Information in Mechanism Design

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Mechanism Design

- Economic agents have private information that is relevant for a social allocation problem.
- Information revelation is voluntary (incentive compatibility).
- A mechanism selects an outcome based on the information reported.

Two key assumptions of the standard model:

- Information is exogenously given at the outset.
- Only payoff relevant information is considered.

In this presentation, we relax these assumptions:

- We allow the agents to acquire additional payoff relevant information.
- We consider the impact of higher order beliefs on optimal mechanisms.

Standard Model

- Agent i has private information about his payoff type $\theta_i \in \Theta_i$.
- Common Prior $p : \Theta \to [0, 1]$
- A social outcome $x \in X$ must be chosen.
- Agent i's preference given by Bernoulli utility function

$$u_i : \Theta \times X \to \mathbb{R}.$$

- Direct mechanism is given by social choice function g

$$g : \Theta \to X$$

Information Acquisition

- Agents acquire costly information:

$$\theta_i \sim F^{\alpha_i}(\theta_i).$$

- Informativeness represented by α_i, cost of α_i is $c_i(\alpha_i)$.

- Individual maximization problem

$$\max_{\alpha_i} \int_{\Theta_i} \int_{\Theta_{-i}} U_i\left(g\left(\theta_i, \theta_{-i}\right), \left(\theta_i, \theta_{-i}\right)\right) dF^{\alpha_i}\left(\theta_i\right) dF^{\alpha_{-i}}\left(\theta_{-i}\right) - c(\alpha_i)$$

Interaction between Mechanism and Information

- Observe that optimal α_i depends on g:
  - Direct effect via allocation given available information
  - Indirect effect through the induced distribution of types $F^{\alpha(g)}(\theta)$.

- Incentive compatibility gives incentives for agents to reveal their information.

- In our view, in many allocation problems, it is important to consider the effects of the mechanism on the realized distribution of types as well.
  - What kinds of mechanisms perform well when both of these effects are accounted for?
  - What form should the optimal trade-off between these two effects take?

Efficient Allocation of a Single Object

- Two bidders
- Interprete θ_i as posterior probability that object is valuable for bidder i.
- Information acquisition: mean preserving spread of θ_i.
- Quasilinear utility:
$$u_i(\theta, y) = q_i v_i(\theta) - y_i,$$
q_i : probability of receiving the object, y_i : monetary transfer
- interdependent values:
$$v_i(\theta) = \theta_i + \gamma\theta_j \qquad \text{(private value: } \gamma = 0\text{)}$$

Private Values

- Efficiency:

$$x_i = 1 \implies \theta_i \geq \theta_j \text{ for all } j \text{ and } \theta_i \geq 0.$$

- Second price auction gives efficient allocation in dominant strategies:

$$U_i(\theta_i) = \mathbb{E}_{\theta_j} \max\{\theta_i - \theta_j, 0\}.$$

- Social surplus:

$$\mathbb{E}_\theta \max\{\theta_i, \theta_j\} = \mathbb{E}_\theta \max\{\theta_i - \theta_j, 0\} + \theta_j$$

  - choice by agent i leads to surplus maximizing choice of α_i.
  - general property of *Vickrey-Clarke-Groves* with private values (Rogerson (1992))

Insert the pictures for the private values case here.

Interdependent Values

- Interdependent values:

$$v_i(\theta) = \theta_i + \gamma\theta_j$$

- Efficiency still requires that

$$x_i = 1 \implies \theta_i \geq \theta_j \text{ for all } j \text{ and } \theta_i \geq 0.$$

- $x_i(\theta_i)$ is increasing in θ_i only if $\gamma \leq 1$: single crossing condition

- ex post (rather than dominant) incentive compatible transfer rule

$$q_i(\theta) = \begin{cases} (1+\gamma)\,\theta_j, & \text{if} \quad \theta_i > \theta_j \\ 0, & \text{if} \quad \text{otherwise} \end{cases}$$

Incentives to Acquire Information

- net utility of agent i:

$$U_i\left(\theta_i\right) = \mathbb{E}_{\theta_j} \max\{\theta_i - \theta_j, 0\},$$

- yet social surplus is given by:

$$\mathbb{E}_{\theta} \max\{\theta_i + \gamma\theta_j, \theta_j + \gamma\theta_i\} = \mathbb{E}_{\theta} \max\{\theta_i - \theta_j, \gamma\left(\theta_i - \theta_j\right)\} + \left(1+\gamma\right)\theta_j$$

- social surplus and private surplus of i (as a function of θ_i) have different slopes if $\theta_i < \theta_j$.

Insert the pictures for interdependent values case here.

- Bergemann and Välimäki (2002) relate the incentives for individual over- and underacquisition of information to supermodualrity of individual payoff functions.

THEOREM Every efficient mechanism induces insufficient (excessive) incentives for information acquisition by agent i if $\Sigma_{j \neq i} u_j (x, \theta)$ is supermodular (submodular) in (x, θ_i) .

- Caveat: Does not necessarily translate to equilibria results.

- Bergemann, Shi and Välimäki (2005) show that the same characterization is valid for the equilibria in a class of auction models.

In particular, there is excessive aquisition if $\gamma > 0$ and insufficient acquisition relative to social efficiency if $\gamma < 0$.

Information Acquisition in Other Contexts

- Principal Agent Models
  - Information gathering prior to signing: Cremer & Khalil (1992).
  - Information acquisition prior to contract offer: Cremer, Khalil, Rochet (1998a, 1998b).
- Committee Decisions
  - Optimal committee size: Persico (2004).
  - Optimal information collection: Gershkov & Szentes (2005).
- Auctions
  - Revenue comparisons in auctions: Persico (2001),
  - Dynamic auction formats: Compte & Jehiel (2004).

Information, Large Types and Robustness

- how does information acquisition, and more generally the possibility of rich private information, influence the design and performance of mechanisms?

- how does rich private information affect standard results in mechanism design?

- how can we obtain robust mechanisms in the presence of large type spaces?

First Price Auction

- two bidders: $i = 1, 2$
- private values, discrete valuations:

$$\theta_i \in \{1/3, \quad 2/3, \quad 3/3\}$$

- discrete bids:

$$b_i \in \{1/6, \quad 2/6, \quad 3/6, \quad 4/6, \quad 5/6, \quad 6/6\}$$

- θ_i, θ_j distributed identically, independently and uniformly:

$p(\theta_i, \theta_j)$	t_j^1	t_j^2	t_j^3	
t_i^1	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	θ_i^1
t_i^2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	θ_i^2
t_i^3	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	θ_i^3
	θ_j^1	θ_j^2	θ_j^3	

- types t_i are payoff types (i.e. identified by θ_i)

- conditional posterior distribution given type:

	θ_j^1	θ_j^2	θ_j^3
$p(\theta_j \mid \theta_i)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Private Information

- via private information (or information acquisition):
  - every i with largest θ_i obtains additional information: t_i', t_i''
  - all others receive no additional information

- posterior belief incorporates additional information:

	θ_j^1	θ_j^2	θ_j^3	$\Pr(t_i \mid \theta_i = 1)$	
$p(\theta_j \mid t_i')$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	"weak" competitor
$p(\theta_j \mid t_i'')$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{1}{3}$	"strong" competitor

Private Information and Large Type Space

- information structure can be represented by a new, larger type space

- i's *type* is $t_i \in T_i$, t_i includes description of
  - *payoff type* (about own payoff function) $\widehat{\theta}_i : T_i \to \Theta_i$
  - *belief type* (about other players types) $\widehat{\pi}_i : T_i \to \Delta(T_{-i})$
  - *type space* is a collection: $\mathcal{T} = \left\{T_i, \widehat{\theta}_i, \widehat{\pi}_i\right\}_{i=1}^{I}$

- importantly:
  - given payoff type may be associated with many belief types
  - given belief type may be associated with many payoff types

Private Information and Large Type Space

- information structure can be represented by a new type space:

$p(\cdot)$	t_j^1	t_j^2	t_j^3	t_j^4	
t_i^1	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	0	θ_i^1
t_i^2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{2}{27}$	$\frac{1}{27}$	θ_i^2
t_i^3	$\frac{1}{9}$	$\frac{2}{27}$	$\frac{1}{27}$	0	θ_i^3
t_i^4	0	$\frac{1}{27}$	0	$\frac{2}{27}$	θ_i^3
	θ_j^1	θ_j^2	θ_j^3	θ_j^3	

- here:
  - given payoff type is associated with many belief types
  - bidding strategy will be contingent on beliefs about weak or strong competitor

Equilibrium Strategies

- in the payoff type space:

$$b_i^*(\theta_i) = \frac{1}{2}\theta_i$$

- in the large type space:

$$b_i^{**}(t_i) = \frac{1}{2}\theta_i, \text{ for } t_i^1, t_i^2$$

but

$$b_i^{**}\left(t_i^3\right) = \frac{2}{6} \neq \frac{3}{6} = b_i^*\left(t_i^4\right).$$

Equilibrium Implications

- lower revenues with more strategic uncertainty

- failure of efficient allocation:

$$b_i\left(t_i^2\right) = b_i\left(t_i^3\right),$$

but

$$\theta_i\left(t_i^2\right) < \theta_i\left(t_i^3\right).$$

Implications of Large Type Space: Revenue Equivalence

- Fang and Morris (2005) reconsider revenue equivalence in private value model:
  - private information of bidder is her valuation,
    but also her posterior belief about other bidders payoff types
  - with additional private information,
    the type of every agent is a multidimensional object
  - revenue equivalence (and revenue ranking)
    between first and second price auction fails
- Che and Kim (2004), Andreoni, Che and Kim (2005) consider subsets of informed bidders

Implications of Large Type Space: Surplus Extraction

- Neeman (2004) reconsiders full surplus extraction results à la Cremer and McLean
  - belief type does not determine payoff type
  - extraction via lotteries fails and agents received positive informational rents
- Neeman and Heifetz (2005) show that generically surplus extraction fails

Robustness

- discussion of robustness is a pervasive theme in mechanism design
  - "non-parametric" (Hurwicz (1972))
  - "do not rely on agents common knowledge", Wilson doctrine (Wilson (1985, 1987))
  - "detail free" (Maskin (1992), Maskin and Dasgupta (2000))
- if optimal solution is too complicated or sensitive, perhaps the original modelling itself was flawed
- does improved modelling of planner's problem endogenously create "robust" features previously assumed

Robust Mechanism Design

- Bergemann and Morris (2004) introduce rich type spaces into mechanism design
  - fix payoff types and social choice function
- for fixed environment there exist many type spaces, where a type specifies:
  - the payoff type and the belief type
- the larger the type space, the harder it is to implement social choice:
  - more “robust” the resulting mechanism will be
  - smallest type space: “payoff type space” standard construction
  - largest type space: “universal type space”

- are ex post incentive constraints necessary for Bayesian implementation on all type spaces?

- yes in "separable" environments and in particular for social choice function

- A mechanism $\mathcal{M}$ and a type space $\mathcal{T}$ is an incomplete information game

- A pure strategy $s_i : T_i \to M_i$

THEOREM. If the environment is separable, there exists a mechanism $\mathcal{M}$ such that for every type space $\mathcal{T}$, there is an equilibrium s of $(\mathcal{M}, \mathcal{T})$ consistent with scc F (i.e., $g(s(t)) \in F\left(\widehat{\theta}(t)\right)$ for all t) if and only if there exists $f \in F$ that is ex post incentive compatible.

Implementation

- Robust Mechanism Design:
  Does there exist $\mathcal{M}$ such that for every $\mathcal{T}$, there is an equilibrium s of $(\mathcal{M}, \mathcal{T})$ consistent with f: $g(s(t)) = f\left(\widehat{\theta}(t)\right), \forall t$

- Robust Implementation:
  Does there exist $\mathcal{M}$ such that for every $\mathcal{T}$, there exists an equilibrium of $(\mathcal{M}, \mathcal{T})$ and every equilibrium is consistent with f.

- multiple equilibria through collusion, shill bidding relevant for practical mechanism design (Ausubel and Milgrom (2004), Yokoo et al. (2004))

- yet gap between pure and practical implementation especially stark
  - complicated augmented mechanism, integer games

Robust Implementation

- Bergemann and Morris (2005a, 2005b) identify conditions for robust implementation in an environment with interdependent preferences

1. Robust implementation is equivalent to implementation in iterated deletion of strictly dominated strategies

2. Robust implementation is possible if and only if not too much interdependence of preferences (see also Chung and Ely (2002))

3. Robust implementation is possible if and only if it is possible in the direct mechanism

Local Robustness

- Bergemann and Schlag (2005)
  - standard optimal monopoly pricing: $q, \quad p(q)$
  - buyer valuation $\theta q,$ seller $cq, \quad q \in [0,1]$,

- model distribution given by F_0 , true distribution given by F:

$$\|F - F_0\| \leq \varepsilon$$

- seller is minimizing regret

- optimal pricing involves a menu in constract to single price without robustness concerns

Mechanism Design without Common Prior

- without any prior expected utility cannot be the criterion anymore
- one possible notion is notion of competitiveness, competitive ration

$$c = \frac{\text{realized revenue under incompelete information}}{\text{feasible revenue with complete information}}$$

- Goldberg, Hartline and Wright (2001) consider sealed bid auction with unlimited supply
- Segal (2003) adds unknown but identically and independently distributed valuations
- Neeman (2003) determines competitive ratio of second price auction

Conclusion

- sequential, dynamic design to improve informational and allocative efficiency
- second best design in the presence of information acquisition
- mechanism design in the presence of synchronization, coordination issues
- robust revenue maximization design