Selling to Intermediaries: Auction Design in a Common Value Model

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Selling to Intermediaries

- a single object is sold to one of N intermediaries
- winning intermediary offers the object to the downstream client with highest willingness to pay (at take-it-or-leave-it price)
- final value is common among all intermediaries
- each intermediary receives private signal about final value
- each signal is willingness to pay of a distinct client
- thus, intermediaries have pure common value: equal to the maximum of the individual signals
- what is the revenue maximizing selling mechanism?

Auction Design in Pure Common Value Model

- more broadly, private signal gives partially information about strength of secondary market
- key feature: conditional on highest signal, the other signals contain no additional information about the common value
- related interpretations where bidders are not intermediaries:

1. a pure common value model
   (as an alternative spefication of the wallet game (Myerson))
2. frictionless resale market with independent private value: interim owner can make take-it-or-leave-it price under complete information

Revenue Maximizing Design

- characterize revenue maximizing auction
- maximal revenue is obtained by strikingly simple mechanism, stated at interim level (given signal of bidder i)

1. constant – signal independent – participation fee
2. constant – signal independent – probability of getting object

- contrast with first, second, optimal auction with private values

Revenue Maximizing Design: Posted Price

- optimal mechanism shares some features with posted price

1. constant – signal independent – price

- it exactly coincides with posted price if

2. constant – signal independent – probability is $1/N$

- necessary and sufficient condition when optimal mechanism reduces exactly to posted price
- if posted price is an optimal mechanism it is inclusive: every bidder with every signal realization is willing to buy

Revenue Maximizing Design: Beyond Posted Price

- in generally aggregate assignment probability may be < 1
- interim probability of getting object is constant and $< 1/N$
- ex post probability for$i$ depends on entire signal profile
- conditionally on allocating the object optimal mechanism:

1. favors bidders with lower signals
2. discriminates against bidder with highest signal

- "winner's blessing" rather than "winner's curse"

A First Argument

- if seller could sell to intermediary without private information (or act as intermediary herself), then she could extract all surplus (expectation of best client value)
- with private information there are two competing forces:

1. would like to sell to intermediary with good client/signal, since has high unconditional willingness to pay
2. would like to sell to intermediary with bad client/signal, since he has less (private) information about best client value

- compromise: sell with equal probability to all intermediaries independent of their clients' values

Contributions: Substantive

- setting where bidders with higher signals have more accurate information about common value;
- arises in market with intermediaries, and many other settings: auctions for resources, IPO's
- countervailing screening incentives, tension between selling to

1. bidder with higher expected value and
2. bidder with less private information

- optimal to screen "less" - with no screening in inclusive limit
- foundation for posted price mechanisms
- new auction format: guaranteed demand auction

Contributions: Methodological

- very few results extend characterization of optimal auctions beyond private value case
- we extend optimal auctions into interdependent values:

1. with private values, “local” incentive constraints are sufficient to pin down optimal mechanism
2. with interdependent values, “global” constraints matter, new arguments are required

Plan of Talk

- model of pure common values
- revenue equivalence and failure of first order approach
- upper bound on revenue
- optimality of posted price mechanism
- construction of optimal mechanism
- implementation as guaranteed demand auction
- relationship to private values, power of optimal auctions

Model

Pure Common Value Model

- N bidders for a single object
- bidder i receives a signal $s_i \in [\underline{s}, \overline{s}] \subset \mathbb{R}_+$
- absolutely continuous cumulative distribution $F(s_i), f(s_i)$
- value is maximum of N independent signals:

$$v(s_1, \ldots, s_N) = \max\{s_1, \ldots, s_N\}$$

- signal distribution $F(s_i)$ induces value distribution $G_N(v)$:

$$G_N(v) = (F(s))^N$$

- value is first order statistic of N independent signals

Utility and Allocation

- bidder i is expected utility maximizer with quasilinear preferences, probability q_i of receiving object and transfers t_i:

$$u_i\left(s, q_i, t_i\right) = v\left(s\right) q_i - t_i$$

- feasibility of auction

$$q_i\left(s\right) \geq 0, \text{ with } \sum_{i=1}^{N} q_i\left(s\right) \leq 1$$

- *ex post* transfer $t_i\left(s\right)$ of bidder i, *interim* expected transfer:

$$t_i\left(s_i\right) = \int_{s_{-i} \in S^{N-1}} t_i\left(s_i, s_{-i}\right) f_{-i}\left(s_{-i}\right) ds_{-i},$$

where

$$f_{-i}\left(s_{-i}\right) = \prod_{j \neq i} f\left(s_j\right)$$

Incentive Compatibility

- bidder i surplus when reporting s_i' while observing s_i:

$$u_i\left(s_i, s_i'\right) \equiv \int_{s_{-i} \in S^{N-1}} q_i\left(s_i', s_{-i}\right) v\left(s_i, s_{-i}\right) f_{-i}\left(s_{-i}\right) ds_{-i} - t_i\left(s_i'\right)$$

- indirect utility given truthtelling is:

$$u_i(s_i) \equiv u_i\left(s_i, s_i\right)$$

- direct mechanism $\{q_i, t_i\}_{i=1}^{N}$ is *incentive compatible (IC)* if

$$u_i(s_i) \geq u_i\left(s_i, s_i'\right), \text{ for all } i \text{ and } s_i, s_i' \in S$$

and *individually rational (IR)* if $u_i(s_i) \geq 0$, for all i and $s_i \in S$

Bidder Surplus and Revenue

- ex-ante bidder surplus is

$$U_i = \int_{s_i \in S} u_i(s_i) f(s_i) \, ds_i$$

- revenue is expected sum of transfers:

$$R = \sum_{i=1}^{N} \int_{s_i \in S} t_i(s_i) f(s_i) \, ds_i$$

- seller maximizes R over all IC and IR direct mechanisms
- probability $q(v)$, $q_i(v)$ object is assigned (to bidder i) given v
- total surplus is

$$TS = \int_{v=\underline{s}}^{\overline{s}} vq(v) g_N(v) \, dv$$

Three Related Papers

- Bulow & Klemperer (AER 1996) study optimal auctions with interdependent values:

1. provide a revenue equivalence theorem (we use/adapt it)
2. optimal allocation is sensitive to marginal value of private information
3. solve for special case when local constraints are sufficient

- crucially, interim monotonicity of the allocation is neither necessary nor sufficient for incentive compatibility
- Bulow & Klemperer (Rand 2002) introduce “maximum game” as common value model for mineral rights

$$v(s_1, \ldots, s_N) = \max\{s_1, \ldots, s_N\}$$

- observe that winner’s curse is so strong that second price auction is revenue dominated by posted price

Three Related Papers

- in "First Price Auctions with General Information Structures: Implications for Bidding and Revenue" (ECTA, 2017), we fix distribution of values $G_N(v)$ and ask how common prior distribution of signals affect surplus and welfare

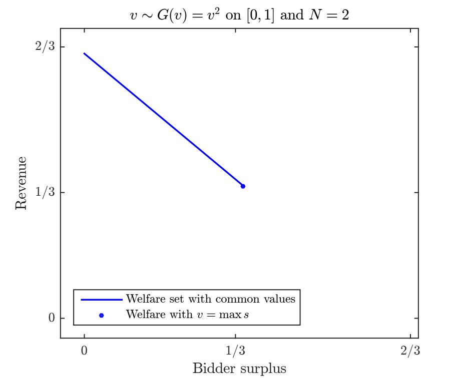
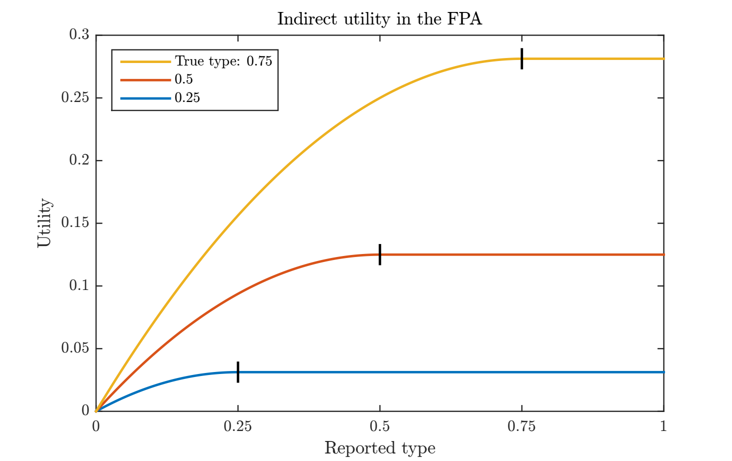


Figure 1: Information and Surplus Distribution

Three Related Papers

- we find

$$F(s(v)) = (G(v))^{1/N}$$



Bounds on Bidder Surplus and Revenue

Review: Revenue Equivalence with Private Value

- standard approach since Myerson (1981) uses the constraint that truthful reporting is a local maximizer for the bidder
- local IC depends on interim probability of getting object

$$q_i(s_i) \equiv \int_{s_{-i}} q_i\left(s_i, s_{-i}\right) f_{-i}\left(s_{-i}\right) ds_{-i}$$

- with private values, logic of envelope theorem implies a small increase in signal (=value) has direct effect only:

$$u_i'\left(s_i\right) = q_i(s_i)$$

Lemma (Revenue Equivalence)

In any incentive compatible direct mechanism indirect utility is

$$u_i\left(s_i\right) = u_i\left(\underline{s}\right) + \int_{x=\underline{s}}^{s_i} q_i\left(x\right) dx.$$

Revenue Equivalence with Maximum Value

- local IC depends on interim probability of getting object if signal s_i is informative about $v(s)$
- $\Leftrightarrow$if signal of i is highest among all bidders:

$$\widehat{q}_i(s_i) \equiv \int_{s_{-i}} \mathbb{I}_{s_i \geq max_{j \neq i} s_j} q_i\left(s_i, s_{-i}\right) f_{-i}\left(s_{-i}\right) ds_{-i}$$

- with maximum value, logic of envelope theorem implies a small increase in signal (=value) has direct effect only if s_i is highest:

$$u_i'\left(s_i\right) = \widehat{q}_i(s_i)$$

Lemma (Revenue Equivalence)

In any incentive compatible direct mechanism indirect utility is

$$u_i\left(s_i\right) = u_i\left(\underline{s}\right) + \int_{x=\underline{s}}^{s_i} \widehat{q}_i\left(x\right) dx.$$

Failure of Sufficiency of Local Constraints

- standard argument would establish that local incentive constraints also support global incentive constraints
- consider policy that assigns object to bidder with lowest signal

$$q_i^{min}(s) = \mathbb{I}_{s_i < min_{j \neq i} s_j},$$

- this would imply that

$$\widehat{q}_i(x) = 0$$

  and thus by the previous lemma:

$$u_i(s_i) = u_i(\underline{s}) + \int_{x=\underline{s}}^{s_i} \widehat{q}_i(x)\, dx = u_i(\underline{s})$$

- optimal auction would leave bidders with zero surplus, would support efficient allocation...but then consider deviation of $s_i = \overline{s}$
- there are additional – global – constraints on $\widehat{q}_i(x)$!

Global Incentive Constraints

- which global constraints matter for optimal revenue?
- previous argument suggest (all) downward deviations
- seller wants to distorts allocation to lower signals
- thus possibly all deterministic downward misreports, and hence all random downward misreports

A Relaxed Problem

- consider a smaller, one-dimensional, family of constraints:
- instead of reporting signal s_i, report a random signal $s_i' < s_i$, drawn from truncated prior on support $[\underline{s}, s_i]$:

$$F\left(s_i'\right)/F\left(s_i\right)$$

- *misreporting a redrawn lower signal*
- analyze a relaxed problem which consists of local and small class of global constraints
- use these constraints to derive:

1. an upper bound on seller revenue
2. a lower bound on bidder utility

A Lower Bound on Bidder Utility

- what are the gains from *misreporting a redrawn lower signal?*
- equilibrium surplus of a bidder with type x is

$$u_i\left(s_i\right) = \int_{x=\underline{s}}^{s_i} \widehat{q}_i\left(x\right) dx$$

- surplus from misreporting the redrawn lower signal

$$\frac{1}{F\left(s_i\right)} \int_{x=\underline{s}}^{s_i} u_i\left(s_i, x\right) f\left(x\right) dx$$

- gains vary depending on realized misreport
  average gains across all misreports are easy to compute

Average Gains from Misreporting

- misreport is redrawn from prior, bidder i is equally likely to fall anywhere in distribution of signals, unconditional on misreport, ex-ante likelihood that i receives object and x is highest signals

$$q_i(x)\, g_N(x)$$

- if highest report is less than s_i, surplus that bidder i obtains from being allocated object is s_i rather than x, so $s_i - x$ is difference between deviator and truthtelling surplus:

$$\frac{1}{F(s_i)} \int_{x=\underline{s}}^{s_i} \left[(s_i - x)\, q_i(x)\, g_N(x) + u_i(x)\, f(x) \right] dx$$

- thus the incentive constraint requires:

$$u_i(s_i) \geq \frac{1}{F(s_i)} \int_{x=\underline{s}}^{s_i} \left[(s_i - x)\, q_i(x)\, g_N(x) + u_i(x)\, f(x) \right] dx$$

Lower Bound As Equality

- notice that
$$u_i\left(s_i\right)=u_i\left(\underline{s}\right)+\int_{x=\underline{s}}^{s_i}\widehat{q}_i\left(x\right)dx$$
- and thus $q_i(x)$ induces $\widehat{q}_i\left(x\right)$, connects local and global constraints
$$u_i(s_i)\geq\frac{1}{F\left(s_i\right)}\int_{x=\underline{s}}^{s_i}\left[\left(s_i-x\right)q_i\left(x\right)g_N(x)+u_i\left(x\right)f\left(x\right)\right]dx$$
it also has to hold as sum across i :
$$u(s)\geq\frac{1}{F\left(s\right)}\int_{x=\underline{s}}^{s}\left[\left(s-x\right)q\left(x\right)g_N(x)+u\left(x\right)f\left(x\right)\right]dx$$
- lowest solution $\underline{u}\left(s\right)$ exists and solves inequality as equality
- can be integrated by parts as
$$\underline{U}=\int_{x\in S}\underline{u}\left(s\right)f\left(s\right)ds=\int_s\left(\int_{x=s}^{\overline{s}}\frac{1-F\left(x\right)}{F\left(x\right)}dx\right)q\left(s\right)g_N\left(s\right)ds$$

A Generalized Virtual Utility Formula

- we obtain our final formula for revenue, which is

$$\overline{R} = TS - \underline{U} = \int_{v} \psi(v)\, q(v)\, g_N(v)\, dv$$

where

$$\psi(v) = v - \int_{x=v}^{\overline{s}} \frac{1 - F(x)}{F(x)} dx,$$

- compare to virtual utility in private value environments:

$$\pi(x) = x - \frac{1 - F(x)}{f(x)}$$

Upper Bound on Revenue

- generalized virtual utility:

$$\psi(x) = x - \int_{y=x}^{\overline{s}} \frac{1 - F(y)}{F(y)} dy,$$

Theorem (Revenue Upper Bound)

In any auction in which the probability of allocation is given by q, bidder surplus is bounded below by $\underline{U}$ and expected revenue is bounded above by $\overline{R}$.

- bound is valid for any allocation policy $q(v)$

Trade-Offs: Efficiency vs. Information Rents

- bound is generated by incentive constraint:

$$u(s) \geq \frac{1}{F(s)} \int_{x=\underline{s}}^{s} \left[(s-x)\, q(x)\, g_N(x) + u(x)\, f(x) \right] dx$$

  and rewriting in terms of $\widehat{q}(x)$ and $q(x)$:

$$\int_{y=\underline{s}}^{s} (s-x)\, q(x)\, f(x)\, dx \leq \int_{x=\underline{s}}^{s} \widehat{q}(x)\, F(x)\, dx$$

- by allocation that favors low-signal bidders by making $\widehat{q}(s)$ as small as possible and $q(s)$ as large as possible:
- increasing $q(x)$ has two competing effects on revenue:

1. increases total surplus generated by auction,
2. generates additional information rents for types greater than x

- it increases value of misreporting a redrawn lower signal

Posted Prices As Optimal Mechanism

Posted Prices

- consider mechanisms where object is always allocated
- pure common values – allocation is therefore socially efficient

Theorem (Revenue Optimality among Efficient Mechanisms)
Among all mechanisms that allocate the object with probability one, revenue is maximized by setting a posted price of

$$p = \int_{v=\underline{s}}^{\overline{s}} v g_{N-1}(v)\, dv, \tag{1}$$

i.e., expected value of object conditional on having lowest signal $\underline{s}$*.*

- posted price is inclusive: all types purchase at p
- all bidders equally likely to receive object: $q_i(v) = 1/N$, $\forall i, v$.
- optimal selling mechanism is attained with constant interim transfer $t = t_i(s_i)$ and probability $q = q_i(s_i)$

Optimality of Posted Price

- next, optimality of posted price among all – possibly inefficient – mechanisms

Corollary (Revenue Optimality of Posted Prices)
A posted price mechanism is optimal if and only if

$$\psi(\underline{s}) = \underline{s} - \int_{\underline{s}}^{\overline{s}} \frac{1 - F(x)}{F(x)} dx \geq 0.$$

If a posted price p is optimal, then it is fully inclusive.

Optimal Mechanism in General

Optimal Mechanism in General

- construct an novel mechanism/game that attains the revenue bound for all distributions
- *guaranteed demand auction* (GDA)
- direct mechanism to implement the bound exists as well
- descending clock (probability) auction implements revenue bound as well

The Guaranteed Demand Game: Rules

- bidder i demands $d_i \in \left[0, \overline{d}\right]$, where $\overline{d}$ is a parameter of game:

$$0 \leq d_i \leq \overline{d} \leq 1/N$$

- let i^* denote identity of bidder with the highest demand:

$$d_{i^*} = \max\{d_1, ..., d_N\} :$$

- if $d_{i^*} > 0$:

1. i^* is allocated object with probability d_{i^*}
2. bidder $j \neq i^*$ receives the object with probability

$$\left(1 - d_{i^*}\right) / \left(N - 1\right) > d_{i^*}$$

- if $d_{i^*} = 0$, then the seller keeps the object

The Guaranteed Demand Game: Properties

- importantly if upper bound on demand $\overline{d}$ is:

$$\overline{d} < 1/N$$

- then conditional on highest demand being positive,

$$d_{i^*} > 0$$

  bidder j is more likely to get object if he doesn't have highest demand:

$$q_j > d_i^* > d_j$$

- each bidder's probability of receiving object is always at least his demand

Equilibrium Strategy

- unique equilibrium has a monotone pure strategy:

$$\sigma(s_i) = \begin{cases} \frac{1}{N}\left(1 - \frac{G_N(\bar{r})}{G_N(s_i)}\right) & \text{if } s_i \geq \bar{r}; \\ 0 & \text{if } s_i < \bar{r}, \end{cases} \tag{2}$$

  where threshold $\bar{r}$ solves:

$$G_N(\bar{r}) = 1 - N\bar{d}. \tag{3}$$

- and demand at $\bar{s}$ satisfies:

$$\sigma(\bar{s}) = \left(1 - \left(1 - N\bar{d}\right)\right)/N = \bar{d}$$

- denote resulting equilibrium utility: $\bar{u}_i(\underline{s})$

The Guaranteed Demand Auction

- turn guaranteed demand game into *guaranteed demand auction* (GDA) by adding entry fees f_i
- each bidder's message consists of a pair: entry decision and demand
- if bidder i decides to enter, he pays f_i to the seller, and object is allocated among bidders who enter guaranteed demand game

Proposition (Equilibrium of Guaranteed Demand Auction)
As long as $f_i \leq \overline{u}_i(\underline{s})$ for all i, it is an equilibrium for all bidders to enter the GDA and make demands according to (2). In equilibrium, bidders are indifferent between their equilibrium demands and all lower demands.

Optimality of Guaranteed Demand Auction

- consider the GDA where the threshold r has zero generalized virtual utility:

$$\psi(r^*) = r^* - \int_{y=r^*}^{\overline{s}} \frac{1 - F(y)}{F(y)} dy = 0$$

- choice of $\overline{d}$

$$\overline{d} = \frac{1 - G_N(r^*)}{N}$$

ensures that bidder makes a positive demand iff bidder has signal greater than r^*

Theorem (Optimality of Guaranteed Demand Auction)
Revenue is maximized with a guaranteed demand auction with maximum demand $\overline{d} = (1 - G_N(r^*))/N$ and a symmetric entry fee which is equal to $\overline{u}_i(\underline{s})$.

Incentive Constraints in Optimal Auction

- in the optimal auction, each bidder is indifferent between his equilibrium bid and any lower bid

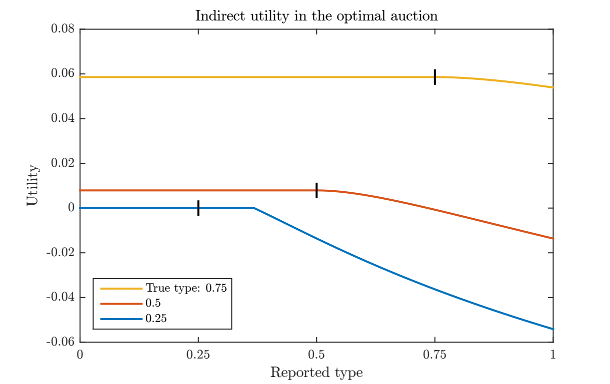


Figure 3: Uniform Downward Incentive Constraints and Winner's Blessing

Incentive Constraints in First Price Auction

- in the first price auction, each bidder is indifferent between his equilibrium bid and any higher bid

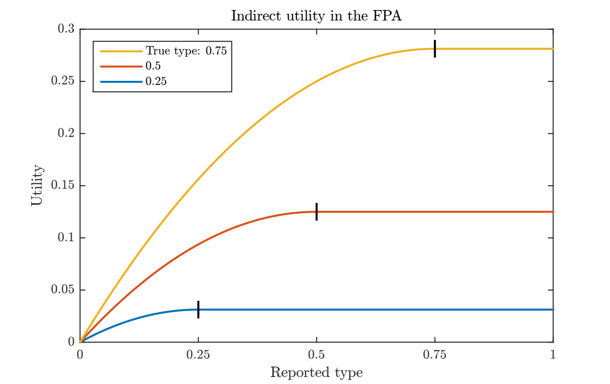


Figure 4: Uniform Upward Incentive Constraints and Winner's Curse

Where Do We use Maximum Game?

- which properties of the maximum game do we use, and where?

1. exact description of local incentive constraint in terms of probability $\widehat{q}_i(v)$
2. exact description of gains of global incentive constraint in terms of $s - x$
3. symmetry among losing bidders

Resale Interpretation

- seller biases the allocation towards those bidders who are likely to want to resell the object in secondary market
- since they have less private information about the resale value than the seller would have to incentivize them to reveal
- $\widehat{q}$ cannot be too low, however, or else bidders would want to deviate by misreporting redrawn lower signals
- constraint boils down to the requirement that $\widehat{q}(x)$ cannot be smaller than the probability that the object is allocated conditional on the highest signal being less than x

Implications

Maximum Game

- Bulow and Klemperer (2002) define "maximum game" and show in second price auction in equilibrium each bidder bids his signal
$$s_i \leq v = max_j\{s_1, ..., s_j, ..., s_N\}$$
- in equilibrium, bidder with highest signal wins the auction and pays second-highest signal
- in fact, it is optimal to bid any amount which is at least your signal, and, in particular, it is optimal to bid your signal
- by contrast, in optimal auction, each bidder is indifferent between reporting his signal and reporting any *lower* signal

Comparison with IPV

- suppose now that the signals are the values, thus independent private value environment:

$$s_i = v_i \leq max_j\{s_1, ..., s_N\}$$

- in second price auction bidding his signal remains optimal
- thus, in second price auction of pure common value environment, each bidder behaves as if his signal is his true private value rather than a signal, and in particular a lower bound on the pure common value
- observation can be generalized

Strategic Equivalence

- consider independent private value model: $v_i(s_1, \ldots, s_N) = s_i$
- denote the set of bidders with high signals

$$H(s) = \left\{ i | s_i = \max_j s_j \right\}$$

- direct mechanism $\{q_i, t_i\}$ is *conditionally efficient* if (i) $q_i(s) > 0$ if and only if $s_i \in H(s)$ and (ii) there exists a cutoff r such that the object is allocated whenever $\max_i s_i > r$.

Proposition

Suppose a direct mechanism $\{q_i, t_i\}$ is incentive compatible and individually rational for the independent private value model in which $v_i(s) = s_i$ and that the allocation is conditionally efficient. Then $\{q_i, t_i\}$ is also incentive compatible and individually rational for the maximum common value model in which $v_i(s) = \max_j \{s_j\}$.

Auctions vs Optimal Mechanism

- Bulow and Klemperer (1996) establish the limited power of optimal mechanisms as opposed to standard auction formats
- revenue of optimal auction with N bidders is strictly dominated by standard absolute auction with $N+1$ bidders
- current common value environment is an instance of general interdependent value setting – with one exception
- virtual utility function—or marginal revenue function—is not monotone due to maximum operator in common value model

A Closer Look at the Virtual Utility

- non-monotonicity leads to an optimal mechanism with features distinct from standard first or second price auction.
- it elicits information from bidder with highest signal but minimizes probability of assigning him the object subject to incentive constraint
- *virtual utility* of each bidder, $\pi_i(s_i, s_{-i})$:

$$\pi_i(s_i, s_{-i}) = \begin{cases} \max_j\{s_j\}, & \text{if} \quad s_i \leq \max\{s_{-i}\}; \\ \max\{s_j\} - \frac{1-F_i(s_i)}{f_i(s_i)}, & \text{if} \quad s_i > \max\{s_{-i}\}. \end{cases}$$

- downward discontinuity in virtual utility indicates why seller wishes to minimize the probability of assigning the object to the bidder with the high signal

Revenue Comparison

- virtual utility of bidder i fails monotonicity assumption even when hazard rate of distribution function is increasing everywhere
- BK (1996) require monotonicity of virtual utility when establishing their main result that an absolute English auction with $N + 1$ bidders is more profitable than any optimal mechanism with N bidders
- revenue ranking does not extend to current auction environment
- compare revenue from optimal auction with N bidders to absolute, English or second-price, auction with $N + K$ bidders
- absolute as there is no reserve price imposed

Reversal in Revenue Comparison

Proposition (Revenue Comparison)

For every $N \geq 1$ and every $K \geq 1$, the revenue from an absolute second-price auction with $N + K$ bidders is strictly dominated by the revenue of an optimal auction with N bidders.

- comparison of *second order statistic* of $N + K$ i.i.d. signals and *first order statistic* of $N + K - 1$ i.i.d. signals
- second order statistic of $N + K$ signals is revenue of absolute second-price auction with $N + K$ bidders.
- by earlier Theorem, optimal mechanism (weakly) exceeds revenue from a posted price set equal to the maximum of $N + K - 1$ signals.
- now, if instead of $N + K$ bidders, the optimal auction only has N bidders, then it is as if only N independent and identical distributed signals are revealed to the N bidders

The Power of Optimal Auctions

- compare the indirect utility in the first price auction and the optimal auction

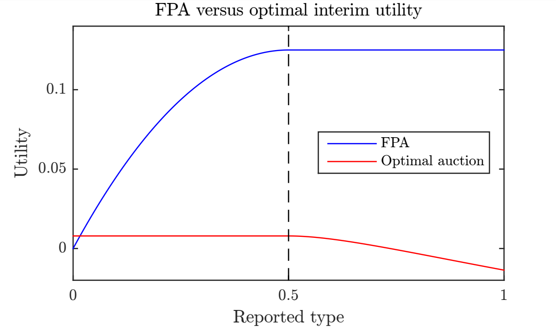


Figure 5: Indirect Utility of Bidder with $s = 1/2$ across two mechanisms

Auctions with Resale

- characterization of the optimal mechanism remains valid if we interpret the model as one where the object is initially sold optimally among N bidders with independent private values
- then offered for resale under complete information
- in contrast to previous work on auctions with resale, such as Gupta and le Brun (1999) and Haile (2003) we analyze optimal mechanism in primary market
- Carroll and Segal (2016) study robust resale mechanisms

Conclusion

- characterized novel revenue maximizing auctions for a class of common value models
- common value models with qualitative feature that values are more sensitive to private information of bidders with more optimistic beliefs
- second interpretation as auction with frictionless resale market
- characterizations of optimal revenue that exist in the literature depend on information rents being smaller for bidders who are more optimistic about value
- qualitative impact is that earlier results found that optimal auctions discriminate in favor of more optimistic bidders
- today: optimal auctions discriminate in favor of less optimistic bidders since they obtain less information rents from being allocated the object

Appendix: Additional Slides

Optimal Auction

- construct an incentive compatible mechanism that exactly achieves the upper bound
- in the direct mechanism, all types are asked to make a fixed payment, a participation fee, that is independent of their type
- no transfers beyond the participation fee are collected
- every type has the same interim expected probability of being allocated the object
- these two features of the optimal mechanism resemble a posted price mechanism
- unlike posted prices, the object is only allocated if the highest realized signal among the bidders exceeds a threshold value
- thus, typically, the probability that the object is assigned to *some* bidder is strictly smaller than one
- second feature distinct from posted prices is that the optimal mechanism discriminates against bidders with higher signals

Uniform Distribution

- family of translated uniform distributions on $[a, a+1], a > 0$.
- marginal revenue function for these distributions is

$$\psi_a(x) = x - \int_{y=x}^{a+1} \left((x-a)^{-\frac{1}{N}} - 1 \right) dx,$$

- lowest marginal revenue is

$$\begin{aligned} \psi_a(a) &= a - \int_{y=a}^{a+1} \left((x-a)^{-\frac{1}{N}} - 1 \right) \\ &= a - \frac{1}{N-1}. \end{aligned}$$

- thus posted price is optimal if

$$a > 1/(N-1)$$