xi_s(w|p))]} = \frac{\left[w - cs + \delta b + k\delta^2 q_0(b)\right] + (k\delta + \lambda s)q_s(w)}{r + \delta}$$

$$\frac{w - cs + (k\delta + \lambda s)[q_0(p) + \chi(\xi_s(w|p)|p)]}{r + \delta + (k\delta + \lambda s)[1 - F(\xi_s(w|p)))]} = \frac{\xi_s(w|p) + k\delta q_0(\xi_s(w|p))}{r + \delta}. \tag{7.3}$$

Since the functions q_0 and χ are known from the BM equilibrium's offer distribution F, this equation determines the function $\xi_s(w|p)$. So $\xi_s(w|p)$ is the root x of

$$\frac{w - cs + (k\delta + \lambda s)[q_0(p) + \chi(x, p)]}{r + \delta + (k\delta + \lambda s)[1 - F(x)]} = \frac{W_0(x)}{r + \delta}$$

$$W_0(x)\{r + \delta + (k\delta + \lambda s)[1 - F(x)]\} - (k\delta + \lambda s)[q_0(p) + \chi(x|p)] - (w - cs)(r + \delta) = 0.$$

The LHS has slope

$$\begin{aligned} & W_0'(x)\{r + \delta + (k\delta + \lambda s)[1 - F(x)]\} - f(x)W_0(x) - (k\delta + \lambda s)\chi'(x|p) \\ = & \frac{r + \delta + (k\delta + \lambda s)[1 - F(x)]\}}{r + \delta + k\delta[1 - F(x)]} - f(x)W_0(x) + (k\delta + \lambda s)f(x)W_0(\omega^{-1}(x)) \end{aligned}$$

$$\frac{d\{\phi(x)[1 - F(x)]\}}{dx} = \phi'(x)[1 - F(x)] - f(x)\phi(x) = \frac{(r + \delta)[1 - F(x)]}{r + \delta + k\delta[1 - F(x)]} - f(x)x - f(x)k\delta q_0(x)$$

is positive using the previous expressions and after some algebra, ϕ and $-\psi$ are increasing. Therefore, under appropriate boundary conditions, there exists a well-defined function $\xi_s(w|p)$. Once this is known, we can solve for $W_s^M(w|p)$ from (7.2).

Notice that

$$W_0(\omega^{-1}(\xi_s(w|p))) = \frac{w + \delta U + k\delta q_0(\omega^{-1}(\xi_s(w|p)))}{r + \delta}$$

Let $\pi_1^M(p)$ denote the steady state profits of a $p-$firm which follows a strategy of ex post Bertrand competition, coupled with optimal wage offers ex ante. This deviant firm will gradually compete against poachers, and its workers will be paid differently depending on their histories of received outside offers. The question is then: can the BM allocation, where no firm responds to outside offers, be supported in a sequential equilibrium by the threat of the employees to raise OJS effort to $s = 1$ for ever, after the firm competes just once? This is possible if the following conditions simultaneously hold:

1. When the firm never competes against poachers, its employee optimally respond with low OJS effort:
$$W_0(w) \geq W_1(w).$$
2. When the firm deviates and matches one outside offer, its workers expect it to do so for ever after, and optimally respond with high OJS effort:
$$W_1^M(w|p) \geq W_0^M(w|p).$$
3. When workers choose low OJS effort, their employer prefers not to match outside offers:
$$\pi_0(p) \geq \pi_1^M(p).$$
4. When workers choose high OJS effort for ever, their employer prefers to match outside offers for ever after:
$$\pi_1^M(p) \geq \pi_1(p).$$

Using the expression in (7.2), we can show that the first two conditions are mutually inconsistent, as long as there is sufficient heterogeneity in firms' productivities: $\underline{p} < \omega(\overline{p})$. (This condition always holds if the least productive firm produces less than the flow value of leisure, $\underline{p} \leq b$, because $\omega(\overline{p}) > b$.) If a worker always finds it optimal not to spend the OJS effort cost c when the firm is not competing, it must find it optimal to do so also when the firm is competing but paying already almost the maximum possible wage. The reason is that, at that point, the competitive posture of the firm does not change significantly the returns to OJS for its workers.

Formally, consider a low-productivity firm $p \leq \omega(\overline{p})$. For every $\varepsilon > 0$ small, in steady state this deviant firm will be employing workers at any wage $w \in [p - \varepsilon, p]$. This is because with probability one offers in that range will arrive from poachers, due to the full support of the BM wage offer distribution in $[\omega(\underline{p}), \omega(\overline{p})]$. But then, for these workers, the continuation value following an outside offer is approximately equal to the first term in (7.1),
$$Q_1^M(w|p) \simeq \int_p^\infty W_0(w') \frac{dF(w')}{1 - F(\xi_1(w|p))}$$
and the chance of a competitive outside offer is approximately
$$1 - F(\xi_1(w|p)) = 1 - F(\xi_1(p - \varepsilon|p)) \simeq 1 - F(p).$$
This worker faces approximately the same returns to OJS as another worker who is employed by a non-deviating firm $p' = \omega^{-1}(p)$ paying the BM wage $\omega(p') = p$. Such a firm exists in the BM allocation by the assumption $p \leq \omega(\overline{p})$. Comparing (3.6) and (4.3), in this case
$$W_s^M(p - \varepsilon|p) \simeq W_s(p - \varepsilon). \qquad (7.4)$$
Finally, observe that
$$\begin{aligned} W_0(w) &\geq W_1(w) \Rightarrow W_0(p) > W_1(p) \\ &\Rightarrow W_0(p - \varepsilon) > W_1(p - \varepsilon) \\ &\Rightarrow W_0^M(p - \varepsilon|p) > W_1^M(p - \varepsilon|p) \end{aligned}$$

for $\varepsilon > 0$ small enough. The first implication holds because the positive cost of OJS $c > 0$ cannot be justified when the firm is already paying (almost) the maximum wage it can possibly pay. If it was justified at such a high wage, $W_0(p) \leq W_1(p)$, then it would be justified at all lower wage levels where the returns to OJS are higher and the cost is the same, violating $W_0(w) \geq W_1(w)$. The second implication holds by continuity. The third implication follows from (7.4). By the last inequality, these workers do not switch to active OJS when their employer starts matching outside offers. So the firm faces no moral hazard problem with these workers, and can compete ex post with impunity.

In fact, we can say more. A worker will respond to the employer's offer-matching strategy by raising OJS effort to $s = 1$ as long as the wage w does not exceed a threshold: $w \leq \bar{w}(p) < p$ defined by

$$W_1^M(\bar{w}(p)|p) = W_0^M(\bar{w}(p)|p).$$

To show that there exists at most one such maximum wage $\bar{w}(p)$ that creates moral hazard problems (one exists for sure from the above argument), let

$$z(w|p) \equiv W_1^M(w|p) - W_0^M(w|p).$$

Taking a derivative

$$\begin{aligned} z'(w|p) &= \frac{1}{r+\delta+(k\delta+\lambda)\left[1-F(\xi_1(w|p))\right]} - \frac{1}{r+\delta+k\delta\left[1-F(\xi_0(w|p))\right]} \\ &\propto k\delta\left[1-F(\xi_0(w|p))\right] - (k\delta+\lambda)\left[1-F(\xi_1(w|p))\right] \\ &= k\delta\left[F(\xi_1(w|p)) - F(\xi_0(w|p))\right] - \lambda\left[1-F(\xi_1(w|p))\right]. \end{aligned}$$

Since $W_0(\cdot)$ is increasing from (3.5), (4.1) implies

$$z(w|p) \leq 0 \Leftrightarrow W_1^M(w|p) \leq W_0^M(w|p) \Leftrightarrow \xi_1(w|p) \leq \xi_0(w|p) \Rightarrow z'(w|p) < 0.$$

So, once z is non-positive, and in fact positive but small enough, it declines. As $z(\bar{w}(p)|p) = 0$ by definition, $z'(\bar{w}(p)|p) < 0$ and then $z(w|p) < 0$ for all $w > \bar{w}(p)$. This shows that there exists at most one threshold wage $\bar{w}(p)$ where the worker stops searching on the job after receiving a sufficiently generous raise by her employer.

The question is, then, whether the OJS reaction by lower-paid workers, with wages below $\bar{w}(p)$, will be strong enough to reduce a deviant firm's profits from the BM level $\pi_0(p)$ to the new level $\pi_1^M(p)$, so as to deter this firm from fighting outside offers.

8. The Firm's Deviation to Bertrand Competition

A high-p firm seldom encounters superior offers in BM equilibrium. If it competes, the extra returns to the worker are small, because firm dominates most competitors. So ex ante the chance of competition and the returns from competition for the worker are small, insufficient to trigger $s = 1$. That is, for $w > \bar{w}(p)$ OJS stops anyway, so if $\omega(p) > \bar{w}(p)$ for p high then firm can match with impunity. BM equilibrium cannot survive.

A fixed cost of OJS is necessary to justify an arrival rate independent of the wage. But then BM equilibrium must break down at high wages. So all we need to show is that for $w > \bar{w}(p)$ OJS ceases anyway.

8.1. Worker Behavior

Suppose now that a firm deviates from the proposed BM equilibrium, and matches outside offers. First, we compute the value to the worker, denoted by $W_s^M(w|p)$, of spending ($s = 1$) or not ($s = 0$) OJS effort in such circumstances. As shown by PVR04, a firm that triggers Bertrand competition following an outside offer provides its employees with a higher value for equal wage. Therefore, this firm may attract workers from other firms even if offering a slightly lower wage, compensated by future wage raises. The potential for raising wages in the future to fend off outside offers depends on the firm's productivity p. Therefore, the value of employment to a worker now depends also directly on p independently of the wage earned, and we write $W_s^M(w|p)$ rather than $W_s^M(w)$. In the BM equilibrium, conversely, each firm p posts a unique wage $\omega(p)$ and does not compete *ex post*, so the wage is a sufficient statistic and we could write $W_s(w)$.

We propose the following strategies to support the BM allocation as an equilibrium. If a firm deviates and competes to retain her own employee who received an outside offer and to poach workers from other firms, the following events ensue. All the workers of the firm, current and future, raise their OJS intensity to $s = 1$ whenever this is a best response to the firm's strategy of matching outside offers. Current workers observe the firm's strategy, or "competitive corporate culture", and future workers learn about it from existing employees at the firm. Any other firms who make an offer respond to the counteroffer by the competing firm with a final counter-counteroffer, so effectively the two firms play a Bertrand game. If, however, at a firm that is competing a worker receives a low outside offer from another firm, so that the worker prefers the current terms of employment, the poacher does not raise its bid.

Let $\xi_s(w|p)$ be the wage at a non-matching firm playing the BM equilibrium that makes the worker indifferent vis-a-vis working at the deviating $p-$firm at wage w.

Notice that, when a firm pays the highest possible wage p and makes no profits out of the worker, whether the firm matches or not future outside offers becomes irrelevant, because at that point the firm can only either ignore outside offers or let the worker go, just like in the BM equilibrium.

When an outside offer arrives to a worker employed at wage w by a firm p that deviates and competes, one of three events can take place:

1. The new offer w' is superior to the current package, made of a promised wage w and future competition: $W_s^M(w|p) = W_0(\xi_s(w|p)) < W_0(w')$. Bertrand competition follows, and the more productive firm wins, because the matching firm can offer at most $W_0(p)$ and the new firm at most $W_0(p')$, and W_0 is increasing. The poacher wins the worker if the poacher has productivity $p' = \omega^{-1}(w') > p$, i.e. if $w' > \omega(p)$, and offers the worker a value $\max \langle W_0(p), W_0(w')\rangle$, namely $W_0(p)$ if the current $p-$employer can respond to $W_0(w')$, and the promised $W_0(w')$ if this is already too high for the current employer. This event has chance $1 - F(\omega(p))$.

2. As above, but $p \geq p' = \omega^{-1}(w')$, so the employer responds and retains the worker at value $W_0(\omega^{-1}(w')) = W_0(p')$, which is the maximum that the incumbent can offer. This event has chance $F(\omega(p)) - F(\xi_s(w|p))$.

3. The new offer is less attractive than the current package, so the current employer does not compete, and then nothing happens because the poacher sticks to the BM equilibrium strategy of not competing: $W_s^M(w|p) \geq W_0(w')$.

Accordingly, we can write the employed worker's Bellman equation for $w < p$:

$$\begin{aligned}(r+\delta) W_s^M(w|p) &= w - cs + \delta U + (k\delta + \lambda s)\int_p^{\infty}[W_0(w') - W_s^M(w|p)]dF(w') \\ &+ (k\delta + \lambda s)[F(p) - F(\omega(p))][W_0(p) - W_s^M(w|p)] \\ &+ (k\delta + \lambda s)\int_{\xi_s(w|p)}^{\omega(p)}[W_0(\omega^{-1}(w')) - W_s^M(w|p)]dF(w')\end{aligned}$$

The first two lines correspond to the first event and the two possibilities for the max. The third line corresponds to the second event. The third event entails no change in a worker's status, so no capital gain or loss. The first term accrues whether the incumbent firm competes or not: those are just offers that are out of the current employer's reach. The extra incentives for the worker to spend effort on OJS come from the second and third terms, in that either the current employer forces a poacher to be more aggressive to win, or the poacher forces the current employer to be more aggressive to retain the worker.

Integrating by parts as before yields

$$\int_p^{\infty} W_0(w')dF(w') = W_0(p)[1 - F(p)] + q_0(p)$$

$$\int_{\xi_s(w|p))}^{\omega(p)} W_0(\omega^{-1}(w'))dF(w') = -W_0(p)[1-F(\omega(p))] + W_0(\omega^{-1}(\xi_s(w|p))))[1-F(\xi_s(w|p)))] + \chi_s(\xi_s(w|p)|p)$$

where, using the envelope theorem,

$$\omega'(p) = l(\omega(p)) = \frac{k}{\{1 + k[1 - F(\omega(p))]\}^2}.$$

we define

$$\int_{\xi_s(w|p)}^{\omega(p)} \frac{W_0'(\omega^{-1}(w'))}{\omega'(\omega^{-1}(w'))}[1-F(w')]dw' = \int_{\xi_s(w|p)}^{\omega(p)} \frac{\{1 + k[1 - F(w')]\}^2 / k[1 - F(w')]}{r + \delta + (k\delta + \lambda s)[1 - F(\omega^{-1}(w'))]} dw' \equiv \chi_s(\xi_s(w|p)|p)$$

a function of parameters and the BM wage offer distribution F.

Collecting all terms, the value to the worker of spending OJS effort $s \in \{0, 1\}$ while working for a competing employer is

$$W_s^M(w|p) = \frac{w - cs + \delta U + (k\delta + \lambda s)\{q_0(p) + \chi_s(\xi_s(w|p)|p) + W_0(\omega^{-1}(\xi_s(w|p)))[1 - F(\xi_s(w|p))]\}}{r + \delta + (k\delta + \lambda s)[1 - F(\xi_s(w|p))]}. \tag{8.1}$$

Notice that $1 - F(\xi_s(w|p))$ is the probability that the worker gets a raise, either internally or externally.

The final step consists of solving for the cutoff wage $\xi_s(w|p)$. Replacing the expressions for $W_s^M(w|p)$ and $W_0(\xi_s(w|p))$ resp. from (8.1) and (3.6) into (4.1) and simplifying terms in U we obtain:

$$\frac{w - cs + (k\delta + \lambda s)\left[q_0(p) + \chi_s(\xi_s(w|p)|p) + W_0(\omega^{-1}(\xi_s(w|p)))[1 - F(\xi_s(w|p))]\right]}{r + \delta + (k\delta + \lambda s)\left[1 - F(\xi_s(w|p))\right]} = \frac{\xi_s(w|p) + k\delta q_0(\xi_s(w|p))}{r + \delta}. \tag{8.2}$$

Since the functions q_0 and χ_s are known from the BM equilibrium's offer distribution F, this equation determines the function $\xi_s(w|p)$. Notice that

$$W_0(\omega^{-1}(\xi_s(w|p))) = \frac{w + \delta U + k\delta q_0(\omega^{-1}(\xi_s(w|p)))}{r + \delta}$$

To show that there exists one such function, let $\phi(x) \equiv x + k\delta q_0(x)$, which is strictly increasing

$$\phi'(x) = 1 - k\delta \frac{1 - F(x)}{r + \delta + k\delta[1 - F(x)]} = \frac{r + \delta}{r + \delta + k\delta[1 - F(x)]} > 0,$$

and

$$\psi_s(x) \equiv \chi_s(x|p) + W_0(\omega^{-1}(x))[1 - F(x)],$$

which is strictly decreasing:

$$\psi_s'(x) = -f(x)\, W_0(\omega^{-1}(x)) < 0.$$

In this notation, (8.2) $\xi_s(w|p)$ is the root x of

$$\phi(x)\left\{r + \delta + (k\delta + \lambda s)\left[1 - F(x)\right]\right\} - (k\delta + \lambda s)\left[\psi_s(x) + q_0(p)\right] = (w - cs)(r + \delta)$$

The LHS is increasing globally as

$$d\{\phi(x)\left[1 - F(x)\right]\}/dx = \phi'(x)\left[1 - F(x)\right] - f(x)\,\phi(x)$$

is positive using the previous expressions and after some algebra, ϕ and $-\psi$ are increasing. Therefore, under appropriate boundary conditions, there exists a well-defined function $\xi_s(w|p)$. Once this is known, we can solve for $W_s^M(w|p)$ from (7.2).

It is clear that a worker will never find it optimal to search actively on the job independently of the wage. In fact, the worker will respond to the employer's offer-matching strategy by raising OJS effort to $s = 1$ as long as the wage w does not exceed a threshold: $w \leq \bar{w}(p) < p$ defined by

$$W_1^M(\bar{w}(p)|p) = W_0^M(\bar{w}(p)|p).$$

To show that there exists at most one such maximum wage $\bar{w}(p)$ that creates moral hazard problems, let

$$z(w|p) \equiv W_1^M(w|p) - W_0^M(w|p).$$

Taking a derivative

$$\begin{aligned} z'(w|p) &= \frac{1}{r+\delta+(k\delta+\lambda)\left[1-F(\xi_1(w|p))\right]} - \frac{1}{r+\delta+k\delta\left[1-F(\xi_0(w|p))\right]} \\ &\propto k\delta\left[1-F(\xi_0(w|p))\right]-(k\delta+\lambda)\left[1-F(\xi_1(w|p))\right] \\ &= k\delta\left[F(\xi_1(w|p))-F(\xi_0(w|p))\right]-\lambda\left[1-F(\xi_1(w|p))\right]. \end{aligned}$$

Since $W_0(\cdot)$ is increasing from (3.5), (4.1) implies

$$z(w|p) \leq 0 \Leftrightarrow W_1^M(w|p) \leq W_0^M(w|p) \Leftrightarrow \xi_1(w|p) \leq \xi_0(w|p) \Rightarrow z'(w|p) < 0.$$

So, once z is non-positive, and in fact positive but small enough, it declines. As $z(\bar{w}(p)|p) = 0$ by definition, $z'(\bar{w}(p)|p) < 0$ and then $z(w|p) < 0$ for all $w > \bar{w}(p)$. This shows that there exists at most one threshold wage $\bar{w}(p)$ where the worker stops searching on the job after receiving a sufficiently generous raise by her employer.

8.2. The Deviant Firm's Profits

Once the firm starts matching outside offers and workers respond by searching at maximum OJS intensity, the wage loses its retention role, because the firm can always offer the minimum wage that attracts the worker, and then raise it when necessary. Therefore, let $R(p)$ denote the reservation wage that the matching firm can offer unemployed workers when the "corporate culture" prescribes searching actively:

$$W_1^M(R(p)|p) = U.$$

The deviant firm generally posts $R(p)$. But, when an employed applicant who is earning w applies, the deviant firm offers w' such that the worker is made indifferent whether to join, $W_1^M(w'|p) = W_0(w)$, namely, $w' = \xi_1^{-1}(w|p)$.

In the long run, the deviant firm has a distribution of wages on $[R(p), p]$, due to the different offer histories of its employees. To compute this (non-normalized) distribution $H(w|p)$ with density $h(w|p)$ on $[R(p), p]$ and firm size $H(p|p)$, we impose that it is stationary.

First, we compute the inflow into wage w. We start with the initial wage $R(p)$, initially offered to all workers. This guarantees an unemployed worker just a value U. Since other firms offer values at least as large as U, they would not accept this low wage $R(p)$ at the deviant firm. So the inflow into $R(p)$ can occur only from unemployment, because employed job applicants earn at least b. Because unemployed workers produce a total of $u\lambda s_0 = uk\delta = k\delta/(1+k)$ hires, and there are m firms, the flow of hires from unemployment for each firm is $k\delta/[m(1+k)]$.

If $R(p) < w < p$, then job applications and hires may arrive also from other firms' employees. To be hired exactly at wage w it must be the case that a worker was previously employed at wage $\xi_1^{-1}(w|p)$. Because there is a measure $g(\xi_1^{-1}(w|p))$ of such workers in the market, who choose OJS intensity $s = 0$, and their applications at rate $k\delta$ land randomly on all m firms, the inflow from other firms is $k\delta g(\xi_1^{-1}(w|p))/m$.

Finally, inflow into wage w can occur from the ranks of the same firm, because of a wage raise from some low wage $w'' < w$ to w following an outside offer w'. This takes place when the outside offer w' is such that $W_1^M(w|p) = W_0(\omega^{-1}(w'))$, where $\omega^{-1}(w')$ is the productivity of the competitor and $W_0(\omega^{-1}(w'))$ is the maximum value that the competitor can offer after Bertrand competition, which develops and is won by the current employer with probability $F(\omega(p)) - F(\xi_1(w''|p))$. Since $W_1^M(w|p) = W_0(\xi_1(w|p))$ by definition, the new offer must be exactly equal to $w' = \omega(\xi_1(w|p))$ and the wage w'' before competition must be such that $W_0(w') > W_1^M(w''|p) = W_0(\xi_1(w''|p))$, or $\xi_1(w''|p) < w' = \omega(\xi_1(w|p))$ to trigger renegotiation to a new wage exactly equal to w without a quit. So all those wages $w'' < \xi_1^{-1}(\omega(\xi_1(w|p))|p)$ are upgraded to w when an outside offer $w' = \omega(\xi_1(w|p))$ arrives. There are $H(\xi_1^{-1}(\omega(\xi_1(w|p))|p))$ workers employed by the firm at such low wages, and each of them locates outside offers that trigger competition ending up with a raise just to w at rate $(k\delta + \lambda) f(\omega(\xi_1(w|p)))$.

Summing up

$$\text{Inflow into } w = \begin{cases} \frac{k\delta}{m(1+k)} & \text{for } w = R(p) \\ m^{-1}k\delta g(\xi_1^{-1}(w|p)) + H(\xi_1^{-1}(\omega(\xi_1(w|p))|p)\,(k\delta + \lambda)\, f(\omega(\xi_1(w|p))) & \text{for } R(p) < w \leq p \\ 0 & \text{otherwise} \end{cases}.$$

Next, we compute the outflow from wage w. This is made of two components: either the worker quits, exogenously at rate δ and endogenously at rate $(k\delta + \lambda)[1 - F(\omega(p))]$ (when receiving an offer from a firm with $p' > p$, namely an offer $w' = \omega(p) > w$) or he stays but gets a raise, at rate $(k\delta + \lambda s)[F(\omega(p)) - F(\xi_1(w|p))]$. Either way, the worker leaves the position w in the within-firm wage distribution. Here OJS effort s can be either 0 or 1 depending on whether the worker is already paid above $\bar{w}(p)$ and very close to the firm's valuation p, so further competition is not useful to the worker, or otherwise.

Summing up:

$$\text{Outflow from } w = \begin{cases} H(R(p)|p)(\delta + k\delta + \lambda) & w = R(p) \\ h(w|p)\{\delta + (k\delta + \lambda)[1 - F(\xi_1(w|p))]\} & \text{for } R(p) < w \leq \bar{w}(p) \\ h(w|p)\{\delta + k\delta[1 - F(\xi_1(w|p))]\} & \text{for } \bar{w}(p) < w \leq p \\ 0 & \text{otherwise} \end{cases}$$

Equating inflow and outflow allows to solve for the atom of workers who were hired straight from unemployment and never got an outside offer so far

$$H(R(p)|p) = \frac{k\delta}{m\,(1+k)\,(\delta + k\delta + \lambda)}$$

and produces, for $w > R(p)$, a differential equation in $h(w|p)$ given the BM equilibrium distributions of offered wages f and paid wages g.

The total steady state undiscounted profits to the firm from the matching deviant strategy are

$$\begin{aligned} \pi_1^M(p) &= H(R(p)|p)[p - R(p)] + \int_{R(p)+}^{p} (p - w)h(w|p)dw \\ &= H(R(p)|p)[p - R(p)] + [H(p|p) - H(R(p)|p)]\,p - \int_{R(p)+}^{p} wh(w|p)dw \end{aligned}$$

simplifying terms and integrating by parts

$$\begin{aligned}
\pi_1^M(p) &= H(p|p)p - H(R(p)|p)R(p) + \int_{R(p)+}^{p} wd\left[H(p|p) - H(w|p)\right] \\
&= H(p|p)p - H(R(p)|p)R(p) - R(p)\left[H(p|p) - H(R(p)|p)\right] - \int_{R(p)}^{p}\left[H(p|p) - H(w|p)\right]dw \\
&= H(p|p)\left[p - R(p)\right] - \int_{R(p)}^{p}\left[H(p|p) - H(w|p)\right]dw \\
&= \int_{R(p)}^{p} H(w|p)dw.
\end{aligned}$$

9. Equilibrium

In order for the BM allocation to be an equilibrium, we require, for all $p \in$supp(J), $w \in [R(p), p]$:

1. When the firm follows the BM equilibrium strategy without matching outside offers, the worker is better off not searching on the job actively:

$$W_0(w) \geq W_1(w).$$

2. When the firm deviates and matches one outside offer, every employee of that firm who is being paid less than $\bar{w}(p)$ is better off raising OJS effort to the level that is optimal if the firm will match for ever.

$$W_1^M(w|p) \geq W_0^M(w|p) \text{ for all } w \leq \bar{w}(p).$$

   Because each worker knows that her co-workers will raise their OJS effort in response to the firm's deviation, she knows that the firm has no longer a credible threat to let workers go and will match all future outside offers.

3. The firm prefers to match outside offers when all workers are searching on the job at high intensity:

$$\pi_1^M(p) \geq \pi_1(p).$$

4. When workers receive offers higher than the BM equilibrium wage $\omega(p)$, their employer prefers to let them go than to match even a single outside offer and then trigger high OJS effort by all current and future employees:

$$\pi_0(p) \geq \pi_1^M(p).$$

The First Inequality. It is clear that the first two inequalities can be satisfied for a set of OJS costs $[\underline{c}(p), \bar{c}(p)]$. Are $\underline{c}(\cdot), \bar{c}(\cdot)$ increasing or decreasing? Notice that in the candidate equilibrium wages are in 1:1 correspondence with productivity. Furthermore, from (3.5) we get $W_0'(w) > W_1'(w)$, so the first inequality is most binding for employees of small, low-productivity, low-wage firm who have the most to gain from upgrading. Therefore, $\underline{c}(p)$ is decreasing in p, and the global requirement is that the inequality holds at the lowest possible equilibrium wage: $W_0(b) > W_1(b)$.

Let

$$\begin{aligned}
\beta(w) &\equiv (k\delta + \lambda) q_1(w) - k\delta q_0(w) \\
&= \int_w^\infty \left\{ \frac{(k\delta+\lambda)[1-F(x)]}{r+\delta+(k\delta+\lambda)[1-F(x)]} - \frac{k\delta[1-F(x)]}{r+\delta+k\delta[1-F(x)]} \right\} dx \\
&= \lambda(r+\delta) \int_w^\infty \frac{1-F(x)}{\{r+\delta+(k\delta+\lambda)[1-F(x)]\}\{r+\delta+k\delta[1-F(x)]\}} dx
\end{aligned}$$

with $\beta'(w) < 0 < \beta(w)$. The inequality $W_0(b) > W_1(b)$ can be written as

$$\frac{b+\delta/r[b+k\delta q_0(b)] + k\delta q_0(b)}{r+\delta} > \frac{b+\delta/r[b+k\delta q_0(b)] + (k\delta+\lambda)q_1(b) - c}{r+\delta}$$

and we obtain the desired lower bound on OJS costs.

$$c \geq \beta(b) \equiv \underline{c} > 0.$$

The Second Inequality. The first inequality implies $W_0(p) \geq W_1(p)$. Since $W_s^M(p|p) = W_s(p)$, this implies $z(p|p) \leq 0$. So a threshold wage $\bar{w}(p) \leq p$ exists, where a worker reduces OJS intensity to $s = 0$ even if her employer competes. We have already proven that this threshold is unique. The second inequality is $z(w|p) \geq 0$ for all p and all $w \in [R(p), \bar{w}(p)]$. As $z(\bar{w}(p)|p) = 0$ by definition, $z'(\bar{w}(p)|p) < 0$ and then $z(w|p) < 0$ for all $w > \bar{w}(p)$. As a consequence, we have to concentrate on the second inequality at the lower end of the wage support: $z(R(p)|p) \geq 0$. This is sufficient for the second inequality to hold for a non-empty set of wages $[R(p), \bar{w}(p)]$.

$$U = W_1^M(R(p)|p) \geq W_0^M(R(p)|p) = W_0(\xi_0(R(p)|p)) = \frac{\xi_0(R(p)|p) + \delta U + k\delta q_0(\xi_0(R(p)|p))}{r+\delta}$$

$$U \geq \frac{R(p) + \delta U + k\delta \{q_0(p) + \psi(\xi_s(w|p)|p) + W_0(\omega^{-1}(\xi_s(w|p)))[1-F(\xi_s(w|p))]\}}{r+\delta+(k\delta+\lambda s)[1-F(\xi_s(w|p))]}$$

equivalently

$$rU = b + k\delta q_0(b) \geq \xi_0(R(p)|p) + k\delta q_0(\xi_0(R(p)|p))$$

or

$$\begin{aligned}
\phi(b) &\geq \phi(\xi_0(R(p)|p)) \\
b &= \xi_1(R(p)|p) > \xi_0(R(p)|p).
\end{aligned}$$

Recall from (4.1) that $\xi_0(w|p)$ is the wage that a worker must earn at a non-matching firm to be indifferent relative to working at wage w at a matching firm of productivity p, in both cases keeping zero OJS effort. Is $\xi_0(R(p)|p)$ increasing or decreasing in p? There are two opposing forces at play. On the one hand, as p rises the matching firm can afford paying a lower reservation wage to unemployed job applicants, so $R(p)$ falls, and $\xi_0(w|p)$ is clearly increasing in w. On the other hand, as p rises the opportunities for future internal raises expand, and the equivalent wage $\xi_0(w|p)$ rises for given w.

The Third Inequality. The third inequality is always true due to a simple dominance argument. Once workers search on the job at high intensity, the firm could keep paying the same wage, until forced to raise it by outside competition. Therefore, the firm would retain the same workers as before, plus a few more at a higher wage rather than losing them altogether to other firms. After re-optimizing on wages, the firm could do even better. So when all workers search actively on the job, the firm is better off matching their outside offers. This is the key to our reputational commitment.

A requirement for the last two inequalities to hold is that $\pi_0(p) \geq \pi_1(p)$: if the firm does not match, active OJS by all workers reduces profits. Again, this follow from a simple dominance argument. If the firm maintains its strategies and workers raise their OJS effort, firm size declines from $l(w)$ to $l_1(w) < l(w)$. After re-optimizing

$$\pi_1(p) = \max_{x\in[b,p]} l_1(x)(p-x) = l_1(x(p))(p-x(p)) < l(x(p))(p-x(p)) \leq \max_{w\in[b,p]} l(w)(p-w) = \pi_0(p).$$

We conclude that we can ignore the third inequality.

The Fourth Inequality. The last inequality is the heart of the matter. Due to the complexity of the computation of the deviant firm's eventual wage distribution H, for now we illustrate only one example.

9.1. Example (in progress)

Similarly to BM, Section 4, let $J(p) = p$ on $p \in [0,1]$, $b = 0$. The FOC for a firm in the BM equilibrium gives a solution for the BM equilibrium:

$$\begin{aligned}
\omega(p) &= \frac{k}{1+k}p^2 \\
F(w) &= \omega^{-1}(w) = \sqrt{\frac{1+k}{k}w} \\
G(w) &= \frac{\sqrt{\frac{1+k}{k}w}}{1+k\left[1-\sqrt{\frac{1+k}{k}w}\right]}
\end{aligned}$$

with upper bound to the wage support $\bar{w} = k/(1+k) < 1$. The total profits of a $p-$firm in the BM equilibrium are

$$\pi_0(p) = \frac{k}{\{1+k(1-p)\}^2}\left(p - \frac{k}{1+k}p^2\right) = \frac{kp}{1+k(1-p)}$$

which rises from 0 to k as p rises from 0 to 1.

To compute the function

$$q_s = \int_w^{\frac{k}{1+k}} \frac{1-\sqrt{\frac{1+k}{k}x}}{r+\delta+(k\delta+\lambda s)\left(1-\sqrt{\frac{1+k}{k}x}\right)}dx$$

change variables to

$$\begin{aligned} y &= 1-\sqrt{\frac{1+k}{k}x} \\ x &= \frac{k}{1+k}(1-y)^2 \\ dy &= -\frac{1}{2}\sqrt{\frac{1+k}{kx}}dx = -\frac{1}{2(1-y)}\frac{1+k}{k}dx \end{aligned}$$

to obtain

$$q_s(w) = \frac{(r+\delta)\ln(r+\delta)-(r+\delta)\ln\left(r+\delta+(k\delta+\lambda s)\left(1-\sqrt{\frac{1+k}{k}w}\right)\right)-\frac{1}{2}(k\delta+\lambda s)\ln\left(\frac{1+k}{k}w\right)}{(\delta+k\delta+\lambda s+r)(k\delta+\lambda s)}$$

with

$$q_s'(w) = \frac{\frac{1}{2}\sqrt{\frac{1+k}{kw}}\frac{r+\delta}{r+\delta+(k\delta+\lambda s)\left(1-\sqrt{\frac{1+k}{k}w}\right)}-\frac{k\delta+\lambda s}{2w}}{(\delta+k\delta+\lambda s+r)(k\delta+\lambda s)}$$

Next

$$\begin{aligned} rU &= b+k\delta q_0(b) \\ W_s(w) &= \frac{w+\delta U+(k\delta+\lambda s)q_s(w)-cs}{r+\delta} \\ W_s(\omega^{-1}(x)) &= \frac{w+\delta U+(k\delta+\lambda s)q_s(\sqrt{\frac{1+k}{k}x})-cs}{r+\delta} \\ \chi_s(x|p) &= \int_x^{\frac{k}{1+k}p^2} \frac{\left\{1+k\left[1-\sqrt{\frac{1+k}{k}y}\right]\right\}^2}{k\left[1-\sqrt{\frac{1+k}{k}y}\right]\left\{r+\delta+(k\delta+\lambda s)[1-\left(\frac{1+k}{k}\right)^{\frac{3}{4}}y^{\frac{1}{4}}]\right\}}dy \end{aligned}$$

$\phi(x) \equiv x+k\delta q_0(x)$, with

$$\phi'(x) = 1-k\delta\frac{1-F(x)}{r+\delta+k\delta[1-F(x)]} = \frac{r+\delta}{r+\delta+k\delta[1-F(x)]} > 0,$$

and $\psi_s(x) \equiv \chi_s(x|p)+W_0(\omega^{-1}(x))[1-F(x)]$, In this notation, (7.3) $x = \xi_s(w|p)$ is the root of

$$[w-cs-x-k\delta q_0(x)]\frac{r+\delta}{k\delta+\lambda s} = \left[x+k\delta q_0(x)-W_0(\omega^{-1}(x))\right]\left[1-\sqrt{\frac{1+k}{k}x}\right]-\chi_s(x|p)-q_0(p)$$

$$\frac{(w-cs-x)(r+\delta)}{k\delta}$$

$$-\frac{(r+\delta)\ln(r+\delta)-(r+\delta)\ln\left(r+\delta+k\delta\left(1-\sqrt{\frac{1+k}{k}x}\right)\right)-\frac{1}{2}k\delta\ln\left(\frac{1+k}{k}x\right)}{\delta+k\delta+r}\left(\frac{r+\delta}{k\delta}-1+\sqrt{\frac{1+k}{k}x}\right)$$

$$=\left[x-W_0(\omega^{-1}(x))\right]\left[1-\sqrt{\frac{1+k}{k}x}\right]-\chi_s(x|p)-k\delta q_0(p)$$

From this function we can compute numerically all the values.

For each $p\in[0,1]$, $w\in[0,1]$, $s\in\{0,1\}$, compute:

1. $q_0(p)$;
2. $\chi_s(x|p)$;
3. $q_0(b)$ and U;
4. $$W_0(\omega^{-1}(x)) = \frac{x+\delta U+k\delta q_0(\omega^{-1}(x))}{r+\delta}$$ $$= \frac{x+\delta U}{r+\delta}+\frac{\ln(r+\delta)-\ln\left(r+\delta+k\delta\left(1-\left(\frac{1+k}{k}\right)^{\frac{3}{4}}x^{\frac{1}{4}}\right)\right)-\frac{1}{2}\frac{k\delta}{r+\delta}\ln\left(\left(\frac{1+k}{k}\right)^{\frac{3}{4}}x^{\frac{1}{4}}\right)}{\delta+k\delta+r}$$
5. $x=\xi_s(w|p)$;

We calibrate at quarterly frequency, and choose $b=0.0001$ (rather than $b=0$ for numerical reasons), $r=0.01$, $\delta=0.04$, $\lambda=s_0=s_1=m=1$. We find $\beta(b)=0.17=\underline{c}$, $\bar{c}=.71$, and the second inequality is most binding for the least productive firm. Most importantly, the fourth inequality, the incentive constraint for the firm not to match outside offers, is most binding for relatively low-productivity firms. For p close to 1 the opportunities for extra raises following Bertrand competition are limited, because the firm has few valid competitors.

10. Conclusions

Competition between firms for a worker employed at one of them is probably a relatively rare phenomenon. The equilibrium search literature has analyzed formal settings where this form of competition is either ruled out by assumption, or allowed and unfolding fully. In this paper we explore reasons why this type of competition, albeit always possible and in the interest of an employer, will not in fact take place in equilibrium. These reasons are all based on workers' moral hazard in on the job search. Employed workers have some control over the effort they can spend to generate outside offers, and this effort is typically not contractible. Therefore, a firm's wage policy affects the incentives not only to accept outside offers, as emphasized by the existing search literature, but also to generate them.

We focus on the "corporate culture" as a reputational device for the firm to commit to let workers go without competing. Workers employed at a firm know that it is unprofitable to spend effort to generate outside offers if their employer will not compete for them. Only *ex post* competition would generate the returns to search that are necessary to justify costly effort. But workers also know that, as soon as the firm falls for the temptation and competes to retain a worker, all the firm's employee will observe this change in "corporate culture" from "loyal" and non-competitive to aggressive and competitive. The change in the firm's stance towards outside offers will provide a coordination device for all workers to raise their OJS effort, anticipating that, once this happens, the firm will no longer have a credible threat not to compete, and the returns to generate outside offers will materialize. Therefore, the firm refuses to match outside offers and accepts the turnover costs of losing valuable employees, even when it could retain them, in order to keep in line the incentives of its workforce, which is watching. This corporate culture maintains high turnover and preserves some of the monopsony power that search frictions provides the firm with. Yet, workers earn moral hazard rents, because wages exceed the reservation value in order to maintain the incentives not to search on the job.

The assumption that firms do not discount payoffs, but only care about average undiscounted profits, is important. In this case, the loss of one specific employee entails no cost to the firm. The moral hazard problem is only 'systemic': if workers search at high intensity for outside offers, the firm ends up with either paying higher wages or employing fewer workers. Only the long-run implications of competing to retain a specific worker matter. This is an assumption that deserves to be relaxed, both here and in the original BM setup.

A likely direction of future research in the search approach to labor markets concerns the origin of workers' rents. Reduced-form bargaining games may find more explicit foundations in the moral hazard problem analyzed in this paper. The worker's bargaining power originates not only from the threat of quitting, but also from the threat of generating outside offers more intensively. Therefore, even if the firm makes take-it-or-leave-it wage offers, it must leave workers some rents as a discipline device.

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