Information Design

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Mechanism Design and Information Design

- **Basic Mechanism Design**:
  - Fix an economic environment and information structure
  - Design the rules of the game to get a desirable outcome
- **Information Design**
  - Fix an economic environment and rules of the game
  - Design an information structure to get a desirable outcome

Mechanism Design and Information Design

- **Basic Mechanism Design**:
  - Can compare particular mechanisms..
    - e.g., first price auctions versus second price auctions
  - Can work with space of all mechanisms...
    - without loss of generality, let each agent's action space be his set of types...revelation principle
    - e.g., Myerson's optimal mechanism
- **Information Design**
  - Can compare particular information structures
    - Linkage Principle: Milgrom-Weber 82
    - Information Sharing in Oligopoly: Novshek-Sonnenschein 82
  - Can work with space of all information structures
    - without loss of generality, let each agent's type space be his set of actions......revelation principle

Information Design: Some Leading Cases

1. Uninformed information designer (or "mediator"):
   - Myerson: "Bayesian games with communication"
   - Incomplete Information Correlated Equilibrium literature of the 1980s and 1990s (Forges 93)
2. One player (a "receiver") and an informed information designer (or "sender")
   - "Bayesian Persuasion": Kamenica-Gentzkow 11 and large and important literature inspired by it
3. Many players *and* an informed information designer
   - Some of our recent theoretical and applied work with various co-authors....
   - ...and this lecture

This Lecture

- a general framework in two slides
- leading examples at length
- applications in brief
- various elaborations if time

Setup

- Maintained environment, fix:
- players 1,...,I; actions $a_i \in A_i$, payoff states Θ;
  utility function:
  $$u_i : A \times \Theta \to \mathbb{R},$$
  common prior on states $\psi \in \Delta(\Theta)$
- Basic Game:
  $$G : (A_i, u_i, \Theta, \psi)_{i=1,\dots,I}$$
- Information Structure S: $(T_i)_{i=1..,I}$ and likelihood function:
  $$\pi : \Theta \to \Delta(T)$$
- *Decision rule* $\sigma : T \times \Theta \to \Delta(A)$, (recommendation)

Decision Rule and Obedience

- *Decision rule*
$$\sigma : T \times \Theta \longrightarrow \Delta(A)$$
- *Decision rule* $\sigma : T \times \Theta \longrightarrow \Delta(A)$ is *obedient* for (G, S) if, for all i, t_i, a_i and a'_i,
$$\begin{aligned} & \sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(a_i, a_{-i}\right), \theta\right) \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi(\theta) \\ \geq \quad & \sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(a'_i, a_{-i}\right), \theta\right) \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi(\theta) ; \end{aligned}$$
- Obedient decision rule σ is a *Bayes correlated equilibrium* (BCE).
- Characterizes the set of implementable decision rules by information designer.

Information Design: Three Interpretations

1. Literal: actual information designer with ex ante commitment: Information designer with payoff $v : A \times \Theta \to \mathbb{R}$ picks a Bayes correlated equilibrium $\sigma \in BCE(G, S)$ to maximize

$$V_S(\sigma) \equiv \sum_{a,t,\theta} \psi(\theta) \pi(t|\theta) \sigma(a|t,\theta) v(a,\theta).$$

2. Metaphorical: e.g., adversarial / worst case
3. Informational robustness: family of objectives characterize set of attainable outcomes

One Uninformed Player: Benchmark Investment Example

- a firm is deciding whether to invest or not:
- binary state: $\theta \in \{B, G\}$, bad or good
- binary action: $a \in \{\text{Invest, Not Invest}\}$
- payoffs

	bad state B	good state G
Invest	-1	x
Not Invest	0	0

  with $0 < x < 1$
- prior probability of each state is $\frac{1}{2}$
- firm is uninformed (so one uninformed player)
- information designer (government) seeks to maximize probability of investment (independent of state)
- leading example of Kamenica-Gentzkow 11

Decision Rule

- p_θ is probability of investment, conditional on being in state θ

	bad state B	good state G
Invest	p_B	p_G
Not Invest	$1 - p_B$	$1 - p_G$

- interpretation: firm observes signal = "action recommendation," drawn according to (p_B, p_G)

Obedience Constraints

- if "advised" to invest, invest has to be a best response:

$$
\begin{aligned}
-\frac{1}{2}p_B + \frac{1}{2}p_G x &\geq 0 \Leftrightarrow \\
p_G &\geq \frac{p_B}{x}
\end{aligned}
$$

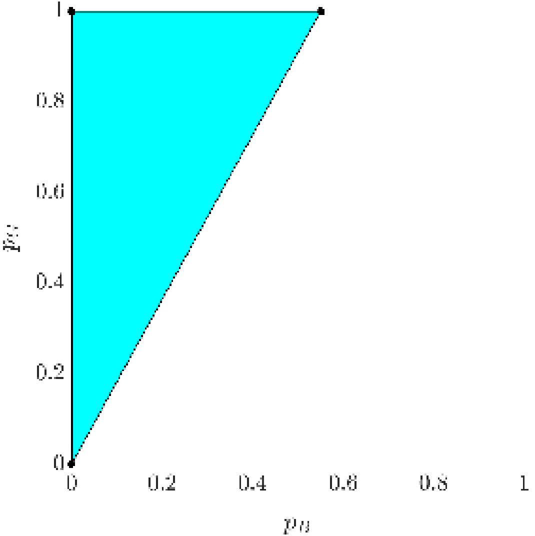
- if "advised" to not invest, not invest has to be a best response

$$
\begin{aligned}
-\frac{1}{2}(1 - p_B) + \frac{1}{2}(1 - p_G)x &\leq 0 \Leftrightarrow \\
p_G &\geq \frac{p_B}{x} + 1 - \frac{1}{x}
\end{aligned}
$$

- because $x < 1$, investment constraint is binding
- always invest ($p_B = 1$ and $p_G = 1$) cannot happen in equilibrium
- the full information equilibrium has invest only in good state ($p_B = 0$ and $p_G = 1$)

Bayes Correlated Equilibria

- equilibrium outcomes (p_B, p_G) for $x = 0.55$



Information Design

- recommendation maximizing the probability of investment:

$$p_B = x, \; p_G = 1$$

- best BCE

	B	G
Invest	x	1
Not Invest	$1-x$	0

- Optimal for government to obfuscate: partially pooling good state and bad state
- Optimal for government to isolate: bad state is set apart

Food for Thought

1. Literal Interpretation: Conflict of interest between information designer and player creates incentive for obfuscation (partial information revelation)
2. Robust Interpretation: Intuitively extremal information structures may not be extremal for outcomes

What do extremal information structures look like?

One Informed Player

- Firm receives a signal which is "correct" with probability $q > 1/2$.
- Formally, the firm observes a signal g or b, with signals g and b being observed with conditionally independent probability q when the true state is G or B respectively:

	bad state B	good state G
bad signal b	q	$1 - q$
good signal g	$1 - q$	q

Obedience Constraints with Informed Player

- p^t_θ is probability of investing in state $\theta \in \{B, G\}$ signal $t \in \{b, g\}$; a decision rule is $\left(p^b_B, p^b_G, p^g_B, p^g_G\right)$.
- Same constraints signal by signal essentially as before...
  - A firm with good signal invests (when told to invest) if

$$p^g_G \geq \frac{1-q}{q}\frac{p^g_B}{x}$$

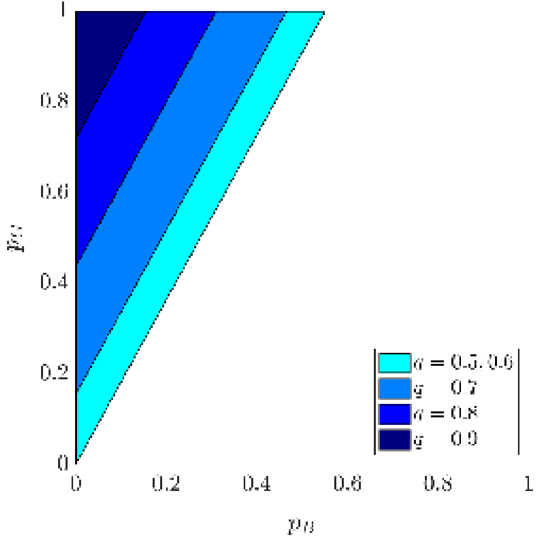
    and not invest (when told to not invest) if

$$p^g_G \geq \frac{1-q}{q}\frac{p^g_B}{x} - \frac{1-q}{qx} + 1.$$

- If private information of firm is sufficiently noisy, $q \leq \frac{1}{1+x}$, binding constraint remains investment constraint.

One Informed Player: Bayes Correlated Equilibrium

equilibrium set for $x = 0.55$



Food for Thought

1. More information limits the ability of the information designer to achieve his objectives
2. But what does "more information" mean in general?
3. in single player case, Blackwell's information order
4. in many player case, something new, Individual Sufficiency "“Bayes Correlated Equilibrium and the Comparison of Information Structures in Games”, Theoretical Economics, 2016,

Many Players: Two Firms

- payoffs almost as before....

$\theta = B$	I	N
I	$-1+\varepsilon$	-1
N	0	0

$\theta = G$	I	N
I	$x+\varepsilon$	x
N	0	0

- ...up to ε term
- assume that information designer (government) wants to maximize the *sum* of probabilities that firms invest....
- if $\varepsilon = 0$, problem is exactly as before firm by firm; doesn't matter if and how firms' signals are correlated
- we will consider what happens when $|\varepsilon| \approx 0$ (so the analysis cannot change very much)
- will now have profile of action recommendations depending on the state
- $\varepsilon > 0$: strategic complements; $\varepsilon < 0$: strategic substitutes

Two Firms: Strategic Complementarities

- If $\varepsilon > 0$, optimal rule is

$\theta = B$	I	N
I	$\frac{x+\varepsilon}{1-\varepsilon}$	0
N	0	$\frac{1-x-2\varepsilon}{1-\varepsilon}$

$\theta = G$	I	N
I	1	0
N	0	0

- the probability of any one firm investing is still about x..
- binding constraints are still investment constraints, slackened by having simultaneous investment...

$$\frac{x+\varepsilon}{1-\varepsilon}(-1+\varepsilon) + x + \varepsilon \geq 0$$

- ....so signals are public

Two Firms: Strategic Substitutes

- If $\varepsilon < 0$, optimal rule is

$\theta = B$	I	N
I	0	$x + \varepsilon$
N	$x + \varepsilon$	$1 - 2x - 2\varepsilon$

$\theta = G$	I	N
I	1	0
N	0	0

- the probability of any one firm investing if the state is bad is still about x....
- binding constraints are still investment constraints, slackened by having minimally correlated investment...

$$(x + \varepsilon)(-1) + x + \varepsilon \geq 0$$

- ....and signals are private

Food for Thought

1. Public information under strategic complementarities / private information under strategic substitutes
2. How does this matter in applications?
3. How about alternative objectives for the information designer?

Application 1 - Information Sharing: Strategic Substitutes

- Classic Question: are firms better off if they share their information?
- Consider quantity competition when firms uncertain about level of demand (intercept of linear demand curve) with symmetry, normality and linear best response; two effects in conflict:
  1. Individual Choice Effect: Firms would like to be as informed as possible about the state of demand
  2. Strategic Effect: Firms would like to be as uncorrelated with each other as possible
- "Robust Predictions in Games with Incomplete Information" , Econometrica, 2013

Application 1 - Information Sharing

- Classic Question: are firms better off if they share their information?
- Consider quantity competition when firms uncertain about level of demand: individual and strategic effects in conflict
- Resolution:

1. If inverse demand curve is flat enough... i.e., small strategic effect...individual choice effect wins and full sharing is optimal
2. If inverse demand curve is very steep...i.e., large strategy effect...strategic effect wins and no sharing of information is optimal
3. In intermediate cases, optimal to have firms observe imperfect information about demand, with conditionally independent signals, and thus signals which are as uncorrelated as possible conditional on their accuracy

Application 2 - Aggregate Volatility

- Classic Question: can informational frictions explain aggregate volatility?
- Consider a setting where each agent sets his output equal to his productivity which has a common component and an idiosyncratic component
- again with symmetry and normality.... common compenent y with variance σ^2; idiosyncratic component x_i with variance τ^2;
- Which information structure maximizes variance of average action?
- "Information and Volatility" Journal of Economic Theory, 2015,

Application 2 - Aggregate Volatility

What information structure maximizes variance of average action?

- critical information structure has a confounding signal without noise:

$$s_i = \lambda x_i + (1 - \lambda) y$$

- variance of average action is maximized when

$$\lambda = \frac{\sigma}{2\sigma + \sqrt{\sigma^2 + \tau^2}} < \frac{1}{2}$$

and maximum variance of average action is

$$\left(\frac{\sigma + \sqrt{\sigma^2 + \tau^2}}{2} \right)^2$$

Application 2 - Aggregate Volatility

What information structure maximizes variance of average action?

- "optimal" information structure has a confounding (c.f., Lucas 72) signal $s_i = \lambda x_i + (1 - \lambda) y$ *without noise...*
- as $\sigma \rightarrow 0$:
  - "optimal" weight on idiosyncratic component goes to 0
  - agents put a lot of weight on their signal in order to put a non-trivial weight on their idiosyncratic component
  - in the limit, the common component becomes a payoff irrelevant but common "sentiments" shock:
- this was actually a non-strategic problem: logic can be extended to strategic setting
- can then be embedded in a richer setting (Angeletos La'O 13)

Application 3 - First Price Auction: Information Shrinking BCE, Adversarial Information Designer

- "First Price Auctions with General Information Structures", Econometrica 2017
- Example: Two bidders and valuations independently and uniformly distributed on the interval $[0, 1]$
- Plot: (expected bidders' surplus, expected revenue) pairs
- green = feasible pairs, blue = unknown value pairs, red = known value pairs

Application 3 - First Price Auction Robust Prediction

1. Known value case (red region) is subset of unknown value case (blue region)
2. Robust Prediction:
   2.1 revenue has lower bound $\approx 1/10$
   2.2 lower bound (w.r.t. first order stochastic dominance) on bids

Application 3: First Price Auction Partial Identification

- We can give explicit lower bound on bids for a given distribution of valuations
- Can therefore give an upper bound on valuations for a given distribution of observed winning bids

Access to Players' Information

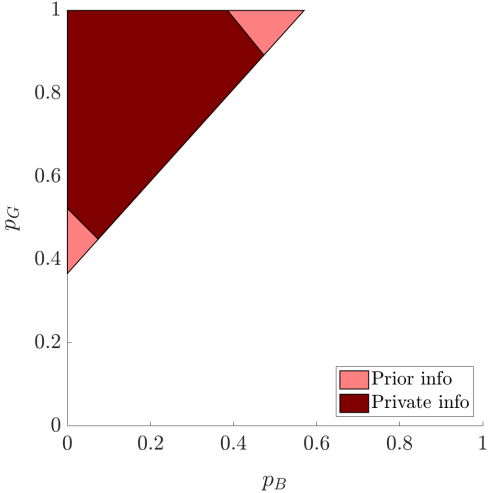
- We want to assume that information designer knows the state θ...
- ...but what should we assume about what information designer knows about players' information? Consider three scenarios:
  1. Omniscient Designer: the designer knows all players' information too...[**maintained assumption so far**]
  2. Communicating Designer: the designer can condition his announcements about the state only on players' reports of their types
  3. Non-Communicating Designer: the designer can tell players about the state but without conditioning on players' information

Back to One Informed Player: Communicating Designer

- Kotolinin et al. 15
- as before, firm observes a signal $t \in T$ and government makes a recommendation to invest p_{θ}^{t} as a function of reported signal t and state θ
- incentive constraint: add truth-telling to obedience
- to insure truth-telling, differences in recommendations must be bounded across states

Communicating Designer

- adding truth-telling constraints...($x = 0.9$, $q = 0.7$)



- communicating (red), omniscient (pink)

Communicating Designer

- if there is a large discrepency in recommendations, then firm has an incentive to misreport his signal
- e.g., at maximum investment BCE (top right), firm with good signal is always told to invest;
- might as well mis-report good signal as bad signal to get information

Non-communicating designer

- firm observes his signal
- government offers a recommendation, independent of the signal, depending on the true state
- In our example, communicating and non-communicating designer can attain the same set of outcomes; Kotolin et al show this in a more general - but still restrictive - class of environments

Taxonomy

	Single Agent	Many Agent Uninformed Designer	Many Agent Informed Designer
Omniscient	.	Bayesian Solution	BCE
Communicating	Kolotilin et al	Communication Equilibrium	.
Non Communicating	KG informed receiver	Strategic Form Correlated Equilibrium	.

Elaborations

1. Other Objectives
   - Ely 15, Arieli 15, Taneva 16
2. Comparing Information
   - many player Blackwell order generalization
3. Concavification and its many player generalizations
   - Kamenica-Gentzkow 11 get a lot of action out of "concavification" (Aumann-Maschler 95); many player generalization harder - Mathevet, Perego and Taneva 16
4. Adversarial Information Design
   - Carroll 15, Taneva et al 16, Kajii-Morris 97
5. Incomplete information correlated equilibrium literature
   - Forges 93
6. Relation to Mechanism Design
   - Myerson 82, 87, 91

Applications and Elaborations

1. Other Objectives
2. Comparing Information
3. Concavification and its many player generalizations
4. Adversarial Information Design
5. Incomplete information correlated equilibrium literature
6. Relation to Mechanism Design

1. Other Objectives

- Suppose the government was interested in maximizing the probability of at least one firm investing
- (Assuming $x > 1/2$) This can always be achieved with probability 1....

$\theta = B$	I	N
I	0	$\frac{1}{2}$
N	$\frac{1}{2}$	0

$\theta = G$	I	N
I	1	0
N	0	0

This is true for $\varepsilon = 0$ and by continuity for $|\varepsilon|$ independent of the sign...

- Compare Ely 15, Arieli 15, Taneva 16

Other Objectives and a Benevolent Information Designer

- In one firm case, if government had the same objective as the firm, he would always give them full information...
- But in the two firm case, a benevolent government maximizing the (joint) profits of the two firms might still manipulate information in order to correct for externalities and coordinate behavior
- In game

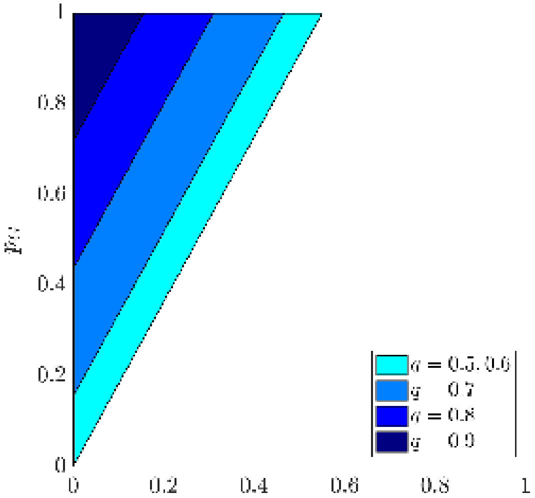
$\theta = B$	I	N
I	$-1 + \varepsilon + z$	-1
N	z	0

$\theta = G$	I	N
I	$x + \varepsilon + z$	x
N	z	0

benevolent government will behave as an investment maximizing government if z is large enough

2. Ordering Information

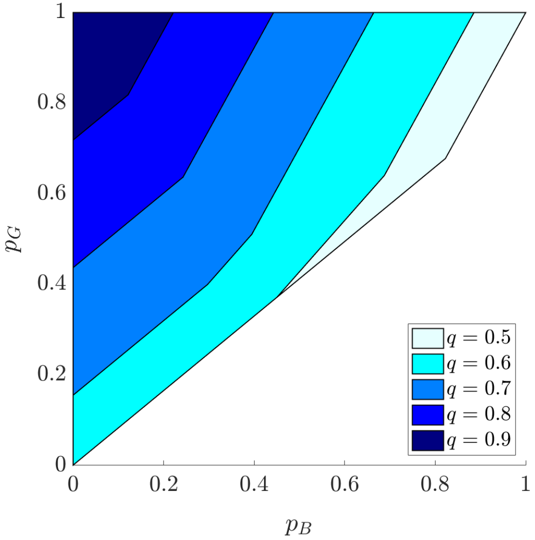
- in one informed player example, more information shrunk attainable outcomes on primitives... for $x = 0.55$:



2. Ordering Information

- Intuition: more information for the player imposes more constraints on the information designer and reduces the set of outcomes she can induce
- Recall Auction Example
- Say that information structure S "is more incentive constrained than" (= more informed than) S' if it gives rise to a smaller set of BCE outcomes than S' in all games
  - in one player case, this ordering corresponds to Blackwell's sufficiency ordering
  - in many player case, corresponds to "individual sufficiency" ordering
- Bergemann-Morris 16, see also Lehrer et al 10 and 11

2. Ordering Information with Many Players



Nice Properties of Individual Sufficiency Ordering

- Reduces to Blackwell in one player case
- Transitive
- Neither implies nor implied by Blackwell on join of players' information
- Two information structures are each individually sufficient for each other if and only if they share the same higher order beliefs about Θ
- S is individually sufficient for S' if and only if giving extra signals to S' equals S plus an appropriate correlation device

3. Concavification

- We described two step procedure for solving information design problem (with one or many players):
  1. Characterize all implementable decision rules
  2. Pick the designer's favorite
- Concavification procedure (with one player) [Aumann-Maschler 95 and Kamenica-Gentzkow 11]
  - Identify information designer's utility for every belief of the single player
  - Identify utility from optimal design by concavification, identifying information design only implicitly
- Many player generalization: Mathevet al 16
- Always nice interpretation, sometimes (but not always) useful in solving information design problem

4. Adversarial Equilibrium Selection

- Suppose that an information designer gets to make a communication $\Phi : T \times \Theta \rightarrow \Delta(M)$; new game of incomplete information (G, S, Φ)
- Write $E(G, S, \Phi)$ for the set of Bayes Nash equilibria of (G, S, Φ) and write $V_S^*(\Phi, \beta)$ for the information designer's utility
- We have been studying the maxmax problem

$$\max_{C} \max_{\beta} V_S^*(\Phi, \beta)$$

  using a revelation principle argument to show that this equals

$$\max_{\sigma \in BCE(G,S)} V_S(\sigma)$$

- The maxmin problem

$$\max_{C} \min_{\beta} V^*(S, \Phi, \beta)$$

  does not have a revelation principle characterization
- Carroll 15, Taneva et al 16, Kajii-Morris 97

5. Incomplete Information Correlated Equilibrium

- Decision rule $\sigma : T \times \Theta \to A$ is *incentive compatible* for (G, S) if, for each i, t_i and a_i, we have

$$\sum_{a_{-i},t_{-i},\theta} u_i\left(\left(a_i, a_{-i}\right), \theta\right) \sigma\left(a|t, \theta\right) \pi\left(t|\theta\right) \psi\left(\theta\right) \tag{1}$$
$$\geq \sum_{a_{-i},t_{-i},\theta} u_i\left(\left(\delta\left(a_i\right), a_{-i}\right), \theta\right) \sigma\left(a|\left(t_i', t_{-i}\right), \theta\right) \pi\left(t|\theta\right) \psi\left(\theta\right);$$

for all t_i' and $\delta_i : A_i \to A_i$.

- Decision rule $\sigma : T \times \Theta \to A$ is *join feasible* for (G, S) if $\sigma\left(a|t, \theta\right)$ is independent of θ, i.e., $\sigma\left(a|t, \theta\right) = \sigma\left(a|t, \theta'\right)$ for each $t \in T$, $a \in A$,and $\theta, \theta' \in \Theta$.
- Solution Concepts:
  - Bayes correlated equilibrium = obedience
  - Communication equilibrium = incentive compatibility (and thus obedience) and join feasibility
  - etc...

6. Mechanism Design and Information Design

- Myerson Mechanism Design:
  - Dichotomy in Myerson (1991) textbook
    - Bayesian games with communication (game is fixed)
    - Bayesian collective choice problems (mechanism is chosen by designer)
  - both combined in Myerson (1982, 1987)
- Truth-telling (honesty) and obedience constraints always maintained
- "information design" = "Bayesian games with communication" − truth-telling + informed information designer/mediator
- compare also informed principal literature

Conclusion and Literature

- Our methodology papers:
  - "Robust Predictions in Incomplete Information Games," Ecta 13
  - "Bayes Correlated Equilibrium and The Comparison of Information Structures," TE 16
  - "Information Design, Bayesian Persuasion and Bayes Correlated Equilibrium," AER P&P 2016
  - Information Design: A Unified Perspective
- Taneva (2016): name "information design" and two player examples
- Kamenica and Gentzkow (2011) and huge follow up literature: "Bayes persuasion" (single player uninformed case)
- Carroll (2016), Kajii and Morris (1997), Methevet, Perego and Taneva (2016)
- Myerson (1991): Bayesian games with communication and "Bayesian collective choice problems" dichotomy
- Kotolin et al (2015)