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Rent Rigidity, asymmetric information, and volatility bounds in labor markets ☆

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ABSTRACT

Two thirds of US unemployment volatility is due to fluctuations in workers' job-finding rate. In search and matching models, aggregate productivity shocks generate such fluctuations: via inputs in the matching technology, they affect the rate at which workers and firms come into contact. Quantitatively, this mechanism has been found to be negligible in a calibrated textbook model, but also more than sufficient if wages are completely rigid. We study a weaker concept of rigidity based on worker rents (wages in excess of the value of unemployment). We show that volatility is subject to an upper bound if worker rents are weakly procyclical, thus at best rigid. Quantitatively, with *Rent Rigidity*, the mechanism accounts for at most 20% of the variance of the job-finding rate. In light of this result we reexamine the question whether asymmetric information on gains from trade amplifies fluctuations. We analyze a series of bargaining solutions, and conclude that asymmetric information at best makes rents rigid. Our analysis provides a unifying perspective on a very lively debate.

1. Introduction

Recent research has shown that about two thirds of US unemployment volatility is due to fluctuations in the rate at which workers find jobs.[1] This has prompted quantitative investigations into mechanisms that give rise to such fluctuations. The search and matching model, the canonical framework of analysis of unemployment, features just such a mechanism. Creating job contacts and overcoming trading frictions requires two essential inputs, unemployed workers and vacant jobs. These costly resources vary over the cycle, generating fluctuations in the rate at which workers come into contact with firms, thereby in the job-finding rate. Shimer (2005) finds that this mechanism generates negligible fluctuations in the job-finding rate in response to productivity shocks in a calibrated textbook search and matching model. Replacing the textbook

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[1] Shimer (2007) argues this fraction is three quarters, Fujita and Ramey (2009) argue for a somewhat lower fraction based on different data and methodology.

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assumption of Nash Bargaining with a fully rigid wage, Hall (2005) finds that the mechanism generates more than enough volatility. Thus, fluctuations in the contact rate can account for the empirical volatility of the job-finding rate if wages are sufficiently but not fully rigid. In this paper we identify the *qualitative properties* of wage setting that deliver sufficient rigidity in this sense. Our answer allows us to shed light on a conjecture put forward by Shimer (2005), namely that private information can induce sufficient wage rigidity.

We introduce a notion of rigidity weaker than full wage rigidity. In frictional labor markets the compensation of a worker has two components: first, an opportunity cost, the value of unemployed job search; second, a rent component, absent in frictionless labor markets. Conceptually, this leads to a distinction between two types of rigidity: *Wage Rigidity* as in Hall (2005), where the sum is acyclical; *Rent Rigidity*, where only the rent is acyclical. The latter is weaker: the unemployed find jobs more easily in booms, raising the opportunity cost of accepting a job offer; hence rigid wages imply countercyclical rents, while rigid rents imply procyclical wages. We study how Rent Rigidity amplifies fluctuations in the job contact rate. Does it suffice to account for the volatility of the job-finding rate? Or must workers rents be countercyclical?

Our first contribution is to show that if worker rents are weakly procyclical, thus at best rigid, then the general equilibrium response of the contact rate between unemployed workers and open vacancies to a productivity shock is subject to an upper bound. Evaluating this volatility bound quantitatively, we find that through the contact rate channel alone, productivity shocks generate at most 20% of the observed cyclical variance of the job-finding rate. This compares to 3% for Nash Bargaining, while Wage Rigidity substantially overshoots 100%. Empirically, the imperfect correlation of job-finding rate and labor productivity indicates that the latter cannot be the only driving force, but contributes about 40%. Thus, unlike in models with Nash Bargaining or Wage Rigidity, under Rent Rigidity the contact rate channel generates fluctuations of the right order of magnitude. But it falls short, and a quantitatively important part of fluctuations in the job-finding rate remains unexplained. The job-finding rate is the product of two factors; the contact rate, and the probability of trade once search frictions are overcome. The latter is a source of employment fluctuations already present in frictionless models. Shimer and Hall study a model with a constant probability of trade, hence their findings concern the specific contribution of variation in inputs in the matching technology to labor market volatility. In the same spirit, our bound applies to the contact rate, but does not rule out that the balance of fluctuations in the job-finding rate is due to variation in the probability of trade.

Our bound implies that all modifications of wage setting that deliver sufficient rigidity, such as Hall and Milgrom's (2008) assumption of strategic bargaining, have a common denominator: in some way they must generate countercyclical worker rents. That is to say, an unemployed worker gains more from a job contact in a recession than in a boom. We discuss the economic forces underlying countercyclical rents in modifications that have been proposed. This sets the stage for our second contribution. Taking up Shimer's conjecture, we pose the question: is private information a potential source of countercyclical rents? We analyze a series of classic bargaining models, and our results indicate that the presence of private information by itself does not give rise to countercyclical worker rents.

The mechanics underlying the volatility bound are as follows. Worker rents represent the capital gain that an unemployed worker realizes upon meeting a firm. Importantly, if rents are rigid, then this capital gain is as large in booms as it is in recessions. Now suppose a positive productivity shock induces firms to raise their recruiting effort, increasing the contact rate. Then the value of unemployed search increases, not because an unemployed worker gains more from a contact with a vacancy, but because contacts are more frequent. This increase in the opportunity cost is transmitted to wages, and absorbs a significant share of additional productivity, a *feedback effect* that limits vacancy creation. For comparison, with Nash Bargaining both worker rents and the contact rate are procyclical. Our bound isolates the limits on volatility imposed by the latter.

The feedback effect vanishes with the workers' gains from market activity. Thus, quantitatively, the size of these gains is a key determinant of our bound. Their size, in turn, depends on the value of non-market activity, derived by the unemployed from unemployment benefits and leisure. Our results rely on Hall and Milgrom's (2008) estimate of this parameter, which utilizes evidence on the Frisch elasticity of labor supply. Hagedorn and Manovskii (2008) obtain sufficient rigidity through a different calibration strategy, rather than wage setting protocol. Maintaining Nash Bargaining, they match the cyclical behavior of aggregate wages with a much larger calibrated value of non-market activity, close to wages. This has prompted criticism, due to the implication that gains from working in the market are minuscule.

To investigate Shimer's conjecture, we introduce private information about gains from trade in labor markets as follows. Upon being matched, the firm privately draws a match-specific productivity component and the worker a match-specific amenity value. The distributions are fixed and independent of (publicly observed) aggregate labor productivity, satisfy standard regularity conditions, but are otherwise unrestricted, affording many degrees of freedom. Our bound allows a *systematic* analysis whether they suffice to attain Wage Rigidity. We study a series of classic models of bargaining in this setting. For each model, we analyze its comparative statics properties, and verify that our bound applies; worker rents are weakly procyclical, hence at best Rent Rigidity but not Wage Rigidity can be attained. The simplicity of this approach affords easy extensions and applications of our comprehensive but certainly not exhaustive analysis (we discuss competitive search shortly).

We start with the monopoly (or monopsony) solution, where a privately informed party receives a take-it-or-leave-it offer. Why are worker rents procyclical in this model? If the firm is making the offer, then its increased willingness to trade in booms leads it to concede higher informational rents to the worker. If the worker is making the wage request, the firm's increased willingness to trade in booms enables the worker to extract higher rents. This basic intuition carries over when we relax the commitment assumptions associated with take-it-or-leave-it offers, and study strategic bargaining, both repeated one-sided offers (Coase conjecture) and alternating offers. Finally, in an environment with two-sided private information we study allocations obtained with the help of a mediator (e.g. an arbitrator in wage negotiations), specifically ex ante incentive efficient allocations which are implementable through (ex post individually rational) sealed-bid mechanisms, and obtain similar results.

Others investigated Shimer's conjecture. Guerrieri (2007) studies competitive search with private information. Simulations for various type distributions show little amplification. Additionally she verifies that her model is subject to our bound, which limits volatility for *any* type distribution. Moen and Rosén (2008) study a setting which combines competitive search and moral hazard. They also verify that the response of the contact rate to changes in aggregate labor productivity is subject to our bound. In addition, they analyze how the labor market responds to changes in the extent of private information. This is related to Kennan (2009) and Menzio (2005b), who study models in which both aggregate labor productivity and the distribution of types fluctuate exogenously, and the two driving forces are assumed to be correlated. In these papers private information plays a dual role. First, as in our setup, the presence of private information changes how the contact rate responds to changes in aggregate labor productivity. Second, fluctuations in private information themselves induce volatility. Our analysis complements these papers, demonstrating why they need both channels to account for observed volatility.

The paper is organized as follows. In Section 2 we introduce the economy. In Section 3 we derive the volatility bound and evaluate it quantitatively. In Section 4 we study comparative statics of rents in specific wage setting models, with a focus on private information. Section 5 provides a qualitatively stronger bound and discusses models that attain it. In Section 6 we discuss implications of on-the-job search. Section 7 concludes.

2. The economy

2.1. Preferences and technology

We extend the model in Shimer (2005) to allow for two-sided private information about gains from trade in the labor market. The economy is populated by a measure 1 of workers and a continuum of firms. All agents are infinitely-lived, risk neutral and discount at rate $r > 0$. Workers can either be employed or unemployed. An unemployed worker searches for a job, and enjoys flow utility b, referred to as the value of non-market activity. Firms can maintain vacancies at flow cost c. The flow of new matches is given by the constant returns to scale matching function $M(u, v)$ where u and v denote unemployment and vacancies, respectively. The ratio $\theta \equiv \frac{v}{u}$ is referred to as labor market tightness. Vacancies contact workers at rate $q(\theta) \equiv M(1/\theta, 1)$ while workers contact firms at rate $m(\theta) \equiv M(1, \theta)$.

Upon being matched, the worker draws a match-specific amenity value z from the distribution F_Z and the firm draws a match-specific productivity component y from F_Y. The former may be private information of the worker, the latter of the firm. Both remain constant until the match dissolves. Means of F_Z and F_Y are normalized to zero. Output of the match is $p + y$, where p denotes aggregate labor productivity. The worker's flow payoff is the sum of amenity z and wage payments. Matches dissolve exogenously at rate δ.

2.2. Driving force

Our goal is to study labor market volatility due to labor productivity shocks. We could calibrate the productivity process and simulate the model. But for our purpose of obtaining an upper bound on volatility, we can simply examine the response to a permanent shock. This works because job creation is driven by the expected present discounted value of profits over the life of a match, which responds more to permanent than to transitory shocks.[2] This has the benefit of delivering a simple formula for the upper bound. Moreover, it comes at virtually no cost, since the (fairly standard) empirical measures of US labor productivity that we choose for our quantitative evaluation are highly persistent.[3]

2.3. Wage determination

Rather than taking various models of wage determination, analyzing one by one their implications for volatility, our approach asks how *qualitative* properties of wage determination limit volatility. For this we need a general definition of a model of wage determination.

Due to search frictions, a match generates quasi-rents. A model of wage determination maps the variables that describe the environment of a match into a decision whether to trade and a division of these rents. Here the assumption that shocks are permanent keeps matters simple: the environment of the match is characterized by the pair of match-specific values y and z, and only two aggregate variables. First, productivity p, constant over the life of the match. Second, the endogenous

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[2] Mortensen and Nagypál (2007) and Hornstein et al. (2005) show this formally. In Brügemann and Moscarini (2007) we consider a two-state Markov process for productivity, and illustrate the role of persistence using the model of take-it-or-leave-it offers by firms to privately informed workers.

[3] As a check, in Brügemann and Moscarini (2007) we simulate a model with rigid rents using Shimer's (2005) calibrated productivity process, and find that the volatility it generates is approximated very well by the response to a permanent shock. This parallels Shimer's finding for the model with Nash Bargaining.

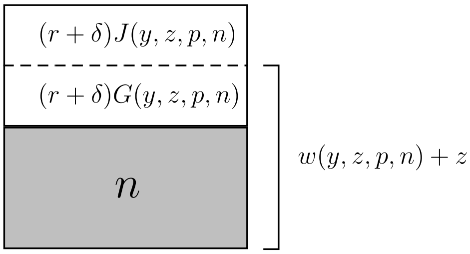

Fig. 1. Model of wage determination.

opportunity cost of the worker, given by the utility of unemployed workers U. Define $n \equiv rU$ as the opportunity cost in flow terms. It is a jump variable and constant along the equilibrium path induced by a permanent shock.

The timing of wage payments over the life of a match does not affect labor market fluctuations. Thus we write the model of wage determination directly in terms of present values of rents going to worker and firm, denoted $G(y,z,p,n)$ and $J(y,z,p,n)$, respectively. Let $x(y,z,p,n)$ be the probability that a match forms.

Definition 1. A *model of wage determination* is a triple of functions $\{G, J, x\}: \mathbb{R}^4 \to \mathbb{R}^2_+ \times [0,1]$ satisfying

$$(r+\delta)\big[G(y,z,p,n) + J(y,z,p,n)\big] = x(y,z,p,n)(y+z+p-n).$$

This definition accommodates models in which wage negotiations occur after the match-specific values y and z are realized, as well as models in which they occur before.

In more familiar notation $W \equiv G + U$ is worker utility. The annuity value of wage payments is $w(y,z,p,n) \equiv (r+\delta)G(y,z,p,n) + n - z$, which would equal the wage if wage payments were constant over the life of the match.

Fig. 1 illustrates this definition, decomposing the total flow value of a match into opportunity cost and rents. The worker's opportunity cost n is shown in gray, that of the firm is zero. Flow rents, shown in white, are comprised of firm and worker rents. Adding n to the latter and subtracting the amenity value z yields the annuity value of wages.

We illustrate the definition through two specific models which serve as benchmarks throughout the paper. Both feature no heterogeneity in y and z around their zero mean, which also implies that there is no private information.

2.3.1. Nash Bargaining

Workers receive a share $\beta \in (0,1)$ of total rents: in flow terms

$$(r+\delta)G(p,n) = x(p,n)\beta(p-n),$$
$$(r+\delta)J(p,n) = x(p,n)(1-\beta)(p-n).$$

The probability of trade is one if forming the match is privately efficient, zero otherwise: $x(p,n) = \mathbb{I}\{p-n \geqslant 0\}$ where $\mathbb{I}$ is an indicator function.

2.3.2. Wage Rigidity

Trade is efficient $x(p,n) = \mathbb{I}\{p-n \geqslant 0\}$. The annuity value of wages w does not respond to changes in p and n, as long as this is consistent with efficient trade:

$$(r+\delta)G(p,n) = \begin{cases} w-n, & p \geqslant w \geqslant n, \\ p-n, & w > p \geqslant n, \\ 0, & \text{otherwise}, \end{cases} \qquad (r+\delta)J(p,n) = \begin{cases} p-w, & p \geqslant w \geqslant n, \\ p-n, & p \geqslant n > w, \\ 0, & \text{otherwise}. \end{cases}$$

The two models are compared in Fig. 2. Each panel replicates Fig. 1 for two aggregate states. Anticipating properties of equilibrium, we assume that the high state exhibits higher productivity $p^H > p^L$, higher opportunity cost $n^H > n^L$, and higher total rents $p^H - n^H > p^L - n^L$. Panel (a) displays Nash Bargaining. Worker rents are a fixed share of total rents, thus inherit their procyclicality. Panel (b) shows Wage Rigidity: the annuity value of wages is constant while the opportunity cost is procyclical, hence worker rents are countercyclical. Panel (c) introduces the idea of Rent Rigidity. Worker rents do not rise in a boom, but the annuity value of wages increases along with the worker's opportunity cost. Thus it is weaker than Wage Rigidity.

To impose some basic structure, we make an assumption which holds for all models of wage determination considered in the paper, including Nash Bargaining and Wage Rigidity. It is stated in terms of ex ante (before matching) expected rents and chance of trade:

$$\mathcal{G}(p,n) \equiv \iint G(y,z,p,n)\,dF_Y(y)\,dF_Z(z),$$

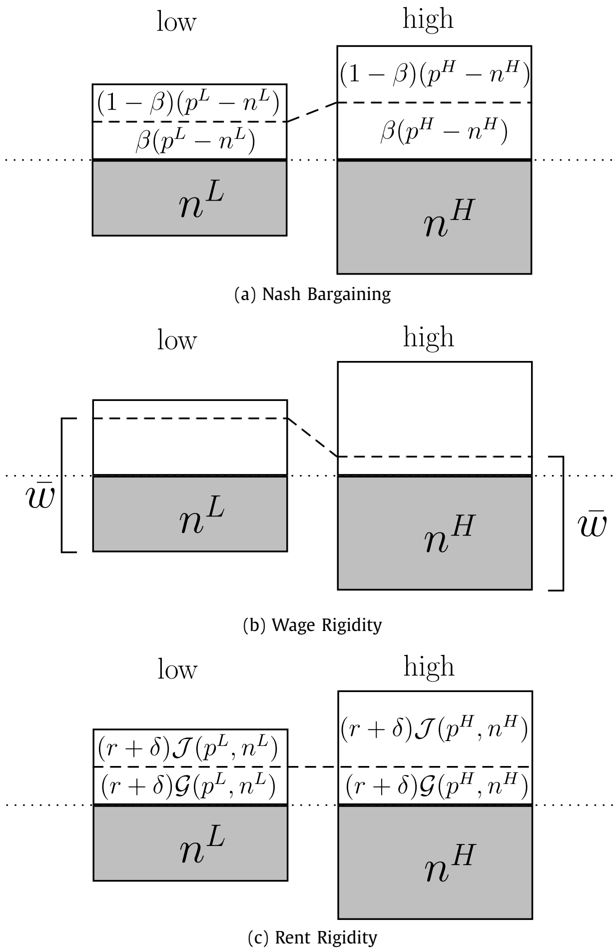

Fig. 2. Wage Rigidity vs. Rent Rigidity.

$$\mathcal{J}(p,n) \equiv \iint J(y,z,p,n)\,dF_Y(y)\,dF_Z(z),$$
$$\xi(p,n) \equiv \iint x(y,z,p,n)\,dF_Y(y)\,dF_Z(z). \tag{1}$$

Assumption 1. The model of wage determination $\{G, J, x\}$ satisfies three properties:

(a) The partial effect of aggregate labor productivity on expected worker rents $\mathcal{G}(p,n)$ is positive:

$$\frac{\partial \mathcal{G}(p,n)}{\partial p} \geqslant 0.$$

(b) The partial effect of the worker's opportunity cost on expected firm rents $\mathcal{J}(p,n)$ is negative:

$$\frac{\partial \mathcal{J}(p,n)}{\partial n} \leqslant 0.$$

(c) The trade probability $x(y,z,p,n)$ depends on p and n only through their difference $p-n$, and is increasing in both match-specific productivity y and amenity z.

Part (a) requires that for a constant opportunity cost n, higher productivity p increases worker rents. Importantly, this does not imply procyclical worker rents, namely, that the *total* derivative $\frac{d\mathcal{G}(p,n)}{dp}$ is positive: under Wage Rigidity the partial effect of p is zero, while worker rents are countercyclical since n increases in booms. Similarly, part (b) does not imply countercyclical firm rents: under Wage Rigidity the partial effect of n is zero, but firm rents are procyclical due to higher p in booms.

Trade is *ex post efficient* if it occurs if and only if $y + z + p - n \geqslant 0$, a property shared by Nash Bargaining and Wage Rigidity. Part (c) immediately follows from ex post efficiency, but is weaker. We interpret this property starting with its first requirement. If p and n increase by the same amount, they shift up the location of the bargaining problem to higher levels of productivity and opportunity cost, but leave average gains from trade $p - n$ unchanged. Part (c) requires that such a shift does not affect the probability of trade. It implies that total rents $\mathcal{J}(p,n) + \mathcal{G}(p,n)$ are only a function of the difference $p - n$.

Let $\bar{y} \equiv \xi(p,n)^{-1} \iint x(y,z,p,n)\, y\, dF_Y(y)\, dF_Z(z)$ denote average match-specific productivity conditional on trade. The second requirement of part (c) is positive selection: $\bar{y} \geqslant 0$, and similarly $\bar{z} \geqslant 0$ for the conditional mean amenity value.[4]

2.4. Equilibrium conditions

Using this definition of a model of wage determination we now write down the equilibrium conditions of the textbook search and matching model.

The model has two endogenous variables: labor market tightness θ and the opportunity cost n. Both immediately jump to their new steady state values in response to a permanent productivity shock.[5] They are determined by the following equilibrium conditions:

$$c = q(\theta)\mathcal{J}(p,n), \tag{2}$$

$$n = b + m(\theta)\mathcal{G}(p,n). \tag{3}$$

The first condition equates the flow cost of a vacancy c to its return, which is the product of the contact rate of vacancies $q(\theta)$ and the capital gain from a contact $\mathcal{J}(p,n)$. The second equation states that the flow value of unemployment n is the sum of the flow value of non-market activity b and the return of search, again the product of contact rate and capital gain. Given (θ, n) as determined by Eqs. (2)–(3), unemployment is governed by the differential equation $\dot{u} = \delta(1-u) - f(\theta,p,n)u$ where

$$f(\theta,p,n) = m(\theta)\xi(p,n) \tag{4}$$

is the job-finding rate. Productivity shocks affect the job-finding rate through two channels. The first is the contribution of search and matching models to the theory of employment fluctuations. In these models trading frictions are overcome through a matching technology with two essential inputs, unemployed workers and vacant jobs. Fluctuations in these two resources generate fluctuations in the contact rate $m(\theta)$. The second channel, fluctuations in the probability of trade $\xi(p,n)$, is more akin to mechanisms found in frictionless models: although frictions have been overcome, employment may not occur, because productivity is too low, or disutility from work too high. Following the literature, we focus on assessing the strength of the contact rate channel.

3. A volatility bound for the contact rate

3.1. Derivation

We derive a bound on the general equilibrium response of the contact rate m to changes in aggregate productivity p. The bound applies if the equilibrium exhibits the following property:

Definition 2 *(Procyclical Worker Rents [PWR]).* Worker rents are *procyclical* in equilibrium if the total effect of aggregate productivity on worker rents is positive:

$$\frac{d\mathcal{G}(p,n)}{dp} = \frac{\partial \mathcal{G}(p,n)}{\partial p} + \frac{\partial \mathcal{G}(p,n)}{\partial n}\frac{dn}{dp} \geqslant 0. \tag{5}$$

Here $\frac{dn}{dp}$ is the equilibrium response of the flow value of unemployment to a change in aggregate productivity, which we refer to as the *feedback effect*. If the inequality in Eq. (5) is reversed, we say that worker rents are countercyclical.

Let $\varepsilon_m \equiv \frac{d\ln m}{d\ln p}$ denote the elasticity associated with the equilibrium response of the contact rate. The goal is to derive an upper bound for ε_m.

We refer to Appendix A the case of a very strong feedback effect, $\frac{dn}{dp} \geqslant 1$. In that case, total rents actually decrease, and so must the firm's expected profits from filling a job, making the contact rate countercyclical. So consider the case of

[4] Recall that the unconditional means of match-specific productivity y and amenity z are zero.

[5] Pissarides (2000), pp. 26–33.

a weaker feedback effect $\frac{dn}{dp} < 1$. We obtain an upper bound on ε_m from the equilibrium condition for the flow value of unemployment (3). Differentiating this equation and exploiting PWR yields

$$\frac{dn}{dp} \geqslant \varepsilon_m m \frac{\mathcal{G}(p,n)}{p} \tag{6}$$

with equality if worker rents are acyclical. This inequality shows that, even if worker rents do not increase in a boom, the opportunity cost n increases nonetheless: while the capital gain from a contact is unchanged, contacts are more likely. This feedback is stronger: (i) the larger the (absolute) response ε_m of the contact rate; (ii) the larger worker rents, as there is more to gain from an increase in the contact rate.

We rewrite Eq. (6) using the following notation. Let $\bar{w}$ be the annuity value of wages conditional on trade, that is $\bar{w} \equiv (r+\delta)\frac{\mathcal{G}(p,n)}{\xi(p,n)} + n - \bar{z}$ (the average amenity value $\bar{z}$ must be deducted from average flow utility to obtain wages). Let $\bar{p} \equiv p + \bar{y}$ denote average labor productivity, the mean of $p + y$ conditional on trade. Eq. (3) can now be written as $\frac{\mathcal{G}(p,n)}{\xi(p,n)} = \frac{\bar{w}-b+\bar{z}}{r+\delta+f}$ and Eq. (6) becomes

$$\frac{dn}{dp} \geqslant \varepsilon_m \frac{f}{r+\delta+f}\frac{\bar{w}-b+\bar{z}}{\bar{p}-\bar{y}} \geqslant \varepsilon_m \frac{f}{r+\delta+f}\frac{\bar{w}-b}{\bar{p}} \tag{7}$$

where f is the job-finding rate defined in Eq. (4). The second inequality in Eq. (7) uses Assumption 1(c): positive selection insures $\bar{y} \geqslant 0$ and $\bar{z} \geqslant 0$.

Using $\frac{dn}{dp} < 1$, the second inequality in Eq. (7) can be written as an upper bound on ε_m:

$$\varepsilon_m < \frac{\bar{p}}{\bar{w}-b}\frac{r+\delta+f}{f} \equiv \bar{\varepsilon}_m. \tag{8}$$

This bound is positive. Recall that ε_m is bounded above by zero when the feedback effect is very strong, $\frac{dn}{dp} \geqslant 1$. Overall, we have established:

Proposition 1 *(Volatility Bound). Under Assumption* 1, *if the equilibrium exhibits PWR, then the steady-state elasticity ε_m of the contact rate w.r.t. to aggregate labor productivity is bounded above by $\bar{\varepsilon}_m$ in* (8), *irrespective of the other details of wage setting.*

The first term $\frac{\bar{p}}{\bar{w}-b}$ of the bound in (8) is the inverse of the worker's flow gains from market activity. The larger are these gains, the larger are worker rents, and the tighter is the bound. The second term $\frac{r+\delta+f}{f}$ summarizes the role of frictions. The ratio of job-finding rate and separation rate $k \equiv \frac{f}{\delta}$ is a standard (inverse) measure of frictions. As discussed below, quantitatively r is small relative to the turnover rates f and δ. Thus $\frac{r+\delta+f}{f} \approx \frac{1+k}{k}$. If frictions are large, k is small and the bound is slack. In the frictionless limit $k \to \infty$ the term $\frac{1+k}{k}$ converges to 1, and the bound is simply given by the first term $\frac{\bar{p}}{\bar{w}-b}$.

PWR is a property of equilibrium, since it involves the feedback effect $\frac{dn}{dp}$. We now define a primitive property of models of wage determination sufficient for PWR. This is critical to the simplicity of our approach: we can verify whether the bound applies by checking only qualitative properties of wage determination, without reference to the search and matching model, thus to the general equilibrium responses. We do just this in Section 4 for many models of wage determination in the presence of private information.

Definition 3 *(Increasing Worker Rents [IWR]).* In a model of wage determination $\{G, J, x\}$, worker rents are *increasing* if

$$\frac{\partial \mathcal{G}(p,n)}{\partial p} \geqslant -\frac{\partial \mathcal{G}(p,n)}{\partial n} \geqslant 0.$$

Lemma 1. *Under Assumption* 1, *IWR implies PWR.*

All proofs not given in the text are in Appendix A.

3.2. Calibration

To evaluate the bound quantitatively, we follow Shimer's (2005) calibration strategy unless explicitly stated otherwise. All rates are quarterly. The discount rate is $r = 0.012$. Shimer infers time series for the job-finding and separation rate from BLS data on unemployment and short term unemployment. The average job-finding rate is $f = 1.82$ while the average separation rate is $\delta = 0.1$.[6]

[6] Shimer (2005) does not distinguish between the *job-finding probability* $F \equiv 1 - e^{-f}$ and the *job-finding rate* f. The empirical average of the former is $F = 1.35$. Shimer targets $f = 1.35$, while the empirical average is $f = 1.82$. See Brügemann (2008).

Table 1
Calibration.

Parameter	Value
r	0.012
f	1.82
δ	0.1
$\frac{b}{\bar{p}}$	0.71
$\frac{\bar{w}}{\bar{p}}$	0.9846
$\hat{\eta}$	0.4

Table 2
Quantitative results.

	ε_m	$\varepsilon_m^2 \frac{\sigma_p^2}{\sigma_f^2}$
Panel A. Benchmark calibration $\frac{b}{\bar{p}} = 0.71$		
Nash Bargaining $\varepsilon_{m,NB}$	1.442	3.2%
Wage Rigidity $\varepsilon_{m,WR}$	43.290	2881%
Volatility Bound $\bar{\varepsilon}_m$	3.866	23.0%
Panel B. Shimer's calibration $\frac{b}{\bar{p}} = 0.4$		
Nash Bargaining $\varepsilon_{m,NB}$	0.689	0.7%
Wage Rigidity $\varepsilon_{m,WR}$	43.290	2881%
Volatility Bound $\bar{\varepsilon}_m$	1.816	5.1%

To quantify average gains from market activity, we need to specify a value of non-market activity. This comprises both unemployment benefits, as well as the value of the additional leisure the unemployed enjoy *vis-à-vis* the employed. Shimer chooses $\frac{b}{\bar{p}} = 0.4$, a value at the upper end of unemployment insurance replacement rates in the US. Hall and Milgrom (2008) improve on this by estimating the value of additional leisure using evidence on the Frisch elasticity of labor supply. We adopt their value $\frac{b}{\bar{p}} = 0.71$ in our benchmark calibration, but also report results for Shimer's value.

To quantify the annuity value of wages $\frac{\bar{w}}{\bar{p}}$, and thereby the size of worker rents, we follow Silva and Toledo (2009) and Hall and Milgrom, who use evidence indicating that recruiting costs are 14% of quarterly pay per hire. To see how this pins down $\frac{\bar{w}}{\bar{p}}$, rewrite Eq. (2) as

$$c \cdot \frac{1}{q(\theta)\xi(p,n)} = \frac{\bar{p} - \bar{w}}{r + \delta}.$$

The left-hand side is the expected cost of recruiting a worker: the flow cost c times the expected duration of a vacancy. Firm flow rents are $\bar{p} - \bar{w}$, and their present value must cover the firm's initial recruiting costs. Setting $\frac{c}{q(\theta)\xi(p,n)} = 0.14\bar{w}$ yields $\frac{\bar{w}}{\bar{p}} = 0.9846$.

The bound does not depend on the matching function elasticity $\eta \equiv \frac{d \ln m}{d \ln \theta}$, but the elasticities implied by the two benchmark models Nash Bargaining and Wage Rigidity do.[7] Thus for comparison we need to calibrate η. Let $\hat{\eta} \equiv \frac{\varepsilon_f}{\varepsilon_\theta}$ denote the ratio of the elasticities of job-finding rate and labor market tightness. Without heterogeneity $\eta = \hat{\eta}$, and $\hat{\eta}$ can be estimated by regressing $\log(f)$ on $\log(\theta)$. In Shimer's data $\hat{\eta} = 0.4$.[8] The calibration is summarized in Table 1.

3.3. Quantitative results

Table 2.A provides the results of the benchmark calibration. The first column shows the elasticities of the contact rate associated with Nash Bargaining $\varepsilon_{m,NB} = 1.442$ and Wage Rigidity $\varepsilon_{m,WR} = 43.290$. The bound is

$$\bar{\varepsilon}_m = \underbrace{\frac{\bar{p}}{\bar{w} - b}}_{3.642} \underbrace{\frac{r + \delta + f}{f}}_{1.062} = 3.866.$$

Frictions are small in the sense that their measure $\frac{r+\delta+f}{f}$ is close to its frictionless limit of one. Hence the bound is primarily determined by the worker's flow gains from market activity. It is clear that Rent Rigidity provides much weaker

[7] They are $\varepsilon_{m,NB} = (\frac{\bar{p}-b}{\bar{p}} \frac{1-\eta}{\eta} + \frac{\bar{w}-b}{\bar{p}} \frac{f}{r+\delta+f})^{-1}$ and $\varepsilon_{m,WR} = \frac{\bar{p}}{\bar{p}-\bar{w}} \frac{\eta}{1-\eta}$.

[8] Shimer obtains $\hat{\eta} = 0.28$ regressing the log of the *job-finding probability* on $\log(\theta)$, see footnote 6.

Table 3
Regressions of job-finding rate on productivity.

Lags	CES	CPS
0	0.179	0.331
3	0.360	0.446
0–4	0.405	0.562

Note. R^2 from regressions of $\log(f)$ on different combinations of contemporaneous and lagged $\log(\bar{p})$. See text for data description.

amplification than Wage Rigidity. Next we investigate what this means for the ability of the contact rate channel to account for productivity-induced fluctuations in the job-finding rate.

First, we review the evidence on fluctuations in the job-finding rate and productivity. Shimer's series for the job-finding rate was discussed earlier. We consider two measures of labor productivity. First, following Shimer, output per person in the nonfarm business sector from the BLS (series PRS85006163). This relies on employment data from the Current Employment Statistics (CES). As unemployment and the job-finding rate are based on the Current Population Survey (CPS), we follow Hagedorn and Manovskii (2009) in replacing CES employment with a CPS measure. Specifically, we use wage and salary employment in private nonagricultural industry (series LNS12032189). The sample period is 1951 to 2003. We obtain cyclical components at the quarterly frequency following Shimer's procedure: we take quarterly averages of all monthly series; then we take logs and HP-filter with smoothing parameter 10^5. From now on the discussion refers to these cyclical components. The standard deviation of the job-finding rate is $\sigma_f = 0.162$, that of unemployment $\sigma_u = 0.191$. Volatility of the two productivity measures is similar, $\sigma_{\bar{p},CES} = 0.020$ and $\sigma_{\bar{p},CPS} = 0.0193$.

As Mortensen and Nagypál (2007) emphasize, not all labor market volatility is due to productivity shocks. Hall and Milgrom (2008) propose a method to determine the fraction that is, and implement it for unemployment. They regress unemployment on various combinations of contemporaneous and lagged labor productivity. At one extreme they use contemporaneous productivity only, at the other they add many lags. They settle on an intermediate case, using only the third lag, at which correlation with unemployment is maximized. In Table 3 we replicate this exercise for the job-finding rate, reporting R^2s for both productivity measures. CPS based productivity attributes a larger fraction of the job-finding rate variance to productivity shocks. With contemporaneous productivity as the only regressor, the two measures indicate contributions of 18% and 33%, respectively. Adopting this as the yardstick for success would make it relatively easy for the model to succeed. But this "success" would simply reflect a distinct but related failure of the model: productivity leads the job-finding rate in the data, while it is well known that the model generates no lead. Adding four lags raises the contributions to 41% and 56%, respectively. The intermediate case yields 36% and 45% percent. We adopt 40% as a conservative yardstick.

Next, we express the elasticities from Table 2.A, and thus the strength of the contact rate channel, in units comparable to the 40% yardstick. To do this, we first discuss the case of an acyclical probability of trade, in order to isolate the role of fluctuations in the contact rate. This encompasses the two benchmark models, Hall (2005) and Shimer (2005). Second, we discuss the interpretation of our results when variation in the probability of trade contributes to fluctuations in the job-finding rate.

With a constant probability of trade $\xi(p,n)$, $\bar{\varepsilon}_m$ is an upper bound on the response of the *job-finding rate* f to *average labor productivity* $\bar{p}$. Thus $\bar{\varepsilon}_m^2 \sigma_{\bar{p}}^2$, where $\sigma_{\bar{p}}^2$ is the variance of observed average labor productivity, is an upper bound on the variance of the job-finding rate. Dividing by the observed variance σ_f^2 yields an upper bound on the fraction of this variance accounted for by productivity shocks. The second column of Table 2.A reports that this fraction is at most 23%. Nash Bargaining and Wage Rigidity yield 3.2% and 2881%, respectively. *Vis-à-vis* the 40% yardstick, the glass is both half full and half empty. On one hand the bound, if attained, has the right order of magnitude, in contrast to both benchmark models. On the other hand, a quantitatively important gap remains between 23% and 40%.

Table 2.B departs from the benchmark calibration, using Shimer's calibration of the value of non-market activity $\frac{b}{\bar{p}} = 0.4$. The bound $\bar{\varepsilon}_m$ is cut in half, hence the contribution to the variance is now at most 5.1%. This explains why papers analyzing models with procyclical worker rents while maintaining Shimer's calibration, such as Guerrieri (2007), find little volatility. At the opposite end of the spectrum, Hagedorn and Manovskii (2008) maintain Nash Bargaining and choose the value of non-market activity to match the cyclical behavior of aggregate wages, arriving at $\frac{b}{\bar{p}} = 0.955$. Mortensen and Nagypál (2007) criticize this calibration strategy, based on its implication that workers' gains from working in the market are minuscule. With this value even the model with Nash Bargaining driven only by productivity shocks generates more than 100% of the variance of the job-finding rate.[9]

[9] This exceeds the 40% yardstick. Hagedorn and Manovskii (2009) apply their calibration strategy with shocks to the value of non-market activity as a second driving force. This yields a lower average value of non-market activity, around 0.93, and a contribution of productivity shocks close to the empirical value.

The flip side of a low value of $\frac{b}{\bar{p}}$ are large worker rents $\mathcal{G}(p,n)$. To show how larger worker rents constrain amplification we revisit the mechanics of the bound, evaluating Eq. (3) at calibrated values (normalizing $\bar{p}=1$):

$$\underset{0.9687}{n} = \underset{0.71}{b} + \underset{1.82}{f}\ \underset{0.1421}{\mathcal{G}(p,n)}.$$

Consider an increase in productivity by 1%, from 1 to 1.01. Suppose the elasticity of the job-finding rate is 4, slightly above the bound. Then the return from search increases by 4% from $1.82 \cdot 0.1421 = 0.2586$ to $1.89 \cdot 0.1421 = 0.2690$. Thus n increases more than p. Hence firm rents fall, contradicting the supposed rise in the job-finding rate.

This discussion has focussed on the contact rate channel, and concluded that accounting for observed volatility of the job-finding rate through a modification of wage setting requires a model of wage determination that delivers countercyclical worker rents. As we elaborate in the next section, both Hall (2005) and Hall and Milgrom (2008) fall in this category.

Our main result is also consistent with a different reconciliation of model and data: worker rents are procyclical, but fluctuations in the probability of trade are quantitatively important.[10] As discussed earlier, while the contact rate channel is specific to search models, the probability of trade channel is more akin to mechanisms found in frictionless models. A procyclical probability of trade enhances volatility in two ways. First, it directly contributes to fluctuations in the job-finding rate. Second, via selection, it implies that volatility of observed average productivity $\bar{p}$ understates volatility of the driving process p.

4. Wage determination and private information

In the previous section we argued that the contribution of fluctuations in the contact rate to labor market volatility is limited if worker rents are procyclical, and thus at best rigid.

In this section we turn to specific models of wage determination. We study their comparative statics, in order to understand under what conditions rents are procyclical, and what mechanisms may deliver countercyclical rents.

Laying the groundwork for the analysis of private information, Section 4.1 studies models of bargaining under complete information. Hall (2005) and Hall and Milgrom (2008) have proposed complete information models that can generate volatility in the job-finding rate beyond what is observed. Our volatility bound implies that this must be due to countercyclical worker rents. This leads us to investigate what mechanisms deliver countercyclical rents in complete information settings.

This sets the stage for our analysis of models of bargaining under private information in Section 4.2, the primary focus of this section. Shimer (2005) conjectured that introducing private information about gains from trade in the labor market can lead to wage rigidity. More generally, private information is a prominent feature of labor markets, so it is important to gain insight into how it may affect labor market volatility. A priori, this appears like a laborious task, both due to the wealth of bargaining protocols that may be relevant in labor markets, and to the need to specify distributions of beliefs which by their nature are difficult to calibrate. Our approach, based on the cyclical behavior of rents, is especially useful here. It leads us to refocus Shimer's conjecture on the following specific question: does the presence of private information give rise to mechanisms that deliver countercyclical worker rents? Once posed this way, it is a question about comparative static properties of bargaining models, which frequently do not depend on the specific distribution of beliefs.

4.1. Wage bargaining with complete information

4.1.1. Strategic bargaining

We discuss a standard alternating offers strategic bargaining game with discounting and risk of breakdown, based on Binmore et al. (1986). The purpose is twofold. First, the game encompasses the bargaining model used in Hall and Milgrom (2008), allowing us to identify their recipe for countercyclical rents. Second, the bargaining protocol we introduce here will be used again for the analysis of private information.

The values y and z are realized before negotiations. In this section they are also common knowledge. Let $i \in \{w, f\}$ stand for {worker, firm}. A public lottery selects party i with chance $o_i \in [0,1]$ to make the first wage offer/request w, where $o_w + o_f = 1$. If an offer is accepted, from then until the (exogenous) end of the match the firm has flow profits $p + y - w$, the worker has flow rents $w + z - n$, the total flow surplus is $y + z + p - n$. If negotiations fail, whether exogenously or because a party opts out, the firm obtains zero and the worker obtains the payoff from returning to unemployment n/r. After an offer by party i, the responding party $j \neq i$ can accept, opt out, or decline. In the last case, negotiations break down with chance $1 - e^{-\phi_j \Delta}$, and continue with chance $e^{-\phi_j \Delta}$, where $\phi_j \geqslant 0$. If they continue, a delay Δ occurs during which party j incurs flow cost γ_j. Party j then counteroffers, and so forth. Payoffs during bargaining are discounted by each party i at rate $\rho_i \geqslant 0$, that we allow to differ from r in order to encompass the notion of bargaining "in virtual time", while $\rho_i = r$ is the case of bargaining in real time.

In subgame perfect equilibrium there is no delay and payoffs are unique. We study the limit of payoffs as $\Delta \to 0$.

[10] Define worker rents conditional on trade $\mathcal{G}_c(p,n) \equiv \mathcal{G}(p,n)/\xi(p,n)$ and write Eq. (3) as $n = b + f(\theta, p, n)\mathcal{G}_c(p,n)$. An argument analogous to the derivation of our bound implies that a procyclical volatility of trade only allows the elasticity of the job-finding rate ε_f to exceed our bound $\bar{\varepsilon}_m$ if *conditional* worker rents are countercyclical. This is a weaker requirement than countercyclical *unconditional* worker rents.

Lemma 2. *The limit as $\Delta \to 0$ of worker rents in subgame perfect equilibrium is given by*

$$G(y,z,p,n) = \begin{cases} \tilde{G}(y,z,p,n), & 0 \leqslant \tilde{G}(y,z,p,n) \leqslant \frac{y+z+p-n}{r+\delta}, \\ 0, & \tilde{G}(y,z,p,n) \leqslant 0, \\ \frac{y+z+p-n}{r+\delta}, & \tilde{G}(y,z,p,n) \geqslant \frac{y+z+p-n}{r+\delta}, \end{cases} \tag{9}$$

where

$$\tilde{G}(y,z,p,n) \equiv \frac{\phi_f + \rho_f}{\phi_w + \phi_f + \rho_w + \rho_f} \frac{y+z+p-n}{r+\delta} + \frac{\gamma_f - \gamma_w - \rho_w \frac{n}{r}}{\phi_w + \phi_f + \rho_w + \rho_f}.$$

Eq. (9) immediately implies the following proposition.

Proposition 2. *The model of wage determination through alternating offers with complete information, discounting and breakdown risk satisfies IWR if and only if* $\frac{\rho_w}{\phi_w+\phi_f+\rho_f} = 0$.

First, consider the case $\frac{\rho_w}{\phi_w+\phi_f+\rho_f} = 0$ and $\gamma_f = \gamma_w$. The model reduces to Nash Bargaining with $\beta = \frac{\phi_f+\rho_f}{\phi_w+\phi_f+\rho_w+\rho_f} \in [0,1]$. Thus IWR holds, and *a fortiori* worker rents are procyclical. With $\gamma_f \neq \gamma_w$ there is a bias favoring the party with lower cost of delay. This affects the level of rents but not comparative statics, hence IWR continues to hold.

Next, suppose $\frac{\rho_w}{\phi_w+\phi_f+\rho_f} > 0$, that is worker discounting during bargaining is not negligible relative to breakdown risk. Loosely speaking, the strength of breakdown risk relative to discounting captures the ability of a party to commit to walk away after having its offer rejected. Now IWR fails. As IWR is sufficient but not necessary for PWR, countercyclical worker rents cannot be ruled out. The special case $\rho_w = r$ and $\phi_w = \delta$, that is real time discounting and breakdown risk equal to the separation rate, is instructive. Here $(r+\delta)\tilde{G}(y,z,p,n) = \frac{(\phi_f+\rho_f)(y+p+z)+(r+\delta)(\gamma_f-\gamma_w)}{\phi_w+\phi_f+\rho_w+\rho_f} - n$, hence flow rents decrease one for one with the opportunity cost n; thus, the annuity value of wages $w = (r+\delta)\tilde{G}(y,z,p,n) + n$ is independent of n.[11] This case reveals an important link between wage rigidity and competitive forces. A worker's threat to keep searching is not credible, thus cannot be exploited to obtain a higher wage; then, with relatively small breakdown risk, wages are insulated from the value of unemployment, and rents may be countercyclical. In contrast, in competitive search models, firms can commit to their offers and use them to attract job applicants. The resulting competition between firms turns worker rents procyclical.[12]

Hall and Milgrom (2008) study this potential source of countercyclical rents quantitatively. The most parsimonious version of the model above has $\rho_f = \rho_w = r$, $\phi_f = \phi_w = \delta$, $\gamma_w = -b$ (the worker receives the value of non-market activity during bargaining), and $\gamma_f = 0$. This version would fail quantitatively. The very mechanism that permits countercyclical rents, namely insulation of wages from competitive forces, creates a dilemma: wages are far below the competitive level, while evidence on recruiting costs indicates that wages are, on average, close to competitive. Rather than in wages, competitive forces manifest themselves in intense recruiting effort, with equilibrium recruiting costs amounting to 150% of quarterly pay, an order of magnitude larger than in the data.[13] Hall and Milgrom escape this dilemma by having firms' cost of delay γ_f as an additional parameter, choosing its value to match a wage level consistent with observed recruiting costs. They find γ_f to be 23% of flow productivity, suggesting this may reflect costs firms face in formulating offers. However, this leaves open the question why making offers is less costly for workers, since only differential costs $\gamma_f - \gamma_w$ matter. Alternatively, they suggest γ_f may capture the cost of capital sitting idle during negotiations. With this modification alone, the model would generates countercyclical rents but substantially overshoot observed unemployment volatility. Hall and Milgrom's calibration avoids this by introducing an additional free parameter that allows breakdown risk to exceed the separation rate δ, choosing it to match observed unemployment volatility.[14]

4.1.2. κ-double auction

In Section 2.3 we introduced Wage Rigidity as a reduced form model of wage determination. Hall (2005) derives Wage Rigidity as an equilibrium outcome of a double auction. Worker and firm simultaneously submit wage proposals w_w and w_f, respectively. Trade occurs if $w_w \leqslant w_f$, at the wage $\kappa w_w + (1-\kappa) w_f$ where $\kappa \in (0,1)$. In this auction, for any $w \in [n,p]$ it is a Nash equilibrium for both worker and firm to propose w. Hall exploits this indeterminacy to obtain

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[11] The condition on ratios $\frac{\rho_w}{\phi_w} = \frac{r}{\delta}$ is sufficient for this result.

[12] With complete information competitive search implies the same reduced form model of wage determination as Nash Bargaining with $\beta = 1-\eta$, hence IWR holds. As mentioned in the introduction, our bound also applies to the competitive search models with private information of Guerrieri (2007) and Moen and Rosén (2008).

[13] Based on our baseline calibration, which in essence is identical to Hall and Milgrom's.

[14] In a model with a constant separation rate, targeting unemployment volatility implies excessive fluctuations in the job-finding rate. To address this point, one can modify Hall and Milgrom's calibration strategy to target the volatility of the job-finding rate, which would yield somewhat lower breakdown risk.

rigid wages. He introduces an equilibrium selection rule, interpreted as a wage norm: as p and n change, the wage remains constant at some level w, as long as w remains in $[n, p]$. Equivalence with Wage Rigidity immediately yields:

Proposition 3. *The model of wage determination through a κ-double auction under complete information, with a wage norm as the equilibrium selection rule, violates IWR.*

Indeterminacy can be exploited to generate countercyclical rents, but creates a difficulty: there are no economic forces determining the rigid wage level w. It could be close to b, leaving most gains from market activity to be absorbed by recruiting cost, or close to competitive. Hall sidesteps this issue, setting it equal to the average wage (over the business cycle) under Nash Bargaining with $\beta = \frac{1}{2}$.

4.2. Wage bargaining under private information

Next we study a series of models of bilateral bargaining under private information. We ask whether the presence of private information can be a source of countercyclical rents. To isolate the role of private information, we shut down the two mechanisms just identified that can deliver countercyclical rents under complete information. In particular, we restrict attention to bargaining models that yield unique equilibrium payoffs, so that countercyclical rents do not arise from multiplicity.

Private information concerns the match-specific productivity component y and the worker's match-specific amenity value z. Their distributions F_Y and F_Z have support $[y_l, y_h]$ and $[z_l, z_h]$, respectively, with $y_l, z_l, y_h, z_h \in \bar{\mathbb{R}}$. We impose a standard regularity condition concerning private information, weaker than monotone (increasing) hazard rates.

Assumption 2 *(Monotone Virtual Valuations).* The "virtual valuations" $y - \frac{1-F_Y(y)}{F_Y'(y)}$ and $z - \frac{1-F_Z(z)}{F_Z'(z)}$ are strictly increasing and continuously differentiable on $[y_l, y_h]$ and $[z_l, z_h]$, respectively.

For each bargaining model considered we show our bound applies by verifying Assumption 1 and IWR, using the following steps. First, we verify Assumption 1(c). As discussed in Section 2.3, this assumption implies that total rents $\mathcal{G}(p,n) + \mathcal{J}(p,n)$ depend on p and n only through their difference $p - n$. In this section we show that this holds for $\mathcal{G}(p,n)$ and $\mathcal{J}(p,n)$ individually, so we can write $\mathcal{G}(p-n)$ and $\mathcal{J}(p-n)$. Next we show that $\mathcal{G}'(p-n) \geqslant 0$ and $\mathcal{J}'(p-n) \geqslant 0$. IWR then follows immediately as

$$\frac{\partial \mathcal{G}(p-n)}{\partial p} = \mathcal{G}'(p-n) = -\frac{\partial \mathcal{G}(p-n)}{\partial n} \geqslant 0.$$

So does the rest of Assumption 1: part (a) is implied by IWR and part (b) follows from

$$-\frac{\partial \mathcal{J}(p-n)}{\partial n} = \mathcal{J}'(p-n) \geqslant 0.$$

4.2.1. Monopoly (take-it-or-leave-it offer)

Using the notation of Section 4.1.1, let $o_i = 1$, so party i makes the first offer. Assume $\phi_j = \infty$. This makes the offer take-it-or-leave-it, since negotiations cannot last beyond the first round. With $\phi_j = \infty$ the extent of discounting during negotiations is irrelevant, so it does not matter whether bargaining occurs in virtual or real time.

This is the model suggested by Shimer (2005) in his conjecture. Whether the offer-making party has private information is irrelevant, as there is no scope for signaling. Effectively, in this model the uninformed party can commit not to consider a counteroffer or make another offer if its first offer is rejected. This commitment sustains its bargaining power, which is tempered by the other party's private information. The game has a unique equilibrium, which is constrained ex ante efficient in that it maximizes the offer-making party's welfare given information asymmetry (Satterthwaite and Williams, 1989).

We analyze the case in which the firm makes the offer: $o_f = 1$ and $\phi_w = \infty$. We establish that our bound applies. By symmetry, the same argument applies to the case $o_w = 1$ and $\phi_f = \infty$. Hence, it applies to any randomization on the first offer when $\phi_f = \phi_w = \infty$, in particular Myerson's Random Dictator mechanism ($o_f = o_w = \frac{1}{2}$). As discussed in Kennan (2009), the latter implements Myerson's (1984) Neutral Bargaining Solution, which is a generalization of the Nash Bargaining Solution to a setting with incomplete information.

A firm of type y offers a wage w_M to the worker, who is then indifferent between accepting and rejecting it when his amenity value is exactly $z_M = n - w_M$. If $z \geqslant z_M$ the offer is accepted, an event with chance $1 - F_Z(z_M)$, and the firm earns flow profits $y + p - w_M = y + z_M + p - n$. Equivalently, the firm chooses the threshold z_M, rather than the wage w_M, to maximize:

$$\left[1 - F_Z(z_M)\right](y + z_M + p - n). \tag{10}$$

The well-known first order condition is

$$y + z_M + p - n = \frac{1 - F_Z(z_M)}{F_Z'(z_M)}. \tag{11}$$

The left-hand side is the gain from trading with an additional worker type. If the firm trades with more types, however, it has to concede higher informational rents to the types it is already trading with. The right-hand side gives the fraction of types that receive higher rents relative to the increase in the probability of trade from reducing z_M.

If Eq. (11) has an interior solution, by Assumption 2 it is unique, differentiable, and the global maximizer. From Eq. (11) it is clear that the optimal threshold is a function only of $y + p - n$, hence we denote it as $z_M(y + p - n)$. Assumption 2 permits finite lower and upper bounds. So $z_M(y + p - n)$ can be at a corner, equal to the lower bound z_l (the offer is accepted for sure) if $y + z_l + p - n \geqslant [F'_Z(z_l)]^{-1}$, or equal to the upper bound z_h (rejection for sure) if $y + z_h + p - n \leqslant 0$. One may expect that corner solutions generate sufficient rigidity to escape the bound. We show this is not the case.

It is now straightforward to map this model of wage determination into the notation of Section 2.3, and check that it satisfies Assumption 1 and IWR:

$$(r+\delta)G(y,z,p,n) = x(y,z,p,n)\big[z - z_M(y+p-n)\big],$$
$$(r+\delta)J(y,z,p,n) = x(y,z,p,n)\big[y + z_M(y+p-n) + p - n\big],$$
$$x(y,z,p,n) = \mathbb{I}\big\{z \geqslant z_M(y+p-n)\big\}. \tag{12}$$

The trade decision x depends on p and n only through their difference $p-n$. Moreover, the same is true for the functions G and J and for their expected counterparts, that we can write as $\xi(p-n)$, $\mathcal{G}(p-n)$ and $\mathcal{J}(p-n)$ from now on. Inspecting the firm's objective in (10), an increase in $y + p - n$ raises the marginal gain from trade associated with lowering the threshold z_M. By a monotone comparative statics argument $z_M(y + p - n)$ is weakly decreasing (and strictly so over the range where the solution is interior). Consulting Eq. (12), this implies that $x(y,z,p,n)$ is non-decreasing in both y and z, and Assumption 1(c) is verified. It remains to show $\mathcal{G}'(p-n)$, $\mathcal{J}'(p-n) \geqslant 0$.

Define the worker's average rents from trading with a firm of type y:

$$(r+\delta)\mathcal{G}(y|p-n) \equiv \int_{z_M(y+p-n)}^{z_h} \big[z - z_M(y+p-n)\big]\,dF_Z(z).$$

This function is differentiable (from one side in knife-edge corner cases), with

$$(r+\delta)\mathcal{G}'(y|p-n) = -z'_M(y+p-n)\big[1 - F_Z\big(z_M(y+p-n)\big)\big] \geqslant 0.$$

The firm expands the range of types it trades with by $-z'_M(y+p-n)$, so informational rents of all worker types that already trade have to increase by exactly this amount. By definition $\mathcal{G}(p-n) = \int \mathcal{G}(p-n|y)\,dF_Y(y)$, so

$$\mathcal{G}'(p-n) = \int \mathcal{G}'(y|p-n)\,dF_Y(y) \geqslant 0.$$

Let $\mathcal{J}(y|p-n)$ denote the maximized value for firm type y. The envelope theorem implies

$$(r+\delta)\mathcal{J}'(y|p-n) = 1 - F_Z\big(z_M(y+p-n)\big).$$

Since the threshold z_M is chosen optimally, the benefit from an increase in $p - n$ is simply the direct effect on the gains from trade for worker types that already trade with the firm. The desired result follows:

$$\begin{aligned}(r+\delta)\mathcal{J}'(p-n) &= \int (r+\delta)\mathcal{J}'(y|p-n)\,dF_Y(y)\\ &= \int \big[1 - F_Z\big(z_M(y+p-n)\big)\big]\,dF_Y(y) = \xi(p-n) \geqslant 0.\end{aligned} \tag{13}$$

This analysis has allowed for corner solutions. If $z_M(y+p-n) = z_l$ (the offer is accepted for sure) the firm pockets higher gains from trade without conceding higher informational rents to the worker. Rent Rigidity, but not Wage Rigidity, is attained. If $z_M(y+p-n) = z_h$ (the offer is rejected for sure) then worker rents are zero, thus again acyclical.

The following proposition summarizes these results.

Proposition 4. *Under Assumption* 2 *the model of wage determination through a take-it-or-leave-it offer by the firm to a privately informed worker has a unique equilibrium outcome which satisfies Assumption* 1 *and IWR. Thus, the volatility bound applies.*

Kennan (2009) studies the Random Dictator mechanism, assuming that only the firm has private information. As shown above, this model is subject to the bound, since the presence of private information alone does not generate countercyclical worker rents. Kennan obtains volatility in excess of the bound by exploiting private information differently. He proposes exogenous fluctuations in the shape of the distribution of types as the main driving force. Specifically, he assumes that the match-specific productivity component can be either high or low, and that the high value is more likely in booms. He assumes that parameters are such that pooling occurs in both booms and recessions: workers always request the wage that makes the low firm type indifferent, hence worker rents are countercyclical.[15]

4.2.2. Alternating offers with one-sided private information

Next, we move away from the case of extreme commitment to a (take-it-or-leave-it) offer, and assume $\phi_i < \infty$. To insure that worker rents are not already countercyclical due to Hall and Milgrom's mechanism, we assume no discounting during negotiations, $\rho_w = \rho_f = 0$. We assume for now that only one party is privately informed. For simplicity, as this makes no difference to the comparative statics of interest, we set costs of delay to zero: $\gamma_w = \gamma_f = 0$.

Rescale time to $\Delta' = (\phi_w + \phi_f)\Delta$. As Δ vanishes, so does Δ'. Let $\beta = \phi_f/(\phi_w + \phi_f)$. Then the risk of breakdown is $1 - e^{-\beta\Delta'}$ after the worker makes a request and the firm rejects it, and $1 - e^{-(1-\beta)\Delta'}$ in the other case. Except for the non-zero outside option of the worker, this is the case analyzed by Menzio (2005a), who extends the Coase conjecture building on Grossman and Perry (1986), and Gul and Sonnenschein (1988). We exploit their results and refer to their articles for terminology.

Again suppose the worker is privately informed, while y is common knowledge. Then, under the standard and plausible restrictions of stationarity of equilibrium strategies and a monotonicity requirement on beliefs, every sequential equilibrium of the bargaining game under one-sided private information implies immediate agreement at the same terms. Menzio shows that the payoff of the firm is the same as if it dealt with the lowest worker type under complete information. Adapting Eq. (9) to compute this payoff yields:

$$(r+\delta)J(y,z,p,n) = (1-\beta)\max\{y + z_l + p - n, 0\} \quad \text{for all } z \in [z_l, z_h]. \tag{14}$$

The firm receives rents only due to strategic bargaining forces emanating from breakdown risk, but extracts no monopoly rents. The latter arise if and only if $\phi_w = +\infty$. Thus firms' rents obtained here do not converge to the monopoly level as $\phi_w \to \infty$. In particular, if $y + z_l + p - n \leqslant 0$, so that there are no gains from trade with the lowest worker type, then firm rents are zero irrespective of the value of ϕ_w.

Trade is ex post efficient, so the ex ante probability of trade is simply $\xi(p-n) = \Pr(y+z+p-n \geqslant 0)$ and Assumption 1(c) is verified. Notice that once again not only the trade decision but also the split of rents depends on p and n only through their difference $p - n$. We are left to verify $\mathcal{G}'(p-n)$, $\mathcal{J}'(p-n) \geqslant 0$. Expected rents equal

$$(r+\delta)\mathcal{J}(p-n) = (1-\beta)\int_{-(z_l+p-n)}^{y_h} (y + z_l + p - n)\,dF_Y(y).$$

Hence

$$(r+\delta)\mathcal{J}'(p-n) = (1-\beta)\big[1 - F_Y(n - p - z_l)\big] \in \big[0, \xi(p-n)\big]. \tag{15}$$

Since trade is ex post efficient, the envelope theorem implies $(r+\delta)\mathcal{J}'(p-n) + (r+\delta)\mathcal{G}'(p-n) = \xi(p-n)$. Together with Eq. (15) this insures $(r+\delta)\mathcal{G}'(p-n) \in [0, \xi(p-n)]$. We conclude:

Proposition 5. *The model of wage determination through alternating offers between an uninformed and an informed party with risk of breakdown and no discounting between bargaining rounds has a unique sequential equilibrium outcome in stationary strategies, monotone beliefs and monotone conjectures in the sense of Menzio (2005a) which satisfies Assumption 1 and IWR. Thus, the volatility bound applies.*

The special case of sequential one-sided offers, the basic Coase conjecture model, is obtained by setting $\phi_f = 0$, which implies $\beta = 0$. Here the firm can safely ignore the worker's offers, since turning them down is not associated with a risk of breakdown.

Menzio (2005b) proposes a model of wage determination that implies rents identical to the Coase conjecture case just discussed, but arising from a different mechanism. Workers are uninformed. The wage requested by a worker in equilibrium is given by the productivity of the lowest firm type: $w = p + y_l$. The firm employs many workers. If the firm were to grant a wage in excess of $p + y_l$ to one worker, all its workers would learn that the firm's type is higher than y_l, prompting them to request higher wages. This spillover of information enables the firm to reject wage requests above $p + y_l$. Since rents are the same as under the Coase conjecture, our bound applies if F_Y is acyclical. Menzio instead assumes that the lower bound of the distribution of firm productivity $p + y_l$ is constant over the cycle. As aggregate productivity p is higher in booms, it follows that the distribution F_Y must change over the cycle. Thus Menzio's approach is similar to Kennan (2009) in that it relies on exogenous fluctuations in private information to generate labor market volatility.

[15] In our notation, both p and F_Y are cyclical. It is straightforward to decompose labor market volatility into three sources: p, F_Y, and an interaction term. In Kennan's calibration fluctuations in F_Y are responsible for most volatility, while the contributions of p and interactions are small.

4.2.3. Two-sided private information: Sealed-bid mechanisms

The theory of strategic bargaining under two-sided private information, whether with one-sided or alternating offers, has not produced unique predictions based on the Perfect Bayes Nash equilibrium notion even with refinements such as stationarity of strategies and monotonicity of beliefs. Thus here it may be possible to exploit multiplicity to generate countercyclical rents. But, as illustrated by the double auction under complete information, private information is not essential for a theory of labor market fluctuations based on multiplicity.

This leads us to maintain our focus on models that deliver unique predictions, but the source of uniqueness differs from Section 4.2.2. We study a bargaining protocol which generalizes the double auction studied by Hall (2005). Trade is mediated by an impartial third party, for example an arbitrator in labor negotiations, who collects once and for all requests from the two parties and implements an outcome according to a pre-defined rule. An example is a sealed-bid mechanism, where trade and wage payments occur only if the worker's request w_w does not exceed the firm's offer w_f, and the resulting wage lies between the two proposals, with some weighting function that depends on the specific bids. This encompasses the κ-double auction with wage rule $\kappa w_w + (1-\kappa) w_f$ adopted by Hall (2005), where the weights are constant. A desirable feature of this particular class of mechanisms, especially in a labor market context, is that they are ex post individual rational (EIR): after proposals have been submitted, the outcome implemented by the mediator makes both parties better off, so neither would prefer to walk away. Games in this class too have, in general, multiple equilibria, but equilibria typically differ in terms of efficiency. Taking a cue from Hall (2005), we restrict attention to mechanisms that implement allocations with desirable efficiency properties. Specifically, we study sealed-bid mechanisms that implement ex ante incentive efficient allocations. This is the source of uniqueness in this section.

The analysis is conveniently split into two parts. First, we study comparative statics of ex ante incentive efficient allocations, verifying IWR and Assumption 1. Second, we discuss implementation through a sealed-bid mechanism that satisfies EIR.

By the revelation principle, we restrict attention to direct mechanisms. Such a mechanism is *ex ante incentive efficient* if it solves a Pareto problem, maximizing $\alpha\mathcal{J}(p,n) + (1-\alpha)\mathcal{G}(p,n)$ subject to *Interim Individual Rationality* (IIR) and *Incentive Compatibility* (IC), for some $\alpha \in [0,1]$. An important special case is maximization of ex ante gains from trade ($\alpha = \frac{1}{2}$). IIR requires a party's rents to be non-negative conditional on its own private information. It is weaker than EIR, a point we return to when discussing implementation.

Throughout we focus on the non-trivial case $y_h + z_h + p - n > 0 > y_l + z_l + p - n$, in which the probability of gains from trade is strictly between zero and one.

The mediator receives reports $\hat{y}$ and $\hat{z}$ by the two parties and enforces a probability of trade $x(\hat{y},\hat{z},p,n)$ and a wage $w(\hat{y},\hat{z},p,n)$. Given a pair of reports $\hat{y},\hat{z}$ and realizations y,z, the worker's and the firm's flow rents are

$$(r+\delta)G(\hat{y},\hat{z},z,p,n) = (z-n)x(\hat{y},\hat{z},p,n) + w(\hat{y},\hat{z},p,n),$$
$$(r+\delta)J(\hat{y},\hat{z},y,p,n) = (p+y)x(\hat{y},\hat{z},p,n) - w(\hat{y},\hat{z},p,n).$$

Expected worker rents are

$$(r+\delta)\mathcal{G}(p,n) = \int_{z_l}^{z_h}\int_{y_l}^{y_h} G(y,z,z,p,n)\,dF_Y(y)\,dF_Z(z),$$

and the combined IC and IIR constraint for the worker is

$$\int_{y_l}^{y_h} G(y,z,z,p,n)x(y,z,p,n)\,dF_Y(y) \geqslant \max\left\{0, \int_{y_l}^{y_h} G(y,\hat{z},z,p,n)x(y,\hat{z},p,n)\,dF_Y(y)\right\}$$

for all $\hat{z}, z$. Firm rents and the combined constraint for the firm are analogous. The following lemma establishes the relevant properties of ex ante incentive efficient allocations.

Lemma 3. *In the ex ante incentive efficient allocation, the probability of trade $x(y,z,p,n)$ satisfies Assumption* 1(c)*, and expected rents can be written as $\mathcal{G}(p-n)$ and $\mathcal{J}(p-n)$. Under Assumption* 2*, $\mathcal{G}'(p-n) \geqslant 0$ and $\mathcal{J}'(p-n) \geqslant 0$.*

As part of the proof, in Appendix A we show that in the case $\alpha \geqslant \frac{1}{2}$ the mediator adopts a trade threshold $z^*(y, p-n)$ implicitly defined by[16]

$$y + z^* + p - n = \frac{2\alpha - 1 + \mu}{\alpha + \mu}\frac{1 - F_Z(z^*)}{F'_Z(z^*)} + \frac{\mu}{\alpha + \mu}\frac{1 - F_Y(y)}{F'_Y(y)}, \tag{16}$$

[16] The case $\alpha \leqslant \frac{1}{2}$ is symmetric.

where μ is the Lagrange multiplier of the combined IC and IIR constraints for both parties, and depends on α and $p-n$. In the extreme case $\alpha=1$ the mediator maximizes firm rents; here IIR is not binding for firms, thus $\mu=0$, and Eq. (16) reduces to the first order condition (11) from firm offer monopoly. For α sufficiently close to one, the constraint remains slack, and the right-hand side of (16) reduces to $\frac{2\alpha-1}{\alpha}\frac{1-F_Z(z^*)}{F'_Z(z^*)}$; lower α mitigates the monopoly distortion and yields more trade. For α sufficiently close to $\frac{1}{2}$, IIR becomes binding for the firm, making the firm's hazard rate $\frac{1-F_Y(y)}{F'_Y(y)}$ a determinant of the threshold. For $\alpha=\frac{1}{2}$ the mediator maximizes ex ante gains from trade as in Myerson and Satterthwaite (1983), and the two hazard rates enter symmetrically. The structure of the threshold rule is similar to monopoly, and so are the comparative statics properties: in response to an increase in the gains from trade $p-n$, the mediator induces more trade, raising informational rents of both parties.

Turning to implementation, ex ante incentive efficiency imposes only IIR. This is too weak in a labor market context, as the worker may want to walk away after learning the wage. Gresik (1996) provides a relatively mild sufficient condition under which an ex ante incentive efficient allocation can be implemented through a sealed-bid mechanism that satisfies EIR. It is more stringent than Assumption 2, but weaker than monotone increasing hazard rates. This generalizes the celebrated finding of Chatterjee and Samuelson (1983) that the $\frac{1}{2}$-double auction implements the ex ante incentive efficient allocation obtained for $\alpha=\frac{1}{2}$ if beliefs are uniform. Importantly, while the mechanism Gresik constructs may have multiple equilibria, it implements a unique ex ante incentive efficient allocation. We summarize:

Proposition 6. *Under two-sided private information satisfying Assumption* 2 *and* $y_h+z_h+p-n>0>y_l+z_l+p-n$*, the ex ante incentive efficient allocation satisfies Assumption* 1 *and IWR. Thus, the volatility bound applies. If in addition hazard rates are monotone increasing, this allocation can be implemented in the unique ex ante efficient* (*and ex post individually rational*) *equilibrium of a sealed-bid mechanism.*

We briefly return to indeterminacy as a source of countercyclical rents. Hall (2005) stresses that all equilibria of the κ-double auction (under complete information) are privately Pareto undominated, allowing his theory to generate rigid wages without invoking unexplained inefficiencies. Hall's result may not be robust to the presence of private information. The κ-double auction still has a multiplicity of equilibria, yet many are inefficient (Leininger et al., 1989), and generically there does not exist an ex ante incentive efficient equilibrium (Satterthwaite and Williams, 1989). Interim incentive efficient equilibria (Pareto undominated after parties learn their own type) exist for an open set of beliefs. However, the set of such equilibria has not been characterized. Hence it is unknown whether multiplicity of such equilibria can be exploited to generate countercyclical rents, or whether the latter requires resorting to interim inefficient equilibria.

5. A stronger volatility bound for the contact rate

Our volatility bound is based entirely on the feedback effect. It ignores the magnitude of the productivity impact on firms' profits, thus on their incentives to create new jobs. Taking the latter into account, we derive a qualitatively stronger bound. We show that the two bounds are quantitatively close in our calibration; thus, the feedback effect emerges as the primary constraint on amplification. The strong bound is useful for analytical purposes, as it is exactly attained by some canonical models of wage determination

5.1. Derivation

Our volatility bound $\bar{\varepsilon}_m$ requires no restriction on $\frac{\partial \mathcal{J}(p,n)}{\partial p}$, the impulse transmitted from a productivity shock to firm rents, and remains valid even as $\frac{\partial \mathcal{J}(p,n)}{\partial p}\to\infty$.

Yet for both Nash Bargaining and Wage Rigidity the impulse is bounded, specifically by the probability of trade. This motivates part (a) of the following regularity condition.

Assumption 3. The model of wage determination $\{G, J, x\}$ satisfies:

(a) The partial effect of aggregate labor productivity p on expected firm rents $\mathcal{J}(p,n)$ is bounded by the probability of trade:

$$(r+\delta)\frac{\partial \mathcal{J}(p,n)}{\partial p}\leqslant \xi(p,n).$$

(b) The trade probability $x(y,z,p,n)$ depends on p and n only through their difference $p-n$ and is weakly increasing in $p-n$.

For Wage Rigidity part (a) holds with equality: $\xi(p,n)=1$ and $(r+\delta)\frac{\partial \mathcal{J}(p,n)}{\partial p}=1$. For Nash Bargaining it holds with slack: $(r+\delta)\frac{\partial \mathcal{J}(p,n)}{\partial p}=(1-\beta)\xi(p,n)$. More generally, if trade is ex post efficient, then $(r+\delta)(\frac{\partial \mathcal{J}(p,n)}{\partial p}+\frac{\partial \mathcal{G}(p,n)}{\partial p})=\xi(p,n)$.

This follows from the envelope theorem: the indirect effect of p through adjustment of the trading decision is zero. In this case part (a) is redundant since it is implied by Assumption 1(a). This implies that part (a) holds for the model of strategic bargaining with one-sided private information analyzed in Section 4.2.2. Despite ex post inefficient trade due to the monopoly distortion, Eq. (13) shows that part (a) also holds (with equality) in the model with take-it-or-leave-it offers by the firm analyzed in Section 4.2.1.

Part (b) strengthens Assumption 1(c), requiring that higher $p - n$ makes trade more likely.

Proposition 7 *(Strong Volatility Bound). Under Assumptions* 1 *and* 3*, if the model of wage determination satisfies IWR, then* $\varepsilon_m \leqslant \hat{\varepsilon}_m$*, where*

$$\hat{\varepsilon}_m \equiv \frac{\bar{p}}{\bar{p}-b}\Bigg(\underbrace{\frac{\bar{w}-b}{\bar{p}-b}\frac{1}{\frac{r+\delta+f}{f}}}_{\text{feedback}} + \underbrace{\frac{\bar{p}-\bar{w}}{\bar{p}-b}\frac{1}{\frac{\hat{\eta}}{1-\hat{\eta}}}}_{\text{impulse}}\Bigg)^{-1}. \tag{17}$$

To facilitate comparison, our first bound can be rewritten as

$$\bar{\varepsilon}_m = \frac{\bar{p}}{\bar{p}-b}\left(\frac{\bar{w}-b}{\bar{p}-b}\frac{1}{\frac{r+\delta+f}{f}}\right)^{-1}.$$

The only difference is the term labeled impulse, which leads to a tighter bound. It arises from the entry condition, Eq. (3). This condition played a limited role for the first bound, but puts an additional limit on volatility under Assumption 3.

The first term of both bounds $\frac{\bar{p}}{\bar{p}-b}$ is the inverse of gains from market activity. Higher gains dampen the response of the contact rate. One can think of gains from market activity as the proximate driving force of the model. A higher *level* of gains implies that a given percentage change in productivity translates into a smaller percentage change in gains from market activity, thus a less variable driving force.

The second term of the strong bound shows how amplification depends on the distribution of these gains between firm and worker. It is the harmonic mean of two terms, weighted by the two parties' shares $\frac{\bar{w}-b}{\bar{p}-b}$ and $\frac{\bar{p}-\bar{w}}{\bar{p}-b}$. The first term $\frac{r+\delta+f}{f}$ is familiar from the first bound and captures the role of frictions in determining the strength of the feedback effect. This effect is stronger if gains of workers are larger, hence the weight $\frac{\bar{w}-b}{\bar{p}-b}$. To understand the second term, note that $\frac{\eta}{1-\eta}$ is the elasticity of the contact rate with respect to firm rents, thus it measures the strength of the impulse transmitted from firms' profits to their job creation, hence to the contact rate. The matching function elasticity η is not directly observed. But since the probability of trade is procyclical, η is bounded by the elasticity of the job-finding rate with respect to tightness $\hat{\eta} = \frac{\varepsilon_f}{\varepsilon_\theta}$. Thus $\frac{\hat{\eta}}{1-\hat{\eta}}$ is an upper bound for $\frac{\eta}{1-\eta}$. This *impulse effect* is weighted by the firm share $\frac{\bar{p}-\bar{w}}{\bar{p}-b}$, because if the level of firms' gains is larger, then the percentage increase of firm rents in a boom is smaller, weakening the impulse.

Effectively, our first bound cannot be attained exactly, since this would require an infinitely strong impulse, that is $\frac{\partial \mathcal{J}(p,n)}{\partial p} = +\infty$. In contrast, the strong bound is attained exactly if both worker rents and the probability of trade are acyclical. We provide two examples of canonical models of wage determination that exactly attain the strong bound; we show below that the two bounds are quantitatively close in our calibration; hence, models that attain the strong bound also come close to attaining our first bound.

First, in the Shapiro and Stiglitz (1984) efficiency wage model, workers receive rents to deter shirking. Specifically, the wage is $w = n + \frac{(r+\delta+q)e}{q}$, where e is a cost of providing effort, and q is the rate at which shirking is detected.[17] The implied rent $\mathcal{G}(p,n) = \frac{e}{q}$ is acyclical if one assumes that the parameters e and q do not respond to changes in p or n.

Next consider the Coase conjecture model discussed in Section 4.2.2. Assume $y_l + z_l + p - n \geqslant 0$, that is gains from trade are non-negative for all realizations of y and z. Then Eq. (14) with $\beta = 0$ (the Coase conjecture case) implies $G(y,z,p,n) = z - z_l$. Thus expected worker rents $(r+\delta)\mathcal{G}(p,n) = \int_{z_l}^{z_h}(z - z_l)\,dF_Z(z) = -z_l$ are acyclical.

5.2. Quantitative evaluation

Table 4.A shows that in the benchmark calibration the strong bound is $\hat{\varepsilon}_m = 3.549$, while the first bound is $\bar{\varepsilon}_m = 3.866$. Mapped into fractions of the variance of the job-finding rate, this corresponds to 19.4% and 23%, respectively. Thus the two bounds are quantitatively close. This is due to the low level of recruiting costs, which implies that firms' share in the gains from market activity is small: $\frac{\bar{p}-\bar{w}}{\bar{p}-b} = 0.053$. Thus the impulse effect is weak, and the feedback effect is the key force limiting amplification. If recruiting costs were higher, the strong bound would be even smaller. This is because $\frac{\hat{\eta}}{1-\hat{\eta}} < \frac{r+\delta+f}{f}$, which implies that, if given equal weight, a weak impulse is actually more severe than the feedback effect.

[17] See Eqs. (3)–(5) in Shapiro and Stiglitz (1984).

Table 4
Strong volatility bound.

	ε_m	$\varepsilon_m^2 \frac{\sigma_p^2}{\sigma_f^2}$
Panel A. Benchmark calibration $\frac{b}{p} = 0.71$		
Volatility Bound $\bar{\varepsilon}_m$	3.866	23.0%
Strong Volatility Bound $\hat{\varepsilon}_m$	3.549	19.4%
Panel B. Shimer's calibration $\frac{b}{p} = 0.4$		
Volatility Bound $\bar{\varepsilon}_m$	1.816	5.1%
Strong Volatility Bound $\hat{\varepsilon}_m$	1.743	4.7%

6. On-the-job search

In deriving our bounds on the cyclical volatility of the job contact rate for the unemployed, we have focussed on unemployed job search. But the flow of job-to-job transitions is comparable in size to the flow out of unemployment (e.g., Fallick and Fleischman, 2004) and is procyclical, with a significant lag (Moscarini and Thomsson, 2007). Accordingly, on-the-job search (OJS) plays a central role in much of the recent literature on labor market fluctuations. OJS has several important implications for our analysis, that we now discuss. Our main message still applies to most existing business cycle models that feature OJS: either amplification is constrained by our bound, or exceeds the bound because rents of workers coming from unemployment are countercyclical.

Formally, our first bound (8) makes use only of the value of unemployment equation, that is (3), and of a property of the free entry condition (2), namely that market tightness θ falls if the value of unemployment n rises faster than p. Hence, OJS can modify our conclusions for three reasons: it changes the calibration; it modifies these properties of the equilibrium conditions; or it introduces transitional dynamics, so that comparative statics across steady states may be less informative of stochastic results. We analyze them in this order.

The first implication of OJS relates to measurement and calibration. Many of the extra vacancies that we measure in aggregate expansions generate job-to-job movements, rather than exits from unemployment. Ignoring OJS leads to underestimate the productivity of vacancies in reducing unemployment. The regression of $\log f$ on $\log(v/u)$ discussed earlier no longer provides a valid estimate of the matching function elasticity η even if the probability of trade is constant, and OJS is likely to induce a downward bias.

It is clear from its definition (8) that our first volatility bound $\bar{\varepsilon}_m$ does not depend on the value of η, thus is not affected by a biased estimate of this parameter. The reason is that for the strength of the feedback effect it is immaterial whether the volatility of the contact rate m originates from a very volatile market tightness θ or from a high matching function elasticity η. A corrected larger value of η, however, does permit volatility closer to the first bound, by relaxing the second, stronger bound $\hat{\varepsilon}_m$ in (17). Indeed, the strong bound converges to the first bound as $\eta \uparrow 1$.

Second, OJS can modify the free entry condition through a composition effect in the pool of job searchers, employed and unemployed, from which firms randomly draw applicants. In aggregate expansions, as unemployment falls, the likelihood of receiving job applications from employed workers increases. If an employed worker can extract all rents from the poaching firm, as for example in an auction model with Bertrand competition among identical firms (Postel-Vinay and Robin, 2002), then the free entry condition depends only on meetings with the unemployed, as in our model without OJS. In most OJS models employed job applicants are more expensive and less profitable to firms. Hence, the composition effect tends to dampen movements in the returns to job creation and (through an appropriately modified free entry condition) labor market volatility. But it does not reverse the property on which our first bound relies, namely that market tightness θ falls if the value of unemployment n rises faster than p. Nagypál (2007) introduces an adverse selection mechanism that makes employed job candidates more attractive to firms, and reverses this composition effect. Combined with the upward revision in the value of η, this mechanism raises the elasticity of the unemployed's job-finding rate f to aggregate productivity, but only by a factor of $0.1376/0.0947 = 1.4530$ relative to her model without OJS (her Table 2). In comparison, our volatility bound permits amplification by a factor of 2.68. Nagypál is able to match observed volatility in f by making worker rents countercyclical through a mechanism unrelated to OJS, namely wage bargaining à la Hall and Milgrom (2008) that we discussed earlier.

Krause and Lubik (2007) present a model where firms can create two types of jobs that differ by creation cost, thus pay different Nash-bargained wages. Unemployed workers can direct their search to job types, and employed workers (but not the unemployed) can control the intensity of their search effort. Volatility in f vastly exceeds our volatility bound due to a different composition effect. When aggregate productivity rises, firms create more jobs of both types, but workers employed in low wage jobs try harder to upgrade to high wage jobs, creating congestion in that sector for the unemployed. The latter, then, pursue more aggressively low wage jobs. As a consequence, the *average* rents $\mathcal{G}$ from a successful job application from unemployment are countercyclical, although rents from working in either type of job are procyclical. This suggests that the amplification their model generates would be reduced if the unemployed could also vary their search intensity.

Given a rent function $\mathcal{G}$, the value of unemployment equation, that is (3), holds in any search model. In some OJS models, however, labor market tightness θ determines the returns from future OJS to newly hired workers, hence it affects rents $\mathcal{G}$

accruing to new hires from unemployment, independently of p and n. The question then becomes whether the total effect of productivity $d\mathcal{G}(p,n,\theta)/dp$ remains positive. Menzio and Shi (2008) solve a tractable equilibrium of a directed job search model with OJS and aggregate productivity shocks. They characterize a Block-Recursive Equilibrium in which unemployed workers apply only to the lowest-paying job. Simulations of their model under their preferred calibration reveal that $\mathcal{G}$ and p are perfectly positively correlated,[18] although $\mathcal{G}$ is relative stable, taming the feedback effect. The volatility in f that they obtain is still constrained by our bound, although close due to the upward revision in the value of η. We conjecture that this result extends to all OJS models where worker rents can be written as $\mathcal{G}(p,n,\theta)$, for example sequential auctions between homogeneous firms (Postel-Vinay and Robin, 2002). The returns from future OJS tend to reinforce the cyclicality of worker rents. OJS increases the value of employment by leaving open the option of job upgrading, and this option is more frequently available in booms. Verifying this conjecture is a task that we leave for future research, following our road map: (i) find conditions on the function $\mathcal{G}(p,n,\theta)$ that imply $d\mathcal{G}(p,n,\theta)/dp \geqslant 0$ through the indirect general equilibrium effects of p on n and θ, (ii) verify these conditions in models of wage determination that feature outside offers.

In the wage posting model with random search of Burdett and Mortensen (1998) OJS generates a non-degenerate distribution of wages for matches of identical productivity. This by itself does not invalidate our approach. It remains possible to analyze the behavior of worker rents across steady-states, and one can show that rents of new hires take the form $\mathcal{G}(p-n,\lambda_1)$, where λ_1 is the arrival rate of offers on the job. Rents increase in both $p-n$ and λ_1. If λ_1 is endogenized as a function of tightness θ, OJS once again reinforces the positive effect of productivity on rents.

The Burdett–Mortensen model presents a different complication, however, which does not arise in the Block-Recursive Equilibrium of Menzio and Shi, but also affects models of OJS with random search and bargaining (Tasci, 2007). Equilibrium market tightness θ is a function of the wage distribution, which is an aggregate state variable that moves slowly in response to permanent productivity shocks. Thus tightness no longer jumps immediately to its new steady state value, and the simple analytical approach we used to derive our bound is not applicable. Here we can no longer presume that comparative statics across steady-states are informative of stochastic properties. But the basic logic underlying our bound can still be useful to illuminate why simulations of specific calibrated models succeed or fail in obtaining plausible amplification. We suggest that any simulation exercise quantify the correlation between rents of new hires and the aggregate impulse.

7. Conclusion

The search and matching model features a mechanism of employment fluctuations absent in frictionless models. Trading frictions are overcome through a matching technology with unemployed workers and vacant jobs as essential inputs, and variation in these two resources over the business cycle generates fluctuations in the rate at which workers contact firms.

How important is this channel quantitatively? It is negligible if wages are Nash-bargained, unless a very different calibration strategy than Shimer's (2005) is adopted. It produces excessive volatility if wages are rigid. With rigid rents it is quantitatively important, but cannot be the only source of fluctuations in the job-finding rate.

Much of the literature instigated by Shimer (2005) has strived to find plausible modifications of wage setting that strengthen the contact rate channel enough to account for *all* fluctuations in the job-finding rate. If one subscribes to this goal, a model with countercyclical worker rents is needed. The presence of private information alone does not appear to be a source of countercyclical rents. Strategic bargaining, and indeterminacy, are ways to obtain countercyclical rents under complete information, but stand in tension with the observation of near competitive wage levels.

Finally, it is plausible that variation in resources devoted to overcoming search frictions is not the only source of fluctuations in the job-finding rate. The probability of trade conditional on having overcome frictions, more akin to margins found in frictionless models, could play a quantitatively important role.

Appendix A. Omitted results and proofs

Lemma A.1. *Under Assumption* 1*, if the equilibrium exhibits PWR, then* $\frac{dn}{dp} \geqslant 1$ *implies* $\varepsilon_m \leqslant 0$*.*

Proof. First, we show that $\frac{dn}{dp} \geqslant 1$ implies countercyclical firm rents, that is $\frac{\partial \mathcal{J}}{\partial p} + \frac{\partial \mathcal{J}}{\partial n}\frac{dn}{dp} \leqslant 0$. This is immediate if the direct effect of p is negative, $\frac{\partial \mathcal{J}}{\partial p} < 0$, since Assumption 1(b) insures that the indirect effect through n is negative as well. So consider $\frac{\partial \mathcal{J}}{\partial p} \geqslant 0$. Assumption 1(c) implies that total rents $\mathcal{J}(p,n) + \mathcal{G}(p,n)$ are a function of only the difference $p-n$, yielding the restriction on derivatives

$$\frac{\partial \mathcal{J}}{\partial p} + \frac{\partial \mathcal{J}}{\partial n} + \frac{\partial \mathcal{G}}{\partial p} + \frac{\partial \mathcal{G}}{\partial n} = 0. \tag{18}$$

Using this restriction the response of total rents can be written as

$$\left(\frac{\partial \mathcal{J}}{\partial p} + \frac{\partial \mathcal{J}}{\partial n}\frac{dn}{dp}\right) + \left(\frac{\partial \mathcal{G}}{\partial p} + \frac{\partial \mathcal{G}}{\partial n}\frac{dn}{dp}\right) = \left(\frac{\partial \mathcal{J}}{\partial p} + \frac{\partial \mathcal{G}}{\partial p}\right)\left(1 - \frac{dn}{dp}\right).$$

[18] We thank Guido Menzio for this calculation.

Assumption 1(a) and $\frac{\partial \mathcal{J}}{\partial p} \geqslant 0$ yield $\frac{\partial \mathcal{J}}{\partial p} + \frac{\partial \mathcal{G}}{\partial p} \geqslant 0$. This shows that total rents are increasing in $p - n$. Since $\frac{dn}{dp} \geqslant 1$ higher p implies lower $p - n$, so total rents are countercyclical. PWR then implies that firm rents are countercyclical.

Log-differentiating (2) yields

$$\frac{1-\eta}{\eta}\varepsilon_m = \frac{p}{\mathcal{J}}\left(\frac{\partial \mathcal{J}}{\partial p} + \frac{\partial \mathcal{J}}{\partial n}\frac{dn}{dp}\right) \tag{19}$$

where $\eta \equiv \frac{d\ln m}{d\ln \theta}$ is the elasticity of the contact rate with respect to labor market tightness. Thus countercyclical firm rents imply $\varepsilon_m \leqslant 0$. □

Proof of Lemma 1. Suppose wage determination satisfies IWR but PWR fails. This is only possible if $\frac{dn}{dp} > 1$. Since $\frac{\partial \mathcal{J}}{\partial n} \leqslant 0$ via Assumption 1(b), this implies $\frac{\partial \mathcal{J}}{\partial p} + \frac{\partial \mathcal{J}}{\partial n}\frac{dn}{dp} \leqslant \frac{\partial \mathcal{J}}{\partial p} + \frac{\partial \mathcal{J}}{\partial n}$. Moreover, (18) together with IWR insures that the right-hand side of this inequality is negative. Thus firm rents are countercyclical. Via (19) this implies $\varepsilon_m \leqslant 0$. But if $\varepsilon_m \leqslant 0$ and PWR fails, then (3) implies $\frac{dn}{dp} \leqslant 0$. This contradicts $\frac{dn}{dp} > 1$. □

Proof of Lemma 2. The proof of no delay and uniqueness of subgame perfect equilibrium payoffs is standard (see Muthoo (1999), Chapter 5), and delivers an intuitive characterization of payoffs through indifference conditions. Let G_w and G_f denote worker rents, depending on whether the worker makes the request or the firm the offer, respectively. These values are determined by the indifference conditions

$$G_f = \max\left\{0, e^{-\phi_w \Delta}\left[e^{-\rho_w \Delta} G_w - \left(1 - e^{-\rho_w \Delta}\right)\left(\frac{\gamma_w}{\rho_w} + \frac{n}{r}\right)\right]\right\},$$
$$\frac{y+z+p-n}{r+\delta} - G_w = \max\left\{0, e^{-\phi_f \Delta}\left[e^{-\rho_f \Delta}\left(\frac{y+z+p-n}{r+\delta} - G_f\right) - \left(1 - e^{-\rho_f \Delta}\right)\frac{\gamma_f}{\rho_f}\right]\right\}.$$

The first condition states that the worker must be indifferent between accepting and rejecting the firm's offer. Acceptance yields rents G_f. The maximum operator reflects that the worker can secure zero rents by opting out. If the worker rejects without opting out, then negotiations continue with probability $e^{-\phi_w \Delta}$. In this event, after the delay, the worker requests G_w and the firm accepts. The delay is associated with a cost $(1 - e^{-\rho_w \Delta})(\frac{\gamma_w}{\rho_w} + \frac{n}{r})$, which comprises the flow cost γ_w as well as the opportunity cost of not taking up the outside option. The second condition is the analogous indifference condition for the firm, where we have written firm rents as the difference between total surplus and worker rents. There is no opportunity cost term since the firm's outside option is zero. As $\Delta \to 0$ both G_f and G_w converge to the value in (9). □

Proof of Lemma 3. The model maps exactly into the setting studied by Myerson and Satterthwaite (1983) and Williams (1987).[19] Myerson and Satterthwaite (pp. 269–270) show that IC implies that expected firm rents satisfy

$$(r+\delta)\mathcal{J}(p,n) = (r+\delta)\mathcal{J}(y_l|p-n) + \int_{z_l}^{z_h}\int_{y_l}^{y_h} \frac{1-F_Y(y)}{F_Y'(y)} x(y,z,p,n)\, dF_Y(y)\, dF_Z(z), \tag{20}$$

where $\mathcal{J}(y_l|p-n)$ are rents of the lowest firm type. Adding the analogous formula for worker rents, and using that total rents are the integral of $(y+z+p-n)x(y,z,p,n)$, yields

$$\begin{aligned}\Gamma[x] &\equiv (r+\delta)\mathcal{G}(z_l|p-n) + (r+\delta)\mathcal{J}(y_l|p-n)\\ &= \int_{z_l}^{z_h}\int_{y_l}^{y_h}\left[y+z+p-n - \frac{1-F_Y(y)}{F_Y'(y)} - \frac{1-F_Z(z)}{F_Z'(z)}\right] x(y,z,p,n)\, dF_Y(y)\, dF_Z(z).\end{aligned}$$

Theorem 1 of Myerson and Satterthwaite implies that under Assumption 2 all IC and IIR constraints are equivalent to the single constraint $\Gamma[x] \geqslant 0$. We analyze the case $\alpha \geqslant \frac{1}{2}$, the results carry over to $\alpha \leqslant \frac{1}{2}$ by symmetry. If $\alpha > \frac{1}{2}$, the higher weight on the firm implies that no rents go to the lowest worker type, and any slack goes to the firm: $\mathcal{G}(z_l|p-n) = 0$ and $(r+\delta)\mathcal{J}(y_l|p-n) = \Gamma[x]$ (Williams (1987), Theorem 2). Using (20), its analog for worker rents, and the formula for $\Gamma[x]$, one can write the objective as

$$\int_{z_l}^{z_h}\int_{y_l}^{y_h}\left[\alpha(y+z+p-n) - (2\alpha-1)\frac{1-F_Z(z)}{F_Z'(z)}\right] x(y,z,p,n)\, dF_Y(y)\, dF_Z(z).$$

[19] They study a seller and a buyer with valuations v_1 and v_2, distributed according to F_1 and F_2, respectively. The following change of variables maps our setting into theirs. The worker is the seller with $v_1 \equiv n - z$, thus $F_1(v_1) \equiv 1 - F_Z(n - v_1)$. The firm is the buyer with $v_2 \equiv p + y$, thus $F_2(v_2) \equiv F_Y(v_2 - p)$.

This formula remains valid if $\alpha = \frac{1}{2}$, with the objective given by ex ante gains from trade.

Theorem 3 of Williams (1987) establishes that, under Assumption 2, the solution to this problem exists and is unique. Since both objective and constraint depend on p and n only through their difference, once again we can write expected rents as $\mathcal{G}(p-n)$ and $\mathcal{J}(p-n)$. The first order condition with respect to the probability of trade implies that $x(y,z,p,n)=1$ if

$$y+z+p-n-\frac{2\alpha-1+\mu(p-n,\alpha)}{\alpha+\mu(p-n,\alpha)}\frac{1-F_Z(z)}{F'_Z(z)}+\frac{\mu(p-n,\alpha)}{\alpha+\mu(p-n,\alpha)}\frac{1-F_Y(y)}{F'_Y(y)}\geqslant 0,$$

and $x(y,z,p,n)=0$ otherwise. This inequality implicitly defines the threshold $z^*(y,p-n)$ in (16). Here $\mu(p-n,\alpha)$ is the multiplier on the constraint $\Gamma[x]\geqslant 0$. Assumption 2 insures that the probability of trade is increasing in both y and z, hence this model of wage determination satisfies Assumption 1(c). For any fixed $\alpha \in [0,1]$, Brügemann and Moscarini (2008) show that $\mathcal{G}'(p-n)\geqslant 0$ and $\mathcal{J}'(p-n)\geqslant 0$. □

Proof of Proposition 7. By Lemma A.1 $\varepsilon_m \leqslant 0$ if $\frac{dn}{dp}\geqslant 1$. Thus consider the case $0\leqslant \frac{dn}{dp}<1$. Assumption 3 and IWR imply

$$\frac{\partial\mathcal{J}}{\partial p}+\frac{\partial\mathcal{J}}{\partial n}\frac{dn}{dp}=\frac{\partial\mathcal{J}}{\partial p}\left(1-\frac{dn}{dp}\right)-\left(\frac{\partial\mathcal{G}}{\partial p}+\frac{\partial\mathcal{G}}{\partial n}\right)\frac{dn}{dp}\leqslant\frac{\xi}{r+\delta}\left(1-\frac{dn}{dp}\right),$$

where the equality uses (18). Combining this with (19) yields

$$\frac{1-\eta}{\eta}\varepsilon_m(r+\delta)\frac{\mathcal{J}}{\xi}\frac{1}{p}\leqslant 1-\frac{dn}{dp}.$$

The definition of $\bar{w}$ implies $(r+\delta)\xi^{-1}\mathcal{J}=\bar{p}-\bar{w}$, so this inequality can be written as

$$\frac{1-\eta}{\eta}\varepsilon_m\frac{\bar{p}-\bar{w}}{\bar{p}}\leqslant 1-\frac{dn}{dp}. \tag{21}$$

Here we also replaced p by $\bar{p}$, appealing to Assumption 1(c) which implies $\bar{y}\geqslant 0$ and thus $\bar{p}\geqslant p$. Adding (7) and (21), and isolating ε_m yields

$$\varepsilon_m\leqslant\frac{\bar{p}}{(\bar{w}-b)\frac{f}{r+\delta+f}+(\bar{p}-\bar{w})\frac{1-\eta}{\eta}}. \tag{22}$$

Now, (4) implies $\varepsilon_f=\eta\varepsilon_\theta+\varepsilon_\xi$, while Assumption 3(b) implies $\varepsilon_\xi\geqslant 0$. Together this insures that $\hat{\eta}=\frac{\varepsilon_f}{\varepsilon_\theta}$ satisfies $\eta\leqslant\hat{\eta}$. Thus (22) implies (17). □

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