Countering the Winner's Curse: Auction Design in a Common Value Model

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November 2018

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Auction Design in A Common Value Model

- a pure common value model
- private signal gives partial information about common value
- key statistical feature:
  higher signals contain more information about common value than lower signals
- today:
  highest signal is sufficient statistic of common value
  $\rightarrow$lower signals carry no additional information
- what is the revenue maximizing selling mechanism?

Revenue Maximizing Design

- characterize revenue maximizing auction
- maximal revenue is obtained by strikingly simple mechanism, stated at interim level (given signal of bidder i)

1. constant – signal independent – participation fee
2. constant – signal independent – probability of getting object

- contrast with first, second, ascending auction with private values

Revenue Maximizing Design: Posted Price

- optimal mechanism shares some features with posted price

1. constant – signal independent – price

- it coincides with posted price if

2. constant – signal independent – probability is $1/N$

- necessary and sufficient condition when optimal mechanism reduces exactly to posted price
- if posted price is an optimal mechanism it is inclusive: every bidder with every signal realization is willing to buy

Revenue Maximizing Design: Beyond Posted Price

- in generally aggregate assignment probability may be < 1
- interim probability of getting object is constant and $< 1/N$
- ex post probability for i depends on entire signal profile
- conditionally on allocating the object optimal mechanism:

1. favors bidders with lower signals
2. discriminates against bidder with highest signal

- "winner's blessing" rather than "winner's curse"

Contributions: Substantive

- setting where bidders with higher signals have more accurate information about common value;
- arises in market with intermediaries, and many other settings: auctions for resources, IPO's
- countervailing screening incentives, tension between selling to

1. bidder with higher expected value and
2. bidder with less private information

- optimal to screen "less" - with no screening in inclusive limit
- foundation for posted price mechanisms

Contributions: Methodological

- very few results extend characterization of optimal auctions beyond private value case
- we extend optimal auctions into interdependent values:

1. with private values, "local" incentive constraints are sufficient to pin down optimal mechanism
2. with interdependent values, "global" constraints matter, new arguments are required

Model

Pure Common Value Model

- N bidders for a single object
- bidder i receives signal $s_i \in [\underline{s}, \overline{s}] \subset \mathbb{R}_+$
- value is maximum of N independent signals:

$$v(s_1, \ldots, s_N) = \max\{s_1, \ldots, s_N\}$$

"maximum signal model"

- absolutely continuous distribution $F(s_i), f(s_i)$
- signal distribution $F(s_i)$ induces value distribution $G_N(v)$:

$$G_N(v) = (F(s))^N$$

- common value is first order statistic of N independent signals

Utility and Allocation

- bidder i is expected utility maximizer with quasilinear preferences, probability q_i of receiving object and transfers t_i:

$$u_i(s, q_i, t_i) = v(s) q_i - t_i$$

- feasibility of auction

$$q_i(s) \geq 0, \text{ with } \sum_{i=1}^{N} q_i(s) \leq 1$$

- *ex post* transfer $t_i(s)$ of bidder i, *interim* expected transfer:

$$t_i(s_i) = \int_{s_{-i} \in S^{N-1}} t_i(s_i, s_{-i}) f_{-i}(s_{-i}) \, ds_{-i},$$

where

$$f_{-i}(s_{-i}) = \prod_{j \neq i} f(s_j)$$

Incentive Compatibility

- bidder i surplus when reporting s_i' while observing s_i:

$$u_i\left(s_i, s_i'\right) \equiv \int_{s_{-i} \in S^{N-1}} q_i\left(s_i', s_{-i}\right) v\left(s_i, s_{-i}\right) f_{-i}\left(s_{-i}\right) ds_{-i} - t_i\left(s_i'\right)$$

- indirect utility given truthtelling is:

$$u_i(s_i) \equiv u_i\left(s_i, s_i\right)$$

- direct mechanism $\{q_i, t_i\}_{i=1}^N$ is *incentive compatible (IC)* if

$$u_i\left(s_i\right) \geq u_i\left(s_i, s_i'\right), \text{ for all } i \text{ and } s_i, s_i' \in S$$

and *individually rational (IR)* if $u_i\left(s_i\right) \geq 0$, for all i and $s_i \in S$

Bidder Surplus and Revenue

- ex-ante bidder surplus is

$$U_i = \int_{s_i \in S} u_i(s_i) f(s_i) \, ds_i$$

- revenue is expected sum of transfers:

$$R = \sum_{i=1}^{N} \int_{s_i \in S} t_i(s_i) f(s_i) \, ds_i$$

- seller maximizes R over all IC and IR direct mechanisms
- probability $q(v)$, $q_i(v)$ object is assigned (to bidder i) given v
- total surplus is

$$TS = \int_{v=\underline{s}}^{\overline{s}} v q(v) g_N(v) \, dv$$

Value and Signal

- earlier we considered first price auction (ECTA 2017)
- fix the distribution of values

$$G_N(v)$$

  and ask how different common prior distribution of signals

$$F(s\,|v)$$

  can affect surplus and welfare in first price auction
- all for general distributions of (interdependent) values:

$$G(v_1, ..., v_N)$$

- today, we specialize to the case of pure common values:

$$G_N(v)$$

Revenue Minimizing Information

- first price auction: $N = 2$ and $G(v) = v^2$, $v \in [0, 1]$

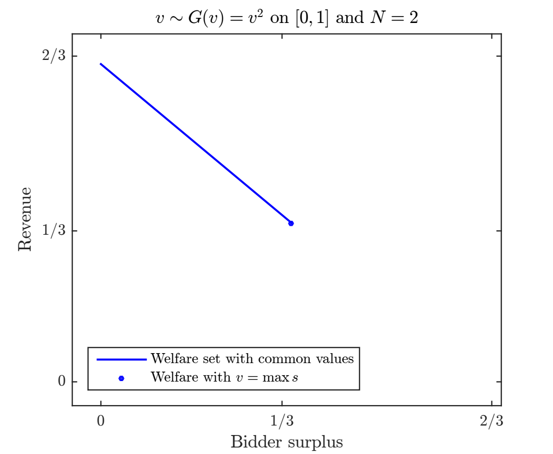

Figure 1: Information and Surplus Distribution

Global Incentive Constraints

- revenue minimizing information is maximum signal model

$$v(s_1, \ldots, s_N) = \max\{s_1, \ldots, s_N\}$$

- we deduce distribution of signals from distribution of values:

$$F(s(v)) = (G(v))^{1/N}$$

- interesting result about structure of incentive constraints
- all upward deviations–relative to unique equilibrium bid–yield equilibrium net utility!
- all upward deviations are binding!

Upward Deviations

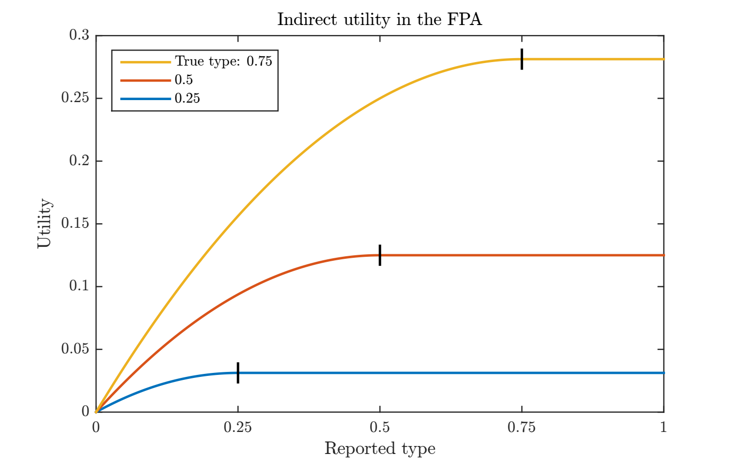

Figure 2: Uniform Upward Incentive Constraints and Winner's Curse

- today: find optimal auction for this information structure

Countering the Winner's Curse

Standard Auction and Monotone Equilibria

- first-price, second-price and ascending auction all have monotone equilibria
- consider bidding in second-price auction:

$$b_i(s_i)$$

- interim expectation of each bidder:

$$\mathbb{E}[v \,|\, s_i]$$

- private signal is sharp lower bound on ex post value of object:

$$s_i \leq v, \; s_i < \mathbb{E}[v \,|\, s_i]$$

- yet, equilibrium bid is given by

$$b_i(s_i) = s_i$$

Adverse Selection and Winner's Curse

- the bidder with the highest signal wins in the second price auction
- conditional on winning the signal s_i turns into a sharp upper bound of the value

$$v = \max\{s_1, ..., s_N\} \leq s_i$$

- this is the winner's curse
- while at the bidding stage it was a sharplower bound!
- adverse selection: winner learns that his signal was more favorable than all other signals

Conditional Expectation and Winner's Curse

- conditional expectation of bidder i given signal s_i

$$\mathbb{E}\left[v\left(s_1,\ldots,s_N\right)\middle|s_i\right] > s_i$$

- conditional expectation of bidder i given signal s_i and that s_i is the highest signal:

$$\mathbb{E}\left[v\left(s_1,\ldots,s_N\right)\middle|s_i = \max\left\{s_1,\ldots,s_N\right\}\right] = s_i$$

- the winner's curse is maximal
- before winning, conditional expectation always exceeds s_i
- after winning, conditional expectation is always exactly s_i

Neutral Selection and Exclusion

- uniform exclusion at threshold r :

$$q_i(s) = \begin{cases} \frac{1}{N} & \text{if } \max s \geq r; \\ 0 & \text{otherwise.} \end{cases}$$

- and a pair of incentive compatible prices:
- an unconditional price:

$$p_u \triangleq r$$

- and a conditional price

$$p_c \triangleq \frac{\int_r^{\overline{s}} \max\{s_{-i}\} \, dF_{-i}(s)}{1 - F^{N-1}(r)},$$

Neutral Selection

- object is sold if and only if at least one bidder is willing to make an unconditional purchase at $p_u = r$.
- If at least one bidder makes unconditional purchase, then *all* bidders get object with uniform probability $1/N$ at price p_c
- with one exception: if only one bidder is willing to make an unconditional purchase, then this bidder gets object at $p_u < p_c$

Proposition (Posted Price Pair)

The posted price pair (p_c, p_u) yields a (weakly) higher revenue than either the inclusive or the exclusive posted price.

- uniform screening among bidders with respect to highest signal
- uniform exclusion among bidders

Advantageous Selection and Winner's Blessing

- there is a fixed reserve price r and a random reserve price $x > r$
- if bidder announces highest signal $s_i > r$, then (i) he receives priority status and (ii) is offered object at price:

$$p = max\left\{x, s_{-i}\right\}$$

- otherwise, any bidder receives the object with probability $1/(N-1)$ if at least one bidder has declared priority status at price:

$$p = max\left\{r, s_{-i}\right\}.$$

Random Reserve Price

- random reserve price is determined by:

$$H(x) = \frac{1}{N}(1 - \frac{F^N(r)}{F^N(x)})$$

- all signals have the same interim probability of receiving the object!

Theorem (Random Reserve Price)

The random reserve price is the revenue maximizing mechanism.

- interim probability of receiving the object is constant in signal s_i
- interim transfer is constant in signal s_i
- all downward incentive constraints are binding

Bounds on Bidder Surplus and Revenue

A Relaxed Problem

- consider a smaller, one-dimensional, family of constraints:
- instead of reporting signal s_i, report a random signal $s_i' < s_i$, drawn from truncated prior on support $[\underline{s}, s_i]$:

$$F\left(s_i'\right)/F\left(s_i\right)$$

- *misreporting a redrawn lower signal*
- analyze a relaxed problem which consists of local and small class of global constraints
- use these constraints to derive:

1. an upper bound on seller revenue
2. a lower bound on bidder utility

A Lower Bound on Bidder Utility

- what are the gains from *misreporting a redrawn lower signal?*
- equilibrium surplus of a bidder with type x is

$$u_i(s_i) = \int_{x=\underline{s}}^{s_i} \widehat{q}_i(x)\, dx$$

- surplus from misreporting the redrawn lower signal

$$\frac{1}{F(s_i)} \int_{x=\underline{s}}^{s_i} u_i(s_i, x)\, f(x)\, dx$$

- gains vary depending on realized misreport
  average gains across all misreports are easy to compute

Posted Prices

- consider mechanisms where object is always allocated
- pure common values – allocation is therefore socially efficient

Theorem (Revenue Optimality among Efficient Mechanisms)
Among all mechanisms that allocate the object with probability one, revenue is maximized by setting a posted price of

$$p = \int_{v=\underline{s}}^{\overline{s}} v g_{N-1}(v)\, dv, \tag{1}$$

i.e., expected value of object conditional on having lowest signal $\underline{s}$.

- posted price is inclusive: all types purchase at p
- all bidders equally likely to receive object: $q_i(v) = 1/N, \forall i, v$.
- optimal selling mechanism is attained with constant interim transfer $t = t_i(s_i)$ and probability $q = q_i(s_i)$

Optimality of Posted Price

- next, optimality of posted price among all – possibly inefficient – mechanisms

Corollary (Revenue Optimality of Posted Prices)
A posted price mechanism is optimal if and only if

$$\psi(\underline{s}) = \underline{s} - \int_{\underline{s}}^{\overline{s}} \frac{1 - F(x)}{F(x)} dx \geq 0.$$

If a posted price p is optimal, then it is fully inclusive.

Incentive Constraints in Optimal Auction

- in the optimal auction, each bidder is indifferent between his equilibrium bid and any lower bid

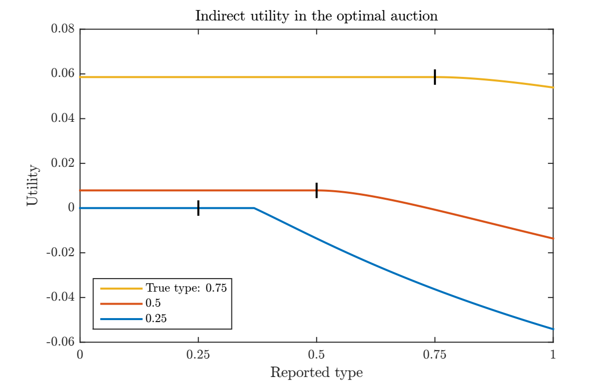

Figure 3: Uniform Downward Incentive Constraints and Winner's Blessing

Incentive Constraints in First Price Auction

- in the first price auction, each bidder is indifferent between his equilibrium bid and any higher bid

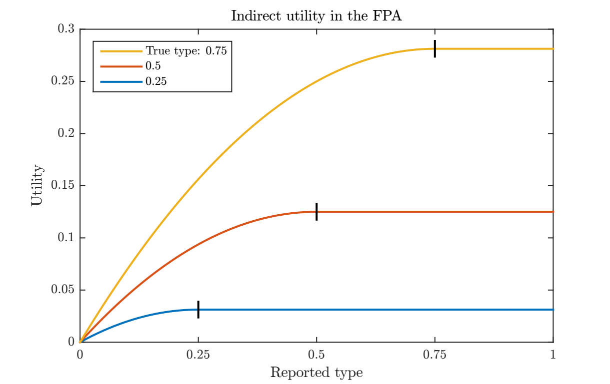

Figure 4: Uniform Upward Incentive Constraints and Winner's Curse

The Power of Optimal Auctions

Auctions vs Optimal Mechanism

- Bulow and Klemperer (1996) establish the limited power of optimal mechanisms as opposed to standard auction formats
- revenue of optimal auction with N bidders is strictly dominated by standard absolute auction with $N + 1$ bidders
- current common value environment is an instance of general interdependent value setting – with one exception
- virtual utility function—or marginal revenue function—is not monotone due to maximum operator in common value model

A Closer Look at the Virtual Utility

- non-monotonicity leads to an optimal mechanism with features distinct from standard first or second price auction.
- it elicits information from bidder with highest signal but minimizes probability of assigning him the object subject to incentive constraint
- *virtual utility* of each bidder, $\pi_i(s_i, s_{-i})$:

$$\pi_i(s_i, s_{-i}) = \begin{cases} \max_j\{s_j\}, & \text{if} \quad s_i \leq \max\{s_{-i}\}; \\ \max\{s_j\} - \frac{1-F_i(s_i)}{f_i(s_i)}, & \text{if} \quad s_i > \max\{s_{-i}\}. \end{cases}$$

- downward discontinuity in virtual utility indicates why seller wishes to minimize the probability of assigning the object to the bidder with the high signal

Revenue Comparison

- virtual utility of bidder i fails monotonicity assumption even when hazard rate of distribution function is increasing everywhere
- BK (1996) require monotonicity of virtual utility when establishing their main result that an absolute English auction with $N+1$ bidders is more profitable than any optimal mechanism with N bidders
- revenue ranking does not extend to current auction environment
- compare revenue from optimal auction with N bidders to absolute, English or second-price, auction with $N+K$ bidders
- absolute as there is no reserve price imposed

Reversal in Revenue Comparison

Proposition (Revenue Comparison)

For every $N \geq 1$ and every $K \geq 1$, the revenue from an absolute second-price auction with $N + K$ bidders is strictly dominated by the revenue of an optimal auction with N bidders.

- comparison of *second order statistic* of $N + K$ i.i.d. signals and *first order statistic* of $N + K - 1$ i.i.d. signals
- second order statistic of $N + K$ signals is revenue of absolute second-price auction with $N + K$ bidders.
- by earlier Theorem, optimal mechanism (weakly) exceeds revenue from a posted price set equal to the maximum of $N + K - 1$ signals.
- now, if instead of $N + K$ bidders, the optimal auction only has N bidders, then it is as if only N independent and identical distributed signals are revealed to the N bidders

The Power of Optimal Auctions

- compare the indirect utility in the first price auction and the optimal auction

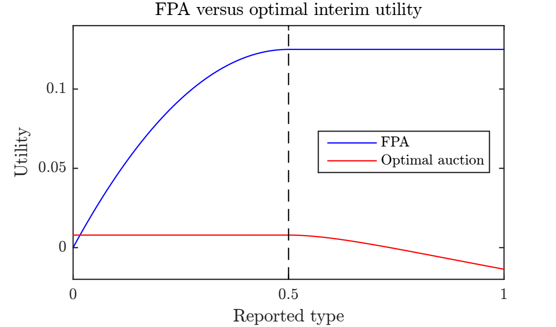

Figure 5: Indirect Utility of Bidder with $s = 1/2$ across two mechanisms

Conclusion

- characterized novel revenue maximizing auctions for a class of common value models
- common value models with qualitative feature that values are more sensitive to private information of bidders with more optimistic beliefs
- second interpretation as auction with frictionless resale market
- characterizations of optimal revenue that exist in the literature depend on information rents being smaller for bidders who are more optimistic about value
- qualitative impact is that earlier results found that optimal auctions discriminate in favor of more optimistic bidders
- today: optimal auctions discriminate in favor of less optimistic bidders since they obtain less information rents from being allocated the object

Appendix: Additional Slides

Optimal Auction

- construct an incentive compatible mechanism that exactly achieves the upper bound
- in the direct mechanism, all types are asked to make a fixed payment, a participation fee, that is independent of their type
- no transfers beyond the participation fee are collected
- every type has the same interim expected probability of being allocated the object
- these two features of the optimal mechanism resemble a posted price mechanism
- unlike posted prices, the object is only allocated if the highest realized signal among the bidders exceeds a threshold value
- thus, typically, the probability that the object is assigned to *some* bidder is strictly smaller than one
- second feature distinct from posted prices is that the optimal mechanism discriminates against bidders with higher signals

Uniform Distribution

- family of translated uniform distributions on $[a, a+1]\,, a > 0.$
- marginal revenue function for these distributions is

$$\psi_a(x) = x - \int_{y=x}^{a+1} \left((x-a)^{-\frac{1}{N}} - 1 \right) dx,$$

- lowest marginal revenue is

$$\begin{aligned} \psi_a(a) &= a - \int_{y=a}^{a+1} \left((x-a)^{-\frac{1}{N}} - 1 \right) \\ &= a - \frac{1}{N-1}. \end{aligned}$$

- thus posted price is optimal if

$$a > 1/(N-1)$$

Optimal Mechanism in General

Optimal Mechanism in General

- construct an novel mechanism/game that attains the revenue bound for all distributions
- *guaranteed demand auction* (GDA)
- direct mechanism to implement the bound exists as well
- descending clock (probability) auction implements revenue bound as well

The Guaranteed Demand Game: Rules

- bidder i demands $d_i \in \left[0, \overline{d}\right]$, where $\overline{d}$ is a parameter of game:

$$0 \leq d_i \leq \overline{d} \leq 1/N$$

- let i^* denote identity of bidder with the highest demand:

$$d_{i^*} = \max\{d_1, ..., d_N\} :$$

- if $d_{i^*} > 0$:

1. i^* is allocated object with probability d_{i^*}
2. bidder $j \neq i^*$ receives the object with probability

$$(1 - d_{i^*}) \,/\, (N - 1) > d_{i^*}$$

- if $d_{i^*} = 0$, then the seller keeps the object

The Guaranteed Demand Game: Properties

- importantly if upper bound on demand $\overline{d}$ is:

$$\overline{d} < 1/N$$

- then conditional on highest demand being positive,

$$d_{i^*} > 0$$

  bidder j is more likely to get object if he doesn't have highest demand:

$$q_j > d_i^* > d_j$$

- each bidder's probability of receiving object is always at least his demand

Equilibrium Strategy

- unique equilibrium has a monotone pure strategy:

$$\sigma\left(s_i\right)=\begin{cases}\frac{1}{N}\left(1-\frac{G_N(\overline{r})}{G_N(s_i)}\right) & \text{if } s_i\geq\overline{r};\\ 0 & \text{if } s_i<\overline{r},\end{cases} \tag{2}$$

where threshold $\overline{r}$ solves:

$$G_N\left(\overline{r}\right)=1-N\overline{d}. \tag{3}$$

- and demand at $\overline{s}$ satisfies:

$$\sigma\left(\overline{s}\right)=\left(1-\left(1-N\overline{d}\right)\right)/N=\overline{d}$$

- denote resulting equilibrium utility: $\overline{u}_i\left(\underline{s}\right)$

The Guaranteed Demand Auction

- turn guaranteed demand game into *guaranteed demand auction* (GDA) by adding entry fees f_i
- each bidder's message consists of a pair: entry decision and demand
- if bidder i decides to enter, he pays f_i to the seller, and object is allocated among bidders who enter guaranteed demand game

Proposition (Equilibrium of Guaranteed Demand Auction)
As long as $f_i \leq \overline{u}_i(\underline{s})$ for all i, it is an equilibrium for all bidders to enter the GDA and make demands according to (2). In equilibrium, bidders are indifferent between their equilibrium demands and all lower demands.

Optimality of Guaranteed Demand Auction

- consider the GDA where the threshold r has zero generalized virtual utility:

$$\psi\left(r^{*}\right)=r^{*}-\int_{y=r^{*}}^{\overline{s}}\frac{1-F\left(y\right)}{F\left(y\right)}dy=0$$

- choice of $\overline{d}$

$$\overline{d}=\frac{1-G_{N}\left(r^{*}\right)}{N}$$

ensures that bidder makes a positive demand iff bidder has signal greater than r^{*}

Theorem (Optimality of Guaranteed Demand Auction)
Revenue is maximized with a guaranteed demand auction with maximum demand $\overline{d}=\left(1-G_{N}\left(r^{*}\right)\right)/N$ *and a symmetric entry fee which is equal to* $\overline{u}_{i}\left(\underline{s}\right)$.

Maximum Game

- Bulow and Klemperer (2002) define "maximum game" and show in second price auction in equilibrium each bidder bids his signal

$$s_i \leq v = max_j\{s_1, ..., s_j, ..., s_N\}$$

  - in equilibrium, bidder with highest signal wins the auction and pays second-highest signal
  - in fact, it is optimal to bid any amount which is at least your signal, and, in particular, it is optimal to bid your signal
  - by contrast, in optimal auction, each bidder is indifferent between reporting his signal and reporting any *lower* signal

Comparison with IPV

- suppose now that the signals are the values, thus independent private value environment:

$$s_i = v_i \leq max_j\{s_1, ..., s_N\}$$

  - in second price auction bidding his signal remains optimal
  - thus, in second price auction of pure common value environment, each bidder behaves as if his signal is his true private value rather than a signal, and in particular a lower bound on the pure common value
  - observation can be generalized

Strategic Equivalence

- consider independent private value model: $v_i(s_1, \ldots, s_N) = s_i$
  - denote the set of bidders with high signals

$$H(s) = \left\{ i | s_i = \max_j s_j \right\}$$

  - direct mechanism $\{q_i, t_i\}$ is *conditionally efficient* if (i) $q_i(s) > 0$ if and only if $s_i \in H(s)$ and (ii) there exists a cutoff r such that the object is allocated whenever $\max_i s_i > r$.

Proposition
Suppose a direct mechanism $\{q_i, t_i\}$ is incentive compatible and individually rational for the independent private value model in which $v_i(s) = s_i$ and that the allocation is conditionally efficient. Then $\{q_i, t_i\}$ is also incentive compatible and individually rational for the maximum common value model in which $v_i(s) = \max_j \{s_j\}$.