Econ 121b: Intermediate Microeconomics

Dirk Bergemann, Spring 2011

Week of 1/8 - 1/14

1 Lecture 1: Introduction

1.1 What's Economics?

This is an exciting time to study economics, even though may not be so exciting to be part of this economy. We have faced the largest financial crisis since the Great Depression. $787 billion has been pumped into the economy in the form of stimulus package by the US Government. $700 billion has been spent on the Troubled Asset Relief Programs for the Banks. The unemployment rate has been high for a long time. The August unemployment rate is 9.7%. Also there has been big debates going on at the same time on health care reform, government deficits, climate change etc. We need answers to all of these big questions and many others. And all of these come under the purview of the discipline of economics (along with other fields of study). But then how do we define this field of study? In terms of subject matter it can be defined as the study of allocation of scarce resources. A more pragmatic definition might be, economics is what economists do! In terms of methodology Optimiz! ation Theory, Statistical Analysis, Game Theory etc characterize the study of Economics. One of the primary goals of economics is to explain human behavior in various contexts, humans as consumer of commodities or decision maker in firms or head of families or politician holding a political office etc. The areas of research extends from international trade, taxes, economic growth, antitrust to crime, marriage, war, laws, media, corruption etc. There are a lot of opportunity for us to bring our way of thinking to these issues. Indeed, one of most active areas of the subject is to push this frontier.

Economists like to think that the discipline follows Popperian methods, moving from Stylized facts to Hypothesis formation to Testing hypothesis. Popperian tradition tells you that hypotheses can only be proven false empirically, not proven true. Hence an integral part of economics is to gather information about the real world in the form of data and test whatever hypothesis that the economists are proposing to be true. What this course builds up, however, is how to come up

with sensible hypotheses that can be tested. Thus economic theory is the exercise in hypothesis formation using the language of mathematics to formalize assumptions (about certain fundamentals of human behavior, or market organization, or distribution of information among individuals etc). Some critics of economics say our models are too simplistic. We leave too many things out. Of course this is true - we do leave many many things out, but for a useful purpose. It is better to be clear about an argument! and focusing on specific things in one model helps us achieve that. Failing to formalize a theory does not necessarily imply that the argument is generic and holistic, it just means that the requirement of specificity in the argument is not as high.
Historically most economists rely on maximization as a core tool in economics, and it is a matter of good practice. Most of what we will discuss in this course follows this tradition: maximization is much easier to work with than alternatives. But philosophically I don't think that maximization is necessary for any work to be considered as part of economics. You will have to decide on your own. My own view is that there are 3 core tools:

- The principle that people respond to incentives
- An equilibrium concept that assumes that absence of free lunches
- A welfare criteria saying that more choices are better

Last methodological point: Milton Friedman made distinction of the field into positive and normative economics:

- Positive economics - why the world is the way it is and looks the way it does
- Normative economics - how the world can be improved

Both areas are necessary and sometimes merge perfectly. But there are often tensions. We will return to this throughout the rest of the class. What I hope you will get out of the course are the following:

- Ability to understand basic microeconomic mechanisms
- Ability to evaluate and challenge economic arguments
- Appreciation for economic way of looking at the world

We now try to describe a very simple form of human interaction in an economic context, namely trade or the voluntary exchange of goods or objects between two people, one is called the seller, the current owner of the object and the other the buyer, someone who has a demand or want for that object. It is referred to as bilateral trading.

1.2 Gains from Trade

1.2.1 Bilateral Trading

Suppose that a seller values a single, homogeneous object at c (opportunity cost), and a potential buyer values the same object at v (willingness to pay). Trade could occur at a price p, in which case the payoff to the seller is $p - c$ and to the buyer is $v - p$. We assume for now that there is only one buyer and one seller, and only one object that can potentially be traded. If no trade occurs, both agents receive a payoff of 0.

Whenever $v > c$ there is the possibility for a mutually beneficial trade at some price $c \leq p \leq v$. Any such allocation results in both players receiving non-negative returns from trading and so both are willing to participate ($p - c$ and $v - p$ are non-negative).

There are many prices at which trade is possible. And each of these allocations, consisting of whether the buyer gets the object and the price paid, is efficient in the following sense:

Definition 1. An allocation is Pareto efficient if there is no other allocation that makes at least one agent strictly better off, without making any other agent worse off.

1.2.2 Experimental Evidence

This framework can be extended to consider many buyers and sellers, and to allow for production. One of the most striking examples comes from international trade. We are interested, not only in how specific markets function, but also in how markets should be organized or designed.

There are many examples of markets, such as the NYSE, NASDAQ, E-Bay and Google. The last two consist of markets that were recently created where they did not exist before. So we want to consider not just existing markets, but also the creation of new markets.

Before elaborating on the theory, we will consider three experiments that illustrate how these markets function. We can then interpret the results in relation to the theory. Two types of cards (red and black) with numbers between 2 and 10 are handed out to the students. If the student receives a red card they are a seller, and the number reflects their cost. If the student receives a black card they are a buyer, and this reflects their valuation. The number on the card is private information. Trade then takes place according to the following three protocols.

1. Bilateral Trading: One seller and one buyer are matched before receiving their cards. The buyer and seller can only trade with the individual they are matched with. They have 5 minutes to make offers and counter offers and then agree (or not) on the price.

2. Pit Market: Buyer and seller cards are handed out to all students at the beginning. Buyers and sellers then have 5 minutes to find someone to trade with and agree on the price to trade.

3. Double Auction: Buyer and seller cards are handed out to all students at the beginning. The initial price is set at 6 (the middle valuation). All buyers and sellers who are willing to trade at this price can trade. If there is a surplus of sellers the price is decreased, and if there is a surplus of buyers then the price is increased. This continues for 5 minutes until there are no more trades taking place.

2 Lecture 2: Choice

In the decision problem in the previous section, the agents had a binary decision: whether to buy (sell) the object. However, there are usually more than two alternatives. The price at which trade could occur, for example, could take on a continuum of values. In this section we will look more closely at preferences, and determine when it is possible to represent preferences by "something handy," which is a utility function.

Suppose there is a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ for some individual decision maker. We are going to assume, in a manner made precise below, that two features of preferences are true.

- There is a complete ranking of alternatives.
- "Framing" does not affect decisions.

We refer to X as a choice set consisting of n alternatives, and each alternative $x \in X$ is a consumption bundle of k different items. For example, the first element of the bundle could be food, the second element could be shelter and so on. We will denote preferences by $\succsim$, where $x \succsim y$ means that "x is weakly preferred to y." All this means is that when a decision maker is asked to choose between x and y they will choose x. Similarly, $x \succ y$, means that "x is strictly preferred to y" and $x \sim y$ indicates that the decision maker is "indifferent between x and y." The preference relationship $\succsim$ defines an ordering on $X \times X$. We make the following three assumptions about preferences.

Axiom 1. *Completeness. For all $x, y \in X$ either $x \succsim y$, $y \succsim x$, or both.*

This first axiom simply says that, given two alternatives the decision maker can compare the alternatives, and will weakly prefer one of the alternatives to the other, or will be indifferent, in case both are weakly preferred to each other.

Axiom 2. *Transitivity. For all triples $x, y, z \in X$ if $x \succsim y$ and $y \succsim z$ then $x \succsim z$.*

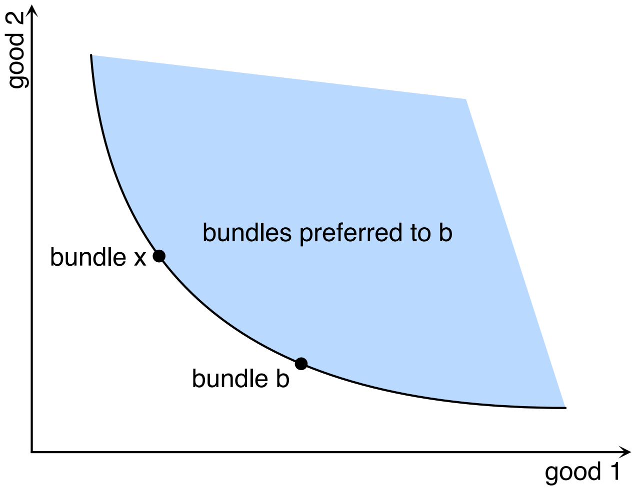

Figure 1: Indifference curve

Very simply, this axiom imposes some level of consistency on choices. For example, suppose there were three potential travel locations, Tokyo (T), Beijing (B), and Seoul (S). If a decision maker, when offered the choice between Tokyo and Beijing, weakly prefers to go to Tokyo, and when given the choice between Beijing and Seoul weakly prefers to go to Beijing, then this axiom simply says that if she was offered a choice between a trip to Tokyo or a trip to Seoul, she would weakly prefer to go to Tokyo. This is because she has already demonstrated that she weakly prefers Tokyo to Beijing, and Beijing to Seoul, so weakly preferring Seoul to Tokyo would mean that their preferences are inconsistent.

But it is conceivable that people might violate transitivity in certain circumstances. One of them is “framing effect”. It is the idea that the way the choice alternatives are framed may affect decision and hence in turn may violate transitivity eventually. The idea was made explicit by an experiment due to Danny Kahneman and Amos Tversky (1984). In the experiment students visiting the MIT-Coop to purchase a stereo for $125 and a calculator for $5 were informed that the calculator is on sale for 5 dollars less at Harvard Coop. The question is would the students make the trip?
Suppose instead the students were informed that the stereo is 5 dollars less at Harvard Coop.
Kahneman and Tversky found that the fraction of respondents who would travel for cheaper calculator is much higher than for cheaper stereo. But they were also told that there is a stockout and the students have to go to Harvard Coop, and will get 5 dollars off either item as compensation, and were asked which item do you care to get money off? Most of them said that they were indifferent. If $x =$

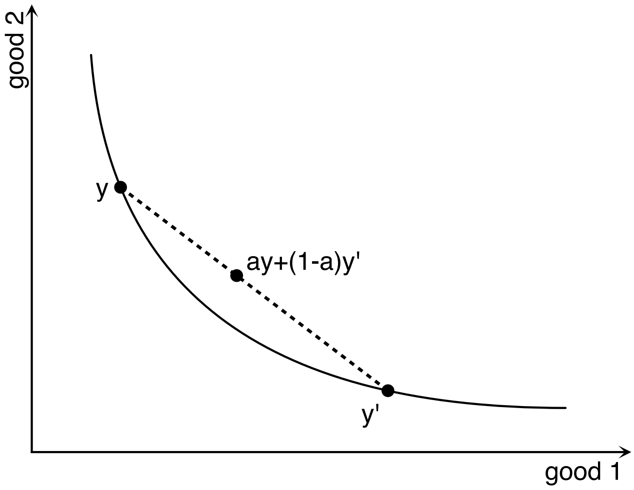

Figure 2: Convex preferences

go to Harvard and get 5 dollars off calculator, y= go to Harvard and get 5 dollars off stereo, z = get both items at MIT. We have $x \succ z$ and $z \succ y$, but last question implies $x \sim y$. Transitivity would imply that $x \succ y$, which is the contradiction. We for the purposes of this course would assume away any such framing effects in the mind of the decision maker.

Axiom 3. *Reflexivity. For all $x \in X$, $x \succsim x$ (equivalently, $x \sim x$).*

The final axiom is made for technical reasons, and simply says that a bundle cannot be strictly preferred to itself. Such preferences would not make sense.

These three axioms allow for bundles to be ordered in terms of preference. In fact, these three conditions are sufficient to allow preferences to be represented by a utility function.

Before elaborating on this, we consider an example. Suppose there are two goods, Wine and Cheese. Suppose there are four consumption bundles $z = (2,2), y = (1,1), a = (2,1), b = (1,2)$ where the two elements of the vector represent the amount of wine or cheese. Most likely, $z \succ y$ since it provides more of everything (i.e., wine and cheese are "goods"). It is not clear how to compare a and b. What we can do is consider which bundles are indifferent with b. This is an indifference curve (see Figure 1). We can define it as

$$I_b = \{x \in X | b \sim x)$$

We can then (if we assume that more is better) compare a and b by considering which side of the indifference curve a lies on: bundles above and to the right are more preferred, bundles below and to the left are less preferred. This reduces

Figure 3: Perfect substitutes (left) and perfect complements (right)

the dimensionality of the problem. We can speak of the "better than b" set as the set of points weakly preferred to b. These preferences are "ordinal:" we can ask whether x is in the better than set, but this does not tell us how much x is preferred to b. It is common to assume that preferences are monotone: more of a good is better.

Definition 2. The preferences $\succsim$ are said to be (strictly) monotone if $x \geq y \Rightarrow x \succsim y$ ($x \geq y, x \neq y \Rightarrow x \succsim y$ for strict monotonicity).[1]

Suppose I want to increase my consumption of good 1 without changing my level of well-being. The amount I must change x_2 to keep utility constant, $\frac{dx_2}{dx_1}$ is the marginal rate of substitution. Most of the time we believe that individuals like moderation. This desire for moderation is reflected in convex preferences. A mixture between two bundles, between which the agent is indifferent, is strictly preferred to either of the initial bundle (see Figure 2).

Definition 3. A preference relation is convex if for all y and y' with $y \sim y'$ and all $\alpha \in [0, 1]$ we have that $\alpha y + (1 - \alpha)y' \succsim y \sim y'$.

While convex preferences are usually assumed, there could be instances where preferences are not convex. For example, there could be returns to scale for some good.

Examples: perfect substitutes, perfect complements (see Figure 3). Both of these preferences are convex.

Notice that indifference curves cannot intersect. If they did we could take two points x and y, both to the right of the indifference curve the other lies on. We would then have $x \succ y \succ x$, but then by transitivity $x \succ x$ which contradicts reflexivity. So every bundle is associated with one, and only one, welfare level.

Another important property of preference relation is continuity.

[1]If $x = (x_1, \ldots, x_N)$ and $y = (y_1, \ldots, y_N)$ are vectors of the same dimension, then $x \geq y$ if and only if, for all i, $x_i \geq y_i$. $x \neq y$ means that $x_i \neq y_i$ for at least one i.

Definition 4. Let $\{x_n\}, \{y_n\}$ be two sequences of choices. If $x_n \succsim y_n$, $\forall n$ and $x_n \to x$, and $y_n \to y$, then $x \succsim y$.

This property guarantees that there is no jump in preferences. When X is no longer finite, we need continuity to ensure a utility representation.

2.1 Utility Functions

What we want to consider now is whether we can take preferences and map them to some sort of utility index. If we can somehow represent preferences by such a function we can apply mathematical techniques to make the consumer's problem more tractable. Working with preferences directly requires comparing each of a possibly infinite number of choices to determine which one is most preferred. Maximizing an associated utility function is often just a simple application of calculus. If we take a consumption bundle $x \in \mathbb{R}_+^N$ we can take a utility function as a mapping from $\mathbb{R}_+^N$ into $\mathbb{R}$.

Definition 5. A utility function (index) $u : X \to \mathbb{R}$ represents a preference profile $\succsim$ if and only if, for all $x, y \in X$: $x \succsim y \Leftrightarrow u(x) \geq u(y)$.

We can think about a utility function as an "as if"-concept: the agent acts "as if" she has a utility function in mind when making decisions.

Is it always possible to find such a function? The following result shows that such a function exists under the three assumptions about preferences we made above.

Proposition 1. *Suppose that X is finite. Then the assumptions of completeness, transitivity, and reflexivity imply that there is a utility function u such that $u(x) \geq u(y)$ if and only if $x \succsim y$.*

Proof. We define an explicit utility function. Let's introduce some notation:

$$B(x) = \{z \in X | x \succsim z\}$$

Therefore $B(x)$ is the set of "all items below x". Let the utility function be defined as,

$$u(x) = |B(x)|$$

where $|B(x)|$ is the cardinality of the set $B(x)$, i.e. the number of elements in the set $B(x)$. There are two steps to the argument:
First part:

$$u(x) \geq u(y) \quad \Rightarrow \quad x \succsim y$$

Second part:
$$x \succsim y \quad \Rightarrow \quad u(x) \geq u(y)$$
First part of proof:

By definition, $u(x) \geq u(y) \quad \Rightarrow \quad |B(x)| \geq |B(y)|$. If $y \in B(x)$, then $x \succsim y$ by definition of $B(x)$ and we are done. Otherwise, $y \notin B(x)$. We will work towards a contradiction.

Since $y \notin B(x)$, we have
$$|B(x) - \{y\}| = |B(x)|$$
Since $y \in B(y)$ (by reflexivity), we have
$$|B(y)| - 1 = |B(y) - \{y\}|$$
Since $|B(x)| \geq |B(y)|$, $|B(x)| > |B(y)| - 1$ and hence,
$$|B(x) - \{y\}| > |B(y) - \{y\}|$$
Therefore, there must be some $z \in X - \{y\}$ such that $x \succsim z$ and $y \not\succsim z$. By completeness: $z \succsim y$. By transitivity: $x \succsim y$. But this implies that $y \in B(x)$, a contradiction. Second part of proof

Want to show: $x \succsim y \quad \Rightarrow \quad u(x) \geq u(y)$.
Suppose $x \succsim y$ and $z \in B(y)$.
Then $x \succsim y$ and $y \succsim z$, so by transitivity $x \succsim z$.
Hence, $z \in B(x)$.
This shows that when $x \succsim y$, anything in $B(y)$ must also be in $B(x)$.
$$B(y) \subset B(x) \quad \Rightarrow \quad |B(x)| \geq |B(y)| \quad \Rightarrow \quad u(x) \geq u(y)$$
This completes the proof. □

In general the following proposition holds:

Proposition 2. *Every (continuous) preference ranking can be represented by a (continuous) utility function.*

This result can be extended to environments with uncertainty, as was shown by Leonard Savage. Consequently, we can say that individuals behave as if they are maximizing utility functions, which allows for marginal and calculus arguments. There is, however, one qualification. The utility function that represents the preferences is not unique.

Remark 1. If u represents preferences, then for any increasing function $f : \mathbb{R} \to \mathbb{R}$, $f(u(x))$ also represents the same preference ranking

In the previous section, we claimed that preferences usually reflect the idea that "more is better," or that preferences are monotone.

Definition 6. The utility function (preferences) are monotone increasing if $x \geq y$ implies that $u(x) \geq u(y)$ and $x > y$ implies that $u(x) > u(y)$.

One feature that monotone preferences rule out is (local) satiation, where one point is preferred to all other points nearby. For economics the relevant decision is maximizing utility subject to limited resources. This leads us to consider constrained optimization.

3 Lecture 3: Maximization

Now we take a look at the mathematical tool that will be used with the greatest intensity in this course. Let $x = (x_1, x_2, \ldots, x_n)$ be a n-dimensional vector where each component of the vector $x_i, i = 1, 2, \ldots, n$ is a non-negative real number. In mathematical notations we write $x \in \mathbb{R}^n_+$. We can think of x as description of different characteristics of a choice that the decision maker faces. For example, while choosing which college to go (among the ones that have offered admission) a decision maker, who is a student in this case, looks into different aspects of a university, namely the quality of instruction, diversity of courses, location of the campus etc. The components of the vector x can be thought of as each of these characteristics when the choice problem faced by the decision maker (i.e. the student) is to choose which university to attend. Usually when people go to groceries they are faced with the problem of buying not just! one commodity, but a bundle of commodities and therefore it is the combination of quantities of different commodities which needs to be decided and again the components of x can be thought of as quantities of each commodity purchased. Whatever be the specific context, utility is defined over the set of such bundles. Since $x \in \mathbb{R}^n_+$, we take $X = \mathbb{R}^n_+$. So the utility function is a mapping $u \colon \mathbb{R}^n_+ \to \mathbb{R}$.

Now for the time being let x be one dimensional, i.e. $x \in R$. Let $f \colon \mathbb{R} \to \mathbb{R}$ be a continuous and differentiable function that takes real numbers and maps it to another real number. Continuity is assumed to avoid any jump in the function and differentiability is assumed to avoid kinks. The slope of the function f is defined as the first derivative of the function and the curvature of the function is defined as the second derivative of the function. So, the slope of f at x is formally defined as:

$$\frac{df(x)}{dx} \triangleq f^{'}(x)$$

and the curvature of f at x is formally defined as:

$$\frac{d^2f(x)}{dx^2} \triangleq f^{''}(x)$$

In order to find out the maximum of f we must first look into the slope of f. If the slope is positive then raising the value of x increases the value of f. So to find out the maximum we must keep increasing x. Similarly if slope is negative then reducing the value of x increases the value of f and therefore to find the maximum we should reduce the value of x. Therefore the maximum is reached when the slope is exactly equal to 0. This condition is referred to as the First Order Condition (F.O.C.) or the necessary condition:

$$\frac{df(x)}{dx} = 0$$

But this in itself doesn't guarantee that maximum is reached, as a perfectly flat slope may also imply that we're at the trough, i.e. at the minimum. The F.O.C. therefore finds the extremum points in the function. We need to look at the curvature to make sure whether the extremum is actually a maximum or not. If the second derivative is negative then it means that from the extremum point if we move x a little bit on either side $f(x)$ would fall, and therefore the extremum is a maximum. But if the second derivative is positive then by similar argument we know that its the minimum. This condition is referred to as the Second Order Condition (S.O.C) or the sufficient condition:

$$\frac{d^2f(x)}{dx^2} \leq 0$$

Now we look at the definitions of two important kind of functions:

Definition 7. (i)A continuous and differentiable function $f\colon \mathbb{R} \to \mathbb{R}$ is (strictly) concave if

$$\frac{d^2f(x)}{dx^2} \leq (<)0.$$

(ii) f is convex if

$$\frac{d^2f(x)}{dx^2} \geq (>)0.$$

Therefore a concave function the F.O.C. is both necessary and sufficient condition for maximization. We can also define concavity or convexity of functions with the help of convex combinations.

Definition 8. A convex combination of two any two points $x^{'}, x^{''} \in \mathbb{R}^n$ is defined as $x_\lambda = \lambda x^{'} + (1-\lambda)x^{''}$ for any $\lambda \in (0, 1)$.

Convex combination of two points represent a point on the straight line joining those two points. We now define concavity and convexity of functions using this concept.

Definition 9. f is concave if for any two points $x^{'}, x^{''} \in \mathbb{R}$, $f(x_\lambda) \geq \lambda f(x^{'}) + (1-\lambda) f(x^{''})$ where x_λ is a convex combination of $x^{'}$ and $x^{''}$ for $\lambda \in (0,1)$. f is strictly concave if the inequality is strict.

Definition 10. Similarly f is convex if $f(x_\lambda) \leq \lambda f(x^{'}) + (1-\lambda) f(x^{''})$. f is strictly convex if the inequality is strict.

If the utility function is concave for any individual then, given this definition, we can understand that, she would prefer to have a certain consumption of x_λ than face an uncertain prospect of consuming either $x^{'}$ or $x^{''}$. Such individuals are called risk averse. We shall explore these concepts in full detail later in the course and then we would require these definitions of concavity and convexity.