

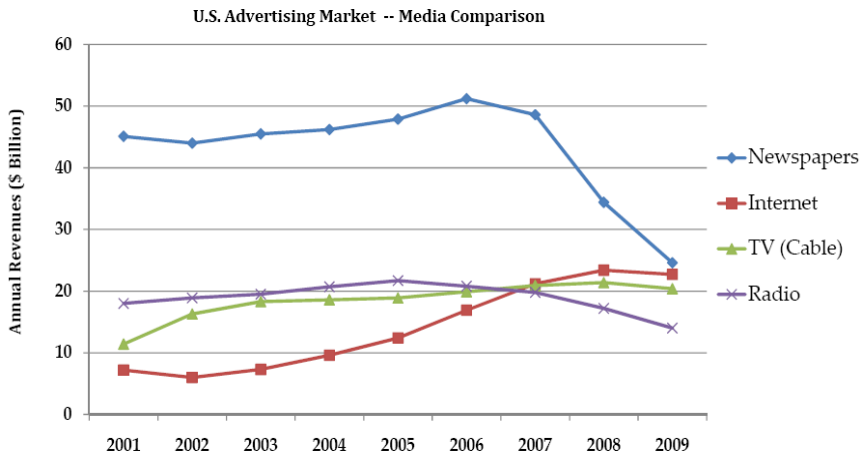
# Targeting in Advertising Markets: Implications for Offline vs. Online Media

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- “Recent” progress in advertising technology:
  - display advertising                      mobile ads
  - sponsored search                      addressable cable
  - social networks                              ...
- Distinctive feature is the ability to target:
  - attribute, demographic targeting;
  - behavioral, contextual targeting.



# Targeting with Many Markets/Products

- to offer a model of targeting in advertising markets in the presence of
  - many distinct advertising markets
  - many distinct advertisers
- we trace out the implications of targeting for:
  - the allocation of advertisement messages;
  - the social value of advertising;
  - the equilibrium price of advertising;
  - the equilibrium revenues of new and old media.

# A Model of Advertising as Matching

- Advertising matches a consumer and a product.
- An advertisement message turns a potential, interested consumer into an actual customer.
- Advertising markets operate under substantial frictions:
  - ① messages may reach the wrong consumer;
  - ② messages may reach the same consumer repeatedly.
- Targeting reduces matching frictions.

# Advertising and Product Markets

- A continuum of distinct advertising markets

$$a \in [0, \infty),$$

representing outlets, channels, websites, searches.

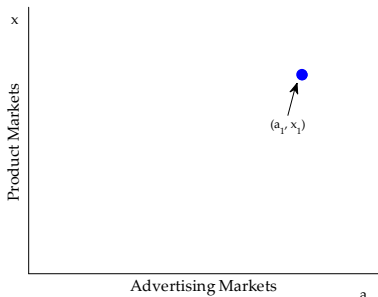
- A continuum of distinct products (= firms),

$$x \in [0, \infty).$$

- A unit mass of consumers with two-dimensional type  $(a, x)$ :
  - each consumer is located in a specific advertising market  $a$ ;
  - each consumer is interested in a specific product  $x$ .

## Consumer Characteristics...

- A consumer is characterized by  $(a, x)$ :
  - ① his location in a specific advertising market  $a$ ,
  - ② his preference for a specific product  $x$



- market structure: joint density  $s(a, x)$  over  $(a, x)$  :

$$\int_0^{\infty} \int_0^{\infty} s(a, x) da dx = 1.$$

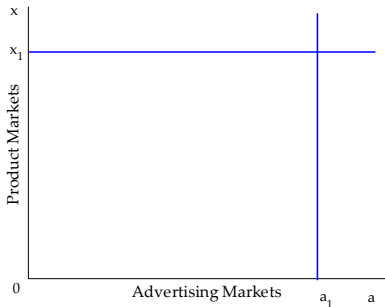
## ... and Market Characteristics

- advertising market  $a$  : distribution over consumer preferences

$$s(x|a) = \frac{s(a, x)}{\int_0^\infty s(a, x') dx'}$$

- firm  $x$ : distribution of its consumers over advertising media:

$$s(a|x) = \frac{s(a, x)}{\int_0^\infty s(a', x) da'}$$





- we maintain the distribution over consumer preferences:

$$s(x) = \int_0^{\infty} s(a', x) da',$$

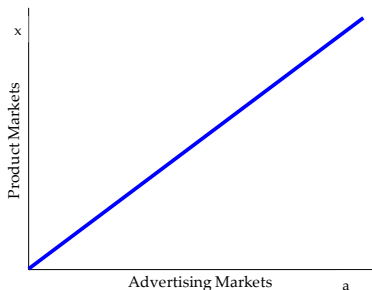
the share  $s(x)$  of each product in the consumer market

- we order  $x$  (without loss of generality) so that:

$$s'(x) < 0,$$

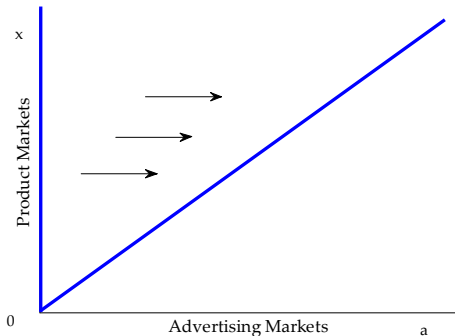
- there are products with a broad audience  $x \approx 0$  and products with a narrow audience  $x \approx \infty$  (the long tail of Anderson (2006))

- we investigate the impact of different distributions of consumers across advertising markets..
- the distribution of consumer across advertising markets range
  - from perfect targeting
  - to zero targeting
- and ask how does an increase in targeting impact the allocation and the price of advertising across media markets



# Impact of Targeting

- an increase in targeting then has two effects:
  - ① consumers move from mass market publications to more specialized, narrower media
  - ② in every media market, the naturally targeted audience has a larger relative population share



# Distribution in Product Markets

- Exponential distribution of consumers' interests:

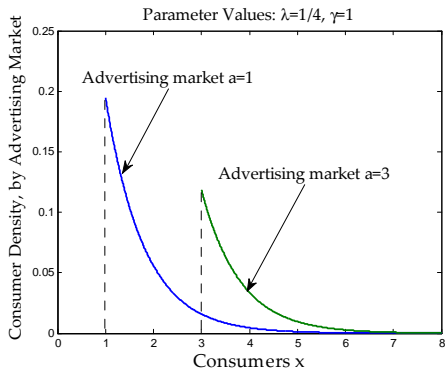
$$s_x := \lambda e^{-\lambda x}.$$

- $\lambda$  measures concentration of consumers in product markets.
- Market shares  $s_x$  are declining in  $x$ .
- Hierarchical structure of products:
  - popularity: bicycles, music, watches, travel destinations;
  - mass vs. niche products, mainstream vs. fringe firms.

# Distribution in Advertising Markets

- Conditional distribution of consumers  $x$  in markets  $a$ :

$$s(x|a) = \gamma e^{-\gamma(x-a)}, \quad \text{for all } 0 < a \leq x.$$

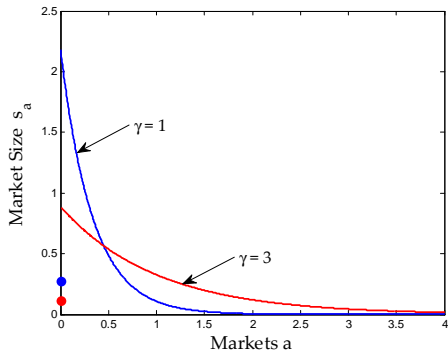


- Distribution across markets is upper triangular (stationary):

$$s(x|a) = 0 \quad \text{for all } x < a,$$

## Size of Advertising Markets

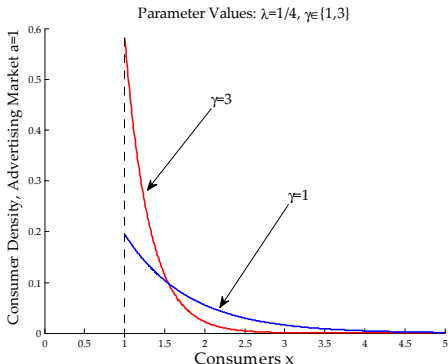
- an increase in the targeting technology  $\gamma$  has a size effect...:



- $\gamma$  measures consumer concentration in advertising markets.
- high  $\gamma \Rightarrow$  the consumers of  $x$  move to nearby markets  $a \approx x$ .

# Composition of Advertising Markets

- ... and an increase in targeting  $\gamma$  has a composition effect:



- $\gamma$  measures consumer concentration in advertising markets.
- high  $\gamma \Rightarrow$  most consumers in  $a$  have nearby preferences  $x \approx a$ .
- a higher  $\gamma$  facilitates targeting.

## Advertising as Random Matching

- Each consumer reads/views/processes  $M$  messages
- A consumer with preference for product  $x$  purchases if and only if she receives a message from firm  $x$
- Firm  $x$  sends  $m_{a,x}$  messages to consumers in market  $a$ .
- Each message is received with **uniform probability** by one of the consumers in advertising market  $a$ :
- It follows that a consumer in advertising market  $a$  receives at least one message from firm  $x$  with probability

$$f(m_{a,x}, s_a) = 1 - \exp(-m_{a,x} / s_a).$$



- an advertising policy of firm  $x$ :

$$\{m_{a,x}\}_{a=0}^x$$

- advertising intensity in advertising market  $a$ :

$$\frac{m_{a,x}}{s_a}$$

- the gross revenue of  $m_{a,x}$  is given by

$$s_{a,x} \cdot f(m_{a,x}, s_a) = s_{a,x} \cdot (1 - \exp(-m_{a,x} / s_a))$$

- an optimal advertising policy seeks to minimize the role of:
  - ① irrelevant messages:  $1 - s_{a,x}$
  - ② duplicating messages:  $\exp(-m_{a,x} / s_a)$

# Competitive Equilibrium

- price of message in advertising market  $a$  is competitive equilibrium price

$$p_a$$

- $M$  is time/attention of consumer devoted to advertisements
- supply of messages  $M_a$  in advertising market  $a$  is given by:

$$M_a = s_a \cdot M$$

- competitive price  $p_a$  equilibrates demand and supply:

$$\int_0^{\infty} m_{a,x}(p_a) dx = M_a.$$

- Each sale generates revenue \$1, firms only differ in size  $s(x)$ .
- Firm  $x$  chooses  $m_{a,x}$  to maximize profit:

$$\pi_{a,x} = \max_{m_{a,x}} \left[ s_{a,x} \cdot \left( 1 - \exp \left( -\frac{m_{a,x}}{s_a} \right) \right) - p_a \cdot m_{a,x} \right].$$

- advertising policies are separable across advertising markets:

$$m_{a,x} = s_a \left( \ln \frac{\gamma + \lambda}{p_a} - (\gamma + \lambda) (x - a) \right)$$

for all  $x \geq a$ .

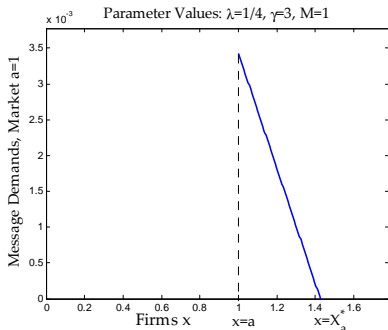
# Competitive Equilibrium

- marginal advertiser in advertising market  $a$  is  $X_a^*$
- The number of active firms is constant across markets  $a$ :

$$X_a^* - a = \sqrt{\frac{2M}{\lambda + \gamma}}$$

- The equilibrium demands are

$$m_{a,x}^* = \gamma \lambda e^{-a\lambda} (X_a^* - x).$$



# Competitive Equilibrium Price

- Equilibrium prices  $p_a^*$  are equalized across advertising market:

$$p_a^* = p^* = (\lambda + \gamma) e^{-\sqrt{2(\lambda + \gamma)M}}, \quad \text{for all } a.$$

- For any  $\gamma > 0$ , all firms advertise somewhere
- positive targeting  $\Rightarrow$  “long tail”.

# The Social Value of Targeting

- an improvement in targeting technology as increase in  $\gamma$
- what is the impact in terms of the social welfare?
  - less irrelevant messages are received
  - more messages are sent by smaller firms

## Proposition (Targeting and Social Welfare)

*As targeting improves the social value of advertising increases.*

- the total number of matches between advertisers and consumers increases
- even, the number of matches of each firm (product) increases

# Targeting and the Profile of Demand

- as the social value of advertising increases, how does the composition in the demand for advertising change?

## Proposition (Targeting and Demand)

*As targeting improves:*

- ① *the large firms purchase less, the small firm purchase more messages (across all markets);*
  - ② *the number of participating firms  $X_a^* - a$  decreases in every advertising market;*
  - ③ *The number of messages per capita  $m_{a,x}^* / s_a$  increases for all  $x < (a + X_a^*) / 2$ .*
- conversely, every firm is present in fewer advertising markets

# Targeting and The Price for Advertising

- as the social value of advertising increases, can (a share of) the increase in value be captured by the media?

## Proposition (Targeting and Price)

*As  $\gamma$  increases, the equilibrium price per message  $p_a^*$  increases if and only if  $\lambda + \gamma < 2/M$ .*

- the equilibrium price is initially increasing in the targeting ability but eventually decreasing
- main trade-off: the messages become more relevant yet eventually to a smaller set of firms and thus the risk of duplication (saturation)
- in hedonic terms: the price per consumer reached is decreasing everywhere.



- Chandra and Kaiser (2010) "Target Advertising in Magazine Markets":
  - advertiser value more homogenous groups of readers (in subscriber characteristics of age, gender, income, etc.)
- Rutz and Bucklin (2010): "From Generic to Branded: A Model of Spillover Dynamics in Paid Search Advertising," compare generic (e.g., "Hotels LA") and branded (e.g., "Hilton Hotels LA") searches
  - find that branded keywords have lower prices than generic keywords "Sheraton Hotel NYC" vs "Hotel NYC";
  - find that long, narrower keywords "Hotels LA Westwood" have lower prices than shorter ones "Hotels LA"

- allow for multi-homing of consumer and thus multiple opportunities for advertiser to match with a customer
- online versus offline media, targeted vs. non-targeted medium
- total exposure to advertising, given by  $M$ , is now divided between media,  $A$  and  $B$  :

$$M_A + M_B = M$$

- suppose firm  $x$  reaches a fraction  $a_x$  of its consumers on medium  $A$ , and a fraction  $b_x$  on medium  $B$ .
- the total fraction of  $s_x$  reached is

$$a_x + b_x - a_x \cdot b_x.$$

- general (offline,  $A$ ) and perfectly targeted (online,  $B$ ) advertising  $(m_x^A, m_{a,x}^B)$ .
- supply in the (single) offline market is  $M_A$ .
- supply in online market  $a$  is  $M_{B,a} := s_a \cdot M_B$ .
- perfectly targeted advertising online:  $\gamma = \infty$
- the relevant online advertising market for firm  $x$  is  $a = x$ .

- large firms ( $x < X^*$ ) are present online and offline
- small firms ( $x > X^*$ ) are present only online

## Proposition (Equilibrium Prices)

- ① *The equilibrium price on the offline medium is given by:*

$$p^* = \lambda \exp(-M_B - \sqrt{2\lambda M_A}).$$

- ② *The equilibrium prices on the online markets are given by:*

$$p_a^* = \begin{cases} \exp(\lambda a - M_B - \sqrt{2\lambda M_A}), & \text{for } a \leq X^*, \\ \exp(-M_B), & \text{for } a > X^*. \end{cases}$$

# The Emergence of the Internet

- the attention/time allocated to online media,  $M_B$ , is increasing; conversely the attention to offline media,  $M_A$  is decreasing
- the segment of firms advertising offline is shrinking as  $x < X^*$ :

$$X^* = \sqrt{\frac{2M_A}{\lambda}}$$

- the price of advertising offline is decreasing faster, linear rather than square root, with an increase in the online media:

$$p^* = \lambda \exp(-M_B - \sqrt{2\lambda M_A}).$$

- in particular, relative to the introduction of competing offline medium where it would be:

$$p^* = \lambda \exp(-\sqrt{2\lambda (M_A + M_B)})$$

- Goldfarb and Tucker (2010): " Search Engine Advertising: Channel Substitution when Pricing Ads to Context" use natural experiment - ambulance-chaser regulations across states.
- when lawyers cannot contact clients by mail, advertising prices per click for search engine advertisements are 5-7% higher. Therefore, online advertising substitutes for offline advertising
- consistent with Chandra and Kaiser (2010) who document the positive valuation of homogenous, targeted audiences; and hence imply differential revenue across media with differential targeting abilities

- A model of targeting in competitive advertising markets.
- Hierarchical framework for product and advertising markets.

Extensions and future directions:

- ① revenue maximization, strategic interaction;
- ② platform competition;
- ③ congestion, consumer preferences over for different ads;
- ④ ad exchanges.