Search, Information, and Prices

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Price Dispersion and Law of One Price

- what determines the distribution of prices under (Bertrand) price competition?
- if firms have common knowledge of the number of competing firms, then either:

1. monopoly: with probability one monopoly price prevails, or
2. oligopoly: with probability one price equal to cost

- yet Varian (1980) observed that "the law of one price is not a law at all": price dispersion is ubiquitous
- classic explanations about equilibrium price dispersion and failure of the law of one price build on firms' uncertainty about the amount of competition, in particular:
- the number of prices consumer has observed: the *price count*

Two Related Issues

1. Price Count Uncertainty

   - lots of possible microfoundations for price count uncertainty, e.g., consumer search, clearinghouses, advertising, spatial price discrimination
   - difficult to say which model is empirically relevant, and measure model parameters (like psychological costs of search)
   - *what can we say about the sale price distribution taking the price count distribution as given?*

Two Related Issues:

2. Informational Model Uncertainty

- nearly all existing models suppose that firms have *no information* about the price count (beyond the ex ante distribution)
- in a (symmetric) static model, resulting expected price is the same as if firms knew the true price count (revenue equivalence) and is a lower bound on what can happen (with intermediate information structures)
- *what are possible outcomes across all informational models about the price count?*

Main Results

1. tight upper bound on the equilibrium sale price distribution (in the sense of first-order stochastic dominance) that holds for any model that rationalizes the price count
2. global upper bound on the effect of monopoly power on price as we depart from perfect competition:
   marginal revenue in the probability μ of monopoly is unbounded at $\mu = 0$.

- *methodological contribution*: predictions about prices given price counts, but not explain where price counts come from
- *empirical test:* can observed sale prices be rationalized by competitive behavior given observed price counts?

Model

Model

- single consumer, unit demand, value $v > 0$
  (can also be interpreted as a continuum of consumers)
  (easily generalized to downward sloping demand)
- firms $N = \{1, \ldots, n\}$, production cost zero
- consumer receives price quotes from a random subset $K \subseteq N$, buys at lowest price
- *price count*: $k = |K|$
- *price count distribution*: $\mu \in \Delta(1, \ldots, n)$
- *symmetry*: all firms equally likely to be quoted given price count (bounds apply but not tight without symmetry)

Information

- an *information structure* (T, π) consists of measurable sets of signals T_i for each firm, and for each set K of quoted firms a joint probability measure on $\prod_{i \in K} T_i$:

$$\pi : 2^N \to \Delta\left(\prod_{i \in K} T_i\right)$$

- when a set K of firms is quoted,
  $\to$ each quoted firm $i \in K$ receives a *signal* t_i, and
  $\to$ any firm j outside of K, $j \in N \setminus K$, does not receive a signal (as the firm is not active)
- conditional probability of $t_K = (t_i)_{i \in K}$ given the set K:

$$\pi\left(t_K \,|K\right)$$

Strategies and Equilibrium

- firm i's strategy: $F_i : T_i \to \Delta([0, v])$
- $F_i(x|t_i)$ is the probability that $p_i \geq x$ given t_i, i.e., an *upper cumulative distribution*
- $K(p)$ is the set of firms charging the lowest price
- if i is quoted and given prices p, firm i's revenue is

$$p_i \frac{\mathbb{I}_{i \in K(p)}}{|K(p)|}$$

- firm i's expected revenue given strategy profile F:

$$R_i(F) = \sum_{k=1}^{n} \frac{\mu(k)}{\binom{n}{k}} \sum_{\{K \subseteq N \,|\, |K|=k\}} \int_{T_K} \int_{[0,v]^K} p_i \frac{\mathbb{I}_{i \in K(p)}}{|K(p)|} F(dp|t) \pi(dt|K)$$

- equilibrium if $R_i(F) \geq R_i(F'_i, F_{-i})$ for all i and F'_i

Sale Price Distribution

- probability good sold by firm i at price greater or equal to x:

$$S_i(x|k) = \frac{1}{\binom{n}{k}} \sum_{\{K \subseteq N | i \in K,\ |K|=k\}} \int_{T_K} \int_{[x,v]^K} \frac{\mathbb{I}_{i \in K(p)}}{|K(p)|} F(dp|t) \pi(dt|K)$$

- *conditional sale price distribution* (given k):

$$S(x|k) = \sum_{i=1}^{n} S_i(x|k)$$

- *unconditional sale price distribution*:

$$S(x) = \sum_{k=1}^{n} \mu(k) S(x|k)$$

- our main question: *what equilibrium sale price distributions are consistent with price count distribution μ?*

Example

Simplest Example: Two Firms

- let's develop intuition with the simplest case with $n = 2$
- price count is 1 with probability μ and 2 with probability $1 - \mu$
- value $v = 1$

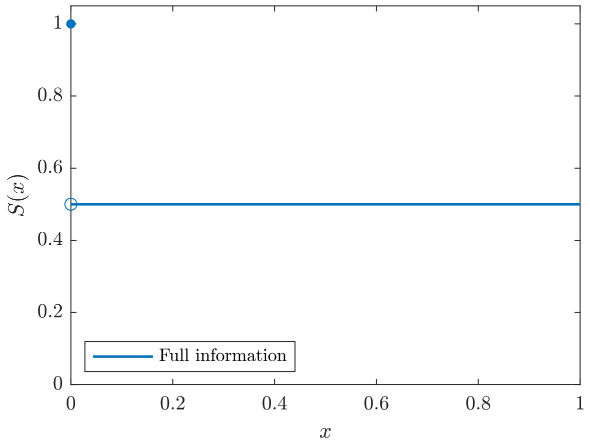
Complete Information

- *equilibrium*:
- monopoly price at 1 when $k = 1$,
- competitive price at 0 when $k = 2$
- *expected price:* μ
- expected price/revenue is linear in probability of single price count/monopoly

Complete Information Sale Price Distribution

- suppose $\mu = 1/2$, sale price distribution

$$S(x) = \Pr(\min p_i \geq x)$$



Zero Information

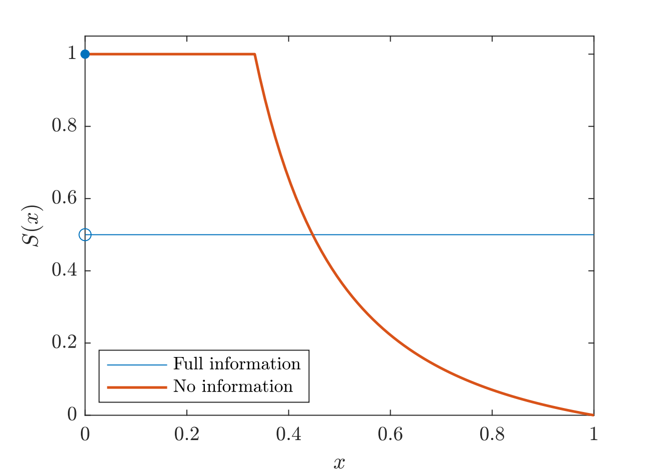
- *equilibrium*
- mixed strategy equilibrium with indifference between monopoly and oligopoly price count
- symmetric $F_i(x)$ such that

$$1 \cdot \frac{\mu}{2} = x\left(\frac{\mu}{2} + (1-\mu)F_i(x)\right)$$
$$\implies F_i(x) = \frac{\mu}{2(1-\mu)}\frac{1-x}{x}$$

  with support $[\mu/(2-\mu), 1]$
- expected sale price: still μ as firms indifferent to pricing at 1

Zero Information Sale Price Distribution

- different sale price distributions, same expected sale price



Public and Partial Information

- zero information increases price dispersion relative to complete information but does not change expected sale price level
- partial information (still public) increase expected sale price
- splitting the market:
  $t_i \in \{1, 2\}$ identifies who might be monopolist

$\pi(t|K = \{1\})$

t_1	
$t_1 = 1$	1

$\pi(t|K = \{2\})$

t_2	$t_2 = 1$
	1

$\pi(t|K = \{1, 2\})$

t_1/t_2	$t_2 = 1$	$t_2 = 2$
$t_1 = 1$	1/2	0
$t_1 = 2$	0	1/2

Figure 1: Public signal represents identity of possible monopolist

Public Information Maximizing the Expected Sale Price

- *equilibrium*: if firm 1 is the "potential monopolist",
- firm 1 has a mass point on $p = 1$
- otherwise, firms randomize on $[\mu, 1]$:

$$F_i(p_i) = \frac{\mu}{p_i}, F_{-i}(p_{-i}) = \frac{\mu(1 - p_{-i})}{(1 - \mu) p_{-i}}$$

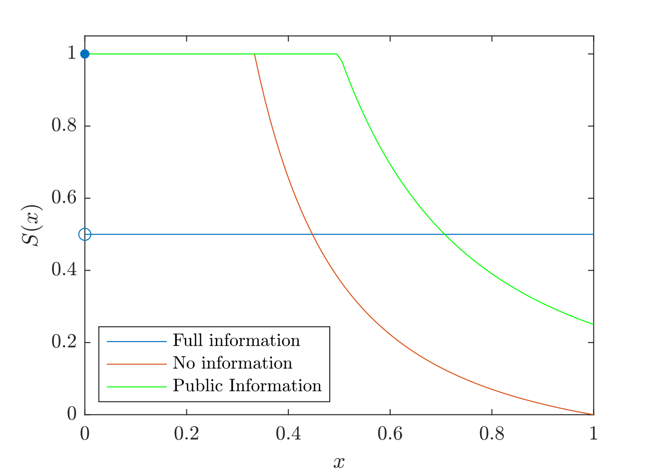
- potential monopolist firm prices higher (naturally)
- "known competitor" firm responds by pricing higher than with zero information
- expected sale price:

$$\mu(2 - \mu) > \mu$$

- increases twice as fast in probability μ of monopoly (near $\mu = 0$)

Public Information And Sale Price Distribution

- sale price distribution of nested markets stochastically dominate zero and complete information sale price distribution



Private Signals Can Drive Prices Higher

- with public information, firms' strategies must have the same support, with firms indifferent between the same prices
- with private information, firms no longer must have the same support
- we will construct a generalized "nested" information structure where each firm may be a "potential monopolist" type or a "known competitor" type
- but now the potential monopolist can always price higher than the known competitor
- so we can make the potential monopolist price even higher; this leads the known competitor to price even higher
- this generalize into an "ordered supports" property in our general construction

Private Information

- consider the following information structure:
- if the price count is 1, active firm gets signal $t_i = 1$
- if the price count is 2, with probability $\alpha \leq 1/2$, one firm gets signal $t_i = 1$, the other gets signal $t_j = 2$; with probability $1 - 2\alpha$, both receive signal $t_i = t_j = 2$:

$\pi(t|K = \{1\})$

t_1	
$t_1 = 1$	1

$\pi(t|K = \{2\})$

t_2	$t_2 = 1$
	1

$\pi(t|K = \{1, 2\})$

t_1/t_2	$t_2 = 1$	$t_2 = 2$
$t_1 = 1$	0	α
$t_1 = 2$	α	$1 - 2\alpha$

Figure 2: Private signal represents lower bound of price count

Information and Equilibrium

- potential monopolist always charges the monopoly price of 1 and known competitor randomizes on interval below 1
- known competitor assigns probability $\alpha/(1-\alpha)$ to other firm being potential monopolist charging price of 1
- strategy of known competitor with support $[\alpha/(2-\alpha), 1]$ is

$$F(x) = \frac{\alpha}{1-2\alpha}\frac{1-x}{x}$$

- monopolist's best response is price of 1 if probability he assigns to being a monopolist

$$\frac{\frac{1}{2}\mu}{\frac{1}{2}\mu + \alpha(1-\mu)} > \frac{\alpha}{1-\alpha}, \text{ or}$$

$$\alpha \leq \alpha^* = \frac{1}{2}\frac{\sqrt{\mu(2-\mu)} - \mu}{1-\mu}$$

Sale Price Maximizing Private Information

- this requires:

$$\alpha \leq \alpha^* = \frac{1}{2}\frac{\sqrt{\mu(2-\mu)}-\mu}{1-\mu}$$

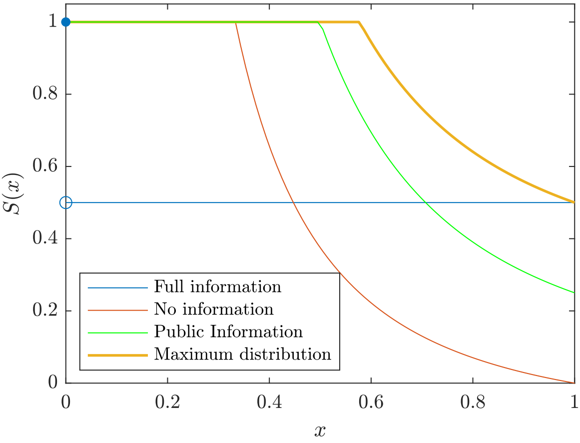
- information structure giving the highest expected price has

$$\alpha = \alpha^*$$

- resulting distribution of sales prices stochastically dominates those from complete, zero, and public information

Private Information Sale Price Distribution

- private information sale price distribution stochastically dominates public information

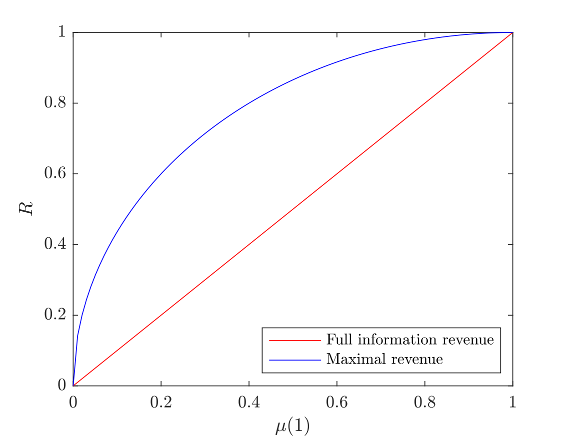


Maximum Firms' Surplus

- expected price is growing rapidly in probability of monopoly:

$$\sqrt{\mu(2-\mu)} > \mu(2-\mu)$$

- revenue has infinite derivative at $\mu = 0$!



Generalizing the Example

- our main result will:

1. establish that this sale price distribution is pointwise higher, i.e., *first-order stochastically dominates* (FOSD), any sale price distribution under any information structure, public or private;
2. generalize for arbitrary price count distributions.

- key properties which will generalize:

1. when the price count is 1, the monopoly price is charged
2. firms' signals will be lower bounds on the price count
3. ordered supports: firms who knows that there are at least $k+1$ price quotes always charge less than firms who think there might be k quotes (generalized "nested" markets)
4. firms indifferent to all downward deviations

Main Result

What Does the Upper Bound Look Like?

- back to the general model and price count distribution

$$\mu = (\mu(1), ..., \mu(n))$$

- ordered supports: there exist

$$1 = x_0 = x_1 > .... > x_n > 0$$

such that each $\overline{S}(x|k)$ has support on interval $[x_k, x_{k-1}]$ and in particular $\overline{S}(x|1)$ puts probability one on value 1.

Specifically...

- cutoffs x_k and distribution are determined by expected number of quotes:

$$Q_m = \sum_{l=1}^{m} l\mu(l) form \geq 1$$

- let

$$x_k = \prod_{m=1}^{k} \left(\frac{Q_{m-1}}{Q_m} \right)^{\frac{m-1}{m}}$$

- let $\overline{S}(x|k)$ be the distribution with support $[x_k, x_{k-1}]$ and

$$\overline{S}(x|k) = \frac{\left(\frac{x_k}{x}\right)^{\frac{k}{k-1}} - \left(\frac{x_k}{x_{k-1}}\right)^{\frac{k}{k-1}}}{1 - \left(\frac{x_k}{x_{k-1}}\right)^{\frac{k}{k-1}}}$$

- let $\overline{S}(x) = \sum_{k=1}^{n} \mu(k)\overline{S}(x|k)$

Main Result

Theorem

Fix a price count distribution μ.

1. *In any information structure $\{T, \pi\}$ and equilibrium F consistent with μ, the distribution of sale prices must be FOSD by $\overline{S}$.*
2. *There exists an information structure and equilibrium in which $\overline{S}$ is the induced sale price distribution.*

- if distribution of sale prices were too high—for example, if all firms priced at the monopoly level—then firms would have an incentive to undercut and thereby gain more sales
- non-trivial bounds on how high sale price distribution can go
- critical equilibrium constraints are those associated with cutting prices, focus on a particular class of deviations

Proof Sketch: Uniform Downward Deviation

- first step: Any $S(\cdot|k)$ and $S(\cdot)$ induced by a distribution and equilibrium must satisfy for all $x \in [0,1]$

$$x \sum_{k=1}^{n} \mu(k) k S(x|k) \leq \int_{y=x}^{v} y S(dy) \qquad (*)$$

- note: randomly chosen firm's equilibrium revenue is

$$\frac{1}{n} \int_{y=0}^{v} y S(dy)$$

- consider deviation: randomly chosen firm i prices at $\min\{p_i, x\}$ when it would have set price p_i; resulting surplus:

$$x \sum_{k=1}^{n} \mu(k) \frac{k}{n} S(x|k) + \frac{1}{n} \int_{y=0}^{x} y S(dy)$$

(more likely that the deviator is quoted, the higher is k)

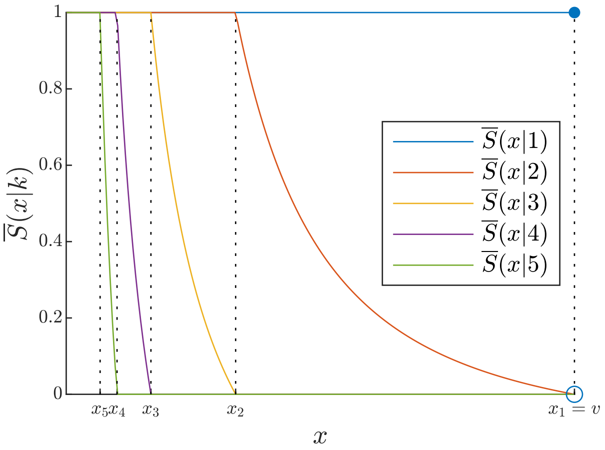
- $(*)$ says that on average, this deviation must be unprofitable

Proof Sketch: Solving for $\overline{S}$

- $\overline{S}$ maximizes sale price distribution pointwise subject to $(*)$
- key properties:

1. ordered supports (if not, can relax constraints by shifting to ordered supports)
2. $(*)$ holds as equality
3. now $(*)$ reduces to first-order differential equations that can be solved inductively on k

Example with μ Uniform on $\{1,\ldots,5\}$



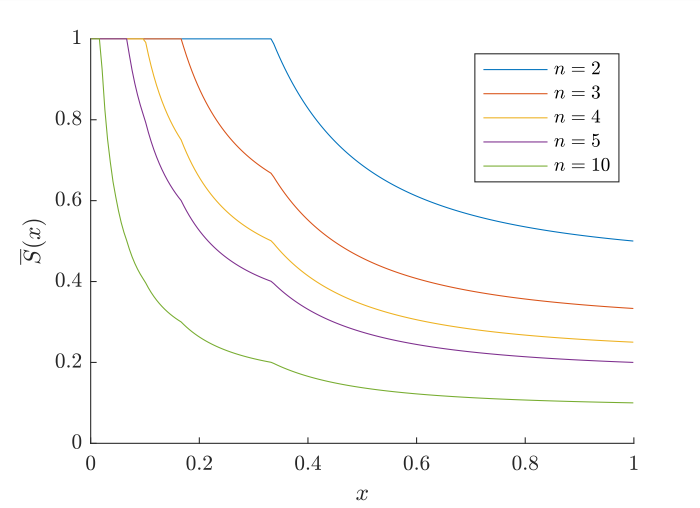
Proof Sketch: Construction of Information Structure

- construct an information structure/equilibrium that attains $\overline{S}$
- signals are $T_i = \{1, \ldots, n\}$
- we independently draw a candidate signal $l \in \{1, \ldots, k\}$ for each agent (according to a distribution $\alpha(l|k)$)
- we discard realizations where all signals are strictly below k, thus the highest signal given k is always k
- pricing strategies are

$$\frac{\left(\frac{x_k}{x}\right)^{\frac{1}{k-1}} - \left(\frac{x_k}{x_{k-1}}\right)^{\frac{1}{k-1}}}{1 - \left(\frac{x_k}{x_{k-1}}\right)^{\frac{1}{k-1}}}$$

- firm with signal k always wins: $\alpha(l|k)$ are chosen to make firms indifferent between choosing lower prices

Sale Price Distributions with Uniform μ



- first order stochastic dominance

Conclusion

Conclusion

two results presented today:

1. derive tight upper bound on the equilibrium distribution of sale prices
2. derive revenue impact of probability of monopoly

paper contains many additional results:

1. sequential search
2. downward sloping demand rather than unit demand
3. relationship to first price auction