Countering the Winner's Curse: Auction Design in a Common Value Model

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Interdependence and Winner's Curse

- interdependence in values across bidders is frequent in auctions

$\rightarrow$ wildcatters bidding for an oil tract ...

$\rightarrow$ investment banks competing for shares in IPO's...

$\rightarrow$ lenders competing in syndicated loan-markets ...

- winning the object is informative about value estimate of competing bidders
- each bidder must carefully account for the interdependence in individual bidding behavior
- winner's curse: unconditional vs conditional expectation

Winner's Curse and Adverse Selection

- consider bidding for a natural resource, such as an oil tract
- richer samples suggest more oil reserves and induce higher bids
- winning means that the other samples' were relatively weak
- a winning bidder therefore faces adverse selection
- the expected value of the tract conditional on winning is less than the unconditional expectation

Winner's Curse and Auction Design

- winner's curse results in bid shading and lower revenues
- how can auction design attenuate the winner's curse...
- how can the resulting selection impact revenue:
  adverse, neutral or advantageous
- today: what is the revenue maximizing selling mechanism?
- prior literature has largely focused on private value

$\rightarrow$ thus a world without winner's curse and selection issues

Auction Design in A Common Value Model

- a pure common value model
- private signal gives partial information about common value
- key statistical feature:
  higher signals contain more information about common value than lower signals
- today:
  $\rightarrow$ highest signal is sufficient statistic of common value
  $\rightarrow$ lower signals carry no additional information

Revenue Maximizing Design

- characterize revenue maximizing mechanism
- maximal revenue is obtained by strikingly simple mechanism, stated at interim level (given signal of bidder i)

1. constant – signal independent – price
2. constant – signal independent – probability of getting object

- contrast with first, second, or ascending auction in an environment with private values

Revenue Maximizing Design: Posted Price

- optimal mechanism shares some features with posted price

1. constant – signal independent – price

- it coincides with posted price if

2. constant – signal independent – probability is $1/N$

- necessary and sufficient condition when optimal mechanism reduces exactly to posted price
- if posted price is an optimal mechanism it is inclusive: every bidder with every signal realization is willing to buy

Revenue Maximizing Design: Beyond Posted Price

- in general, aggregate assignment probability is < 1
- interim probability of getting object is constant and $< 1/N$
- ex post probability for i then depends on entire signal profile
- conditionally on allocating the object optimal mechanism:

1. favors bidders with lower signals
2. discriminates against bidder with highest signal

- "winner's blessing" rather than "winner's curse"
- advantageous rather than adverse selection

Contributions: Substantive

- setting where bidders with higher signals have more accurate information about common value;
- arises in market with intermediaries, and many other settings: auctions for resources, IPO's
- countervailing screening incentives, tension between selling to

1. bidder with higher expected value and
2. bidder with less private information

- optimal to screen "less" - with no screening in inclusive limit
- foundation for posted price mechanisms

Contributions: Methodological

- very few results extend characterization of optimal auctions beyond private value case
- we extend optimal auctions into interdependent values:

1. with private values, “local” incentive constraints are sufficient to pin down optimal mechanism
2. with interdependent values, “global” constraints matter, new arguments are required

Model

Common Value Model

- N bidders compete for a single object
- bidder i receives signal s_i :

$$s_i \in [\underline{s}, \bar{s}] \subset \mathbb{R}_+$$

  according to absolutely continuous distribution $F(s_i), f(s_i)$
- common value is the maximum of N independent signals:

$$v(s_1, \ldots, s_N) \triangleq \max\{s_1, \ldots, s_N\}$$

- "maximum signal model"
- signal distribution $F(s_i)$ induces value distribution $G_N(v)$:

$$G_N(v) = (F(s))^N$$

- common value is first-order statistic of N independent signals

Two Interpretations

- maximum signal model

$$v(s_1, \ldots, s_N) = \max\{s_1, \ldots, s_N\}$$

- two leading interpretations:

1. common value model with informational implications:
    - higher signal realizations contain more information about common value than lower signal realizations
    - specifically, conditional on highest signal, the other signals contain no additional information about the common value
    - drilling/sampling for mineral rights (Bulow and Klemperer (2002))

Two Interpretations

- maximum signal model

$$v(s_1, \ldots, s_N) = \max\{s_1, \ldots, s_N\}$$

- two leading interpretations:

2. private value model of intermediary (dealer) market

   - each intermediary bidder receives the signal (sample) about the downstream trading opportunities
   - final sale in downstream market is open to all intermediaries
   - IPO, syndicated loan-markets, inter-dealer markets (Viswanathan and Wang (2004))

Utility and Allocation

- bidder i is expected utility maximizer with quasilinear preferences, probability q_i of receiving object and transfers t_i:

$$u_i\left(s, q_i, t_i\right) = v\left(s\right) q_i - t_i$$

- feasibility of auction

$$q_i\left(s\right) \geq 0, \text{ with } \sum_{i=1}^{N} q_i\left(s\right) \leq 1$$

- *ex post* transfer $t_i\left(s\right)$ of bidder i, *interim* expected transfer:

$$t_i\left(s_i\right) = \int_{s_{-i} \in S^{N-1}} t_i\left(s_i, s_{-i}\right) f_{-i}\left(s_{-i}\right) ds_{-i},$$

where

$$f_{-i}\left(s_{-i}\right) = \prod_{j \neq i} f\left(s_j\right)$$

Incentive Compatibility

- bidder i surplus when reporting s_i' while observing s_i:

$$u_i\left(s_i, s_i'\right) \equiv \int_{s_{-i} \in S^{N-1}} q_i\left(s_i', s_{-i}\right) v\left(s_i, s_{-i}\right) f_{-i}\left(s_{-i}\right) ds_{-i} - t_i\left(s_i'\right)$$

- indirect utility given truthtelling is:

$$u_i(s_i) \equiv u_i\left(s_i, s_i\right)$$

- direct mechanism $\{q_i, t_i\}_{i=1}^N$ is *incentive compatible (IC)* if

$$u_i\left(s_i\right) \geq u_i\left(s_i, s_i'\right), \text{ for all } i \text{ and } s_i, s_i' \in S$$

- *... is individually rational (IR)* if $u_i\left(s_i\right) \geq 0$, for all i and $s_i \in S$

The Winner’s Curse

Warm-Up: Second Price Auction

- second-price auction in maximum signal model:

$$b_i(s_i)$$

- bid of bidder i is based on his interim expectation:

$$\mathbb{E}[v(s_1, ..., s_N) \,|s_i]$$

- signal s_i is sharp lower bound on ex post (realized) value:

$$s_i \leq v(s_1, ..., s_N),$$

- signal s_i is lower bound for interim expectation of value:

$$s_i < \mathbb{E}[v(s_1, ..., s_N) \,|s_i]$$

Winner's Curse in Second Price Auction

- bidder with highest signal wins in second price auction
- equilibrium bid is given by:

$$b_i(s_i) = s_i$$

- bids as-if private value s_i, not common value $\max\{s_1, ..., s_N\}$
- conditional on winning, signal s_i turns into sharp upper bound:

$$v(s_1, ..., s_N) = \max\{s_1, ..., s_N\} \leq s_i$$

- this is the curse:

1. when bidding, s_i is sharp lower bound of expectation of value
2. when winning, s_i is sharp upper bound of expectation of value

Winner's Curse and Adverse Selection

- adverse selection:
  winner learns his signal is most favorable of all signals
- selection as winner is adverse information to winner
- magnitude of adverse selection is controlled by change in expectation from ex-interim to ex-post:

1. when bidding, s_i is sharp lower bound of expectation of value
2. when winning, s_i is sharp upper bound of expectation of value

- structure of information controls strength of winner's curse
- winner's curse lowers bids, thus lowers revenue of auctioneer
- maximal winner's curse is quantified by minimal revenue
  (in any given auction format)

An Aside:
Magnitude of Winner's Curse

Magnitude of the Curse

- can we quantify the winner's curse ?
- can we identify maximal winner's curse which generates minimal revenue?
- how does it relate to the structure of private information of bidders?
- making it operational
- consider all possible information structures for a fixed distribution of values,
- thus look at all Bayes correlated equilibria of the auction (ECTA, 2017)

Information and Winner's Curse

- fix a distribution of (common) values with N bidders:

$$G_N(v)$$

- ask how different common prior distribution of signals:

$$F(s\,|\,v)$$

  impact bidding and revenue for fixed distribution $G_N(v)$
- maximum signal model: an example of information structure, others are wallet game, afiliated mineral rights model, etc.

Revenue Minimum

- "Revenue Guarantee Equivalence" (AER forthcoming) finds:

1. equivalence: the maximum signal model attains the same revenue in all standard auctions: first-price, second-price, ascending auction, etc.
2. guarantee: the maximum signal model generates the lowest revenue across all information structures in every standard auction

- sharp revenue guarantee through maximum signal model ...
- ... across all standard auction formats
- revenue minimizing is winner's curse maximizing:

$$v(s_1, \dots, s_N) = \max\{s_1, \dots, s_N\}$$

A Visualization

- standard auction (with no reserve prices) with two bidders
- revenue and bidders surplus in all information structures

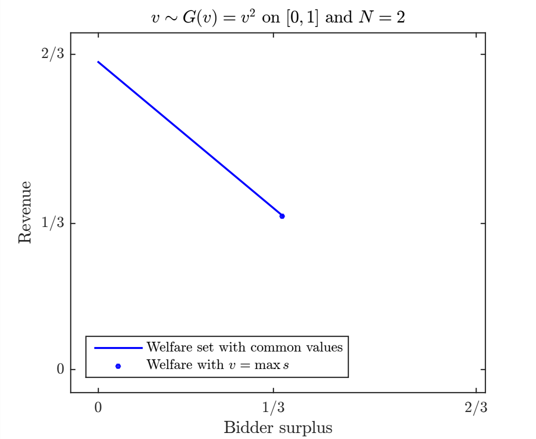

Figure 1: Revenue and Bidder Utility across All Information Structures

Structure of Incentive Constraints

- structure of incentive constraints in maximum signal model
- all upward deviations–relative to unique equilibrium bid–yield the equilibrium net utility
- all upward deviations are binding:

$$b' \in [b_i(s_i), b_i(\overline{s})], \quad \forall s_i \in [\underline{s}, \overline{s}]$$

- global rather than local inventive constraints matter, everywhere!
- global constraints matter in all standard auction formats!

Upward Deviations

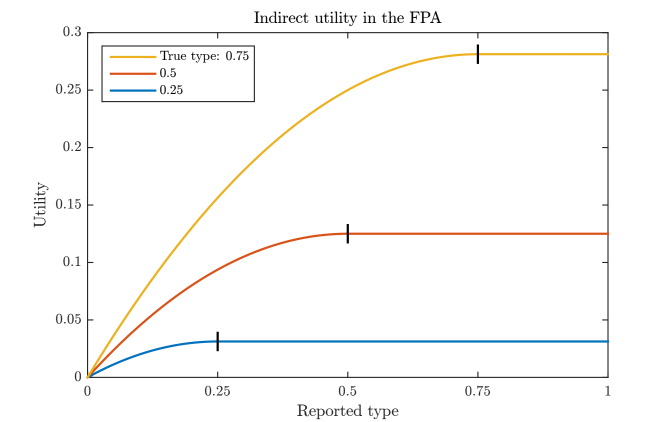

Figure 2: Uniform Upward Incentive Constraints and Winner's Curse

- counter the curse: find optimal auction

Counter the Curse

Adverse Selection and Winner's Curse

- assigning object to highest bidder conveys (too) much information to the winner
- adverse selection: winner learns that his signal was more favorable than all other signals
- winning bid is depressed by adverserial selection of winner
- what about neutral selection of winner?
- a neutral (symmetric) selection must be a random allocation among the bidders
- event of winning does not convey any additional information to the winner

Neutral Selection: Inclusive Posted Price

- a specific neutral selection
- every bidder receives the object with equal probability $1/N$
- every winning bidder is charged a posted price

$$p \triangleq \int_{s_{-i}} v\left(\underline{s}, s_{-i}\right) f_{-i}\left(s_{-i}\right) ds_{-i}$$

- even bidder with lowest signal, $s_i = \underline{s}$, is willing to buy at p,
- thus p is inclusive, does not exlude any signal s_i for any i

Revenue Improvement I

- how does inclusive posted price fare?

Proposition

The inclusive posted price yields a (weakly) higher revenue than absolute first-price, second-price or ascending price auction.

- Bulow-Klemperer (2002) establish second-price auction ranking
- notable features of inclusive posted price

1. random allocation–rather than deterministic allocation
2. constant allocation in signal – rather than increasing in signal
3. no selection on either signal or value, thus no screening

Neutral Selection and Exclusion

- exclusion–not selling the object when the value is low–may increase the revenue
- in private value environments it famously does: Myerson (1981)
- can neutral selection be maintained with exclusion?

Two Tier Price Mechanism

- uniform exclusion at a threshold r:

$$q_i(s) = \begin{cases} \frac{1}{N} & \text{if } \max s \geq r; \\ 0 & \text{otherwise.} \end{cases}$$

- supported by two-tier price:

1. a preferred price (unconditional sale):

$$p_u \triangleq r,$$

2. a standard price (conditional sale):

$$p_c \triangleq \frac{\int_r^{\bar{s}} \max\{s_{-i}\}\, dF_{-i}(s)}{1 - F^{N-1}(r)} > r = p_u,$$

$\Leftrightarrow$ right censored first order statistic of $N-1$ samples

Two-Tier Price Mechanism

- object is sold if and only if at least one bidder is willing to make an unconditional purchase at

$$p_u = r$$

- then *all the remaining* bidders get object with probability $1/N$ at price

$$p_c \triangleq \frac{\int_r^{\bar{s}} \max\{s_{-i}\}\, dF_{-i}(s)}{1 - F^{N-1}(r)}$$

- with one exception... if more than one bidder requests unconditional purchase, then all bidders get object at p_c

Revenue Improvement II

Proposition (Two-Tier Pricing)

A two-tier pricing (p_c, p_u) yields a (weakly) higher revenue than any other inclusive or exclusive posted price.

- standard price p_c could be offered equivalently as random price:

$$p \triangleq \max\{r, s_{-i}\}$$

- resulting mechanism is ex-post incentive compatible and ex-post individually rational
- but neither as dominant strategy!

Implications of Two-Tier Price

- uniform screening among bidders with respect to highest signal
- uniform exclusion among bidders'
- winning at generates winner's blessing:

$$\mathbb{E}[v(s_1, ..., s_N)\,|s_i] < \mathbb{E}[v(s_1, ..., s_N)\,|s_i, x_i > 0]$$

- two-tiered pricing similar to syndicated loan arrangement: one for lead lender, and one for all syndicate lenders
- turned from adverse to neutral selection
- now turn from neutral to to advantageous selection!

Revenue Improvement III

- there is a fixed reserve price r and a random reserve price $x > r$
- if bidder i reports highest signal $s_i > r$, then:

1. he receives priority status,
2. he is offered object at price:

$$p \triangleq max\{x, s_{-i}\}$$

- otherwise, other bidders receive object with probability

$$1/(N-1),$$

- if at least one bidder has declared priority status and pay price:

$$p \triangleq \max\{r, s_{-i}\} = v(s_1, ..., s_N).$$

Random Reserve Price

- reserve price r^* is smallest solution to:

$$x - \int_{y=x}^{\overline{s}} \frac{1 - F(y)}{F(y)} dy = 0$$

- distribution of random reserve price is:

$$H^*(x) = \frac{1}{N}(1 - \frac{F^N(r)}{F^N(x)})$$

- resulting mechanism is interim incentive compatible and ex-post individually rational
- higher signal guarantee higher probability of getting the object

Final Revenue Improvement

- additional revenue from the bidder with the highest signal

Theorem (Random Reserve Price)

The random reserve price mechanism (r^*, H^*) *is a revenue maximizing mechanism.*

- interim probability of receiving object is constant in signal s_i
- interim transfer is constant in signal s_i
- advantageous selection
- all downward incentive constraints are binding!

A Visualization

- with random reserve price, each bidder is indifferent between his equilibrium bid and any lower bid

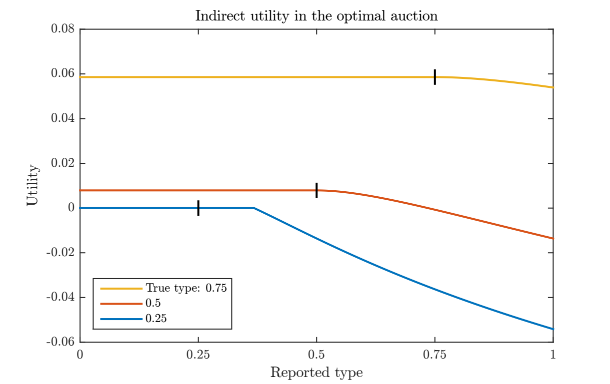

Figure 3: Uniform Downward Incentive Constraints

A Study in Contrasts

- optimal vs standard mechanisms
- exactly flip the orientation of the constraints, and more...

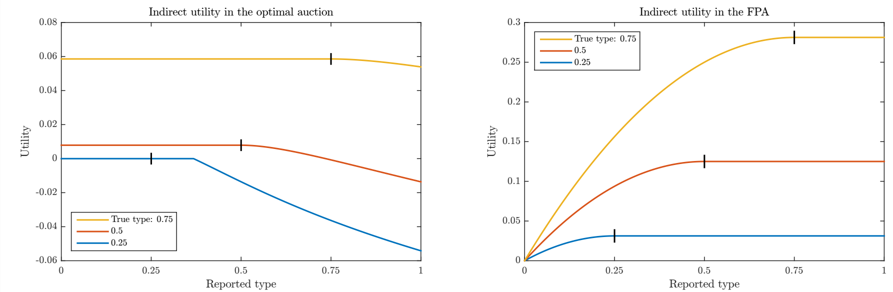

Figure 4: Uniform Downward vs Upward Incentive Constraints

Bounds on
Bidder Surplus and Revenue

A New Problem

- how to establish the optimality of the mechanism?
- evidently, the local constraints are binding, but many others, non-local constraints are binding as well
- thus, we need to consider local as well global constraints
- but which ones?
- analyze a relaxed problem which consists of local and small class of global constraints
- use these constraints to derive:

1. an upper bound on seller revenue
2. a lower bound on bidder utility

A Relaxed Problem

- consider a smaller–one-dimensional–family of constraints:
- instead of reporting signal s_i, report a random signal

$$s_i' < s_i,$$

drawn from truncated prior on support $[\underline{s}, s_i]$:

$$F\left(s_i'\right)/F\left(s_i\right)$$

- *misreporting a redrawn lower signal*

A Lower Bound on Bidder Utility

- what are the gains from *misreporting a redrawn lower signal?*
- equilibrium surplus of a bidder with type x is
  –from envelope condition of local constraints:

$$u_i(s_i) = \int_{x=\underline{s}}^{s_i} \widehat{q}_i(x)\, dx$$

- surplus from misreporting the redrawn lower signal

$$\frac{1}{F(s_i)} \int_{x=\underline{s}}^{s_i} u_i(s_i, x)\, f(x)\, dx$$

- gains vary depending on realized misreport
  average gains across all misreports are easy to compute

Average Gains from Misreporting

- misreport is redrawn from prior, bidder i is equally likely to fall anywhere in distribution of signals, unconditional on misreport, ex-ante likelihood that i receives object and x is highest signals

$$q_i(x)\, g_N(x)$$

- if highest report is less than s_i, surplus that bidder i obtains from being allocated object is s_i rather than x, so $s_i - x$ is difference between deviator and truthtelling surplus:

$$\frac{1}{F(s_i)} \int_{x=\underline{s}}^{s_i} \left[(s_i - x)\, q_i(x)\, g_N(x) + u_i(x)\, f(x)\right] dx$$

- thus the incentive constraint requires:

$$u_i(s_i) \geq \frac{1}{F(s_i)} \int_{x=\underline{s}}^{s_i} \left[(s_i - x)\, q_i(x)\, g_N(x) + u_i(x)\, f(x)\right] dx$$

Lower Bound As Equality

- lower bound of bidder's surplus through small class of deviations:

$$u_i(s_i) \geq \frac{1}{F(s_i)} \int_{x=\underline{s}}^{s_i} \left[(s_i - x)\, q_i(x)\, g_N(x) + u_i(x)\, f(x)\right] dx$$

- inequality hold as sum across all i :

$$u(s) \geq \frac{1}{F(s)} \int_{x=\underline{s}}^{s} \left[(s - x)\, q(x)\, g_N(x) + u(x)\, f(x)\right] dx$$

- lowest solution $\underline{u}(s)$ exists and solves inequality as equality
- monotonic operator on increasing functions has unique smallest fixed point by Knaster-Tarski fixed point
- can be integrated by parts as

$$\underline{U} = \int_{x \in S} \underline{u}(s)\, f(s)\, ds = \int_{s} \left(\int_{x=s}^{\overline{s}} \frac{1 - F(x)}{F(x)} dx \right) q(s)\, g_N(s)\, ds$$

A Generalized Virtual Utility Formula

- with the lower bound on bidder surplus:

$$\underline{U} = \int_{x \in S} \underline{u}(s) f(s)\, ds = \int_s \left(\int_{x=s}^{\overline{s}} \frac{1 - F(x)}{F(x)} dx \right) q(s)\, g_N(s)\, ds$$

- we obtain our final formula for revenue, which is

$$\overline{R} = TS - \underline{U} = \int_v \psi(v)\, q(v)\, g_N(v)\, dv$$

where

$$\psi(v) = v - \int_{x=v}^{\overline{s}} \frac{1 - F(x)}{F(x)} dx,$$

- compare to virtual utility in private value environments:

$$\pi(x) = x - \frac{1 - F(x)}{f(x)}$$

Upper Bound on Revenue

- generalized virtual utility:

$$\psi(x) = x - \int_{y=x}^{\bar{s}} \frac{1 - F(y)}{F(y)} dy,$$

Theorem (Revenue Upper Bound)

In any auction in which the probability of allocation is given by q, bidder surplus is bounded below by $\underline{U}$ and expected revenue is bounded above by $\overline{R}$.

- bound is valid for any allocation policy $q(v)$

Corollary (Random Reserve Price)

The random reserve price mechanism attains the revenue upper bound.

Posted Price
As Optimal Mechanism

Posted Prices

- consider mechanisms where object is always allocated
- pure common values – allocation is therefore socially efficient

Theorem (Revenue Optimality among Efficient Mechanisms)

Among all mechanisms that allocate the object with probability one, revenue is maximized by setting a posted price of

$$p = \int_{v=\underline{s}}^{\overline{s}} v g_{N-1}(v)\, dv,$$

i.e., expected value of object conditional on having lowest signal $\underline{s}$*.*

- posted price is inclusive: all types purchase at p
- all bidders equally likely to receive object: $q_i(v) = 1/N, \forall i, v$.
- optimal selling mechanism is attained with constant interim transfer $t = t_i(s_i)$ and probability $q = q_i(s_i)$

Optimality of Posted Price

- next, optimality of posted price among all
  – possibly inefficient – mechanisms

Corollary (Revenue Optimality of Posted Prices)

A posted price mechanism is optimal if and only if

$$\psi(\underline{s}) = \underline{s} - \int_{\underline{s}}^{\overline{s}} \frac{1 - F(x)}{F(x)} dx \geq 0.$$

If a posted price p is optimal, then it is fully inclusive.

The Power of Optimal Auctions

Auctions vs Optimal Mechanism

- Bulow and Klemperer (1996) establish the limited power of optimal mechanisms as opposed to standard auction formats
- revenue of optimal auction with N bidders is strictly dominated by standard absolute auction with $N + 1$ bidders
- current common value environment is an instance of general interdependent value setting – with one exception
- virtual utility function—or marginal revenue function—is not monotone due to maximum operator in common value model

A Closer Look at the Virtual Utility

- non-monotonicity leads to an optimal mechanism with features distinct from standard first or second price auction.
- it elicits information from bidder with highest signal but minimizes probability of assigning him the object subject to incentive constraint
- *virtual utility* of each bidder, $\pi_i(s_i, s_{-i})$:

$$\pi_i(s_i, s_{-i}) = \begin{cases} \max_j\{s_j\}, & \text{if} \quad s_i \leq \max\{s_{-i}\}; \\ \max\{s_j\} - \frac{1-F_i(s_i)}{f_i(s_i)}, & \text{if} \quad s_i > \max\{s_{-i}\}. \end{cases}$$

- downward discontinuity in virtual utility indicates why seller wishes to minimize the probability of assigning the object to the bidder with the high signal

Revenue Comparison

- virtual utility of bidder i fails monotonicity assumption even when hazard rate of distribution function is increasing everywhere
- BK (1996) require monotonicity of virtual utility when establishing their main result that an absolute English auction with $N + 1$ bidders is more profitable than any optimal mechanism with N bidders
- revenue ranking does not extend to current auction environment
- compare revenue from optimal auction with N bidders to absolute, English or second-price, auction with $N + K$ bidders
- absolute as there is no reserve price imposed

Reversal in Revenue Comparison

Theorem (Revenue Comparison)

For every $N \geq 1$ and every $K \geq 1$, the revenue from an absolute second-price auction with $N + K$ bidders is strictly dominated by the revenue of an optimal auction with N bidders.

- comparison of *second order statistic* of $N + K$ i.i.d. signals and *first order statistic* of $N + K - 1$ i.i.d. signals
- second order statistic of $N + K$ signals is revenue of absolute second-price auction with $N + K$ bidders.
- by earlier Theorem, optimal mechanism (weakly) exceeds revenue from a posted price set equal to the maximum of $N + K - 1$ signals.

Revenue Comparison: Continued

- but pure common value of the object is not affected by number of bidders, it is as if the remaining K signals are simply not disclosed, but the N participating bidders still form the expectation over the $N + K - 1$ signals.
- now, if instead of $N + K$ bidders, the optimal auction only has N bidders, then it is as if only N independent and identical distributed signals are revealed to the N bidders
- thus an attainable revenue for the seller is to offer the object at random to a bidder at a posted price set equal to the maximum of $N + K - 1$ signals

Conclusion

- characterized novel revenue maximizing auctions for a class of common value models
- common value models with qualitative feature that values are more sensitive to private information of bidders with more optimistic beliefs
- second interpretation as auction with intermediary/resale market
- countering the winner's curse
- optimal auctions discriminate in favor of less optimistic bidders since they obtain less information rents from being allocated the object