Learning and Commitment in Incentive Contracts[1]

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Introduction

Partnerships between economic agents commonly endure many periods. The commitment of the partners is documented in a contract, which may exist only implicitly or may be written explicitly. Contracts typically don't cover the entire life-span of the partnership. Insurance policies are typically renewed every year, managerial contracts often extend for several years and only tenure appointments involve the (remaining) professional life span. It is a question of both theoretical interest and practical importance to determine the contractual time horizon and to analyze how the time horizon influences the incentives provided by the contract.

The purpose of this paper is to develop an explicit model in which the optimal dynamic design of the contractual regime of partnerships can be analyzed. The essential advantage of a long-term contract over a short term contract resides in its ability to *commit* to certain acts in the future. This paper will argue that the *very* element which constitutes the advantage of a long-term contract can turn into a *liability*. In the presence of uncertainty, the contracting partners improve over time their knowledge about the value of the partnership. As they learn more about the nature of their match, they may decide to continue or else one of the partners, possibly both, may want to dissolve the partnership and seek for a better match. Thus a long-term contract allows for efficient incentives between the current contracting partners, but it also prevents the individuals from searching for more favorable conditions elsewhere.

The paper is written in the context of a simple two period moral hazard model where one partner is the risk-neutral principal and the other is the risk-averse agent. The model has two features which sets it apart from the standard static or repeated moral hazard model. First, the early information the principal receives about the agent's performance is noisier than the later arriving information. Second, principal and agent are initially uncertain about the value of their partnership. More specifically, they don't know how the technology of the agent performs in the context of the task assigned by the principal. As they receive information about the agent's performance they will become more informed. In

1 I would like to thank George Mailath, Andrew Postlewaite and Rafael Rob for their criticism and helpful suggestions.

The intertemporal contract policy will in fact display a strong relation between short-term contracts and strong incentive based reward schemes. And conversely the period-by-period incentives will be relatively weaker in long-term contracts.

The study of agency relationships has, for the most part, focused on single period arrangements.[2] Recently, however, a number of analyses have investigated the implications of extending the model to multi-period settings. Holmström (1982) argued very early for dynamic considerations regarding incentive problems. In the case of managerial incentive problems he tied the incentive issue to the learning process about the managerial ability. A more recent treatment of this issue appears in Fudenberg, Holmström, and Milgrom (1990). The main theme of their work is the question: under which sufficient conditions can short-term contracts constitute an efficient incentive scheme and successfully implement a long-term relationship. Their central conditions are (1) that the agent can access a bank on equal terms with the principal and (2) that recontracting takes place with common knowledge about technology and preferences. It is precisely condition (2) which is not met by our model. The principal will not know the distribution function of the final outcome once the agent has chosen his effort level, since the current signal will provide only partial information.

Malcomson and Spinnewyn (1988) show that if the short-term contract can punish the agent sufficiently, then long-term contracts do not need to improve on short-term contracts. The importance of long-term contracts, finally, for the consumption smoothing of the agent when the agent's access to borrowing is constrained has been emphasized by Rey and Salanie (1987).

The next section of the paper develops the structure of the model. In the third section we study in some detail the properties of the short-term contract in a noisy environment and analyze how the optimal incentive system and the induced actions change as the environment gradually becomes less informative. The next section presents the main results on the optimal choice between short-term and long-term contracts. The last section discusses some extensions and concludes.

The Model

The model presented in the first subsection is essentially a static principal-agent model repeated over time. In the second subsection we introduce the noisy learning environment.

2 See Holmström (1979), Shavell (1979), and Grossman/Hart (1983).

Principal and Agent

The principal owns an investment project with a finite lifespan. For simplicity of exposition it is assumed that there are two periods, $t = 1,2$. The realization of the project requires the assistance of an agent. In each period t for which the agent is under contract he chooses an *action* $e_t \in [\underline{e}, \overline{e}] = E \subset I\!R$. The principal cannot directly observe effort e_t, and hence the agent's action cannot be used to determine the agent's payoff. The principal can, however, observe the *final outcome* x_t of the action e_t. The outcome x_t is, without loss of generality, the payoff of the principal gross of what is paid to the agent. The final outcomes x_1 and x_2 are only realized at the end of the investment project, which is $t = 2$. The index t of the outcome x_t traces the outcome to the generating effort e_t. In addition, the principal can observe in each period t and without delay a *contemporaneous signal* y_t which is stochastically related to the final outcome x_t. We assume for simplicity that both, contemporaneous signal y_t and final outcome x_t, can take only the following realizations:

$$x_t \in \{x_L, x_H\} \text{ and } y_t \in \{y_L, y_H\}.$$

The outcome x_t is modeled as a random variable conditional to the agent's effort with a probability distribution, common knowledge to both principal and agent:

$$p_L(e_t) = p(e_t) = \quad Pr(x_t = x_L | e_t),$$

$$p_H(e_t) = 1 - p(e_t) = Pr(x_t = x_H | e_t),$$

The contemporaneous signal y_t is a noisy garbling conditional on x_t, to the details of which will be dealt with in the next subsection. In the tradition of the first-order-approach to moral hazard models, it is assumed that the probability of the low outcome x_L is decreasing, convex and twice differentiable in the effort e:[3]

$$p'(\cdot) < 0, \qquad p''(\cdot) > 0.$$

We call $p(\cdot)$ simply the technology of the agent. The technology $p(e_t)$ is time independent and uncorrelated over time. Throughout the exposition we shall use $\{p(e_t), 1\text{-}p(e_t)\}$ and $\{p_L(e_t), p_H(e_t)\}$ interchangeably. Since $p(e_t)$ completely characterizes the probability distribution over x_t, we shall refer to $p(e_t)$, whenever we compare two technologies or discuss properties of the technology. Since many long-term projects exhibit some form of time dependence, we note that all arguments remain valid for a general valuation function $f(x_1, x_2)$ of the outcomes x_1

3 The first-order approach has been developed by Grossman and Hart (1983) and Rogerson (1985).

and x_2 as long as $f(\cdot)$ is increasing and concave.[4] Moreover all arguments presented in the paper extend naturally to the situation when $p(e_{t+1})$ is only *conditionally* independent of $p(e_t)$.

We say that technology *b dominates* technology *a*, if the probability of a low outcome x_L is smaller under technology *b* than under technology *a* for all effort levels *e*: $p_b(e) \leq p_a(e)$. If the difference between technology *a* and *b* increases convexly as a function of the effort *e*, then we call *b convex dominant* over *a*.

Definition 1. (Convex dominant)
The *technology b* convexly dominates *technology a* if

(*i*) $p_a(\underline{e}) = p_b(\underline{e})$,

(*ii*) $p_b'(e) \leq p_a'(e)$,

(*iii*) $p_b''(e) \leq p_a''(e)$ for all $e \in E$.

We shall denote the relation of convex dominance by $\succ_c$, so that $b \succ_c a$ means that *b* convexly dominates *a*. The ordering $\succ_c$ reflects the intuition that existing differences in abilities or technologies become more apparent as the intensity of effort increases. Condition (*i*) merely states that if the lowest possible effort level is chosen, then the probability distribution over outcomes is independent of the technology. One important consequence of convex dominance is that the difference in expected profits between two technologies *a,b* which can be ranked by $\succ_c$, with $b \succ_c a$, is increasing and convex in *e*:

$$(p_b(e) - p_a(e))x_L + (p_a(e) - p_b(e))x_H.$$

The agent is risk averse and effort is costly for him:

$$U(c) - C(e).$$

The agent's utility function is additively separable over time. The utility function $U(\cdot)$ is twice differentiable, strictly increasing and strictly concave. The effort function $C(\cdot)$ is also twice differentiable, strictly increasing and strictly convex. The principal is risk neutral. Both, agent and principal, maximize expected utility over the two periods and there is no discounting between the periods.

There are many competing agents and their reservation utility $\overline{U}$ is determined through the market. The principal can offer choose between different *contractual regimes* γ. The regime $\gamma \in \{s, l\}$ can be a sequence of short-term contracts *s* with

4 Suppose the successful completion of the inital project phase x_1 is a necessary prerequisite to accomplish anything in the final phase x_2 at any level, then we would have $f(x_L, x_L) = f(x_L, x_H) < f(x_H, x_L) < f(x_H, x_H)$.

possibly different agents or a single long-term contract l. We index γ as γ_t and identify the contemporaneous elements of the contractual agreement. The contract can be conditioned on y_t and x_t and is legally enforceable.

Learning and Uncertainty

Each agent on the market is endowed with some idiosyncratic qualification, which could be beneficial for his performance. Principal and agent are, however, initially uncertain whether the agent's idiosyncratic ability can be made productive for the success of the *specific* investment project for which the agent is contracted. If the agent's qualification matches the project's specificity than we say that he operates under technology b, if not, then he operates under technology a, where $b \succ_c a$. The common prior beliefs of agent and principal are given by:

$$\Pr(\theta = a) = \alpha$$

$$\Pr(\theta = b) = \beta = 1\text{-}\alpha.$$

The prior beliefs are identical for all agents and, in consequence, the principal is initially indifferent among the competing agents. The expected probability of an outcome x_t given a belief α is simply:

$$\Pr(x_t = x_L | e_t, \alpha) = p_\alpha(e_t) = \alpha\, p_a(e_t) + (1-\alpha) p_b(e_t),$$

$$\Pr(x_t = x_H | e_t, \alpha) = 1 - p_\alpha(e_t) = \alpha\left(1 - p_a(e_t)\right) + (1-\alpha)\left(1 - p_b(e_t)\right).$$

It is easy to show that for more optimistic beliefs α', with $\alpha' < \alpha$, the technology $p_{\alpha'}(\cdot)$ convexly dominates the technology $p_\alpha(\cdot)$.

The principal and his agent receive new information about the true technology only through the signal y_t and the outcome x_t. Before we describe how the players learn about the true technology we first have to clarify how the noisy signal y_t which arrives earlier conveys information about the final outcome x_t which arrives later. From an informational viewpoint the question is: how reliable is the signal y_t? Suppose the final outcome will be x_L, what is the probability that we will be correctly informed by the earlier corresponding signal y_L? We define $q \in [0,1]$ as the conditional probability that the outcome x_t is indicated earlier by the associated signal y_t,

$$q \equiv \Pr(y_L | x_L) = \Pr(y_H | x_H).$$

The parameter q indicates the noisiness of the first period signal. For $q = 1$, the correlation between outcome and signal is perfect and hence the signal is as informative as the outcome. For $q = \frac{1}{2}$, the correlation between outcome and signal is zero and consequently the signal contains no information.[5] The Markov matrix $Q = [q_{ij}]$ induced through the parameter of noise q,

$$Q = \begin{bmatrix} q & 1-q \\ 1-q & q \end{bmatrix},$$

garbles the more precise information which the principal will only receive in the second period into a noisy signal which is received in the first period. If $p(e_1)$ and $1\text{-}p(e_1)$ are the probabilities for x_L and x_H determined by the effort level e_1, then the probabilities $q(e_1)$ and $1\text{-}q(e_1)$ for the first period signals, y_L and y_H, respectively are given by:

$$\big(q(e_1) \quad 1-q(e_1)\big) = \big(p(e_1) \quad 1-p(e_1)\big) \cdot \begin{bmatrix} q & 1-q \\ 1-q & q \end{bmatrix},$$

and we have:

$$\begin{aligned} q_L(e_1) &= q(e_1) = 1 + 2qp(e_1) - p(e_1) - q, \\ q_H(e_1) &= 1 - q(e_1) = p(e_1) + q - 2qp(e_1). \end{aligned} \tag{2.1}$$

For all practical purposes it will be enough to restrict q to $q \in [\frac{1}{2}, 1]$, since the absolute correlation between signal and outcome is the same under $q = \frac{1}{2} + z$ and $q = \frac{1}{2} - z$, where $z \in [0, \frac{1}{2}]$. We note that the signal y_t has no independent informational content beyond garbling x_t and loses therefore all value as soon as x_t becomes observable. With this understanding we shall refer to signal y_1 simply as *the* signal y, since y_2 appears by definition in the same period as the outcome x_2 and is therefore informationally redundant.

We complete the noisy environment by describing what principal and agent can learn about the later outcome x_t and the true technology θ of the agent by choosing the first period effort level e_1 and observing the signal y_i. Given the noise q and the effort level e_1, the conditional probabilities of the final outcome x_1 are given by:

$$\Pr(x_j | y_i, e) \qquad \text{for } i = L, H, \;\; j = L, H, \;\; \forall e.$$

5 The reader may be cautioned that the informativeness of a signal is not necessary equal to the value of the signal. While the principal, if given the choice between receiving y_i and x_i in the *same* period would always prefer x_i and thus attach a higher value to x_i. In the *sequential* framework here the incremental value of y_i in the first period may however be higher for the principal than the incremental value of receiving x_i in the second period.

Similarly the posterior probabilities of the agent's technology are conditional probabilities on the signal y_i and the effort level e_I:

$$\alpha(y_i,e)=Pr(a|y_i,e) \quad for\, i=L,H,\ \forall e,$$
$$\beta(y_i,e)=Pr(b|y_i,e) \quad for\, i=L,H,\ \forall e. \tag{2.2}$$

Since the action choice of the agent will influence posterior probabilities on the agent's technology and therefore the future terms of his contract or his dismissal, the intertemporal effort profile will depend on the interaction between current and future incentives *and* the optimal learning policy.

At the end of this section we should mention that all the results generalize to $T > 2$, but no additional insights seem to arise for a finite number of periods. Similarly all results extend to the case of a risk-averse principal, as long as she is less risk-averse than the agent.

The Optimal Short-Term Contract

The aim of this section is twofold. First, it presents some results of independent interest on the optimal contract under noise. Second, it prepares us for the analysis of the optimal choice among short-term and long-term contracts by establishing efficiency and comparative statics for the case of the short-term contract. For that purpose we recall some standard result for the reader when $q = 1$ in subsection 3.1. The noisy contracting is then studied in subsection 3.2.

The Contract for $q = 1$

The principal's problem can be described as follows. Let A be the set of pairs, $a = (s,e)$, of incentive schemes s and effort levels e such that, under s, the agent will be willing to work for the principal and will find it optimal to choose e. We call a the set of *implementable allocations*. The principal then chooses $a^* \in A$ so as to maximize:

$$\max_{(e,s)} p_L(e)\,(x_L - s_L) + p_H(e)\,(x_H - s_H), \tag{3.1}$$

subject to the individual rationality constraint,

$$(\text{IR}) \quad p_L(e)U(s_L) + p_H(e)U(s_H) - C(e) \geq \overline{U}, \tag{3.2}$$

and the incentive compatibility constraint,

$$\text{(IC)} \quad e \in \arg\max_{e \in E,} p_L(e)U(s_L) + p_H(e)U(s_H) - C(e). \tag{3.3}$$

We omit in this section the time index as we are looking at a single effort choice. Moreover as $q = 1$, there is no need to distinguish between the signal y_i and the outcome x_i. The payment s_i is contingent on the outcome x_i. We establish next the first order conditions for the optimal contract. In a first step we replace the incentive compatibility constraint with the local first order condition. The implied relaxation of the constraint set is justified under the monotone likelihood ratio condition and the convexity of the distribution function, conditions which are satisfied by the assumptions of our model. Differentiating with respect to e we obtain:

$$p_L'(e)[U(s_L) - U(s_H)] - C'(e) = 0. \tag{3.4}$$

Since $C'(e)$ increases and $|p_L'(e)|$ decreases in e, equation (3.4) implies that a high effort level e can only be sustained through a correspondingly high difference in the contingent payments s_L and s_H. We assign the Lagrange multiplier λ to the individual rationality constraint (3.2) and the multiplier μ to (3.4). The first order conditions for the optimal contract are derived with respect to the contractual payments and the effort level of the agent:

$$(s_L) \quad -p_L(e) + \lambda p_L(e)U'(s_L) + \mu p_L'(e)U'(s_L) = 0,$$

and

$$(s_H) \quad -p_H(e) + \lambda p_H(e)U'(s_H) - \mu p_L'(e)U'(s_H) = 0.$$

The initial order condition with respect to the agent's effort level is, after using (3.4),

$$(e) \quad p_L'(e)[(x_L - s_L) - (x_H - s_H)] + \mu\{p_L''(e)[U(s_L) - U(s_H)] - C''(e)\} = 0.$$

Some first observations can be made at this point. Given a certain reservation utility $\overline{U}$ and a desired implemented effort level e, the corresponding utilities and implicitly the payments are given by:

$$U(s_L) = \overline{U} + C(e) + \frac{(1 - p_L(e))C'(e)}{p_L'(e)}, \tag{3.5}$$

and

$$U(s_H) = \overline{U} + C(e) - \frac{p_L(e)C'(e)}{p_L'(e)}. \tag{3.6}$$

We define $\Delta U \equiv U(s_H) - U(s_L)$, where ΔU indicates how much the agent has to be exposed to risk in order to have the proper work incentives. The difference between $U(s_H)$ and $U(s_L)$ is strictly positive and is given by:

$$\Delta U = U(s_H) - U(s_L) = -\frac{C'(e)}{p_L'(e)} > 0. \tag{3.7}$$

The utilities awarded to the agent under (3.5) and (3.6) reflect in their decomposition the cost of setting incentives. In any event the agent is compensated for his reservation utility level $\overline{U}$ and the cost he incurred for undertaking effort e, which is $C(e)$. We define $s(e)$ as the *pure compensation payment* which renumerates the agent at the level of his reservation value and his expended effort:

$$U(s(e)) \equiv \overline{U} + C(e), \tag{3.8}$$

where $s(e)$ is increasing and convex in e. The partial alignment of the agent's objective with the principal's is governed by payments and induced utilities beyond $\overline{U}+C(e)$. Let us give

Definition 2. The *contingent utility* $I(s_i)$ to be received in state i is given by:

$$I(s_i) \equiv U(s_i) - \overline{U} - C(e), \qquad i = L, H. \tag{3.9}$$

The contingent utility is designed to give the agent the proper incentives at the margin, with

$$I(s_L) = \frac{(1 - p_L(e))C'(e)}{p_L'(e)} < 0, \text{ and } I(s_H) = -\frac{p_L(e)C'(e)}{p_L'(e)} > 0. \tag{3.10}$$

For a marginal increase in e the agent has to be rewarded in the magnitude of the marginal costs $C'(e)$. Since the payment is contingent on x_i, and since $I(s_H) > 0 > I(s_L)$ the marginal shift in probability from x_L to x_H affected by an increase in e and indicated by $p_L'(e) < 0$, determines how much of the necessary compensation can be realized through the marginal change in probabilities. For given marginal costs $C'(e)$, the contingent utility $I(s_H)$ can be smaller the larger the marginal change $|p_L'(e)|$ in the probability. Finally, in order to sustain the incentives, the contingent utility has to be higher the smaller the probability is that the associated event occurs. We notice that

$$\Delta U = U(s_H) - U(s_L) = I(s_H) - I(s_L) = \Delta I,$$

where the expected value of the contingent utilities is given by

$$p(e)I(s_L)+(1-p(e))I(s_H)=0. \tag{3.11}$$

In consequence, the participation constraint is always met exactly and for completeness we give the expressions for the Lagrange multipliers as:

$$\lambda=\frac{p_L(e)}{U'(s_L)}+\frac{p_H(e)}{U'(s_H)},$$

and

$$\mu=\frac{p_L(e)p_H(e)}{p_L'(e)}\left(\frac{1}{U'(s_L)}-\frac{1}{U'(s_H)}\right),$$

from which we can immediately infer that $\lambda>0$ as well as $\mu>0$. The stage is now set to analyze the optimal contract in the noisy environment.

The Noisy Single Period Contract

The transition from the standard model to the noise model requires some modifications. First, the set of *implementable allocations* depends on the level of noise q and is denoted by $A(q)$. Second, the probability of the occurrence of signal y_i no longer coincides with the probability of the outcome x_i. In consequence, the noisy contract in which the payment s_i is induced by the signal y_i has different properties from the noiseless contract discussed in the previous section. We derived the noisy probabilities $q_i(e)$ in (2.1). The principal's problem is given by:

$$\max_{e,s_i} p_L(e)x_L-q_L(e)s_L+p_H(e)x_H-q_H(e)s_H, \tag{3.12}$$

subject to the individual rationality constraint,

$$\text{(IR)}\quad q_L(e)U(s_L)+q_H(e)U(s_H)-C(e)\geq\overline{U}, \tag{3.13}$$

and the incentive compatibility constraint,

$$\text{(IC)}\quad e\in\underset{e\in E}{\arg\max}\, q_L(e)U(s_L)+q_H(e)U(s_H)-C(e). \tag{3.14}$$

We establish the first order conditions for the optimal contract as before. In a first step we will write the incentive compatibility constraints by local first order conditions. Differentiating with respect to e we obtain:

$$\left(2qp_L'(e)-p_L'(e)\right)\left[U(s_L)-U(s_H)\right]-C'(e)=0. \tag{3.15}$$

We notice immediately that for $q=0.5$ the optimal contract cannot exert any effort at all. Since the probability of any signal y_i occurring is independent of the effort level chosen, all information is completely diluted and pure noise. As the precision increases with q, the contingent utility difference necessary to sustain a given effort level e decreases. We assign the Lagrange multiplier λ to the individual rationality constraint (3.13) and the multiplier μ to (3.15). The first order conditions for the optimal contract are derived first with respect to the contractual payments and then with respect to the effort level by the agent.

$$(s_L) \quad -q_L(e)+\lambda q_L(e)U'(s_L)+\mu q_L'(e)U'(s_L)=0,$$

and

$$(s_H) \quad -q_H(e)+\lambda q_H(e)U'(s_H)-\mu q_L'(e)U'(s_H)=0,$$

and the first order condition with respect to the agent's effort level

$$(e) \; p_L'(e)(x_L-x_H)+q_L'(e)(s_H-s_L)+\mu\left\{p_L''(e)\left[U(s_L)-U(s_H)\right]-C''(e)\right\}=0.$$

Given a certain reservation utility $\overline{U}$ and a desired implemented effort level e, the corresponding utilities and implicitly the payments are now given by:

$$U(s_L)=\overline{U}+C(e)+\frac{q_H(e)C'(e)}{q_L'(e)}, \tag{3.16}$$

and

$$U(s_H)=\overline{U}+C(e)-\frac{q_L(e)C'(e)}{q_L'(e)}. \tag{3.17}$$

We first examine in proposition 1-3 how the set of utilities, and hence the contractual payments of implementable allocations, behaves as a function of the effort level e to be implemented. Proposition 4 analyses the cost of the contract and proposition 5 determines the optimal implementable allocation.

Proposition 1. *If the effort level e to be implemented increases then*:

(*i*) $\quad \partial U(s_L)/\partial e<0,$

(*ii*) $\quad \partial U(s_H)/\partial e>0.$

Proof. By differentiating we obtain,

$$\frac{\partial U(s_L)}{\partial e}=\frac{(p_L'(e)C''(e)-p_L''(e)C'(e))[(1-q)(4p_L(e)q-2q)-p_L(e)+q]}{(2qp_L'(e)-p_L'(e))^2}<0,$$

and symmetrically we have

$$\frac{\partial U(s_H)}{\partial e}=\frac{(p_L''(e)C'(e)-p_L'(e)C''(e))[(1-q)(2q-1-4p_L(e)q)+p_L(e)]}{(2qp_L'(e)-p_L'(e))^2}>0.$$

and since for all $p_L(e)\in(0,1)$ and for all $q\in(0.5,1)$,
$[(1-q)(4p_L(e)q-2q)-p_L(e)+q]>0$ as well as
$[(1-q)(2q-1-4p_L(e)q)+p_L(e)]>0$ hold, the results follow.

One would have expected that the utility in the state H, induced through s_H would be required to grow convexly in e so as to offset the convexly increasing costs of effort. Since the probability of a good signal, $q_H(e)$ is also increasing in e, the curvature of $U(s_H)$ as a function of the implemented effort level e depends on the exact interplay of $p(e)$ and $C(e)$. Since the agent is only concerned with the expected value of $U(s_H)$, a greater chance of obtaining $U(s_H)$ also improves his incentives towards a greater provision of effort. The more interesting issue is the change in the contingent utilities relative to the utility provided by the pure compensation. We have the following result

Proposition 2. *If the effort level e to be implemented increases then:*

(*i*) $\partial I(s_L)/\partial e<0$,

(*ii*) $\partial \Delta I/\partial e>0$.

Proof. By differentiating we obtain

$$\frac{\partial I(s_L)}{\partial e}=\frac{(p_L'(e)C''(e)-p_L''(e)C'(e))[(1-q)(4p_L(e)q-2q)-p_L(e)+q]}{q_L'(e)^2}+$$
$$\frac{p_L'(e)^2C'(e)(4q-4q^2-1)}{p_L'(e)^2}<0$$

and symmetrically we have

$$\frac{\partial I(s_H)}{\partial e} = \frac{\left(p_L''(e)C'(e) - p_L'(e)C''(e)\right)\left[(1-q)(2q-1-4p_L(e)q) + p_L(e)\right]}{q_L'(e)^2} + \frac{p_L'(e)^2 C'(e)\left(4q - 4q^2 - 1\right)}{q_L'(e)^2}$$

which can be positive or negative. Finally

$$\frac{\partial \Delta U}{\partial e} = \frac{\partial \Delta I}{\partial e} = \frac{p_L''(e)C'(e) - p_L'(e)C''(e)}{p_L'(e)^2(2q-1)} > 0,$$

which concludes the proof.

The intuition developed earlier in regard to the movement of $U(s_H)$ as a function of e then carries over to the contingent utility $I(s_H)$ which does not need to increase for higher effort levels. Since higher effort levels already increase the probability of a good event, it may be enough to lower the pure incentive payment for the occurrence of a bad state, which will always be decreased when a higher effort level is to be implemented. However, the importance of the incentive pay increases as measured by the payment differences between the good and the bad states: $\Delta U = U(s_H) - U(s_L) = I(s_H) - I(s_L) = \Delta I$, with $\partial \Delta I / \partial e > 0$.

So far we have analyzed the properties of implementable allocations as the effort e to be implemented changes. The comparative statics were formulated in the utility space of the agent rather than in the space of contractual payments of the principal. We shall keep this focus for the moment and analyze how the properties of implementable allocations change as the environment becomes noisier. We then turn to the properties of the fee schedule and finally determine the optimal contract s^* effort level e^*.

Suppose for the moment we would hold the implemented effort level e constant while the noise increases as the precision q decreases. The informational consequence for the principal is that she will more often receive a positive signal when the final outcome is negative, or inversely, she will register a negative signal although the eventual outcome will be to her satisfaction. Agent and principal then recognize that the signal y_i becomes less responsive to the agent's effort and, in consequence, high effort appears to be less desirable to the agent. The last resort for the principal is to make the point more forceful by increasing the relative difference in contingent utility.

Proposition 3. *As the noise decreases, the implementation of a constant effort level e requires:*

(i) $\partial U(s_L)/\partial q > 0, \qquad \partial^2 U(s_L)/\partial q^2 < 0,$

(ii) $\partial U(s_H)/\partial q < 0, \qquad \partial^2 U(s_H)/\partial q^2 > 0.$

Proof. By differentiating we obtain

$$\frac{\partial U(s_L)}{\partial q} = -\frac{C'(e)p_L'(e)}{q_L'(e)^2} > 0, \tag{3.18}$$

and symmetrically for the high effort payment we have

$$\frac{\partial U(s_H)}{\partial q} = \frac{C'(e)p_L'(e)}{q_L'(e)^2} < 0. \tag{3.19}$$

The utility increase in the low state is concave in

$$\frac{\partial^2 U(s_L)}{\partial q^2} = \frac{4C'(e)p_L'(e)^2}{q_L'(e)^3} < 0, \tag{3.20}$$

whereas the utility decrease in the high state is convex in q

$$\frac{\partial^2 U(s_H)}{\partial q^2} = -\frac{4C'(e)p_L'(e)^2}{(q_L'(e))^3} > 0. \tag{3.21}$$

It remains to mention that $\partial U(s_L)/\partial q = -\partial U(s_H)/\partial q$ for all q and all e. The concavity of $U(s_L)$, like the convexity of $U(s_H)$, underlines the fact that the difference in the contingent utility grow ever faster as the noise increases. The coincidence of signal and outcome decrease linearly in (1-q) which implies that the probability of receiving a "truthful" signal decreases in a convex manner. Moreover the marginal cost of increasing e increases as the environment becomes noisier:

$$\frac{\partial^2 U(s_L)}{\partial e \partial q} = -\frac{p_L'(e)C''(e) - p_L''(e)C'(e)}{q_L'(e)^2} > 0$$

and

$$\frac{\partial^2 U(s_H)}{\partial e \partial q} = \frac{p_L'(e)C''(e) - p_L''(e)C'(e)}{q_L'(e)^2} < 0.$$

Before we give the next result we need to establish a set of claims which relate the payment space s_i to the utility space U(s_i). The lemma which summarizes the results relies primarily on the concavity of the utility and the property of mean preserving spreads. For the purpose of the lemma we shall phrase one of the general properties of the contingent utilities as shown in (3.11) as

Condition 1. (Zero expected utility)
A quadruple (s, s_L, s_H, p) *satisfies the zero expected utility condition if*

$$pI(s_L)+(1-p)I(s_H)=0. \tag{3.22}$$

where $I(s_i)=U(s_i)-U(s)$.

The lemma is proved for a given utility function $U(\cdot)$ and some probability $p\in[0,1]$, which for purpose of the lemma need not to have any relationship to the effort level e. We can now state

Lemma 1. *For any quadruples* (s,s_L,s_H,p) *and* (s',s'_L,s'_H,p') *satisfying condition* 1, *the following properties are true*:

$$\begin{aligned}(i)\quad & \text{for } s>s', \quad I(s_H)=I(s'_H) \text{ and } I(s_L)=I(s'_L),\\ & ps_L+(1-p)s_H-s>ps'_L+(1-p)s'_H-s';\end{aligned} \tag{3.23}$$

$$\begin{aligned}(ii)\quad & \text{for } s=s', \quad I(s'_H)-I(s'_L)> I(s_H)-I(s_L), I(s_L)\geq I(s'_L), \text{ and}\\ & I(s_H)\leq I(s'_H),\ p's'_L+(1-p')s'_H>ps_L+(1-p)s_H;\end{aligned} \tag{3.24}$$

$$\begin{aligned}(iii)\quad & \text{for } s=s', \quad I(s'_L)-I(s_L), \text{ and } I(s'_H)-I(s'_L)\geq I(s_H)-I(s_L),\\ & p's'_L+(1-p')s'_H>ps_L+(1-p)s_H.\end{aligned} \tag{3.25}$$

The proof of lemma 1 is given in the Appendix. Here we state the claims verbally. Claim (*i*) says that if the contingent utility is maintained at high compensation levels, then the cost of the contingent payments beyond the pure compensation rises as the compensation rises. Claim (*ii*) states that if the spread of the contingent utility rises in both directions, i.e. downwards and upwards, then the expected costs increase. Claim (*iii*), finally, states that even if the difference in contingent utility remains constant, as soon $I(s_L)$ increases the expected costs increase.

With the support of lemma 1 we can translate the results of the concave utility program of the agent into the linear cost minimization program of the principal. We define $S(e,q)$ as the expected cost of the contract to the principal if he wants to implement effort level e at the noise level q:

$$S(e,q)=q_L(e)s_L+q_H(e)s_H. \tag{3.26}$$

The net profit for the principal is denoted by $\Pi(e,q)$ and given by:

$$\Pi(e,q)=p_L(e)x_L+p_H(e)x_H-S(e,q). \tag{3.27}$$

Before we come to the optimal contract choice let us first state some properties under any implementable allocation $(e,s)\in A(q)$.

Proposition 4. *The contract S(e,q) has the following cost structure*:

(*i*) $\partial S(e,q)/\partial e>0,$

(*ii*) $\partial^2 S(e,q)/\partial e^2>0,$

(*iii*) $\partial S(e,q)/\partial q<0,$

(*iv*) $\partial S^2(e,q)/\partial q^2\le 0$ *for* $p\in[0,\tilde{p}]$, *where* $\tilde{p}\in[0,\frac{1}{2})$,

(*v*) $\partial S^2(e,q)/\partial q^2>0$ *for* $p\in(\tilde{p},1]$, *where* $\tilde{p}\in[0,\frac{1}{2})$.

Proof.

(*i*) By (3.8) $s'(e)>0$ and from proposition 2 it follows that, $\partial\Delta I/\partial e>0$, which implies by lemma 1 that, $\partial S/\partial e>0$.

(*ii*) By (3.8) $s''(e)>0$ and by lemma 1 the convexity is transferred to $S(e,q)$.

(*iii*) Since $\partial\Delta I/\partial q<0$ and $\partial U(\bar{s}(e))/\partial q=0$, it follows by the concavity of $U(\cdot)$ that $\partial S(e,q)/\partial q<0$.

(*iv*) and (*v*). We write the payments s_L and s_H as a function of the inverse of the utility function $U(\cdot)$, so that

$$s_L=U^{-1}\left[\overline{U}+C(e)+\frac{q_H(e)C'(e)}{q_L'(e)}\right].$$

If $U(\cdot)$ is increasing and concave, then $U^{-1}(\cdot)$ is increasing and convex. By differentiating twice s_i with respect to q we obtain

$$\frac{\partial^2 s_L}{\partial q^2}=\left(U_L^{-1}\right)''\left[\frac{C'(e)p_L'(e)}{q_L'(e)^2}\right]^2-\left(U_L^{-1}\right)'\left[\frac{4C'(e)p_L'(e)^2}{q_L'(e)^3}\right]>0,$$

and

$$\frac{\partial^2 s_H}{\partial q^2}=\left(U_H^{-1}\right)''\left[\frac{C'(e)p_L'(e)}{q_L'(e)^2}\right]^2-\left(U_H^{-1}\right)'\left[\frac{4C'(e)p_L'(e)^2}{q_L'(e)^3}\right]>0.$$

As we know how the individual payments move we can infer how the contractual costs evolve:

$$\frac{\partial^2 s(e,q)}{\partial q^2}=\frac{\partial^2 s_L}{\partial q^2}q_L(e)+\frac{\partial^2 s_H}{\partial q^2}q_H(e)+\frac{\partial^2 s_L}{\partial q^2}(4p_L(e)-2)+\frac{\partial^2 s_H}{\partial q^2}(2-4p_L(e))$$

from which the statements follow.

We should point out that the convexity of the pure compensation $s(e)$ is only reinforced by the concavity of the utility function $U(\cdot)$ which implies that the expected cost of a contract increases as the difference in the contingent utilities is maintained at increasingly higher levels. It is worthwhile to note that $\tilde{p}=0$ is possible, which means that as the noise decreases the cost of implementing a given e decreases in a convex fashion. We now have the following proposition concerning the influence of the noise on the optimal short-term contract and the induced effort level.

Proposition 5. *As the noise increases,*

(*i*) *the optimal effort level e^* decreases and,*

(*ii*) *the incentives become stronger: $U(s_H^*)-U(s_L^*)$ increases.*

Proof.

(*i*) According to proposition 4, the optimal effort choice e^* is uniquely determined through the first order conditions

$$p_L'(e)(x_L-x_H)=\frac{\partial s(e,q)}{\partial e}.$$

Since $S(e,q)$ is strictly increasing in (1-q), e^* is strictly decreasing in (1-q).

(*ii*) It follows from (*i*) that $(e^*)'(q)>0$. If we denote by $p_L(e^*(q))=p^*(q)$ the optimal induced distribution as a function of q then we have

$$(p^*)'(q)=2q^*(q)-1+(2q-1)(p^*)'(q)(e^*)'(q). \tag{3.28}$$

For $p^*(q)\in[0,\tilde{p})$ with $\tilde{p}>\frac{1}{2}$, this implies (*i*) $(p^*)'(q)<0$ and (*ii*) $p^*(q)\in[\tilde{p},1]$, we have $(p^*)'(q)\geq 0$. Notice that it is possible that $\tilde{p}=1$, in which case the latter situation is not relevant at all. Since $U(s_H)-U(s_L)=I(s_H)-I(s_L)$, we continue our argument with I. It will be enough to prove the claim for (*i*), the argument for (*ii*) is just the reverse of the one for (*i*). Following from condition 1, we have $pI(s_L)+(1-p)\ (s_H)=0$ or

$$\frac{1-p^*(q)}{p^*(q)}=\frac{|I(s_L)|}{I(s_H)} \tag{3.29}$$

As the noise increases with (1-q), the probability ratio in (3.29) has to decrease by (3.28) which in turn means a change in the ratio of the incentive payments. The decrease can come as $|I(s_L)|$ decreases or as $|I(s_L)|$ increases, but also $I(s_H)$ increases. According to proposition 2 and 3 a noise induced decrease in e cannot be accompanied by a decrease in $|I(s_L)|$, hence both $|I(s_L)|$ and $I(s_H)$ have to increase which concludes the proof.

The optimal strategy of the principal in response to a less informative environment is then twofold. While she will increase the incentives for the agent, the cost of doing so increase too much to maintain the agent at the prior effort level and thus the principal will lower the implemented effort level. Her consequences are described in

Proposition 6. *As the noise increases, the principal-agent relationship becomes less efficient and the profit to the principal decreases.*

Proof. The first-best efficient solution is given by

$$p'(e)(x_L - x_H) = s'(e)$$

where $s(e)$ is the payment to cover reservation utility and cost of effort as defined in (3.8). Since $S(e,q) > s(e)$ for all levels of q, the second-best solution always involves lower effort levels. Furthermore, they decrease with an increase in noise. Lower effort levels decrease the gross profit and higher noise levels increase the cost of implementation, hence the net profits to the principal decrease with more noise.

We conclude this section with finding in regard to the influence of the technology of an agent on the optimal contract. Suppose we compare the optimal contract implemented under two different technologies a and b, where technology a increasingly dominates technology b, or $a \succ_c b$ as in Definition 1. The question is whether the dominant technology will allow the principal to elicit a higher effort level and benefit through higher net profits.

Proposition 7. *If* $a \succ_c b$, *then* $e_a^* > e_b^*$ *and* $\Pi_a(e_a^*, q) > \Pi_b(e_b^*, q)$.

Proof. Since the expected returns are strictly higher under a than under b, it is enough to show that for all e,

$$S_a(e,q) \le S_b(e,q),$$

where $S_j(\cdot)$ is the cost of implementing (e,q) under technology j. According to the dominance property, $p_a'(e^b) \leq p_b'(e^b)$, which implies using (3.7) that $I_a(s_H) - I_a(s_L) \leq I_b(s_H) - I_b(s_L)$. Notice that the pure compensation $s(e)$ is the same under technology a and b. We have either (*i*) $I_b(s_L) > I_a(s_L)$ or (*ii*) $I_b(s_L) \leq I_a(s_L)$. According to lemma 1 (*iii*), this implies for (*i*) directly that $S_a(e_b, q) < S_b(e_b, q)$. On the other hand, since $p_b(e) \geq p_a(e)$, we can apply lemma 1 (*ii*) in the case of (*ii*), which leads us again to the conclusion that $S_a(e_b, q) < S_b(e_b, q)$.

It is easy to verify that the statement would need some modification if the ordering of the technologies is merely stochastic dominance, i.e. technology a dominates technology b if $p_a(e) \leq p_b(e)$ under all effort levels e. The net profit would still be higher under a than under b, however the assertion that $e_a \geq e_b$ is no longer true. The reason is that the effort level is determined by the marginal incentive costs which could be higher almost everywhere for a dominant technology.

Short vs. Long Term Contract

The optimal long-term contracts in the *stationary* and *stochastic environments* are analyzed in the first subsection. The optimal choice between short-term and long-term contract in the stochastic environment is presented in the second subsection.

The Optimal Long-Term Contract

We will first describe the optimal long-term contract when effort is only chosen once and in the first period. We will establish the main result in regard to the benefits of a long-term contract due to improved incentives in proposition 8. This result is extended to the general stochastic environment when effort is chosen in the first as well as in the second period. We will restrict our formal discussion to the situation in which the agent is allowed to leave the contract after the first period. In consequence, the principal has to observe two participation constraints. First, the agent has to be willing to participate in the long-term contract over the entire life-span of the contract and hence an *intertemporal participation constraint* has to be satisfied. As the agent is allowed to leave the contract after the first period, he must be willing to continue with the principal in the second period under all contingencies induced by the signal y_i in the first period. We call the associated constraints in the principal's program the *contingent participation constraints*.

In the first period, principal and agent observe a signal y_i. The signal y_i is a random draw conditional on the realization of the outcome x_j. The signal y_i is received by the players *prior* to the outcome x_j. The principal proposes a long-term contract l to the agent by which she commits to make payments in the first and second period contingent upon y_i and y_i and x_j respectively. A contract l is specified by

$$l = \{s_L, s_H, s_{LL}, s_{LH}, s_{HL}, s_{HH}\}$$

where the first subscript refers to the realization of y_i and the second (if available) to x_j. The principal's objective function is

$$\max_{e, s_i, s_{ij}} \sum_{i=L,H} [p_i(e)x_i - q_i(e)s_i] - q \sum_{i=L,H} p_i(e)s_{ii} - (1-q) \sum_{i=L,H} p_i(e)s_{-i,i}$$

The notation $s_{-i,i}$ documents the events $\{y_L, x_H\}$ and $\{y_H, x_L\}$ when signal y_i and final outcome x_j do not correspond. The principal's optimal choice of e, s_i and s_{ij} is subject to participation and incentive constraints. The intertemporal participation constraint is,

$$\text{(IR)} \quad \sum_{i=L,H} q_i(e)U(s_i) + q \sum_{i=L,H} p_i(e)U(s_{ii}) + (1-q) \sum_{i=L,H} p_i(e)U(s_{-i,i}) - C(e) \geq 2\overline{U},$$

whereas the contingent participation constraints are:

$$\text{(IR}_L\text{)} \quad \frac{qp_L(e)}{q_L(e)} U(s_{LL}) + \frac{(1-q)p_H(e)}{q_L(e)} U(s_{LH}) \geq \overline{U},$$

and

$$\text{(IR}_H\text{)} \quad \frac{(1-q)p_L(e)}{q_H(e)} U(s_{HL}) + \frac{qp_H(e)}{q_H(e)} U(s_{HH}) \geq \overline{U}.$$

Since the payments in the second period can be contingent on the first period's signal y_i the second period's individual rationality constraints have to hold separately for the two events y_L and y_H. Finally, the effort level e is chosen by the agent in the first period so as to maximize his intertemporal utility, given the proposed contract l,

$$\text{(IC)} \quad e \in \arg\max_{e \in E} \sum_{i=L,H} q_i(e)U(s_i) + q \sum_{i=L,H} p_i(e)U(s_{ii}) + (1-q) \sum_{i=L,H} p_i(e)U(s_{-i,i}) - C(e)$$

We shall write the incentive compatibility constraint directly in its first order condition:

$$-p_L'(e)\left\{\begin{array}{c}(2q-1)[U(s_H)-U(s_L)]+q[U(s_{HH})-U(s_{LL})]\\ +(1-q)[U(s_{LH})-U(s_{HL})]\end{array}\right\}=C'(e). \qquad (4.1)$$

We observe again that, conditional upon satisfying the individual rationality constraints, all that matters in eliciting effort is the difference in utility derived from s_L and s_H (or s_{LL} and s_{LH}, or s_{HL} and s_{HH}, for that matter). The contingent difference $\Delta U(s)$ has more weight the closer the contingency itself is probabilistically related to the effort choice, which is apparent after rewriting the incentive compatibility constraint to

$$-p_L'(e)\left\{\begin{array}{c}(2q-1)[U(s_H)-U(s_L)+U(s_{HH})-U(s_{LL})]+\\ (1-q)[U(s_{HH})-U(s_{LL})+U(s_{LH})-U(s_{HL})]\end{array}\right\}=C'(e). \qquad (4.2)$$

From (4.2) one can infer that if $q=\frac{1}{2}$ and hence the signal y_i is pure noise, $U(s_L)-U(s_H)$ carries no weight, and all the incentives have to be provided through payments in the second period where the final outcome x_j contains infinitely more information. On the other hand if $q = 1$, then the signal y_i is as informative as the outcome x_j and payments based only on the signal y_i contribute as much to the incentives as payments based on the outcome x_j. It is interesting to observe that the payments s_{LH} and s_{HL} do not contribute at all to the incentives if $q = 1$. For $q < 1$, these payments work as correctives, since signal and outcome don't need to correspond. In the case of $s_{HL} < s_{LH}$, which holds for $q < 1$, it is better to receive an initial bad signal but end up with a high outcome than receive a high signal initially but ultimately realize a low outcome.

We will now develop the first-order conditions for the long-term contract. The Lagrange multiplier λ is associated with the intertemporal participation constraint, λ_L and λ_H with the contingent participation constraints respectively and μ with the incentive compatibility constraint. The first-order conditions for the optimal contractual payments in the first period are given by:

$$(s_L) \quad q_L(e)(\lambda U'(s_L)-1)+\mu q_L'(e)U'(s_L)=0, \qquad (4.3)$$

and

$$(s_H) \quad q_H(e)(\lambda U'(s_H)-1)-\mu q_L'(e)U'(s_H)=0. \qquad (4.4)$$

The first-order conditions for the optimal contractual payments in the second period are given by:

$$(s_{LL})\quad p_L(e)\left(\lambda U'(s_{LL})+\frac{\lambda_L U'(s_{LL})}{q_L(e)}-1\right)+\mu p'_L(e)U'(s_{LL})=0, \tag{4.5}$$

$$(s_{LH})\quad p_H(e)\left(\lambda U'(s_{LH})+\frac{\lambda_L U'(s_{LH})}{q_L(e)}-1\right)-\mu p'_L(e)U'(s_{LH})=0, \tag{4.6}$$

$$(s_{HL})\quad p_H(e)\left(\lambda U'(s_{HL})+\frac{\lambda_H U'(s_{HL})}{q_H(e)}-1\right)+\mu p'_L(e)U'(s_{HL})=0, \tag{4.7}$$

$$(s_{HH})\quad p_H(e)\left(\lambda U'(s_{HH})+\frac{\lambda_H U'(s_{HH})}{q_H(e)}-1\right)-\mu p'_L(e)U'(s_{HH})=0. \tag{4.8}$$

The first-order conditions for the long-term contract which only needs to satisfy the intertemporal participation constraint are obtained by setting $\lambda_L = \lambda_H = 0$. The profit function is denoted by

$$\Pi(q,\gamma) \tag{4.9}$$

which is a function of the noise q and the contractual regime, where $\gamma = s,l$. We recall that the relationship will endure for two periods and γ indicates only whether the relationship is managed through a sequence of short-term contracts with $\gamma = s$ or through a long-term contract with $\gamma = l$.

Proposition 8. $\Pi(q,l)-\Pi(q,s)>0$ *and* $e_l^*-e_s^*>0$.

Proof. It is sufficient to show that for any effort level e which is implemented by a cost minimizing short-term contract, we can find an implementation in a long-term contract so that $S(q,s,e) > S(q,l,e)$ holds. Suppose then that the short-term regime

$$s=\{\hat{s}_L,\hat{s}_H,\hat{s}\}$$

implements e cost-minimizing, where $\hat{s}$ is the non-contingent payment made to the agent in the second period. We construct a long-term contract

$$l=\{s_L,s_H,s_{LL},s_{LH},s_{HL},s_{HH}\}$$

which implements e at lower costs. Define $\hat{\Delta}\equiv U(\hat{s}_H)-U(\hat{s}_L)$ and

$$U(s)\equiv q_L(e)U(\hat{s}_L)+q_H(e)U(\hat{s}_H).$$

We set $s_L = s_H = s$, and $s_{LL} = s_{LH} = \hat{s}$ and claim that with

$$q\left(U(s_{HH})-U(\hat{s})\right)+(1-q)\left(U(\hat{s})-U(s_{HL})\right)=(2q-1)\hat{\Delta},$$

we obtain a less expensive contract, which satisfies the remaining constraints. By setting $I(s_{HH})=\left(U(s_{HH})-U(\hat{s})\right)$ and $I(s_{HL})=U(\hat{s})-U(s_H)$, the payments s_{HH} and s_{HL} have to satisfy the following two equations

$$qI(s_{HH})+(1-q)I(s_{HL})=(2q-1)\hat{\Delta},$$

which is necessary to maintain the incentive compatibility constraint (4.2) and

$$qp_H(e)I(s_{HH})+(1-q)p_L(e)I(s_{HL})=0,$$

which is the contingent participation constraint. Solving these equations we get

$$I(s_{HL})=-\frac{p_H(e)}{1-q}(2q-1)\hat{\Delta}, \text{ and } I(s_{HH})=\frac{p_L(e)}{q}(2q-1)\hat{\Delta}.$$

Since $I(\hat{s}_L)=-q_H(e)\hat{\Delta}<I(s_{HL})$ and $I(\hat{s}_H)=q_L(e)\hat{\Delta}>I(s_{HH})$, we know that the required utility difference $I(\hat{s}_H)-I(\hat{s}_L)>I(s_{HH})-I(s_{HL})$ is higher in the short-term contract. Finally since $s<\hat{s}$, we can apply lemma 1 (*i*) to conclude that $S(l,q,e)<S(s,q,e)$. It then follows that $e_l^*>e_s^*$ since the expected returns are noise independent.

The construction of an improving contract in the proof is an illustration of the relative strength of the incentives based on final outcomes. Note that we only imposed incentives on the contingent payments following the revelation of the signal y_H and hence did not even use the full strength of the long-term contract. The prevalence of the outcomes x_j over the signals y_i in the establishment of the incentives is documented by:

Proposition 9. *The optimal long-term contract without contingent participation constraints partially defers punishment and reward:*

(*i*) $s_{HH}=s_{LH}>s_H$,

(*ii*) $s_{LL}=s_{HL}<s_L$.

Proof. By (4.3) and (4.4), we get

$$U'(s_L)=\frac{p_L(e)+\frac{1-q}{2q-1}}{\lambda p_L(e)+\lambda\frac{1-q}{2q-1}+\mu p_L'(e)}, \tag{4.10}$$

and

$$U'(s_H)=\frac{p_H(e)+\frac{1-q}{2q-1}}{\lambda p_H(e)+\lambda\frac{1-q}{2q-1}-\mu p_L'(e)}. \tag{4.11}$$

Since the optimal contract when there are no contingent participation constraints amounts to setting $\lambda_L=\lambda_H=0$, by (4.5) and (4.7), we have $s_{LL}=s_{HL}$, and

$$U'(s_{LL})=\frac{p_L(e)}{\lambda p_L(e)+\mu p_L'(e)}, \tag{4.12}$$

similarly we conclude from (4.6) and (4.8) that $s_{LH}=s_{HH}$ and

$$U'(s_{HH})=\frac{p_H(e)}{\lambda p_H(e)-\mu p_L'(e)}. \tag{4.13}$$

We can then conclude, since $\lambda>0$ and $\mu>0$ following from (4.10) and (4.12), that $s_L>s_{LL}=s_{HL}$ and following from (4.11) and (4.13) that $s_H<s_{LH}=s_{HH}$.

The ranking of the payments may change when we require in addition that the contingent participation constraints should be satisfied. The latter may severely limit the possibility of punishment in the second period. As to the reward scheme after the inclusion of the contingent participation constraint, we have the following corollary from proposition 9 and the incentive compatibility (4.2):

Corollary 1. *The agent receives a strictly positive rent in state H in the second period.*

We turn our attention to the general case in which the agent undertakes effort in both periods. The agent and the principal receive an effort-related signal y_i and the principal either continues her relationship with the agent or recontracts with a new agent. As the principal considers whether or not she should employ the specific agent for a second period, y_i generates not only information about the effort of the first period, but also about the true technology under which the agent operates. The conditional posterior probabilities,

$$\begin{aligned}\alpha(y_i,e)&=\Pr(a|y_i,e) && i=L,H,\\ 1-\alpha(y_i,e)&=\beta(y_i,e)=\Pr(b|y_i,e) && i=L,H,\end{aligned} \tag{4.14}$$

of the agent's technology are in general dependent on the signal y_i and the effort level e. In consequence, the principal has to incorporate embed the new information in the remaining part of the contract. The expected technology $p_{\alpha(y_i,e)}(\cdot)$ of the agent in the second period is then a function of y_i and the first period effort level e:

$$p_{\alpha(y_i,e)}(\cdot)=\alpha(y_i,e)p_a(\cdot)+\beta(y_i,e)p_b(\cdot), \qquad i=L,H.$$

The general two period problem is then given by:

$$\begin{aligned}\max_{e,e^k,s_i,s_{ij}^k} \sum_{i=L,H} p_i(e)x_i - q_i(e)s_i - q\sum_{i=L,H} p_i(e)\left\{\sum_{j=L,H} p_j^i(e^i)(x_j - s_{ij}^i)\right\}\\ -(1-q)\sum_{i=L,H} p_i(e)\left\{\sum_{j=L,H} p_j^{-i}(e^{-i})(x_j - s_{ij}^{-i})\right\}\end{aligned} \tag{4.15}$$

Notice that we index the second period technology $p_j^i(\cdot)$ as well as the action e^i chosen under it by the random realization of y_i in the first period. The contingent effort $\{e_i\}$ and the contingent technology $\{p^i(\cdot)\}$ indicate that agent and principal will learn more about the value of their partnership when the first signal y_i is realized. The incentive compatibility of the contract insures that the agent chooses the effort level as agreed and hence principal and agent make the same inference after observing y_i. The indices of the contractual payments s_i or s_{jk}^i refer to the contemporaneous elements in the subscripts, y_i or x_1, x_2 and to past signal y_i in the superscript. The intertemporal participation constraint is given by:

$$\begin{aligned}\sum_{i=L,H} q_i(e)U(s_i) - C(e) + q\sum_{i=L,H} p_i(e)\left\{\sum_{j=L,H} p_j^i(e^i)U(s_{ij}^i) - C(e^i)\right\}\\ +(1-q)\sum_{i=L,H} p_i(e)\left\{\sum_{j=L,H} p_j^{-i}(e^{-i})U(s_{ij}^{-i}) - C(e^{-i})\right\} \geq 2\overline{U}\end{aligned} \tag{4.16}$$

The second period contingent participation constraints are given by:

$$\left\{qp_L(e)\sum_{i=L,H} p_i^L(e^L)U(s_{Li}^L) + (1-q)p_L(e)\sum_{i=L,H} p_i^L(e^L)U(s_{Hi}^L)\right\}\Big/ q_L(e) - C(e^L) \geq \overline{U} \tag{4.17}$$

for y_L, and the participation constraint contingent on y_H is given by:

$$\left\{(1-q)p_L(e)\sum_{i=L,H} p_i^H(e^H)U(s_{Hi}^H) + qp_H(e)\sum_{i=L,H} p_i^H(e^H)U(s_{Li}^H)\right\}\Big/ q_H(e) - C(e^H) \geq \overline{U}. \tag{4.18}$$

The participation constraints (4.17) and (4.18) are, as "last-period" constraints, similar to the static constraints. In particular, the first order conditions which determine the actual effort levels e^L and e^H involve the conditional probabilities induced through the first period effort e only as constants. We omit the explicit representation here. Given the contract proposal l, which involves the contingent payments:

$$l = \left\{s_L, s_H, s_{LL}^L, s_{LH}^L, s_{HL}^L, s_{HH}^L, s_{LL}^H, s_{LH}^H, s_{HL}^H, s_{HH}^H\right\}$$

and the effort choices e^L and e^H, the expected utility derived from e can be represented compactly. By setting

$$EV\left(e_j^i\right) = \sum_{i=L,H} p_i(e)\left[p_j^i\left(e^i\right)U\left(s_{ij}^i\right) - C\left(e^i\right)\right],$$

and correspondingly

$$EV\left(e_j^{-i}\right) = \sum_{i=L,H} p_i(e)\left[p_j^{-i}\left(e^{-i}\right)U\left(s_{ij}^{-i}\right) - C\left(e^{-i}\right)\right],$$

we can write the incentive compatibility constraint for e as,

$$e \in \arg\max_{e \in E} \sum_{i=L,H} q_i(e)U(s_i) - C(e) + q \sum_{i=L,H} EV\left(e_j^i\right) + (1-q) \sum_{i=L,H} EV\left(e_j^{-i}\right), \qquad (4.19)$$

It is easy to see that the first order condition for e in the general model is similar to the one introduced earlier. The first order condition implies then directly that proposition 8, which indicates the superiority of the long-run regime as opposed to the short-run regime, remains true in the general setting.

Intertemporal Effort Choice

While we recognized that different effort levels may induce different second period technologies, we did not yet investigate systematically how the possibility of improved knowledge may effect the choice between short and long-term contracts. Proposition 7 informed us that a more valuable technology in the sense of convex dominance results in an optimal short-term contract which yields a higher effort level and, consequently, higher profits. In the dynamic situation the question is how much learning is dynamically efficient.

Prior to that question we have to know how to generate the desired degree of information. The prior belief of the agent and his technology was given by $\Pr(\theta = a) = \alpha$. If the technology $p_\alpha(\cdot)$ is a probabilistic mixing of the two technologies a and b, then the posterior beliefs $\Pr(\theta | i, e)$ are affected by the signal y_i and the effort level e. Thus, for example, the principal should be more inclined to believe that the true technology is a if she observes a low signal despite a high effort level chosen by the agent. In fact, we have the following

Lemma 2. *The conditional probabilities* $\alpha(y_L, e)$ *and* $1-\alpha(y_H, e)$ *are increasing in e*:

$$\frac{\partial \Pr(a|y_L, e)}{\partial e} > 0, \quad \frac{\partial \Pr(b|y_H, e)}{\partial e} > 0. \tag{4.20}$$

Proof. Through the increasing dominance ordering, we can set $p_a(e) = p_b(e) + c(e)$, with $c(e) \geq 0, c'(e) \geq 0$ and $c''(e) > 0$. The conditional probability $\Pr(a|x_L, e)$ under prior belief α and noise q is given by

$$\alpha(y_L, e) = \Pr(a|y_L, e) = \frac{\alpha(1 + 2q[p_b(e) + c(e)] - (p_b(e) + c) - q)}{[1 + 2qp_b(e) - p_b(e) - q] + \alpha[2qc(e) - c(e)]} > \alpha, \tag{4.21}$$

which is increasing in e:

$$\frac{\partial Pr(a|y_L, e)}{\partial e} = \frac{\alpha(1-\alpha)[c'(e)(1 - p_b(e) - q + 2p_b(e)q) - p_b'(e)(2qc(e) - c(e))]}{[1 + 2qp_b(e) - p_b(e) - q] + \alpha[2qc(e) - c(e)]^2} > 0.$$

To show the monotonicity of the conditional probability $\beta(y_H, e)$ it will prove convenient to show that the ratio of the conditional probabilities is increasing in e,

$$r(y_H, e) = \frac{\beta(y_H, e)}{\alpha(y_H, e)} = \frac{(1-\alpha)(p_b(e) + q - 2qp_b(e))}{\alpha(p_b(e) + c(e) + q - 2q(p_b(e) + c(e)))}$$

which is sufficient for the claim. We have

$$\frac{\partial r(y_H, e)}{\partial e} = \frac{(1-\alpha)(2q-1)[[c'(e)(p_b(e) + q - 2p_b(e)q) + p_b'(e)(2qc(e) - c(e))]]}{\alpha(p_b(e) + c(e) + q - 2q(p_b(e) + c(e)))^2}. \tag{4.22}$$

The numerator is positive at $e = \underline{e}$, since $c(\underline{e}) \geq 0$ and we remain with $c'(e)[p_b(e) + q - 2p_b(e)q] > 0$. It is then enough to show that the numerator is increasing in e. But when we omit the constant coefficients, we obtain after differentiating once more

$$c''(e)(p_b(e) + q - 2p_b(e)q) + p_b''(2qc(e) - c(e)) > 0,$$

and hence the ratio $r(y_H, e)$ is increasing in e by (4.22).

The posterior beliefs about the agent as given according to (4.20) indicate that any signal y_i tends to reinforce the beliefs of the principal in the sense that a low signal leaves the principal with $\alpha(y_L, e) > \alpha$, whereas a high signal changes her belief to $\beta(y_H,e)>\beta$. While the probability that the agent provides a low technology conditional on y_L increases with e, a principal who signed a short-term contract is almost entirely interested in the conditional belief following the signal y_H.

Proposition 10. *The principal retains the agent after observing y_H and dismisses the agent after observing y_L in the short-term regime.*

Proof. According to lemma 2, we have $\alpha(y_L,e)>\alpha$ for all e, which implies that for the technologies associated with $\alpha(y_L,e)$ and α, respectively, $\alpha \succ_c \alpha(y_L,e)$. According to proposition 7, this implies that the principal can obtain higher net profits with α then with $\alpha(y_L,e)$ for all e. Since she is not forced to continue to employ the agent of the last period in the short-term regime, her optimal action is to recruit a new agent with technology α and fire the old one with technology $\alpha(y_L,e)$. If the positive signal y_H is received, then $\alpha(y_H,e)>\alpha$ for all e and the conclusion follows again from proposition 7.

The principal will fire the agent upon observation of y_L independent of the effort level e, since she can recruit a new agent with technology $p_\alpha(\cdot)$ which dominates the technology of the previous agent following (4.20). While she retains the agent after having observed y_H at any effort level, the higher the posterior beliefs $\beta(y_H,e)$ are, the higher are her expected *gross* profits. An increase in the posterior belief that the agent operates with an efficient technology has two implications for the optimal contract and the elicited effort level. The more obvious effect is the increase in expected *gross* profits due to a superior technology. But, equally important, an increase in the posterior beliefs $\beta(y_H,e)$ assures the agent that the matching between his abilities and the required task is successful. In consequence, the incentive scheme can be more aggressive, since he responds more favorably to success contingent payments. Hence the conclusion in proposition 7 that an increase in $\beta(y_H,e)$ is associated with higher effort levels and higher *net* profits.

The optimal dynamic learning policy of the principal is, however, sensitive to the contractual regime. From an informational point of view, the principal's interest in the short-term regime focuses exclusively on maximizing the posterior belief $\beta(y_H,e)$. Since the principal will not continue the relationship with the current agent after observing y_L, her policy is to create optimal condition for recontracting after the occurrence of y_H. Lemma 2 then indicates that the optimal policy will require more incentives in the first period than we could expect in a static situation. In contrast to the short-term regime, the long-term regime has to take into account the occurrence of y_L. Since maximizing $\beta(y_H, e)$ requires an increase in e, the principal is more hesitant to follow this policy since it simultaneously decreases $\beta(y_L, e)$ following lemma 2. The dilemma for the principal in the long-term regime is the polarization in beliefs. While the realization of y_H induces more incentives and more effort in the future, since the motivation of the agent becomes easier, the realization of y_L decrease the expected value of any future effort, and impedes the principal in motivating the agent by contingent payments.

Before we state the central result we need to establish some preliminaries. We call any implementable triple (e, e^L, e^H) an *intertemporal effort profile* or *plan*. The profit associated with the plan (e, e^L, e^H) and implemented under a cost minimizing incentive scheme is denoted by

$$\Pi(e, e^L, e^H | q, \gamma), \quad \text{for } \gamma = s, l.$$

The periodic profit associated with the plan (e, e^L, e^H) and likewise implemented under a cost minimizing incentive scheme is denoted by:

$$\Pi(e, e^L, e^H | q, \gamma_t), \quad \text{for } \gamma = s, l, \ t = 1,2.$$

The intertemporal profit is given as the sum of first and second period profits,

$$\Pi(e, e^L, e^H | q, \gamma) = \Pi(e, e^L, e^H | q, \gamma_1) + \Pi(e, e^L, e^H | q, \gamma_2), \quad \text{for } \gamma = s, l.$$

which is partially separable,

$$\Pi(e, e^L, e^H | q, \gamma) = \Pi(e | q, \gamma_1) + \Pi(e, e^L, e^H | q, \gamma_2), \quad \text{for } \gamma = s, l.$$

The expected profit in the second period is given by:

$$\Pi(e, e^L, e^H | q, \gamma_2) = q_L(e)\Pi(e, e^L | q, \gamma_1) + q_H(e)\Pi(e, e^H | q, \gamma_2), \quad \text{for } \gamma = s, l.$$

We denote the net profit of the principal as a function of the noise q, the contractual regime γ, and the first period effort level e as:

$$\Pi(q, \gamma, e) = \max_{e^L, e^H} \Pi(e^L, e^H | q, \gamma, e), \quad \text{for } \gamma = s, l.$$

The following lemma provides the starting point for our next result.

Lemma 3. *The second period profits* $\Pi(q, \gamma_2, e)$ *conditional on any effort level* e *have the property*:

$$\Pi(q,s_2,e) > \Pi(q,l_2,e).$$

Proof. We note first that the long-term contract carries over some of the incentives for the first-period effort in the second period, so that the payments in the second period are contingent on the realization of the outcome x_i:

$$s_{iL}^L \neq s_{jL}^L, s_{iH}^L \neq s_{jH}^L, s_{iL}^H \neq s_{jL}^H, s_{iH}^H \neq s_{jH}^H, \quad \textit{for } i,j = L,H, \textit{ and } i \neq j.$$

Suppose for the moment that the principal always rehires the agent in the short-term regime, so that the employed technology is the same under the short-term regime as under the long-term regime. Since x_1 and x_2 are independent random variables, it then follows by the concavity of the utility function that state by state, e^L and e^H are implemented at less cost under the short-term contract s_2 than under the long-term contract l_2. By the concavity of the principal's maximization program it then follows that $e^L(s_2) > e^L(l_2)$ and $e^H(s_2) > e^H(l_2)$ and that $\Pi(e^L|q,s_2,e) > \Pi(e^L|q,l_2,e)$ as well as $\Pi(e^H|q,s_2,e) > \Pi(e^H|q,l_2,e)$. It remains to add that following proposition 10 the principal employs in the short-term regime a convex dominant technology α in the state following y_L as compared to the long-term regime $\alpha(y_L,e)$, which only reinforces the implications above.

The next proposition compares the optimal effort levels of the short-term and long-term contracts in the stationary environment and the learning environment. If the environment is stationary, then the principal's belief about the agent does not change from period to period and is given by $\hat{\alpha}$. The optimal first period effort level e under the short-term and long-term contracts are given by $\hat{e}_{s_1} < \hat{e}_{l_1}$, and the intertemporal profits are $\hat{\Pi}(q,l) > \hat{\Pi}(q,s)$ following proposition 8. For the purpose of this proposition all variables and values referring to the stationary environment are specified by "^".

Proposition 11. *The optimal intertemporal learning policy* $(e_{s_1}^*, e_{l_1}^*)$ *reduces the underinvestment in the short-term contract:*

$$e_{l_1}^* - e_{s_1}^* < \hat{e}_{l_1} - \hat{e}_{s_1},$$

and

$$\hat{e}_{s_1} < e_{s_1}^*, e_{l_1}^* < \hat{e}_{l_1}.$$

Proof. Following stationarity $\hat{\Pi}(q,s_2,e) = \hat{\Pi}(q,s_2)$ for all e and optimality:

$$\frac{\partial\hat{\Pi}(q,s,e)}{\partial e}=\frac{\partial\hat{\Pi}(q,s_1,e)}{\partial e}+\frac{\partial\hat{\Pi}(q,s_2,e)}{\partial e}=\frac{\partial\hat{\Pi}(q,s_1,e)}{\partial e}=0.$$

In the learning environment $\hat{\Pi}(q,s_2,e)$ changes in e, since different values of e imply different technologies in the second period because of different posteriors. Since the principal is insured in the short-term regime against $\alpha(y_L,e)$, higher first period effort levels increase her second period profit as long as they increase $1-\alpha(y_H,e)$, which is the case by convex dominance,

$$\frac{\partial\Pi(q,s_2,e)}{\partial e}>0.$$

Since

$$\frac{\partial\hat{\Pi}(q,s_1,e)}{\partial e}=\frac{\partial\Pi(q,s_1,e)}{\partial e},\qquad \forall e,$$

and

$$\frac{\partial\hat{\Pi}(q,s_2,e)}{\partial e}<\frac{\partial\Pi(q,s_2,e)}{\partial e},\qquad \forall e,$$

optimality in the stochastic environment requires $e^*_{s_1}>\hat{e}_{l_1}$. For the long-term regime the optimality condition in the stationary environment is

$$\frac{\partial\hat{\Pi}(q,l,e)}{\partial e}=\frac{\partial\hat{\Pi}(q,l_1,e)}{\partial e}+\frac{\partial\hat{\Pi}(q,l_2,e)}{\partial e}=0,$$

where we showed in proposition 8 that

$$\frac{\partial\hat{\Pi}(q,l_2,e)}{\partial e}<0.$$

In the stochastic environment the situation deteriorates, since the posterior beliefs, $\alpha(y_L,e)$ and $\alpha(y_H,e)$, increase and decrease in e respectively. But since $p_{\alpha(y_i,e)}(\cdot)$ is a martingale and the cost of the contract are convex in e we have

$$0>\frac{\partial\hat{\Pi}(q,l_2,e)}{\partial e}>\frac{\partial\Pi(q,l_2,e)}{\partial e}.$$

According to optimality the learning environment then requires $e^*_{l_1}<\hat{e}_{l_1}$, which proves the claim.

The difference in the optimal effort levels of the short-term and the long-term regimes decreases as a result of the intertemporal learning policy. Under the short-term regime the principal can appropriate the gains from learning while she insulates herself from the potential losses by firing the agent. The first period choice of effort in the long-term regime is distorted since any increase in e, which would be beneficial if y_H is received is thwarted by the simultaneous and detrimental increase in $\alpha(y_L, e)$ which we documented in lemma 2.

As the environment becomes noisier the long-term regime becomes more attractive for two reasons. The signal y_i becomes less informative and thus the short-term incentives very inefficient. Simultaneously and again for the non-informativeness of y_i the posteriors $\alpha(y_L, e)$ and $\beta(y_H, e)$ become very flat and hence the value of learning decreases. The next result is then the generalization of proposition 8 and we omit the proof.

Proposition 12. *The optimal choice between short-term and long-term contracts is monotone in q and there exists $\bar{q} \in \left(\frac{1}{2}, 1\right]$ such that:*

(i) $\Pi(q,l) > \Pi(q,s)$ *for* $q \in \left[\frac{1}{2}, \bar{q}\right)$,

(ii) $\Pi(q,s) > \Pi(q,l)$ *for* $q \in (\bar{q}, 1]$.

Proposition 12 allows the long-term contract to be optimal over the entire range of q. If the benefits from learning are very limited, as it is the case when $p_a(\cdot)$ and $p_b(\cdot)$ are very similar, then the benefits of diversifying the incentives over many periods may be dominant throughout.

Renegotiation. The analysis assumed that the principal can commit to a contract that will not be renegotiated. Suppose now that we would allow the contract to be renegotiated after the first period effort has been chosen and before the realization of the final outcome.[6] The parties could then renegotiate after the effort choice e and the signal reception y_i. At that point there is no longer an incentive-based reason to let the final payment be contingent on x_l, since efficient risk sharing prescribes that the agent bears no risk at all. In consequence, the optimal long-term contract will not be implemented as initially planned. The reason is easy to

6 Cf. Fudenberg and Tirole (1990), Ma (1990, 1991), and Matthews (1991). Fudenberg, Holmström, and Milgrom (1990) show that the possibility of renegotiation has no consequences when principal and agent know each other's preferences over contracts at every potential recontracting date. The condition is not satsfied here where the agent's action has long-term consequences and influences the signal today *and* the final outcome tomorrow.

see. The agent would foresee complete insurance and would have no incentive to provide a high effort level. The renegotiation constraint then implies in general that the parameter space for which the long-term contract is optimal is reduced, since the renegotiation constraint decreases implicitly the value of commitment.

Conclusion

The aim of this paper was to analyze the optimal choice of dynamic contracts in a noisy environment in which principal and agent learn over time how valuable their partnership is.

The advantages of a long-term contract were most pronounced in a very noisy environment in which early signals did not contain too much relevant information about the final outcomes. Agent and principal are at the outset uncertain about the quality of their match and learn more about it as they receive signals. Positive signals confirmed the value of their partnership, whereas low signals led both agent and principal to suspect that the agent is not suited for the task. If the principal committed herself earlier to a long-term contract, then bad news is unfortunate for her in a double sense. She will not only expect lower returns on any effort provided by the agent, but she will also have to expend more to align the agent's interest with her own.

The optimal contract balanced the trade-off between the flexibility of a short-term contract in hiring decisions with the efficient incentive system due to commitment in the long-term contract.

The noise of the environment influenced the optimal contract choice through two channels. At a high noise level the contemporaneous signal gave only limited information about the agent's action and was hence only of limited use for offering efficient incentives to the agent. Simultaneously, the noise constrained the opportunity of the principal to infer from the signal the true technology of the agent. The value of learning, and hence the flexibility offered by the short-term contract, decreases then complementary as the noise increases. While the paper treats the noise of the environment as exogenous, an interesting variation of the model would result in transforming the level of noise into a choice variable of the agent.

The results obtained here were derived with just two occasions for moral hazard. The present model can be regarded as the final stage of a finite multi-period model and exactly the same conclusions are reached by working backwards sequentially to earlier periods.

The principal-agent model is a small model of optimal organization. The question of how organizations are designed optimally in a noisy environment so as to solve their tasks efficiently and still respond to arising contingencies in a flexible manner is, while beyond the scope of this paper, an interesting field of future research.

Appendix

The Appendix provides the proof of lemma 1, which translates the comparative static results of the utility space into the space of transfer payments.

Proof of lemma 1.
(*i*) According to condition 1 we have

$$\frac{U(s_H)-U(s)}{U(s)-U(s_L)}=\frac{U(s'_H)-U(s')}{U(s')-U(s'_L)}=\frac{p}{1-p}, \tag{5.1}$$

then following the concavity of the utility function we have $s_H - s > s'_H - s'$ as well as $s - s_L > s' - s'_L$. We also have

$$\frac{U(s'_L)-U(s')}{s'_L-s'}-\frac{U(s'_H)-U(s')}{s'_H-s'}>\frac{U(s_L)-U(s)}{s_L-s}-\frac{U(s_H)-U(s)}{s_H-s},$$

from which we can infer

$$\frac{s'_L-s'}{U(s'_L)-U(s')}-\frac{s'_H-s'}{U(s'_H)-U(s')}>\frac{s_L-s}{U(s_L)-U(s)}-\frac{s_H-s}{U(s_H)-U(s)}.$$

Multiplying by $U(s'_H)-U(s')$ and using (5.1), we get

$$(s'_L-s')\frac{p}{1-p}-(s'_H-s')<(s_L-s)\frac{p}{1-p}-(s_H-s),$$

from which the result follows directly.

(*ii*) It is enough to show that the claim holds in the two following cases: (*a*) $I(s_H)=I(s'_H)$ and $I(s_L)>I(s'_L)$ and (*b*) $I(s_H)<I(s'_H)$ and $I(s_L)=I(s'_H)$. We start with (*a*). According to condition 1 we have

$$\frac{U(s_H)-U(s)}{U(s)-U(s'_L)}=\frac{p'}{1-p'}<\frac{p}{1-p}=\frac{U(s_H)-U(s)}{U(s)-U(s_L)}, \tag{5.2}$$

with $p'<p$. Because of the identity of s_H and s'_H and (5.2) we can derive

$$\frac{U(s_H)-U(s'_L)}{U(s_H)-U(s_L)}=\frac{p}{p'}. \tag{5.3}$$

We notice that the claim to be proven can be reduced to

$$\frac{s_H - s_L'}{s_H - s_L} = \frac{p}{p'}, \tag{5.4}$$

Due to the concavity of the utility function we have

$$\frac{U(s_H) - U(s_L')}{s_H - s_L'} > \frac{U(s_H) - U(s_L)}{s_H - s_L},$$

which following (5.3) directly implies (5.4). Condition (*b*) can be stated as

$$\frac{U(s_H') - U(s)}{U(s) - U(s_L)} = \frac{p'}{1-p'} > \frac{p}{1-p} = \frac{U(s_H) - U(s)}{U(s) - U(s_L)},$$

with $p' > p$ and can be reduced similary to our procedure under (*a*) to

$$\frac{U(s_H') - U(s_L)}{U(s_H) - U(s_L)} = \frac{1-p}{1-p'}, \tag{5.5}$$

and the claim can be restated as

$$\frac{s_H' - s_L}{s_H - s_L} = \frac{1-p}{1-p'},$$

which follows again directly from the concavity of the utility function and (5.5).

(*iii*) It is sufficient to prove the claim for $I(s_H') - I(s_L') = I(s_H) - I(s_L)$. Condition (*iii*) implies that

$$\frac{U(s_H') - U(s)}{U(s) - U(s_L')} = \frac{p'}{1-p'} > \frac{p}{1-p} = \frac{U(s_H) - U(s)}{U(s) - U(s_L)}, \tag{5.6}$$

with $p' > p$, where (5.6) can be reduced to

$$\frac{U(s_H') - U(s_H)}{U(s_H') - U(s_L')} = \frac{U(s_H') - U(s_H)}{U(s_H) - U(s_L)} = p' - p.$$

The concavity of the utility function then implies that

$$\frac{U(s_H') - U(s_H)}{U(s_H') - U(s_L')} < \frac{s_H' - s_H}{s_H' - s_L'} < \frac{s_H' - s_H}{s_H - s_L}. \tag{5.7}$$

We can restate (3.25) as

$$p'(s_H' - s_L') - p(s_H - s_L) < s_H' - s_H.$$

which follows directly from (5.7).

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