Information Acquisition in Interdependent Value Auctions

joint with Xianwen Shi and Juuso Välimäki

24th June 2008

Introduction

- role of private information in mechanism design
- agents have private information that is relevant for (efficient) allocation
- designer defines mechanism to elicit private information
- information revelation is voluntary (incentive compatibility)

Information Acquisition

- key assumption in mechanism design literature:
  - private information is exogenously given
- our paper allows information to be privately acquired:
  - social value of information
  - equilibrium value of information
- examples: oil tracts & license auctions
  - private information acquired through costly investment
  - interdependent values

Ex-ante and Ex-post Efficiency

- each agent privately decides to acquire information:
  - ex-ante
  - covertly
- information structure is endogenous
  - ex-post mechanism affects incentives to acquire information ex-ante
  - spectrum licenses: lottery vs. auction
- is it possible to design mechanisms that perform well:
  - ex-ante
  - ex-post

Current Paper

- information acquisition in ex-post efficient mechanisms
- generalized Vickrey-Clarke-Groves (VCG) mechanism
- whether and how equilibrium information acquisition differs from the social optimum
- how the difference depends on:
  - the strength of the interdependence
  - the number of informed bidders

Related Literature I

- private values setting
- Stegeman (1996) considers second price auctions
- Bergemann and Välimäki (2002) consider general allocation problems
- each agent receives in equilibrium his marginal contribution
- each agent has correct incentives to acquire information

Related Literature II

- information aggregation and costly information acquisition
  - Milgrom (1981): Vickrey auction
  - Jackson (2003): informational efficiency is not robust to cost of information
- interdependent values setting:
  - Maskin (1992) considers second price auction
  - Bergemann and Välimäki (2002) consider general allocation problems
  - given decisions of other agents (locally), individual incentives are socially excessive (insufficient) if valuations are positively (negatively) dependent

Main Results

- provide a comparison of equilibrium level and social optimal level of information
  - information decisions are strategic substitutes
  - positive dependence: equilibrium information is socially excessive
- difference between socially optimal and equilibrium level decreases if
  - more agents acquire information
  - level of positive dependence decreases

Model

- auction setting with interdependent values
- single object and I bidders
- value to bidder i is linear in bidders' signals $\{\theta_i\}_{i=1}^{I}$:

$$u_i(\theta_i, \theta_{-i}) = \theta_i + \alpha \sum_{j \neq i} \theta_j,$$

  where $0 \leq \alpha \leq 1$ measures interdependence
- quasilinear utility:

$$u_i(\theta) - t_i,$$

  where t_i is monetary transfer

Information

- θ_i's are i.i.d. from a common prior F with support $[\underline{\theta}, \overline{\theta}]$ and

$$\mu = \mathbb{E}[\theta_i]$$

- private information θ_i unknown ex ante
- binary information decision:
  - if bidder i acquires information, i privately observes θ_i
  - otherwise, i's information is given by prior F
- information cost $c > 0$

Allocation

- two-stage game:
  - information acquisition stage
  - bidding stage
- direct revelation mechanism $\{q_i, t_i\}_{i=1}^{I}$
- generalized Vickrey-Clarke-Groves mechanism:

$$y_i = \max_{j \neq i} \{\theta_j\}$$

then the allocation rule is

$$q_i(\theta_i, \theta_{-i}) = \begin{cases} 1 & \textit{if} \quad \theta_i > y_i \\ 0 & \textit{if} \quad \theta_i < y_i \end{cases},$$

and the payment rule

$$t_i(\theta_i, \theta_{-i}) = \begin{cases} u_i(y_i, \theta_{-i}) & \textit{if} \quad \theta_i > y_i \\ 0 & \textit{if} \quad \theta_i < y_i \end{cases}.$$

Two Bidder Example

- two bidders: *i* and *j*

$$u_i\left(\theta_i, \theta_j\right) = \theta_i + \alpha\theta_j$$

  with $\alpha \in (0, 1)$.

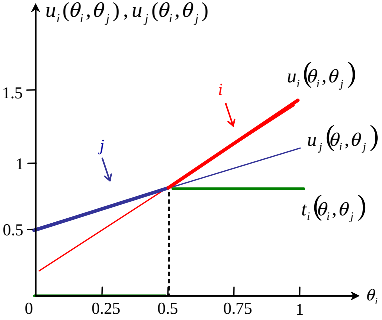
- in the generalized VCG mechanism the allocation is

$$q_i\left(\theta_i, \theta_{-i}\right) = \mathbf{1}\left\{\theta_i \geq \theta_j\right\}$$

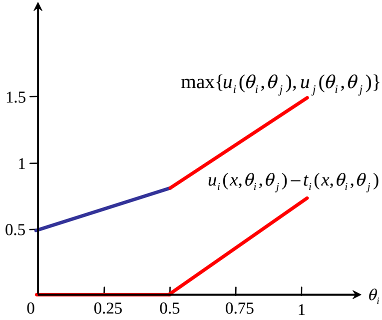
  and the transfer is

$$t_i\left(\theta_i, \theta_{-i}\right) = \left(\theta_j + \alpha\theta_j\right) \cdot \mathbf{1}\left\{\theta_i \geq \theta_j\right\}$$

Social and Private Payoffs



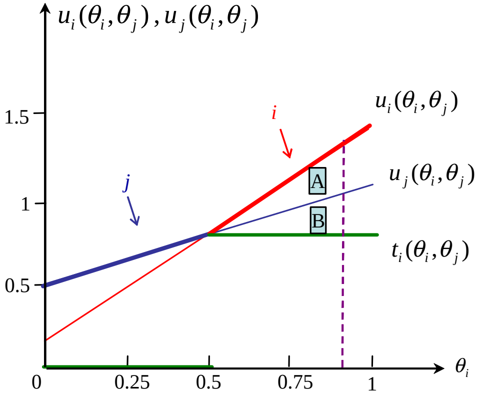
VCG Allocation and Transfer



Social and Private Payoff

$$u_i(\theta_i,\theta_j) = \theta_i + 0.5\theta_j,\ t_i(\theta_i,\theta_j) = 1.5\theta_j,\quad \theta_j = 0.5$$

Social and Private Incentives



Social Gain from Information: A
Private Gain from Information: A+B

$$u_i\left(\theta_i,\theta_j\right) = \theta_i + 0.5\theta_j,\ t_i\left(\theta_i,\theta_j\right) = 1.5\theta_j,\ \ \theta_j = 0.5,\ \theta_i = 0.9$$

Social Efficient Policy: Notation

- set of informed agents: $\{1, 2, ..., m\}$
- set of uninformed agents: $\{m + 1, ..., l\}$
- marginally informed agent: m
- bidder h has highest signal among agents $1, 2, ..., m - 1$:

$$\theta_h \triangleq \max\{\theta_1, ..., \theta_{m-1}\}.$$

Socially Efficient Information Policy

- Δ_m^* is expected social gain of marginal informed bidder m:

$$\begin{aligned}\Delta_m^* &= \mathbb{E}_\theta\left[\left(u_m(\theta) - u_h(\theta)\right) \cdot \mathbf{1}(\theta_m \geq \theta_h \geq \mu)\right] \\ &\quad + \mathbb{E}_\theta\left[\left(u_m(\theta) - u_l(\theta)\right) \cdot \mathbf{1}\left(\theta_m \geq \mu > \theta_h\right)\right]\end{aligned}$$

- Δ_m^* is the difference between:
  - social value when allocation incorporates information θ_m
  - social value without incorporating information θ_m
- define

$$y_m = \max\{\theta_h, \mu\} = \max_{j \neq i} \theta_j,$$

then we have using linearity

$$\Delta_m^* = (1-\alpha)\,\mathbb{E}_{\theta_m, y_m}\left[(\theta_m - y_m) \cdot \mathbf{1}(\theta_m \geq y_m)\right]$$

Social Efficient Policy

- social efficient policy $s_m^* \in \{0, 1\}$:
- $s_m^* = 1$ if it is efficient to to acquire information
- $s_m^* = 0$ otherwise

Proposition

The socially efficient policy s_m^* is given by

$$s_m^* = \begin{cases} 0 & \text{if} \quad \Delta_m^* < c \\ 1 & \text{if} \quad \Delta_m^* \geq c \end{cases} .$$

Δ_m^* is strictly decreasing in m and α.

Equilibrium Value of Information

- $\hat{\Delta}_m$: expected private gain of bidder m from information about θ_m

$$\begin{aligned} \hat{\Delta}_m &= \mathbb{E}_\theta \left[\left(u_m\left(\theta_m, \theta_{-m}\right) - u_h\left(y_m, \theta_{-m}\right)\right) \cdot \mathbf{1}(\theta_m \geq y_m) \right] \\ &= \mathbb{E}_{\theta_m, y_m} \left[\left(\theta_m - y_m\right) \cdot \mathbf{1}(\theta_m \geq y_m) \right] \end{aligned}$$

- $\hat{\Delta}_m$ is the difference between:
  - private payoff of allocation that incorporates information θ_m
  - private payoff of allocation without incorporating θ_m

Equilibrium Information Policy

Proposition

The equilibrium policy in the pure strategy equilibrium is given by

$$\widehat{s}_m = \begin{cases} 0 & \textit{if} \quad \hat{\Delta}_m < c \\ 1 & \textit{if} \quad \hat{\Delta}_m \geq c \end{cases} .$$

$\hat{\Delta}_m$ is strictly decreasing in m and constant in α for all m.

Welfare Analysis

Theorem

For all m,

1. *private gains are higher than social gains of information;*
2. *information decisions are strategic substitutes;*
3. *unique pure strategy equilibrium displays socially excessive information acquisition;*
4. *the difference* $\hat{\Delta}_m - \Delta_m^*$ *is increasing in* α.

- with positive dependence, equilibrium information is socially excessive
  - the number of informed bidders in equilibrium is larger than in a planner's solution
- information decisions are strategic substitutes in both equilibrium and social optimum

Mixed Strategy Equilibrium

- symmetric equilibrium
- restrict social program to choose the same probability of acquiring information for all bidders
  - concentrate solely on the informational externalities
  - ignore coordination problems arising due to mixing
- comparison between social and equilibrium level of information continues to hold with symmetric solutions
  - σ^*: socially optimal probability of acquiring information
  - $\hat{\sigma}$: equilibrium probability of acquiring information

Mixed Strategy Equilibrium

- $\Delta^* (\sigma)$: expected social gain of additional informed bidder

$$\Delta^* (\sigma) = \sum_{m=1}^{I} \binom{I-1}{m-1} \sigma^{m-1} (1-\sigma)^{I-m} \Delta^*_m$$

- $\hat{\Delta} (\sigma)$: individual gain if other bidders acquire information with probability σ

Proposition

For all $\sigma^* \in (0,1)$, $\sigma^* < \hat{\sigma}$.

Nonlinear Interdependence

- question:
  - can we generalize results in the linear setting to a nonlinear environment?
- no-crossing condition
  - the ranking of any two bidders is unaffected by the private information of a third bidder
- example: linear signal model with constant absolute risk aversion utility

Basic Setup

- general nonlinear valuation functions

$$u_i : \left[\underline{\theta}, \overline{\theta}\right]^I \rightarrow \mathbb{R}$$

- symmetric: $\forall \theta$,θ', if θ' is a permutation of θ and $\theta_i = \theta'_j$, then

$$u_i\left(\theta\right) = u_j(\theta')$$

- single-crossing property

$$\theta_i \geq \theta_j \Rightarrow u_i\left(\theta\right) \geq u_j\left(\theta\right)$$

- positive interdependence

$$\frac{\partial u_i\left(\theta\right)}{\partial \theta_j} > 0, \ \forall i, j, \forall \theta.$$

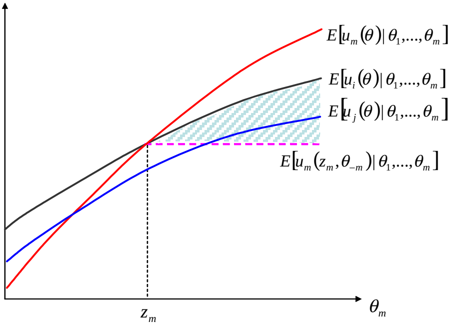
No-Crossing Condition

- valuations $\{u_i(\theta)\}_{i=1}^{I}$ satisfy the no-crossing condition if for all m and all $i, j \neq m$:

$$
\begin{aligned}
\exists\theta_m \text{ s.th.} \mathbb{E}\left[u_i(\theta)\,|\theta_1,...,\theta_m\right] > & \quad \mathbb{E}\left[u_j(\theta)\,|\theta_1,...,\theta_m\right] \quad \Rightarrow \\
\forall\theta_m \text{ s.th. } \mathbb{E}\left[u_i(\theta)\,|\theta_1,...,\theta_m\right] > & \quad \mathbb{E}\left[u_j(\theta)\,|\theta_1,...,\theta_m\right]
\end{aligned}
$$

- this condition is important to ensure $\Delta_m^* < \hat{\Delta}_m$:
  - if violated, the information of agent m may be socially valuable in determining allocation between i and j without agent m ever getting the object
  - agent m will have very weak incentive to acquire information even though it would be socially valuable
  - social gain from information about θ_m may exceed private gain

Excessive Private Incentives



- no-crossing: curves $\mathbb{E}[u_i(\theta)|\theta_1,...,\theta_m]$ and $\mathbb{E}\left[u_j(\theta)|\theta_1,...,\theta_m\right]$ do not cross
- single-crossing: curve $\mathbb{E}[u_m(\theta)|\theta_1,...,\theta_m]$ crosses both $\mathbb{E}[u_i(\theta)|\theta_1,...,\theta_m]$ and $\mathbb{E}\left[u_j(\theta)|\theta_1,...,\theta_m\right]$ only once
- difference between private and social incentives: shaded area

Results

Theorem

If the no-crossing condition is satisfied then

1. *the private gain from information is higher than social gain from information* $\left(\hat{\Delta}_m \geq \Delta_m^*\right)$*;*
2. *information decisions are strategic substitutes* $\left(\hat{\Delta}_{m-1} \geq \hat{\Delta}_m\right)$*;*
3. *unique pure strategy equilibrium displays socially excessive information acquisition.*

The Role of Positive Interdependence

- we identified sufficient conditions for excessive equilibrium information
  - private incentives $>$ social incentives
  - strategic substitutes
- question
  - positive interdependence $\Rightarrow$ excessive equilibrium information?
  - not true in general

Insufficient Private Incentives

- value of object is determined by the K highest signals.

$$u_i(\theta) = \theta_i + \alpha \sum_{k=1}^{K} y_{ik}$$

- example: license to operate in K markets
  - bidder i's signal reveals the profitability of market i
  - choose to operate in the K markets with highest potential

Privately versus Socially Pivotal Signals

- privately vs. socially pivotal signals
  - privately pivotal: determine the winner of the license
  - socially pivotal: determine which market to operate
  - a signal could be socially pivotal but not privately pivotal
- findings:
  - information decision remain strategic substitutes
  - equilibrium level of information is socially insufficient.

Strategic Complements

- local comparison may not extend to equilibrium comparison
- strategic complements $\Rightarrow$ multiple equilibria
- despite positive interdependence, an equilibrium of the game may display a lower level of information acquisition than the social optimum

Strategic Complements

- two bidders, $i \in \{1, 2\}$, compete for an object
- linear payoff structure: $u_i(\theta_i, \theta_j) = \theta_i + \frac{1}{2}\theta_j$
- types θ_i, θ_j are independently drawn from $U[-5, 1]$
- efficient allocation: assign the object to bidder *i* if

$$\mathbb{E}[u_i(\theta)] > \max\left\{0, \mathbb{E}\left[u_j(\theta)\right]\right\},$$

  otherwise retain the object
- information decisions are strategic *complements*
- for small *c* the efficient policy asks both bidders to acquire information, but in one of the two pure strategy equilibria, both bidders remain uninformed

Conclusion

- with interdependent values equilibrium information differs from social optimum.
- extensions:
  - multi-unit auction setting
  - negative interdependence: too low incentives
- future research questions:
  - how should a planner correct the incentives? participation fees, randomization?
  - revenue maximizing design
  - sequential information design
  - information acquisition in double auctions with large number of traders