Information Design: A Unified Perspective

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Information Design

- there are fundamentally two different ways to influence the behavior of economic agents

1. by changing their payoffs/incentives, this is the realm of mechanism design
2. by changing their information, this is the realm of information design

- information design analyzes how through the choice of information provided, the designer can influence the individually optimal behavior
- even when the designer has no ability to change outcomes or force players to choose particular actions

Information Design and Applications

- digital information is widely used in allocation of services and commodities
- many digital platform and service providers operate as information designers
- platform services:
- uber provide recommendations in terms of matchers for drivers and customers
- netflix provides recommendation for viewer using past and concurrent information
- traffic navigation apps such as waze or google maps offer route recommendations

Mechanism Design and Information Design

Mechanism Design:

- Fix an economic environment (payoff function, actions, states) and an information structure
- Design the rules of the game to get a desirable outcome

Information Design

- Fix an economic environment (payoff function, actions, states) and rules of the game
- Design an information structure to get a desirable outcome

Mechanism Design and Information Design

Mechanism Design:

- can compare particular mechanisms..
  - e.g., first price auctions versus second price auctions
- can work with space of all mechanisms...
  - let each agent's action space be his set of types–revelation principle, Myerson's optimal mechanism

Information Design

- can compare particular information structures
  - Linkage Principle: Milgrom-Weber 82
  - Information Sharing in Oligopoly: Novshek-Sonnenschein 82
- can work with space of all information structures
- let each agent's type space be his set of actions–revelation principle

Information Design: Some Leading Cases

1. Uninformed information designer (or "mediator"):
   - Myerson: "Bayesian games with communication"
   - Incomplete Information Correlated Equilibrium literature of the 1980s and 1990s (Forges 93)
2. Informed information designer ("sender") and one player ("receiver")
   - "Bayesian Persuasion": Brocas-Carrillo 07, Kamenica-Gentzkow 11 and large and important literature inspired by it
3. Informed information designer and many players with private information
   - Some of our recent theoretical and applied work with various co-authors....
   - ...and this lecture

This Lecture

- a general framework in two slides
- leading examples at length
- applications in brief
- various elaborations if time

Setup

- maintained environment, fix:
- players 1,...,I; actions $a_i \in A_i$, payoff states Θ; utility function:

$$u_i : A \times \Theta \rightarrow \mathbb{R},$$

  common prior on states $\psi \in \Delta(\Theta)$
- basic game:

$$G : (A_i, u_i, \Theta, \psi)_{i=1,\ldots,I}$$

- information structure S: $(T_i)_{i=1..,I}$ and likelihood function:

$$\pi : \Theta \rightarrow \Delta(T)$$

Information as Recommendation

- by revelation principle it suffices to convey information in terms of recommended action a_i for agent i :
- *decision rule*
$$\sigma : T \times \Theta \rightarrow \Delta(A)$$
- action a_i is recommended to player i by information designer as a function of state of the world θ and signal profile t
- omniscient information designer (to be relaxed later)
- follow action recommendation a_i has to be in the interest of agent i : obedience condition

Decision Rule and Obedience

- *decision rule*

$$\sigma : T \times \Theta \to \Delta(A)$$

- *decision rule* σ is *obedient* in (G, S) if, $\forall i$, t_i, a_i and a_i',

$$\begin{aligned} & \sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(a_i, a_{-i}\right), \theta\right) \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi(\theta) \\ \geq & \sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(a_i', a_{-i}\right), \theta\right) \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi(\theta); \end{aligned}$$

- define *Bayes correlated equilibrium* (BCE) as joint distribution $\mu(a, \theta)$ that can be induced by an obedient decision rule σ

$$\mu(a, \theta) = \sum_t \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi(\theta);$$

- characterizes the set of implementable decision rules by information designer.

Information Design: Three Interpretations

1. Literal: actual information designer with ex ante commitment: Information designer with payoff

$$v : A \times \Theta \to \mathbb{R}$$

   picks a Bayes correlated equilibrium $\sigma \in BCE\left(G, S\right)$ to maximize

$$V_S\left(\sigma\right) \equiv \sum_{a,t,\theta} \psi\left(\theta\right) \pi\left(t|\theta\right) \sigma\left(a|t,\theta\right) v\left(a,\theta\right).$$

2. Metaphorical: used by the analyst as a tool:
   for example, we might be interested in finding an upper bound (across information structures) on the aggregate variance of output in a given economy with idiosyncratic and common shocks to agents' productivity
3. Informational robustness: family of objectives characterize set of attainable outcomes by changes in the information structure alone

One Uninformed Player: Benchmark Investment Example

- a firm is deciding whether to invest or not:
- binary state: $\theta \in \{B, G\}$, bad or good
- binary action: $a \in \{\text{Invest, Not Invest}\}$
- payoffs with $0 < x < 1$

	bad state B	good state G
Invest	-1	x
Not Invest	0	0

- prior probability of each state is $\frac{1}{2}$
- firm is uninformed (so one uninformed player)
- information designer (government) seeks to maximize probability of investment (independent of state)
- leading example of Kamenica-Gentzkow 11

Decision Rule

- uninformed receiver, $T = \varnothing$, thus decision rule is simply

$$\sigma : \Theta \rightarrow \Delta\left(A\right)$$

- probability of investment p_θ conditional on state θ :

$$\sigma\left(I \left|\theta\right.\right) \triangleq p_\theta$$

thus

$\sigma\left(\cdot \left|\theta\right.\right)$	bad state B	good state G
Invest	p_B	p_G
Not Invest	$1 - p_B$	$1 - p_G$

- interpretation: firm observes signal = "action recommendation," drawn according to $\left(p_B, p_G\right)$

Obedience Constraints

- if "advised" to invest, invest has to be a best response:

$$\begin{aligned} -\frac{1}{2}p_B + \frac{1}{2}p_G x &\geq 0 \Leftrightarrow \\ p_G &\geq \frac{p_B}{x} \end{aligned}$$

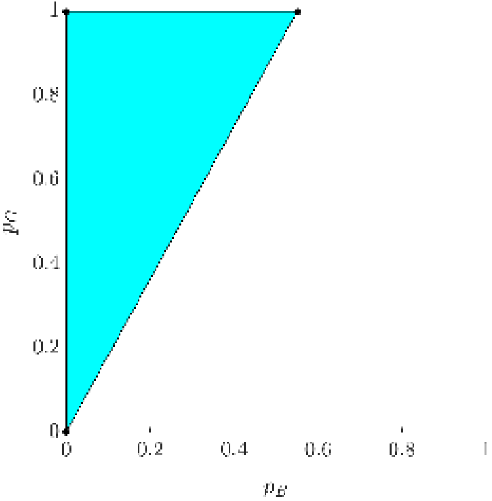
- if "advised" to not invest, not invest has to be a best response

$$\begin{aligned} -\frac{1}{2}(1 - p_B) + \frac{1}{2}(1 - p_G)x &\leq 0 \Leftrightarrow \\ p_G &\geq \frac{p_B}{x} + 1 - \frac{1}{x} \end{aligned}$$

- because $x < 1$, investment constraint is binding
- always invest ($p_B = p_G = 1$) can't be equilibrium
- full information equilibrium invests in good state only ($p_B = 0,\ p_G = 1$)
- zero information equilibrium never invests ($p_B = p_G = 1$)

Impact of Information Design

- all Bayes correlated equilibrium outcomes (p_B, p_G) for $x = 0.55$



- always invest ($p_B = p_G = 1$) is not a BCE

Information Design

- recommendation maximizing the probability of investment:

$$p_B = x,\ p_G = 1$$

- best BCE

	B	G
Invest	x	1
Not Invest	$1-x$	0

- optimal for government to obfuscate:
  partially pooling good state and bad state
- optimal for government to isolate:
  bad state is set apart

Food for Thought

1. Literal Interpretation: Conflict of interest between information designer and player creates incentive for obfuscation (partial information revelation)
2. Robust Interpretation: Intuitively extremal information structures (zero information, complete information) may not be extremal for outcomes

What do extremal information structures look like?

One Informed Player

- firm receives signal "correct" with probability $q > 1/2$
- firm observes a signal g or b with conditionally independent probability q when true state is G or B respectively:

	bad state B	good state G
bad signal b	q	$1 - q$
good signal g	$1 - q$	q

- probability p_θ^t of invest given $\theta \in \{B, G\}$ and $t \in \{b, g\}$
- decision rule σ is

$$\left(p_B^b, p_G^b, p_B^g, p_G^g\right)$$

Obedience Constraints with Informed Player

- same constraints signal by signal essentially as before...
- a firm with good signal invests (when told to invest) if

$$p_G^g \geq \frac{1-q}{q}\frac{p_B^g}{x}$$

and not invest (when told to not invest) if

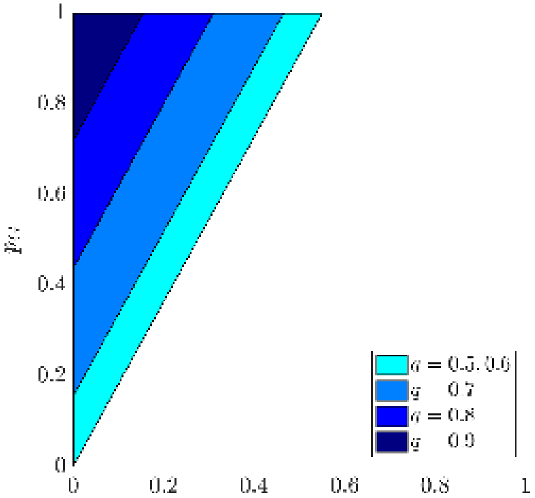
$$p_G^g \geq \frac{1-q}{q}\frac{p_B^g}{x} - \frac{1-q}{qx} + 1.$$

- if private information of firm is sufficiently noisy, $q \leq \frac{1}{1+x}$, binding constraint remains investment constraint
- what happens to aggregate probability to invest across signals:

$$p_G = qp_G^g + (1-q)\,p_G^b$$

One Informed Player: Bayes Correlated Equilibrium

- equilibrium set for $x = 0.55$
- private information reduces control of designer



Food for Thought

1. More information limits the ability of the information designer to achieve his objectives
2. But what does "more information" mean in general?
3. in single player case, Blackwell's information order of Sufficiency
4. in many player case, a many player generalization, Individual Sufficiency
   in "Bayes Correlated Equilibrium and the Comparison of Information Structures in Games", Theoretical Economics, 2016,

Many Players: Two Firms

- payoffs almost as before....

$\theta = B$	I	N
I	$-1+\varepsilon$	-1
N	0	0

$\theta = G$	I	N
I	$x+\varepsilon$	x
N	0	0

- ...up to ε term for coordinated investment
- assume that information designer (government) wants to maximize the *sum* of probabilities that firms invest....
- if $\varepsilon = 0$, problem is exactly as before firm by firm; doesn't matter if and how firms' signals are correlated
- what happens if $|\varepsilon| \approx 0$ (so analysis can't change much)
- profile of action recommendations depends on state
- $\varepsilon > 0$: strategic complements; $\varepsilon < 0$: strategic substitutes

Two Firms: Strategic Complementarities

- If $\varepsilon > 0$, optimal decision rule σ is

$\theta = B$	I	N
I	$\frac{x+\varepsilon}{1-\varepsilon}$	0
N	0	$\frac{1-x-2\varepsilon}{1-\varepsilon}$

$\theta = G$	I	N
I	1	0
N	0	0

- probability of any one firm investing is still about x..
- binding constraints are still investment constraints, slackened by having simultaneous investment...

$$\frac{x+\varepsilon}{1-\varepsilon}\left(-1+\varepsilon\right)+x+\varepsilon \geq 0$$

- ....so signals are public

Two Firms: Strategic Substitutes

- If $\varepsilon < 0$, optimal decision rule σ is

$\theta = B$	I	N
I	0	$x + \varepsilon$
N	$x + \varepsilon$	$1 - 2x - 2\varepsilon$

$\theta = G$	I	N
I	1	0
N	0	0

- probability of any one firm investing if the state is bad is still about x....
- binding constraints are still investment constraints, slackened by having minimally correlated investment...

$$(x + \varepsilon)(-1) + x + \varepsilon \geq 0$$

- ....and signals are private

Food for Thought

1. Public information under strategic complementarities
2. Private information under strategic substitutes
3. How does this matter in applications? Algorithmic aspect of information design emphasized in Dughmi (2017)
4. How about alternative objectives for the information designer?

Application 1 - Information Sharing: Strategic Substitutes

- Classic Question: are firms better off if they share their information?
- Consider quantity competition when firms uncertain about level of demand (intercept of linear demand curve) with symmetry, normality and linear best response;
- Two effects in conflict:

1. Individual Choice Effect: Firms would like to be as informed as possible about the state of demand
2. Strategic Effect: Firms would like to be as uncorrelated with each other as possible

- "Robust Predictions in Games with Incomplete Information" , Econometrica, 2013

Application 1 - Information Sharing

- Classic Question: are firms better off if they share their information?
- Consider quantity competition when firms uncertain about level of demand: individual and strategic effects in conflict

- Resolution:

1. If inverse demand curve is flat enough... i.e., small strategic effect...individual choice effect wins and full sharing is optimal
2. If inverse demand curve is very steep...i.e., large strategy effect...strategic effect wins and no sharing of information is optimal
3. In intermediate cases, optimal to have firms observe imperfect information about demand, with conditionally independent signals, and thus signals which are as uncorrelated as possible conditional on their accuracy

Application 2 - Aggregate Volatility

- Classic Question: can informational frictions explain aggregate volatility?
- Consider a setting where each firm sets its output equal to its productivity which has idiosyncratic and common component

$$x_i + y$$

- again with symmetry and normality
- idiosyncratic component x_i with variance τ^2;
- common component y with variance σ^2;
- which information maximizes variance of average action?
- "Information and Volatility" Journal of Economic Theory, 2015,

Application 2 - Aggregate Volatility

What information structure maximizes variance of average action?

- critical information structure:
  confounding signal without noise:

$$s_i = \lambda x_i + (1-\lambda)\, y$$

- variance of average action is maximized when

$$\lambda = \frac{\sigma}{2\sigma + \sqrt{\sigma^2 + \tau^2}} < \frac{1}{2}$$

  and maximum variance of average action is

$$\left(\frac{\sigma + \sqrt{\sigma^2 + \tau^2}}{2}\right)^2$$

Application 2 - Aggregate Volatility

What information structure maximizes variance of average action?

- "optimal" information structure has a confounding (Lucas 72) signal $s_i = \lambda x_i + (1 - \lambda) y$ *without noise...*

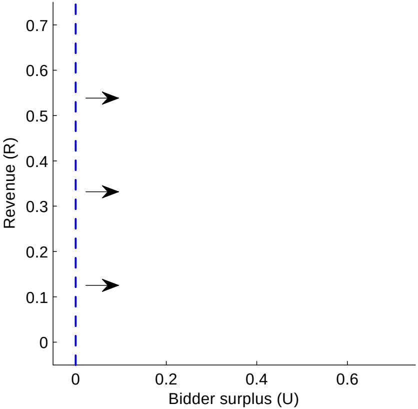
- as variance of common component $\sigma \to 0$:

1. "optimal" weight on idiosyncratic component goes to 0
2. agents put a lot of weight on their signal in order to put a non-trivial weight on their idiosyncratic component
3. in the limit, the common component becomes a payoff irrelevant but common "sentiments" shock

Application 3 - First Price Auction: Information Shrinking BCE, Adversarial Information Designer

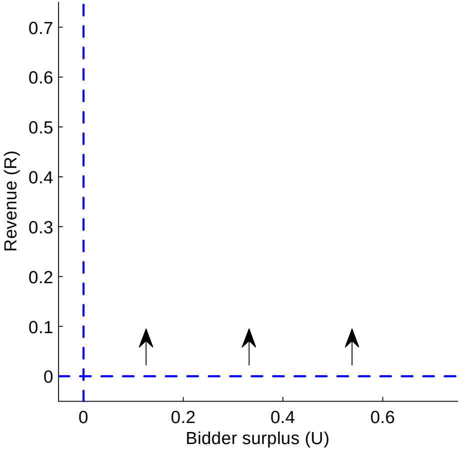
- "First Price Auctions with General Information Structures", Econometrica 2017
- what can we say about revenue and bidder utility across all information structures
- example: Two bidders and valuations independently and uniformly distributed on the interval $[0, 1]$

A Different Approach: Identify Bounds



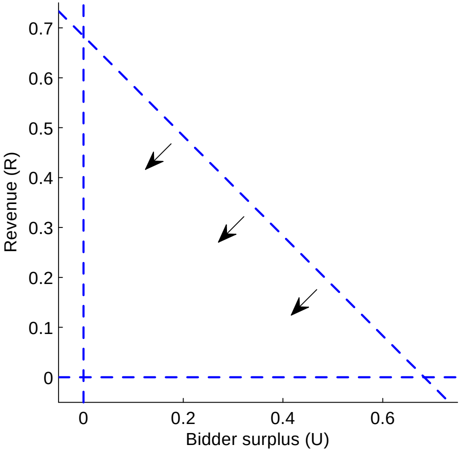
- nonnegative bidder surplus

A Few Bounds



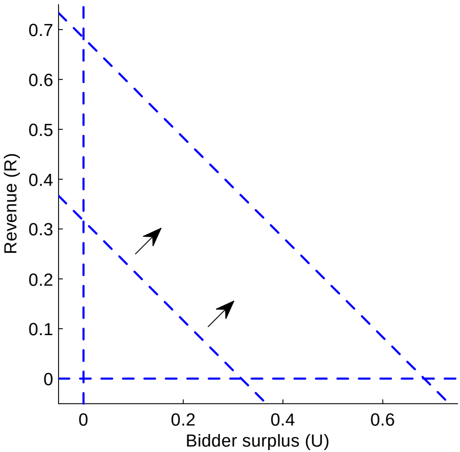
- nonnegative revenues

A Few Bounds



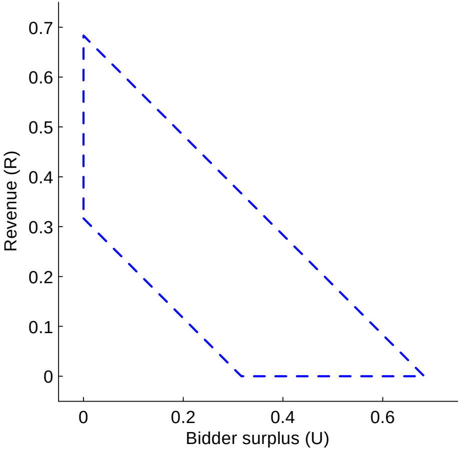
- efficient social surplus: always give the object to the bidder with the highest valuation

A Few Bounds



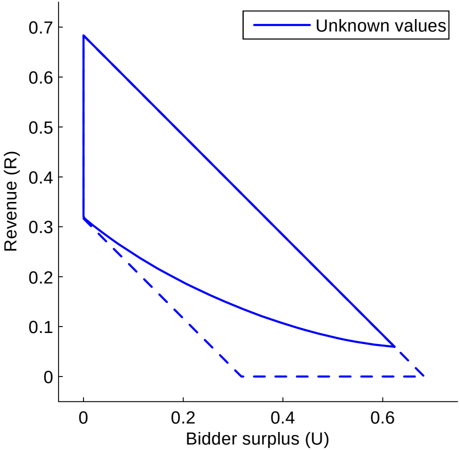
- least efficient allocation: always give the object to the bidder with the lowest valuation

Surplus Trapezoid



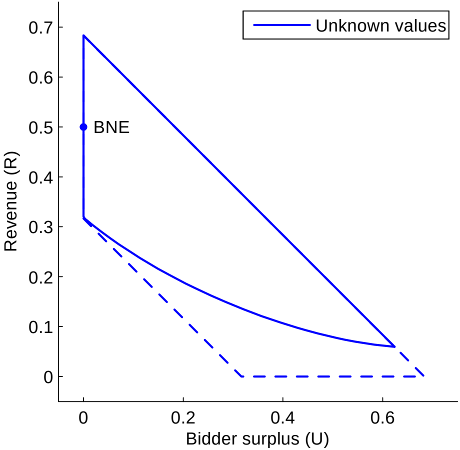
- we only considered feasibility and participation constraints

Incentives Imposes Restrictions: Unknown Values



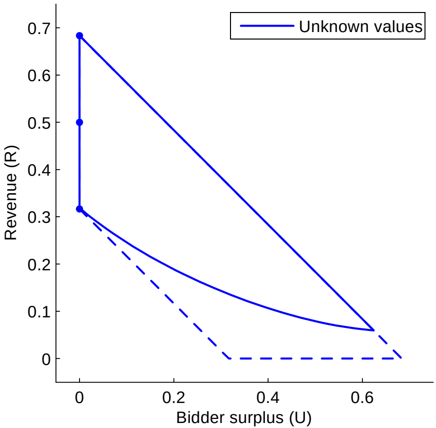
- incentive constraints (optimal bidding) adds new constraints, raises revenue for the seller

Zero Information Equilibrium



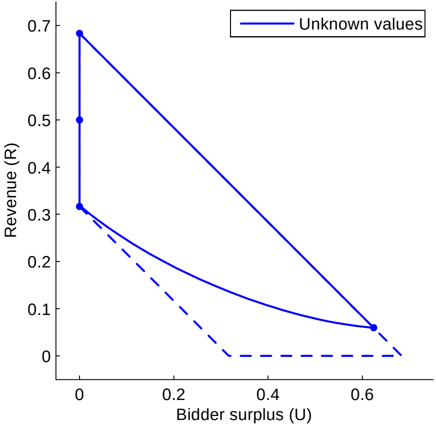
- bidders compete with zero information, bid expected value, inefficient allocation

Almost Zero Information Equilibria



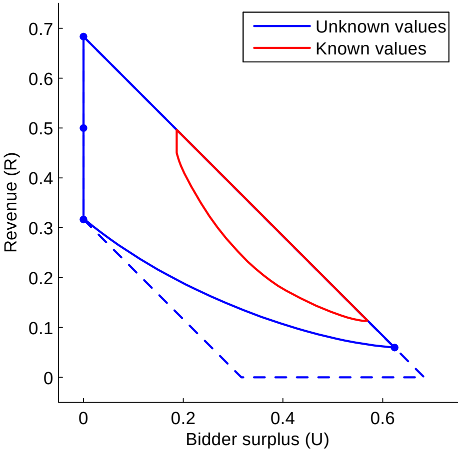
- with limited information, bidder surplus remains at zero, but allocation can attain most or least efficient outcome
- (information) rent requires information

Maximal Bidder Surplus Equilibrium



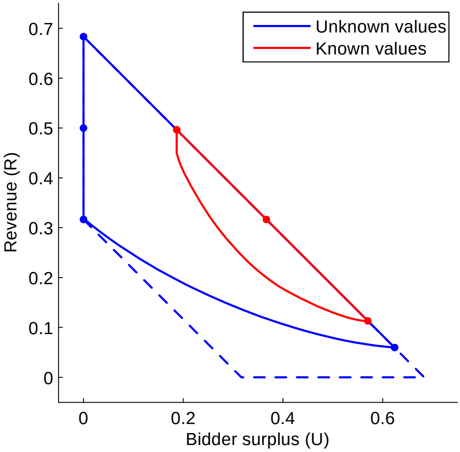
- information leads to efficiency
- efficiency and information rent appear as complements
- exclude low revenue and low efficiency outcomes

Information Generates Incentives: Known Values



- each bidder knows his own value v_i,
  but may still be uncertain about value of bidder j,
  or what bidder j knows about bidder i

Maximal and Minimal Revenue



- exact construction of maximal revenue (and minimal revenue)
- contrast with bound construction by Syrgkanis and Tardos (2014)

Literature

Our methodology papers:

- "Robust Predictions in Incomplete Information Games," ECTA 13
- "Bayes Correlated Equilibrium and The Comparison of Information Structures," TE 16
- "Information Design, Bayesian Persuasion and Bayes Correlated Equilibrium," AER P&P 2016
- Information Design: A Unified Perspective

Our application papers:

- "Limits of Price Discrimination," AER 15
- "Information and Volatility," JET 15
- "First Price Auctions with General Information Structure," ECTA 17

Conclusion: Information Design

1. Literal: actual information designer with ex ante commitment
2. Metaphorical: e.g., adversarial / worst case
3. Informational robustness: family of objectives characterize set of attainable outcomes