Third-Degree Price Discrimination Versus Uniform Pricing

Dirk Bergemann
Yale University

Francisco Castro
UCLA

Gabriel Weintraub
Stanford University

INFORMS November 2020

Introduction

- third-degree price discrimination (Pigou, 1920) occurs when different segments of consumers can be offered different prices
- third-degree price discrimination is powerful tool to increase revenue (Bergemann, Brooks, Morris, 2015)
- compare the revenue of the optimal third-degree price discrimination policy against a uniform pricing policy
- uniform pricing policy offers the same price to all segments of the market
- establishes that for a broad class of third-degree price discrimination problems with concave revenue functions and common support:
- a uniform price is guaranteed to achieve one half of the optimal monopoly profits

Limits of Result

- revenue bound holds for any arbitrary number of segments and segment sizes
- revenue bounds holds for arbitrary prices that the seller would use with third-degree price discrimination
- these conditions are tight and weakening either common support or concavity leads to arbitrarily poor revenue comparisons

Model

- monopolist selling to K different customer segments
- segment k has share α_k of market:

$$\alpha_k \geq 0, \quad \sum_{k=1}^{K} \alpha_k = 1$$

- if monopolist offers price p_k to segment k, then profit is

$$R_k(p_k) \triangleq p_k \cdot (1 - F_k(p_k)) \ ,$$

- $F_k(\cdot)$ distribution function with support in $\Theta_k \subset \mathbb{R}_+$.
- total profit with segmented prices

$$\boldsymbol{p} = (p_1, \ldots, p_K)$$

is

$$\Pi(\boldsymbol{p}) = \sum_{k=1}^{K} \alpha_k R_k(p_k)$$

Third Degree Price Discrimination vs Uniform Price

- third degree price discrimination: in each segment k, the monopolist sets price $p_k^\star$:

$$p_k^\star \in \arg\max_{p \in \Theta_k} R_k(p)$$

- market segmentation:

$$\Pi^M(\alpha, R) \triangleq \sum_{k=1}^{K} \alpha_k R_k(p_k^\star)$$

- uniform price across all segments:

$$\Pi^U(\alpha, R) \triangleq \max_{p \in \cup_{k=1}^{K} \Theta_k} \sum_{k=1}^{K} \alpha_k R_k(p)$$

- we analyze the ratio across α and R

$$\min_{\alpha, R} \frac{\Pi^U(\alpha, R)}{\Pi^M(\alpha, R)}$$

Concave Profit Function

- assume that profit functions $R_k(\cdot)$ are concave in p for all k
- supports are $\Theta_k = \Theta$ for all k and Θ is a closed and bounded interval in $\mathbb{R}_+$

Theorem (Uniform Price is Half-Approximation)

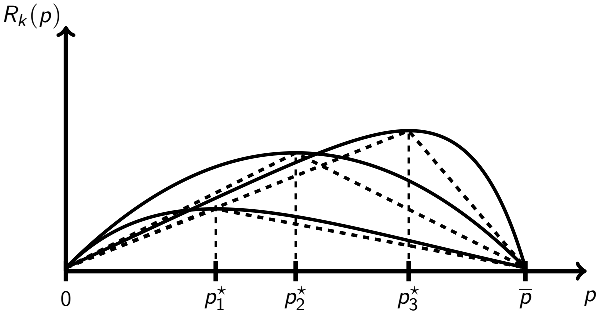
$$\Pi^U \geq \frac{1}{2}\Pi^M.$$

- Theorem 1 provides a fundamental guarantee of uniform pricing compared to optimal third-degree price discrimination
- Theorem 1 also suggests two simple and appealing ways of selecting the price:

1. select p^U among $p_k^\star$
2. set $p^U = \overline{p}/2$

A Proof in Three Pictures

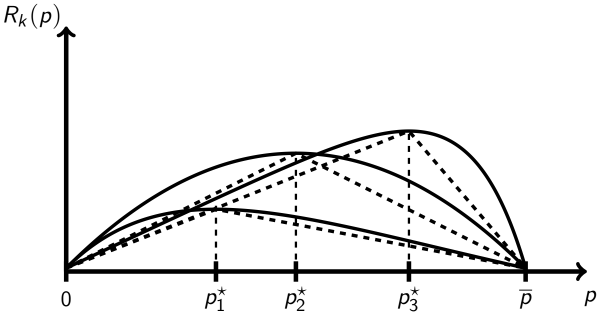
- let $r_k \triangleq \alpha_k R_k(p_k^\star)$, note $\Pi^M = \sum_{k=1}^K r_k$.
- by concavity, lower bounded by triangular shaped function



First Picture

- triangular shaped function is given by:

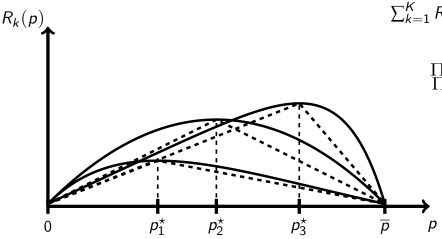
$$R_k^L(p) \triangleq \begin{cases} \frac{r_k}{p_k^\star} \cdot p & \text{if } p \in [0, p_k^\star] \\ \frac{r_k}{\overline{p}-p_k^\star} \cdot (\overline{p} - p) & \text{if } p \in [p_k^\star, \overline{p}] \end{cases}$$



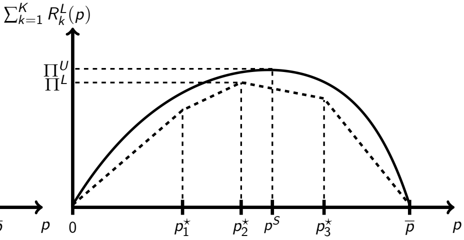
Second Picture

- summing up over all segments gives a concave, piecewise linear function:

$$\Pi^L = \max_p \sum_{k=1}^{K} R_k^L(p)$$

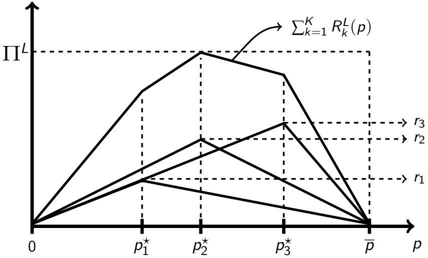


(a)



(b)

Third Picture



$$\Pi^L \cdot \overline{p} \geq \int_0^{\overline{p}} \sum_{k=1}^{K} R_k^L(p)dp = \sum_{k=1}^{K} \int_0^{\overline{p}} R_k^L(p)dp = \sum_{k=1}^{K} \frac{r_k \cdot \overline{p}}{2},$$

- $R_k^L(p)$ is triangle-shaped and thus the area below its curve equals $r_k \cdot /2$
- dividing both sides in the expression above by $\overline{p}$

An Application to Dynamic Pricing: Sequential Screening

- sequential screening as in Courty and Li (1999), Kraehmer and Strausz (2015), BCW (2020)
- seller offers a menu of contracts that incentivize buyers to self-select into different contracts in period 1, transaction price (and quantity) are determined in period 2
- uniform price $\approx$ self-selection only on the basis of transaction price in period 2

Theorem

In any ex-post individually rational sequential screening mechanism, the optimal static contract delivers a 1/2-approximation for seller's dynamic profits.

Necessity of Common Support

- finite interval Θ_k for each segment $k \geq 1$

Theorem

Suppose that the revenue functions for all classes are concave with finite support, then the optimal uniform price only deliver an arbitrarily small profit guarantee as the number of segments increases.

- by construction:

$$R_k(p) = \begin{cases} p & \text{if } p \in [0, v_k] \\ \frac{v_k}{\varepsilon_k}(v_k + \varepsilon_k - p) & \text{if } p \in [v_k, v_k + \varepsilon_k] \end{cases}$$

and let

$$v_k = \frac{1}{K - k + 1}$$

and

$$\varepsilon_k \in (0, v_{k+1} - v_k)$$

Non-Concave Environments

- distribution F is regular if and only if $\phi(p)$ is non-decreasing:

$$\phi(p) \triangleq p - \frac{1 - F(p)}{f(p)}$$

- revenue function is concave in quantity q if and only if F is regular

Theorem

There exist regular distributions with common and bounded support for which the optimal uniform price delivers an arbitrarily small profit guarantee as the number of segments increases.

- by construction of truncated exponential distribution in every segement
- truncated exponential distribution are regular

Next Questions

- today we considered approximation of third-degree price discrimination
- second-degree price discrimination: segmentation by quantity or quality
- a special case of second-degree price discrimination is mutli-item monoply:
- what are optimal policies, what are approximation results remain wide open with second-degree price discrimination