The Limits of Price Discrimination

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Introduction: A classic economic issue ...

- a classic issue in the analysis of monpoly is the impact of discriminatory pricing on consumer and producer surplus
- if monopolist has additional information beyond the aggregate distribution of valuations (common prior), he can discriminate among segments of the aggregate market using the additional information about consumers' valuations
- a monopolist engages in third degree price discrimination if he uses additional information - beyond the aggregate distribution - about consumer characteristics to offer different prices to different segments

...information and segmentation...

- with additional information about the valuations of the consumers
  seller can match/tailor prices
- additional information leads to segmentation of the population
- different segments are offered different prices
- what are then the possible (consumer surplus, producer surplus) pairs (for some information)?
- in other words, what are possible welfare outcomes from *third degree price discrimination*?

... and a modern issue

- if market segmentations are exogenous (location, time, age), then only specific segmentations may be of interest,
- but, increasingly, data intermediaries collect and distribute information, and in consequence segmentations become increasingly endogeneous, choice variables
- for example, if data is collected directly by the seller, then as much information about valuations as possible might be collected, consumer surplus is extracted
- by contrast, if data is collected by an intermediary, to increase consumer surplus, or for some broader business model, then the choice of segmentation becomes an instrument of design
- implications for privacy regulations, data collection, data sharing, etc....

A Classical Economic Problem: A First Pass

- Fix a demand curve
- Interpret the demand curve as representing single unit demand of a continuum of consumers
- If a monopolist producer is selling the good, what is producer surplus (monopoly profits) and consumer surplus (area under demand curve = sum of surplus of buyers)?

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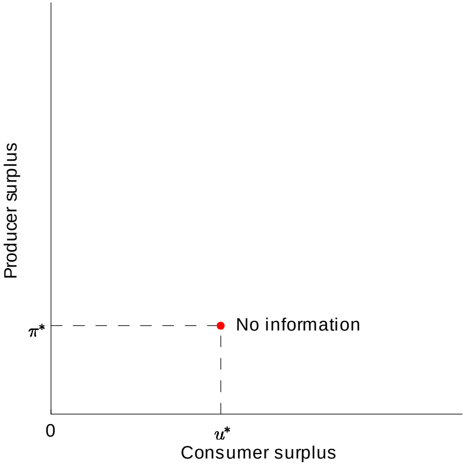
- If a monopolist producer is selling the good, what is producer surplus (monopoly profits) and consumer surplus (area under demand curve = sum of surplus of buyers)?
- If the seller cannot discriminate between consumers, he must charge uniform monopoly price

- Write u^* for the resulting consumer surplus and π^* for the producer surplus ("uniform monopoly profits")



Perfect Price Discrimination

- But what if the producer could observe each consumer's valuation perfectly?
- Pigou (1920) called this "first degree price discrimination"
- In this case, consumer gets zero surplus and producer fully extracts efficient surplus $w^* > \pi^* + u^*$

First Degree Price Discrimination

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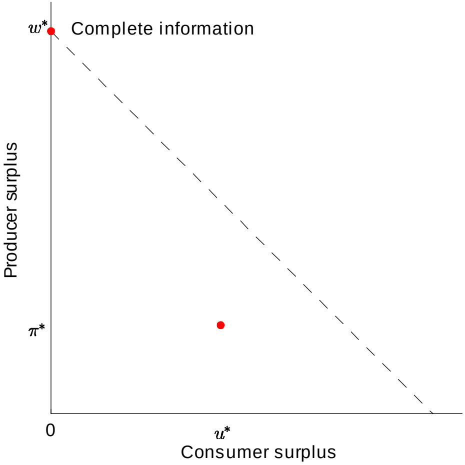

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- What can happen?
- A large literature (starting with Pigou (1920)) asks what happens to consumer surplus, producer surplus and thus total surplus if we segment the market in particular ways

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    - What could happen to consumer surplus, producer surplus and thus total surplus for all possible ways of segmenting the market?

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- Our main result
  - A complete characterization of all (consumer surplus, producer surplus) pairs that can arise...

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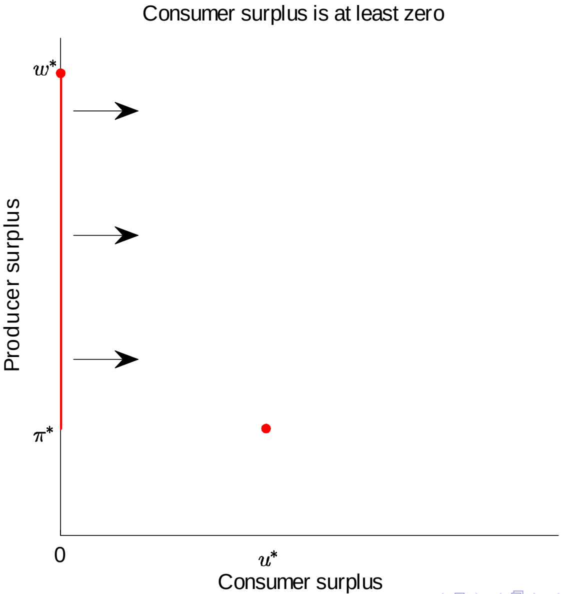
Payoff Bounds: Voluntary Participation



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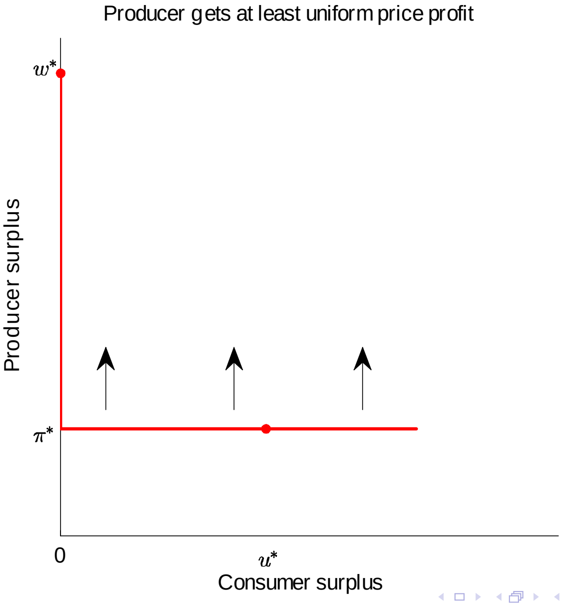
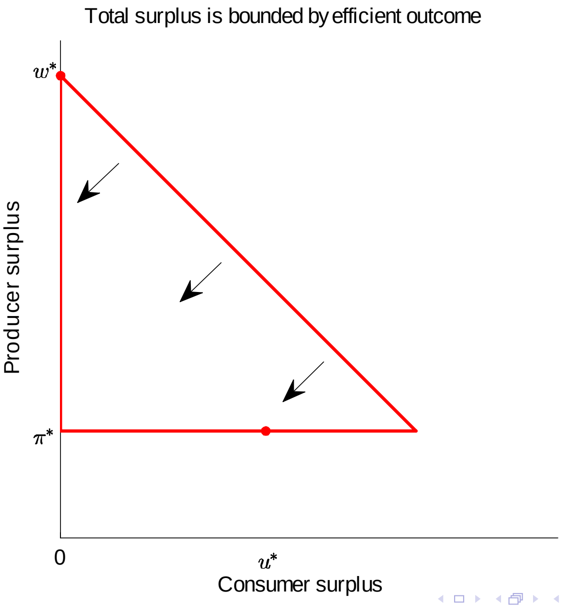
Payoff Bounds: Nonnegative Value of Information



Three Payoff Bounds

1. Voluntary Participation: Consumer Surplus is at least zero
2. Non-negative Value of Information: Producer Surplus bounded below by uniform monopoly profits π^*
3. Social Surplus: The sum of Consumer Surplus and Producer Surplus cannot exceed the total gains from trade

Payoff Bounds: Social Surplus



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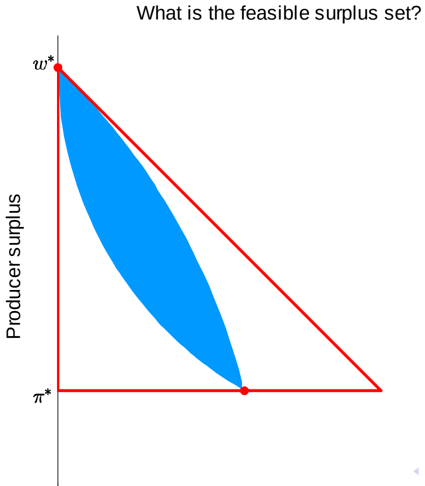
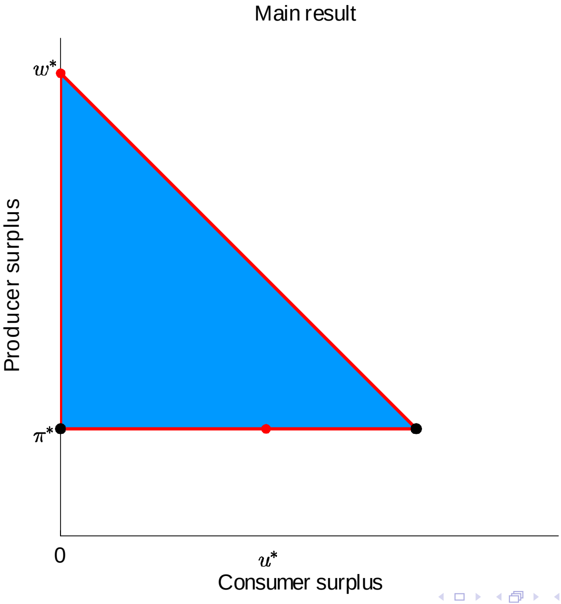


Main Result: Payoff Bounds are Sharp



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1. a *consumer surplus maximizing segmentation* where
    1. the producer earns uniform monopoly profits,
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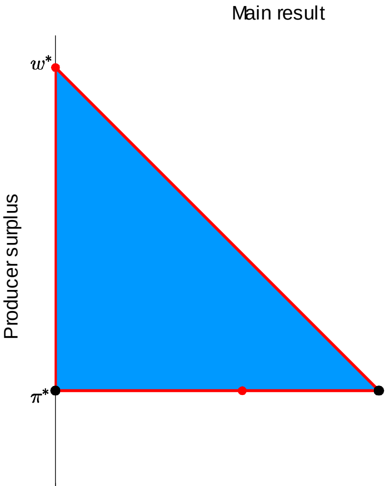
    3. and the consumers attain the difference between efficient surplus and uniform monopoly profit.
2. a *social surplus minimizing segmentation* where
    1. the producer earns uniform monopoly profits,
    2. the consumers get zero surplus,
    3. and so the allocation is very inefficient.

- convex combination of any pair of achievable payoffs as binary segmentation between constituent markets
- it suffices to obtain the vertices of the surplus triangle



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   - The Relation to the Classical Literature on Third Degree Price Discrimination, including results for output and prices

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- A solution concept, "Bayes correlated equilibrium," characterizes what could happen in (Bayes Nash) equilibrium for all information structures
- Advantages:
  - do not have to solve for all information structures separately
  - nice linear programming characterization

Papers Related to this Agenda

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3. Linear Normal Symmetric
    1. Stylised applications within continuum player, linear best response, normally distributed games with common values (aggregate uncertainty) ("Robust Predictions in Incomplete Information Games", Econometrica 2013)
    2. "Information and Volatility" (with Tibor Heumann): economy of interacting agents, agents are subject to idiosyncratic and aggregate shocks, how do shocks translate into individual, aggregate volatility, how does the translation depend on the information structure?
    3. "Market Power and Information" (with Tibor Heumann): adding endogeneous prices as supply function equilibrium

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  where x_k is the proportion of consumers with valuation v_k
- set of possible markets X is the K-dimensional simplex,

$$X \triangleq \left\{ x \in \mathbb{R}_+^K \,\middle|\, \sum_{k=1}^{K} x_k = 1 \right\}.$$

- the price v_i is *optimal* for a given market x if and only if

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- each X_i is a convex polytope in the probability simplex

Aggregate Market

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- define the uniform monopoly price for aggregate market x^*:

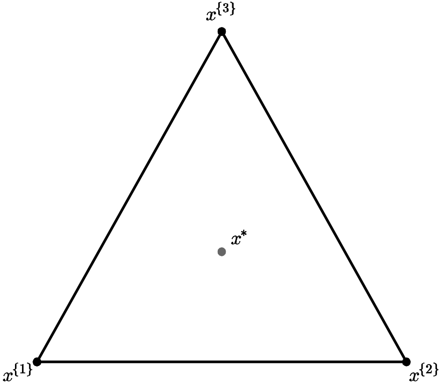
$$p^* = v_{i^*}$$

such that:

$$v_{i^*} \sum_{j \geq i^*} x_j \;\geq\; v_k \sum_{j \geq k} x_j, \quad \forall k$$

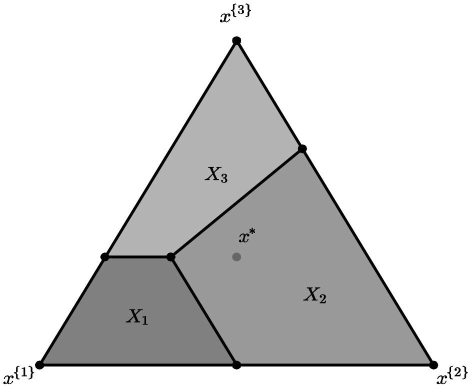
A Visual Representation: Aggregate Market

- given aggregate market x^* as point in probability simplex
- here $x^* = (1/3, 1/3, 1/3)$ uniform across $v \in \{1, 2, 3\}$



A Visual Representation: Optimal Prices and Partition

- composition of aggregate market $x^* = (x_1^*, ..., x_k^*, ..., x_K^*)$ determines optimal monopoly price: $p^* = 2$



Segmentation of Aggregate Market

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- a *pricing strategy* for segmentation σ specifies a price in each market in the support of σ,
$$\phi : \mathbf{supp}(\sigma) \to \Delta\{v_1, ..., v_K\},$$

- consider the uniform market with three values
- a segmentation of the uniform aggregate market into three market segments:

	$v = 1$	$v = 2$	$v = 3$	weight
market 1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
market 2	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$
market 3	0	1	0	$\frac{1}{6}$
total	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

- the segments of the aggregate market form a joint distribution over market segmentations and valuations

	$v = 1$	$v = 2$	$v = 3$
market 1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{9}$
market 2	0	$\frac{1}{18}$	$\frac{1}{9}$
market 3	0	$\frac{1}{6}$	0

Signals Generating this Segmentation

- additional information (signals) can generate the segmentation
- likelihood function

$$\lambda : V \to \Delta(S)$$

- in the uniform example

λ	$v = 1$	$v = 2$	$v = 3$
signal 1	1	$\frac{1}{3}$	$\frac{2}{3}$
signal 2	0	$\frac{1}{6}$	$\frac{1}{3}$
signal 3	0	$\frac{1}{2}$	0

Segmentation into "Extremal Markets"

- this segmentation was special

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$\{1, 2, 3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
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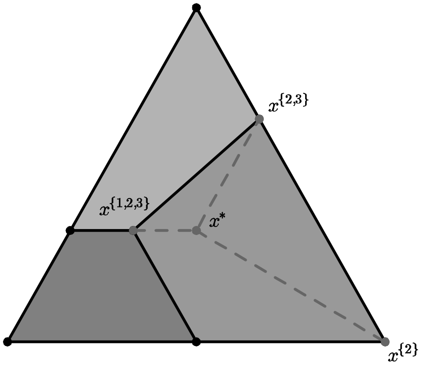
- price 2 is optimal in all markets
- in fact, seller is always indifferent between all prices in the support of every market segment, "unit price elasticity"

- extremal segment x^S: seller is indifferent between all prices in the support of S



- *an* optimal policy: always charge lowest price in the support of every segment:

	$v = 1$	$v = 2$	$v = 3$	price	weight
$\{1, 2, 3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	1	$\frac{2}{3}$
$\{2, 3\}$	0	$\frac{1}{3}$	$\frac{2}{3}$	2	$\frac{1}{6}$
$\{2\}$	0	1	0	2	$\frac{1}{6}$
total	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		1

- another optimal policy: always charge highest price in each segment:

	$v=1$	$v=2$	$v=3$	price	weight
$\{1,2,3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	3	$\frac{2}{3}$
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Extremal Market: Definition

- for any support set $S \subseteq \{1, ..., K\} \neq \varnothing$, define market x^S :

$$x^S = \left(...., x_k^S, ... \right) \in X,$$

with the properties that:

1. no consumer has valuations outside the set $\{v_i\}_{i \in S}$;
2. the monopolist is indifferent between every price in $\{v_i\}_{i \in S}$.

- for every S, this uniquely defines a market

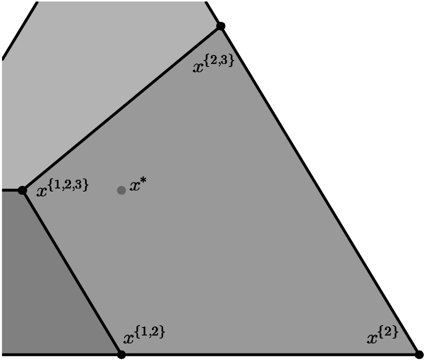
$$x^S = \left(...., x_k^S, ... \right) \in X$$

- writing $\underline{S}$ for the smallest element of S, the unique distribution is

$$x_k^S \triangleq \begin{cases} \frac{v_{\underline{S}}}{v_k} - \sum_{k'>k} x_{k'} & \text{if} \quad k \in S \\ 0, & \text{if} \quad k \notin S. \end{cases}$$

- for any S, market x^S is referred to as *extremal market*

- extremal markets



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  - both $x + \varepsilon\left(x^S - x\right)$ and $x - \varepsilon\left(x^S - x\right)$ are contained in X_{i^*} for small enough $\varepsilon > 0$

- $\mathcal{S}^*$ is subset of subsets $S \subseteq \{1, ..., i^*, ..., K\}$ containing i^*

Lemma (Extremal Segmentation)

X_{i^*} is the convex hull of $\left(x^S\right)_{S \in \mathcal{S}^*}$

Sketch of Proof:

- pick any $x \in X$ where price v_{i^*} is optimal (i.e., $x \in X_{i^*}$) but there exists k such that valuation v_k arises with strictly positive probability (so $x_k > 0$) but is not an optimal price
- let S be the support of x
- now we have
  - $x^S \neq x$
  - both $x + \varepsilon\left(x^S - x\right)$ and $x - \varepsilon\left(x^S - x\right)$ are contained in X_{i^*} for small enough $\varepsilon > 0$
- so x is not an extreme point of X_{i^*}

Remainder of Proof of Main Result

- Split x^* into *any* extremal segmentation
- There is a pricing rule for that one segmentation that attains *any* point on the bottom of the triangle, i.e., producer surplus π^* anything between 0 and $w^* - \pi^*$.
- The rest of the triangle attained by convexity

Pricing Rules

A pricing rule specifies how to break monopolist indifference

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- So minimum pricing rule maximizes consumer surplus (bottom right corner of triangle)
- So maximum pricing rule minimizes total surplus (bottom left corner of triangle)

Theorem (Minimum and Maximum Pricing)

1. *In every extremal segmentation, minimum and maximum pricing strategies are optimal;*
2. *producer surplus is π^* under every optimal pricing strategy;*
3. *consumer surplus is zero under maximum pricing strategy;*
4. *consumer surplus is $w^* - \pi^*$ under minimumpricing strategy.*

A Simple "Direct" Construction

We first report a simple direct construction of a consumer surplus maximizing segmentation (bottom right hand corner):

1. first split:
   1. We first create a market which contains all consumers with the lowest valuation v_1 and a constant proportion q_1 of valuations greater than or equal to v_2
   2. Choose q_1 so that the monopolist is indifferent between charging price v_1 and the uniform monopoly price v_{i^*}
   3. Note that v_{i^*} continues to be an optimal price in the residual market
2. Iterate this process

A Simple "Direct" Construction

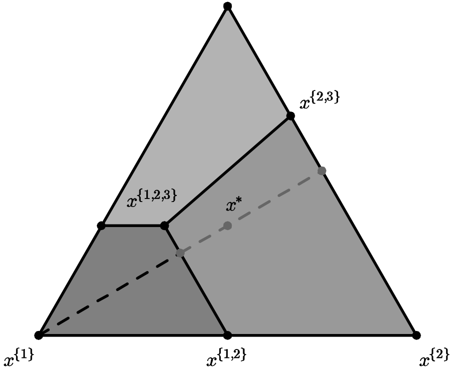
We first report a simple direct construction of a consumer surplus maximizing segmentation (bottom right hand corner):

1. first split:
2. Iterate this process
3. thus at round k,
    1. first create a market which contains all consumers with the lowest remaining valuation v_k and a constant proportion q_k of valuations greater than or equal to v_{k+1}
    2. Choose q_k so that the monopolist is indifferent between charging price v_k and the uniform monopoly price v_{i^*} in the new segment
    3. Note that v_{i^*} continues to be an optimal price in the residual market

In our three value example, we get:

	$v = 1$	$v = 2$	$v = 3$	price	weight
first segment	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{2}{3}$
second segment	0	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{1}{3}$
total	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		1

A Simple "Direct" Construction



Advice for the Consumer Protection Agency?

- Allow producers to offer discounts (i.e., prices lower the uniform monopoly price)
- Put enough high valuation consumers into discounted segments so that the uniform monopoly price remains optimal

A Dual Purpose Segementation: Greedy Algorithm

1. Put as many consumers as possible into extremal market $x^{\{1,2,...,K\}}$
2. Generically, we will run out of consumers with some valuation, say, v_k
3. Put as many consumers as possible into residual extremal market $x^{\{1,2,...,K\}/\{k\}}$
4. Etc....

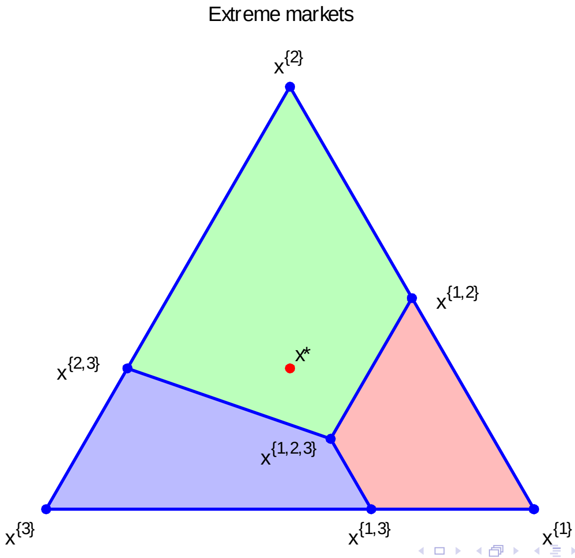
- In our three value example, we get first:

	$v = 1$	$v = 2$	$v = 3$	weight
$\{1, 2, 3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
$\{2, 3\}$	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
total	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1

- Then we get

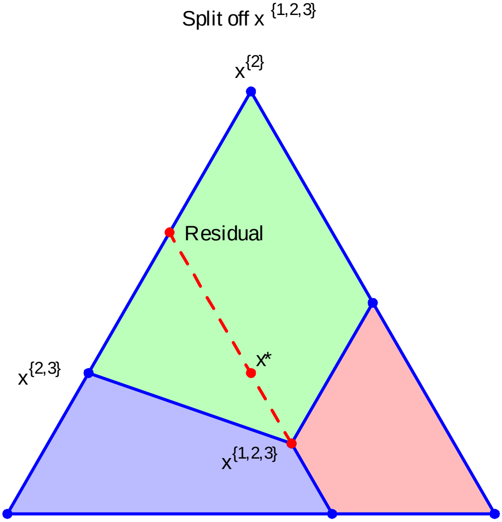
	$v = 1$	$v = 2$	$v = 3$	weight
market 1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
market 2	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$
market 3	0	1	0	$\frac{1}{6}$
total	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

- extremal markets $x^{\{\cdots\}}$



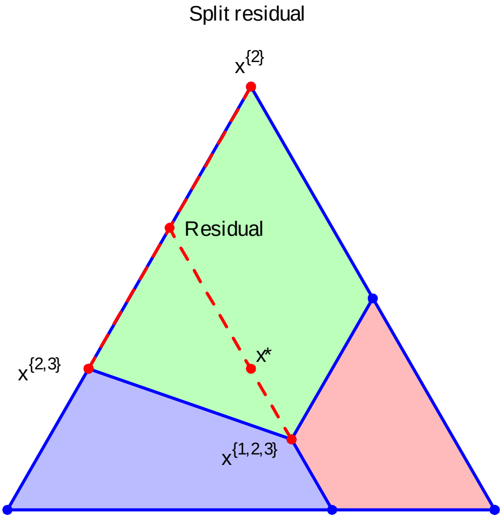
A Visual Proof: Splitting into Extremal Markets

- splitting the aggregate market x^* into extremal markets $x^{\{\dots\}}$



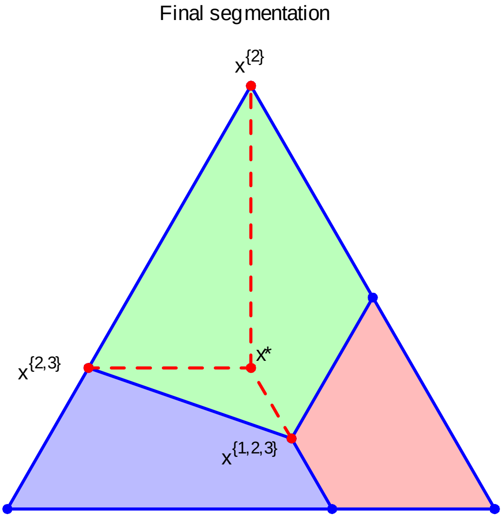
A Visual Proof: Splitting and Greedy Algorithm

- splitting greedily: maximal weight on the maximal market



A Visual Proof: Extremal Market Segmentation

- splitting the aggregate market x^* into extremal market segments all including $p^* = 2$



Surplus Triangle

- minimal and maximal pricing rule maintained π^*
- first degree price discrimination resulted in third vertex

Theorem (Surplus Triangle)

There exists a segmentation and optimal pricing rule with consumer surplus u *and producer surplus* π *if and only if* (u, π) *satisfy* $u \geq 0$, $\pi \geq \pi^*$ *and* $\pi + u \leq w^*$

- convexity of information structures allows to establish the entire surplus triangle

Continuous Demand Case

- All results extend
- Main result can be proved by a routine continuity argument
- Constructions use same economics, different math (differential equations)
- Segments may have mass points

Third Degree Price Discrimination

- classic topic:
  - Pigou (1920) *Economics of Welfare*
  - Robinson (1933) *The Economics of Imperfect Competition*
- middle period: e.g.,
  - Schmalensee (1981)
  - Varian (1985)
  - Nahata et al (1990)
- latest word:
  - Aguirre, Cowan and Vickers (AER 2010)
  - Cowan (2012)

Existing Results: Welfare, Output and Prices

- examine welfare, output and prices
- focus on two segments
- price rises in one segment and drops in the other if segment profits are strictly concave and continuous: see Nahata et al (1990))
- Pigou:
    - welfare effect = output effect + misallocation effect
    - two linear demand curves, output stays the same, producer surplus strictly increases, total surplus declines (through misallocation), and so consumer surplus must strictly decrease
- Robinson: less curvature of demand $(-\frac{p \cdot q''}{q'})$ in "strong" market means smaller output loss in strong market and higher welfare

Our Results (across all segmentations)

- Welfare:
  - Main result: consistent with bounds, anything goes
  - Non first order sufficient conditions for increasing and decreasing total surplus (and can map entirely into consumer surplus)
- Output:
  - Maximum output is efficient output
  - Minimum output is given by *conditionally efficient* allocation generating uniform monopoly profits as total surplus (note: different argument)
- Prices:
  - all prices fall in consumer surplus maximizing segmentation
  - all prices rise in total surplus minimizing segmentation
  - prices might always rise or always fall *whatever* the initial demand function (this is sometimes - as in example - consistent with weakly concave profits, but not always)

- our results concerned a special "screening" problem: each consumer has single unit demand

Beyond Linear Demand and Cost

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- we study what drives our results by seeing what happens as we move towards general screening problems by adding a little non-linearity
- corresponds to Pigou's "second degree price discrimination", i.e., charging different prices for different quantities / qualities

Re-interpret our Setting and adding small concavity

- Our main setting: Consumer type v consuming quantity $q \in \{0, 1\}$ gets utility $v \cdot q$

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- Note that efficient allocation for all types is 1

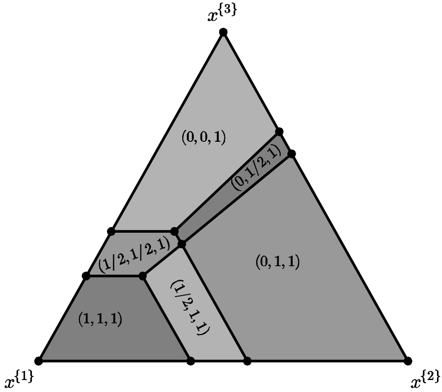
Three Types and Three Output Levels

- Suppose $v \in \{1, 2, 3\}$; $q \in \left\{0, \frac{1}{2}, 1\right\}$
- Always efficient to have allocation of 1
- Note that in this case, utilities are given by

	0	$\frac{1}{2}$	1
1	0	$\frac{1}{2} + \varepsilon$	1
2	0	$1 + \varepsilon$	2
3	0	$\frac{3}{2} + \varepsilon$	3

- contract $q = (q_1, q_2, q_3)$ specifies output level for each type
- six contracts which are monotonic and efficient at the top:
  - $(0, 0, 1)$, $\left(0, \frac{1}{2}, 1\right)$, $(0, 1, 1)$, $\left(\frac{1}{2}, \frac{1}{2}, 1\right)$, $\left(\frac{1}{2}, 1, 1\right)$ and $(1, 1, 1)$
- Now we can look at analogous simplex picture
- Illustrates geometric structure in the general case

- richer partition of probability simplex

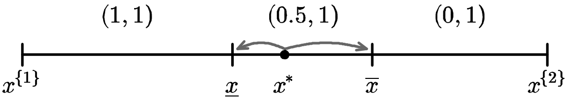


- additional allocations beyond binary appear as optimal

Two Types and Three Output Levels

- Now restrict attention to $v \in \{1, 2\}$
- probability simplex becomes unit interval
- denote by x probabilit of low valuation:

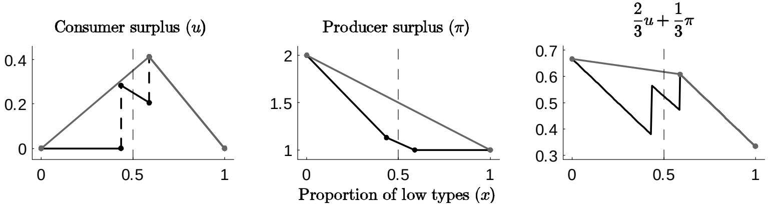
$$x \triangleq \Pr(v = 1)$$



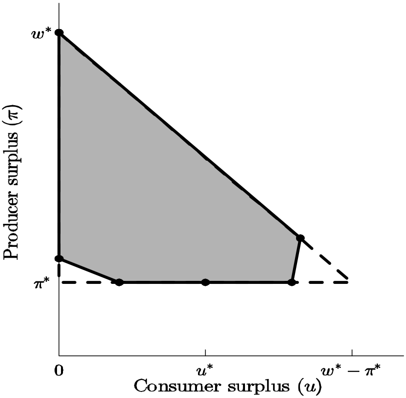
- extremal markets are $\underline{x}$ and $\overline{x}$

Surplus and Concavified Surplus

- Now it is natural to plot consumer surplus and producer surplus as a function of x, the probability of type 1



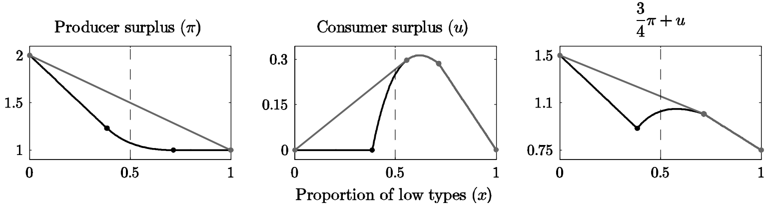
- Now solving for feasible (consumer surplus, producer surplus pairs) for $x = \frac{1}{2}$ comes from concavifying weighted sums of these expressions



- Now allow any $q \in [0, 1]$
- If x is the proportion of low types, the optimal contract is now:

$$\tilde{q}(x) = \begin{cases} 0, & \text{if} \quad x \leq \frac{1}{2+4\varepsilon} \\ \frac{1}{2} - \frac{1}{8\varepsilon}\left(2 - \frac{1}{x}\right), & \text{if} \quad \frac{1}{2+4\varepsilon} \leq x \leq \frac{1}{2-4\varepsilon} \\ 1, & \text{if} \quad x \geq \frac{1}{2-4\varepsilon} \end{cases}$$

Two Types, Continuous Output



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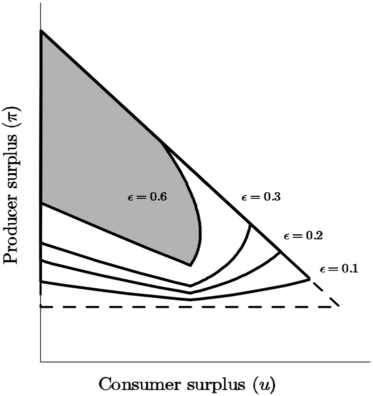


Bottom Line

1. The set of prior distributions of types where it is possible to attain bottom left and bottom right corner will shrink fast as the setting gets more complex
2. As long as there are a finite set of output levels,
    1. There is an analogous restriction to extreme points of best response regions of the simplex (geometric approach translates)
    2. The "bottom flat" survives: there is an open set of information rents consistent with principal getting uninformed profit
3. With continuum output levels
    1. The "bottom flat" goes
    2. Multiple information rents consistent with other levels of consumer profit, approaching the triangle continuously as we approach a linear case

1. Kamenica and Gentzkow (2010): Suppose that a sender could commit (before observing his type) to cheap talk signals to send to a receiver. What would he send?
2. de facto, this is what happened in Aumann and Maschler (1995) repeated games with one sided information who showed sender "concavifies" payoffs
3. We can solve for feasible surplus pairs by this method if the "sender" were a social planner maximizing a arbitrary weighted sum of consumer and producer surplus and the "receiver" were the monopolist
4. Very helpful in two type case, implicit in many type case

Many Player Version

- robust predictions research agenda....
- the set of all outcomes that could arise in Bayes Nash equilibrium in given "basic game" for all possible information structures = "Bayes correlated equilibria"
    - "The comparison of information structures in games: Bayes correlated equilibrium and individual sufficiency" (general theory)
    - "Robust predictions in games with incomplete information games" (applications in symmetric continuum player linear best response games, Ecta (2013))
- seller problem here is single player application
- this paper is by-product of many player application:
    - Bergemann, Brooks and Morris: "Extremal Information Structures in First Price Auction"

- First price auction
- Bidder i's valuations drawn according to cdf F_i
- Lower bound on interim bidder surplus of bidder with valuation v is

$$\underline{u}_i(v) = \max_b (v - b) \prod_{j \neq i} F_j(b)$$

- Lower bound on ex ante expected surplus of bidder i is

$$\underline{U}_i = \int_{v=0}^{1} \underline{u}_i(v) f_i(v)\, dv$$

- Upper bound on expected revenue is total expected surplus minus each bidder's surplus lower bound
- Claim: there is an information structure where these bounds are attained in equilibrium

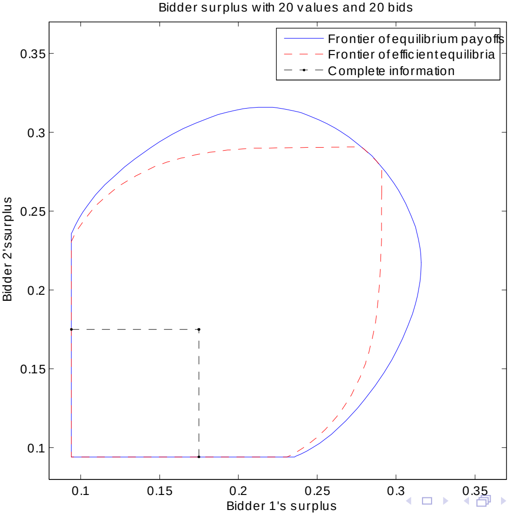
Auction Teaser: Information Structure Attaining the Lower Bound

- Tell each bidder if he has the highest value or not
- Losing bidders bid their values and lose (undominated strategy)
- Winning bidder's "uniform monopoly profit" (maximum profit if he knows nothing about the losing bid) is now the lower bound $\underline{U}_i$
- Our main result states that we can provide (partial) information to the winner about highest losing bid in just such a way that he is still held down to his uniform monopoly profit and always wins

Two Bidders: Information and Revenue

- 2 bidders, valuations uniform on $[0, 1]$
- Ex ante expected surplus is $\frac{2}{3}$
- No information:
    - bid $\frac{1}{2}v$, each bidder surplus $\frac{1}{6}$, revenue $\frac{1}{3}$
- Complete information = Bertrand:
    - each bidder surplus $\frac{1}{6}$, revenue $\frac{1}{3}$
- Our intermediate information structure:
    - each bidder surplus $\frac{1}{12}$, revenue $\frac{1}{2}$

- distribution of bidders (surplus) and implications for revenue equivalence, ...



Conclusion

- It is feasible and interesting to see what happens under many information structures at once.
- This methodology generates striking new answers for classical economic questions
- In mechanism design we design the payoffs of the game, assuming the information structure is fixed
- In information design , we design the information received by the players, assuming the game is fixed.

Do We Care about Extremal Segmentations?

- extremal segmentations are "extreme"...
- might not arise exogenously....
- but suppose someone could choose segments endogenously?

Endogenous Segmentations and a Modern Perspective

- extremal segmentations are "extreme"
- might not arise exogenously
- but suppose someone could choose segments endogenously?
- Google knows everyone's values of everything (pretty much)
- Google wants to "do no evil"
- Operationalization of "do no evil": report noisy signals of values to sellers in such a way that sellers choose to price discriminate in a way that attains efficiency and gives all the efficiency gains to consumers