Countering the Winner’s Curse: Auction Design in a Common Value Model

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Interdependence and Winner’s Curse

- interdependence in values across bidders is frequent in auctions
  - wildcatters bidding for an oil tract ...
  - investment banks competing for shares in IPO’s...
  - lenders competing in syndicated loan-markets ...

- winning the object is informative about value estimate of competing bidders
- each bidder must carefully account for the interdependence in individual bidding behavior
- winner’s curse: unconditional vs conditional expectation
• consider bidding for a natural resource, such as an oil tract
• richer samples suggest more oil reserves and induce higher bids
• winning means that the other samples’ were relatively weak
• a winning bidder therefore faces adverse selection
• the expected value of the tract conditional on winning is less than the unconditional expectation
Winner’s Curse and Auction Design

- winner’s curse results in bid shading and lower revenues
- how can auction design attenuate the winner’s curse...
- how can the resulting selection impact revenue: adverse, neutral or advantageous
- today: what is the revenue maximizing selling mechanism?
- prior literature has largely focused on private value

→ thus a world without winner’s curse and selection issues
Auction Design in A Common Value Model

- a pure common value model
- private signal gives partial information about common value
- key statistical feature:
  higher signals contain more information about common value than lower signals
- today:
  → highest signal is sufficient statistic of common value
  → lower signals carry no additional information
• characterize revenue maximizing mechanism
• maximal revenue is obtained by strikingly simple mechanism, stated at interim level (given signal of bidder $i$)

1. constant – signal independent – price
2. constant – signal independent – probability of getting object

• contrast with first, second, or ascending auction in an environment with private values
Revenue Maximizing Design: Posted Price

- optimal mechanism shares some features with posted price

1. constant – signal independent – price

- it coincides with posted price if

2. constant – signal independent – probability is $1/N$

- necessary and sufficient condition when optimal mechanism reduces exactly to posted price
- if posted price is an optimal mechanism it is inclusive: every bidder with every signal realization is willing to buy
in general, aggregate assignment probability is $< 1$
interim probability of getting object is constant and $< 1/N$
ex post probability for $i$ then depends on entire signal profile
conditionally on allocating the object optimal mechanism:

1. favors bidders with lower signals
2. discriminates against bidder with highest signal

• “winner’s blessing” rather than “winner’s curse”
• advantageous rather than adverse selection
Contributions: Substantive

- setting where bidders with higher signals have more accurate information about common value;
- arises in market with intermediaries, and many other settings: auctions for resources, IPO’s
- countervailing screening incentives, tension between selling to
  1. bidder with higher expected value and
  2. bidder with less private information
  - optimal to screen “less” - with no screening in inclusive limit
  - foundation for posted price mechanisms
Contributions: Methodological

- very few results extend characterization of optimal auctions beyond private value case
- we extend optimal auctions into interdependent values:
  1. with private values, “local” incentive constraints are sufficient to pin down optimal mechanism
  2. with interdependent values, “global” constraints matter, new arguments are required
Model
Common Value Model

- $N$ bidders compete for a single object
- bidder $i$ receives signal $s_i$:
  \[ s_i \in [s, \bar{s}] \subset \mathbb{R}_+ \]
  according to absolutely continuous distribution $F(s_i), f(s_i)$
- common value is the maximum of $N$ independent signals:
  \[ v(s_1, \ldots, s_N) \triangleq \max \{ s_1, \ldots, s_N \} \]
- “maximum signal model”
- signal distribution $F(s_i)$ induces value distribution $G_N(v)$:
  \[ G_N(v) = (F(s))^N \]
- common value is first-order statistic of $N$ independent signals
Two Interpretations

- maximum signal model

$$v(s_1, \ldots, s_N) = \max\{s_1, \ldots, s_N\}$$

- two leading interpretations:

1. common value model with informational implications:

   - higher signal realizations contain more information about common value than lower signal realizations
   - specifically, conditional on highest signal, the other signals contain no additional information about the common value
   - drilling/sampling for mineral rights (Bulow and Klemperer (2002))
Two Interpretations

- maximum signal model

\[ v(s_1, \ldots, s_N) = \max \{s_1, \ldots, s_N\} \]

- two leading interpretations:

2. **private value model** of intermediary (dealer) market

   - each intermediary bidder receives the signal (sample) about the downstream trading opportunities
   - final sale in downstream market is open to all intermediaries
   - IPO, syndicated loan-markets, inter-dealer markets

(Viswanathan and Wang (2004))
Utility and Allocation

- bidder $i$ is expected utility maximizer with quasilinear preferences, probability $q_i$ of receiving object and transfers $t_i$:

$$u_i(s, q_i, t_i) = v(s) q_i - t_i$$

- feasibility of auction

$$q_i(s) \geq 0, \quad \text{with} \quad \sum_{i=1}^{N} q_i(s) \leq 1$$

- ex post transfer $t_i(s)$ of bidder $i$, interim expected transfer:

$$t_i(s_i) = \int_{s_{-i} \in S^{N-1}} t_i(s_i, s_{-i}) f_{-i}(s_{-i}) ds_{-i},$$

where

$$f_{-i}(s_{-i}) = \prod_{j \neq i} f(s_j)$$
Incentive Compatibility

• bidder $i$ surplus when reporting $s'_i$ while observing $s_i$:

$$u_i(s_i, s'_i) \equiv \int_{s_{-i} \in S^{N-1}} q_i(s'_i, s_{-i}) \vee (s_i, s_{-i}) f_{-i}(s_{-i}) \, ds_{-i} - t_i(s'_i)$$

• indirect utility given truthtelling is:

$$u_i(s_i) \equiv u_i(s_i, s_i)$$

• direct mechanism $\{q_i, t_i\}_{i=1}^N$ is incentive compatible (IC) if

$$u_i(s_i) \geq u_i(s_i, s'_i), \text{ for all } i \text{ and } s_i, s'_i \in S$$

• ... is individually rational (IR) if $u_i(s_i) \geq 0$, for all $i$ and $s_i \in S$
The Winner’s Curse
• second-price auction in maximum signal model:

$$b_i(s_i)$$

• bid of bidder $i$ is based on his interim expectation:

$$\mathbb{E}[v(s_1, ..., s_N) | s_i]$$

• signal $s_i$ is \textbf{sharp lower bound} on ex post (realized) value:

$$s_i \leq v(s_1, ..., s_N),$$

• signal $s_i$ is lower bound for interim expectation of value:

$$s_i < \mathbb{E}[v(s_1, ..., s_N) | s_i]$$
Winner’s Curse in Second Price Auction

- bidder with highest signal wins in second price auction
- equilibrium bid is given by:
  \[ b_i(s_i) = s_i \]
- bids as-if private value \( s_i \), not common value \( \max\{s_1, \ldots, s_N\} \)
- conditional on winning, signal \( s_i \) turns into sharp upper bound:
  \[ v(s_1, \ldots, s_N) = \max\{s_1, \ldots, s_N\} \leq s_i \]
- this is the curse:
  
  1. when bidding, \( s_i \) is sharp lower bound of expectation of value
  2. when winning, \( s_i \) is sharp upper bound of expectation of value
Winner’s Curse and Adverse Selection

- adverse selection:
  winner learns his signal is most favorable of all signals
- selection as winner is adverse information to winner
- magnitude of adverse selection is controlled by change in expectation from ex-interim to ex-post:

1. when bidding, $s_i$ is sharp lower bound of expectation of value
2. when winning, $s_i$ is sharp upper bound of expectation of value

- structure of information controls strength of winner’s curse
- winner’s curse lowers bids, thus lowers revenue of auctioneer
- maximal winner’s curse is quantified by minimal revenue (in any given auction format)
An Aside:

Magnitude of Winner’s Curse
Magnitude of the Curse

• can we quantify the winner’s curse?
• can we identify maximal winner’s curse which generates minimal revenue?
• how does it relate to the structure of private information of bidders?
• making it operational
• consider all possible information structures for a fixed distribution of values,
• thus look at all Bayes correlated equilibria of the auction (ECTA, 2017)
Information and Winner’s Curse

- fix a distribution of (common) values with $N$ bidders:
  \[ G_N(v) \]

- ask how different common prior distribution of signals:
  \[ F(s | v) \]

impact bidding and revenue for fixed distribution $G_N(v)$

- maximum signal model: an example of information structure, others are wallet game, affiliated mineral rights model, etc.
“Revenue Guarantee Equivalence” (AER forthcoming) finds:

1. equivalence: the maximum signal model attains the same revenue in all standard auctions: first-price, second-price, ascending auction, etc.

2. guarantee: the maximum signal model generates the lowest revenue across all information structures in every standard auction

- sharp revenue guarantee through maximum signal model ...
- ... across all standard auction formats
- revenue minimizing is winner’s curse maximizing:

\[ v(s_1, \ldots, s_N) = \max \{s_1, \ldots, s_N\} \]
• standard auction (with no reserve prices) with two bidders
• revenue and bidders surplus in all information structures

Figure 1: Revenue and Bidder Utility across All Information Structures
Structure of Incentive Constraints

- structure of incentive constraints in maximum signal model
- all upward deviations—relative to unique equilibrium bid—yield the equilibrium net utility
- all upward deviations are binding:
  \[ b' \in [b_i(s_i), b_i(\bar{s})], \quad \forall s_i \in [s, \bar{s}] \]
- global rather than local inventive constraints matter, everywhere!
- global constraints matter in all standard auction formats!
Figure 2: Uniform Upward Incentive Constraints and Winner’s Curse

- counter the curse: find optimal auction
Counter the Curse
Adverse Selection and Winner’s Curse

- assigning object to highest bidder conveys (too) much information to the winner
- adverse selection: winner learns that his signal was more favorable than all other signals
- winning bid is depressed by adverserial selection of winner
- what about neutral selection of winner?
- a neutral (symmetric) selection must be a random allocation among the bidders
- event of winning does not convey any additional information to the winner
Neutral Selection: Inclusive Posted Price

- a specific neutral selection
- every bidder receives the object with equal probability $1/N$
- every winning bidder is charged a posted price

$$p \triangleq \int_{s_i} v(s, s_{-i}) f_{-i}(s_{-i}) \, ds_{-i}$$

- even bidder with lowest signal, $s_i = s$, is willing to buy at $p$,
- thus $p$ is inclusive, does not exclude any signal $s_i$ for any $i$
Revenue Improvement I

- how does inclusive posted price fare?

Proposition

The inclusive posted price yields a (weakly) higher revenue than absolute first-price, second-price or ascending price auction.

- Bulow-Klemperer (2002) establish second-price auction ranking
- notable features of inclusive posted price

1. random allocation—rather than deterministic allocation
2. constant allocation in signal – rather than increasing in signal
3. no selection on either signal or value, thus no screening
Neutral Selection and Exclusion

• exclusion—not selling the object when the value is low—may increase the revenue

• in private value environments it famously does: Myerson (1981)

• can neutral selection be maintained with exclusion?
Two Tier Price Mechanism

- uniform exclusion at a threshold $r$:

$$q_i(s) = \begin{cases} \frac{1}{N} & \text{if } \max s \geq r; \\ 0 & \text{otherwise}. \end{cases}$$

- supported by two-tier price:

1. a preferred price (unconditional sale):

   $$p_u \triangleq r,$$

2. a standard price (conditional sale):

   $$p_c \triangleq \int_r^{\bar{s}} \max \{ s_i \} \, dF_{-i}(s) \frac{1}{1 - F_{N-1}(r)} > r = p_u,$$

   $\iff$ right censored first order statistic of $N - 1$ samples
Two-Tier Price Mechanism

• object is sold if and only if at least one bidder is willing to make an unconditional purchase at

\[ p_u = r \]

• then all the remaining bidders get object with probability \( 1/N \) at price

\[
p_c \triangleq \frac{\int_r^s \max \{ s_i \} \, dF_i(s)}{1 - F^{N-1}(r)}
\]

• with one exception... if more than one bidder requests unconditional purchase, then all bidders get object at \( p_c \)
Proposition (Two-Tier Pricing)

A two-tier pricing \((p_c, p_u)\) yields a (weakly) higher revenue than any other inclusive or exclusive posted price.

- standard price \(p_c\) could be offered equivalently as random price:
  \[ p \triangleq \max \{ r, s_{-i} \} \]
- resulting mechanism is ex-post incentive compatible and ex-post individually rational
- but neither as dominant strategy!
Implications of Two-Tier Price

- uniform screening among bidders with respect to highest signal
- uniform exclusion among bidders’
- winning at generates winner’s blessing:
  \[ E[v(s_1, ..., s_N) | s_i] < E[v(s_1, ..., s_N) | s_i, x_i > 0] \]
- two-tiered pricing similar to syndicated loan arrangement: one for lead lender, and one for all syndicate lenders
- turned from adverse to neutral selection
- now turn from neutral to to advantageous selection!
• there is a fixed reserve price $r$ and a random reserve price $x > r$
• if bidder $i$ reports highest signal $s_i > r$, then:

1. he receives priority status,
2. he is offered object at price:
\[ p \triangleq \max \{ x, s_i \} \]

• otherwise, other bidders receive object with probability
\[ 1/(N - 1), \]

• if at least one bidder has declared priority status and pay price:
\[ p \triangleq \max \{ r, s_i \} = v(s_1, ..., s_N). \]
Random Reserve Price

• reserve price $r^*$ is smallest solution to:

$$x - \int_{y=x}^{\bar{s}} \frac{1 - F(y)}{F(y)} dy = 0$$

• distribution of random reserve price is:

$$H^*(x) = \frac{1}{N}(1 - \frac{F^N(r)}{F^N(x)})$$

• resulting mechanism is **interim** incentive compatible and **ex-post** individually rational

• higher signal guarantee higher probability of getting the object
Final Revenue Improvement

- additional revenue from the bidder with the highest signal

Theorem (Random Reserve Price)

The random reserve price mechanism \((r^*, H^*)\) is a revenue maximizing mechanism.

- interim probability of receiving object is constant in signal \(s_i\)
- interim transfer is constant in signal \(s_i\)
- advantageous selection
- all downward incentive constraints are binding!
• with random reserve price, each bidder is indifferent between his equilibrium bid and any lower bid

Figure 3: Uniform Downward Incentive Constraints
A Study in Contrasts

- optimal vs standard mechanisms
- exactly flip the orientation of the constraints, and more...

Figure 4: Uniform Downward vs Upward Incentive Constraints
Bounds on
Bidder Surplus and Revenue
A New Problem

- how to establish the optimality of the mechanism?
- evidently, the local constraints are binding, but many others, non-local constraints are binding as well
- thus, we need to consider local as well global constraints
- but which ones?
- analyze a relaxed problem which consists of local and small class of global constraints
- use these constraints to derive:

1. an upper bound on seller revenue
2. a lower bound on bidder utility
A Relaxed Problem

- consider a smaller–one-dimensional–family of constraints:
- instead of reporting signal $s_i$, report a random signal
  \[ s'_i < s_i, \]
  drawn from truncated prior on support $[s, s_i]$:
  \[ F(s'_i) / F(s_i) \]
- misreporting a redrawn lower signal
A Lower Bound on Bidder Utility

- what are the gains from misreporting a redrawn lower signal?
- equilibrium surplus of a bidder with type $x$ is
  - from envelope condition of local constraints:
    $$u_i(s_i) = \int_{x=s_i}^{\hat{q}_i(x)} dx$$
- surplus from misreporting the redrawn lower signal
  $$\frac{1}{F(s_i)} \int_{x=s_i}^{u_i(s_i, x)} f(x) dx$$
- gains vary depending on realized misreport
  average gains across all misreports are easy to compute
Average Gains from Misreporting

- misreport is redrawn from prior, bidder $i$ is equally likely to fall anywhere in distribution of signals, unconditional on misreport, ex-ante likelihood that $i$ receives object and $x$ is highest signals

$$q_i(x) g_N(x)$$

- if highest report is less than $s_i$, surplus that bidder $i$ obtains from being allocated object is $s_i$ rather than $x$, so $s_i - x$ is difference between deviator and truthtelling surplus:

$$\frac{1}{F(s_i)} \int_{x=s}^{s_i} [(s_i - x) q_i(x) g_N(x) + u_i(x) f(x)] \, dx$$

- thus the incentive constraint requires:

$$u_i(s_i) \geq \frac{1}{F(s_i)} \int_{x=s}^{s_i} [(s_i - x) q_i(x) g_N(x) + u_i(x) f(x)] \, dx$$
Lower Bound As Equality

- lower bound of bidder’s surplus through small class of deviations:
  \[ u_i(s_i) \geq \frac{1}{F(s_i)} \int_{s_i}^{s_i} [(s_i - x) q_i(x) g_N(x) + u_i(x) f(x)] \, dx \]

- inequality hold as sum across all \( i \):
  \[ u(s) \geq \frac{1}{F(s)} \int_{x=s}^{s} [(s - x) q(x) g_N(x) + u(x) f(x)] \, dx \]

- lowest solution \( u(s) \) exists and solves inequality as equality
- monotonic operator on increasing functions has unique smallest fixed point by Knaster-Tarski fixed point
- can be integrated by parts as
  \[ U = \int_{x \in S} u(s) f(s) \, ds = \int_s^s \left( \int_{x=s}^{s} \frac{1 - F(x)}{F(x)} \, dx \right) q(s) g_N(s) \, ds \]
A Generalized Virtual Utility Formula

• with the lower bound on bidder surplus:

\[ U = \int_{x \in S} u(s) f(s) \, ds = \int_s \left( \int_{x=s}^s \frac{1 - F(x)}{F(x)} \, dx \right) q(s) g_N(s) \, ds \]

• we obtain our final formula for revenue, which is

\[ \bar{R} = TS - U = \int_v \psi(v) q(v) g_N(v) \, dv \]

where

\[ \psi(v) = v - \int_{x=v}^{\bar{s}} \frac{1 - F(x)}{F(x)} \, dx, \]

• compare to virtual utility in private value environments:

\[ \pi(x) = x - \frac{1 - F(x)}{f(x)} \]
• generalized virtual utility:

\[ \psi(x) = x - \int_{y=x}^{\bar{s}} \frac{1 - F(y)}{F(y)} \, dy, \]

Theorem (Revenue Upper Bound)

In any auction in which the probability of allocation is given by \( q \), bidder surplus is bounded below by \( \underline{U} \) and expected revenue is bounded above by \( \overline{R} \).

• bound is valid for any allocation policy \( q(v) \)

Corollary (Random Reserve Price)

The random reserve price mechanism attains the revenue upper bound.
Posted Price
As Optimal Mechanism
Posted Prices

• consider mechanisms where object is always allocated
• pure common values – allocation is therefore socially efficient

Theorem (Revenue Optimality among Efficient Mechanisms)
Among all mechanisms that allocate the object with probability one, revenue is maximized by setting a posted price of

\[ p = \int_{\bar{s}}^s v g_{N-1}(v) \, dv, \]

i.e., expected value of object conditional on having lowest signal \( \bar{s} \).

• posted price is inclusive: all types purchase at \( p \)
• all bidders equally likely to receive object: \( q_i(v) = 1/N, \forall i, v \).
• optimal selling mechanism is attained with constant interim transfer \( t = t_i(s_i) \) and probability \( q = q_i(s_i) \)
Optimality of Posted Price

- next, optimality of posted price among all possibly inefficient mechanisms

Corollary (Revenue Optimality ofPosted Prices)

A posted price mechanism is optimal if and only if

$$\psi(s) = s - \int_s^\bar{s} \frac{1 - F(x)}{F(x)} \, dx \geq 0.$$ 

If a posted price $p$ is optimal, then it is fully inclusive.
The Power of Optimal Auctions
• Bulow and Klemperer (1996) establish the limited power of optimal mechanisms as opposed to standard auction formats.

• Revenue of optimal auction with $N$ bidders is strictly dominated by standard absolute auction with $N + 1$ bidders.

• Current common value environment is an instance of general interdependent value setting – with one exception.

• Virtual utility function—or marginal revenue function—is not monotone due to maximum operator in common value model.
A Closer Look at the Virtual Utility

- non-monotonicity leads to an optimal mechanism with features distinct from standard first or second price auction.
- it elicits information from bidder with highest signal but minimizes probability of assigning him the object subject to incentive constraint
- virtual utility of each bidder, $\pi_i(s_i, s_{-i})$:

$$
\pi_i(s_i, s_{-i}) = \begin{cases} 
  \max_j \{s_j\}, & \text{if } s_i \leq \max \{s_{-i}\}; \\
  \max \{s_j\} - \frac{1-F_i(s_i)}{f_i(s_i)}, & \text{if } s_i > \max \{s_{-i}\}.
\end{cases}
$$
- downward discontinuity in virtual utility indicates why seller wishes to minimize the probability of assigning the object to the bidder with the high signal
• virtual utility of bidder \( i \) fails monotonicity assumption even when hazard rate of distribution function is increasing everywhere

• BK (1996) require monotonicity of virtual utility when establishing their main result that an absolute English auction with \( N + 1 \) bidders is more profitable than any optimal mechanism with \( N \) bidders

• revenue ranking does not extend to current auction environment

• compare revenue from optimal auction with \( N \) bidders to absolute, English or second-price, auction with \( N + K \) bidders

• absolute as there is no reserve price imposed
Theorem (Revenue Comparison)

For every \( N \geq 1 \) and every \( K \geq 1 \), the revenue from an absolute second-price auction with \( N + K \) bidders is strictly dominated by the revenue of an optimal auction with \( N \) bidders.

- comparison of second order statistic of \( N + K \) i.i.d. signals and first order statistic of \( N + K - 1 \) i.i.d. signals
- second order statistic of \( N + K \) signals is revenue of absolute second-price auction with \( N + K \) bidders.
- by earlier Theorem, optimal mechanism (weakly) exceeds revenue from a posted price set equal to the maximum of \( N + K - 1 \) signals.
• but pure common value of the object is not affected by number of bidders, it is as if the remaining $K$ signals are simply not disclosed, but the $N$ participating bidders still form the expectation over the $N + K - 1$ signals.

• now, if instead of $N + K$ bidders, the optimal auction only has $N$ bidders, then it is as if only $N$ independent and identical distributed signals are revealed to the $N$ bidders

• thus an attainable revenue for the seller is to offer the object at random to a bidder at a posted price set equal to the maximum of $N + K - 1$ signals
Conclusion

- characterized novel revenue maximizing auctions for a class of common value models
- common value models with qualitative feature that values are more sensitive to private information of bidders with more optimistic beliefs
- second interpretation as auction with intermediary/resale market
- countering the winner’s curse
- optimal auctions discriminate in favor of less optimistic bidders since they obtain less information rents from being allocated the object