Part I

A Complete Foundation

1

Intensionality

Intensionality in logic and language is a phenomenon that has been recognized for over two millennia, and still there is no adequate theory for it. My investigations of intensionality will begin with an elementary inquiry into the origins of intensionality in natural language. Some surprisingly simple arguments will expose defects in what is today the leading treatment of intensionality, the multiple-operator approach. The best representation of intensionality, it will turn out, is one that explicitly appeals to properties, relations, and propositions. In this, the theory of PRPs is seen to be undeniably part of logic.

6. Intensional Abstraction

Consider the following intuitively valid argument:

(I) Whatever x believes is necessary.
Whatever is necessary is true.
∴ Whatever x believes is true.

Suppose that 'is necessary' and 'is true' are treated as 1-place predicates and 'believes', as a 2-place predicate.* Then, the above argument can be represented as valid in any standard quantifier logic:

$$
\text{(I}'\text{)}\quad
\begin{array}{l}
(\forall y)(xB^2y \supset N^1y) \\
\underline{(\forall y)(N^1y \supset T^1y)} \\
\therefore\ (\forall y)(xB^2y \supset T^1y).
\end{array}
$$

Now in theoretical matters, if a currently accepted theory can be easily and naturally employed to account for new data, then other

* In this work when I mention linguistic expressions I will usually follow the convenient convention of autonymous use, by which a simple expression names itself and a concatenation of simple expressions names their concatenation. But where clarity demands, I will shift to the use of single quotes; when I do this, I will sometimes take the liberty to use them for the kind of variable quotation achieved more properly by Quinean corner quotes. I reserve double quotes for use as scare quotes.

things being equal it is desirable to do so. In the science of logic, the currently accepted theory includes quantifier logic. By treating 'is necessary' and 'is true' as 1-place predicates and 'believes' as a 2-place predicate, we can easily and naturally account for the validity of (I) in a currently accepted theory, namely, quantifier logic. Other things being equal, it is therefore desirable to do so.[1]

Now consider another intuitively valid argument, where A is any formula:

(II) Whatever x believes is true.
x believes that A.
$\therefore$ It is true that A.

Suppose, as was just suggested, that we do treat 'is true' as a 1-place predicate and 'believes' as a 2-place predicate. In this case, we seem to be left with no alternative but to parse the second and third lines of (II) as follows:

$$\underbrace{x}\ \underbrace{\text{believes}}\ \underbrace{\text{that } A}$$
$$\underbrace{\text{It is true}}\ \underbrace{\text{that } A}$$

where 'that A' is counted as a singular term syntactically. I do not wish to beg any questions here about the philosophical treatment of 'that'-clauses. For this reason, I will introduce a philosophically neutral notation. I have in mind the bracket notation introduced by Quine for somewhat similar purposes (§35 *Word and Object*). For the moment I leave open what semantical significance the bracket notation will have, and the possibility of indirectly defining the bracket notion will also be left open here.[2] When this bracket notation is adopted, (II) can be naturally represented as follows:

(II′) $(\forall y)(xB^2y \supset T^1y)$
$xB^2[A]$
$\therefore\ T^1[A]$.

The conclusion of (II′) is straightforwardly derivable from the two premises by an application of universal instantiation (UI) and modus ponens (MP), two rules of inference valid in standard quantifier logic. Thus, one can bring argument (II) within the scope of standard quantifier logic simply by adopting the hypothesis that 'that'-clauses are singular terms representable with the bracket notation. To successfully represent (II), one needs no new logical principles, and one needs no knowledge about the logic of expressions occurring within $[A]$. It would seem, therefore, that relative to the framework of quantifier logic, (II′) is the simplest way to represent (II). Thus, on the assumption that the logic for the new singular terms $[A]$ can be satisfactorily worked out, I conclude that it is desirable to treat 'that'-clauses in natural language as singular terms that may be represented by means of the bracket notation.[3]

Summing up, I conclude that it is desirable to treat 'is true' and 'is necessary' as 1-place predicates, 'believes' as a 2-place predicate, and 'that'-clauses as (defined or undefined) singular terms. (This conclusion is just desideratum 16 from §4.)

7. Quantifying-in

Consider the following argument:

(III) x believes that he believes something.
∴ There is someone v such that x believes that v believes something.

There is a reading according to which (III) is intuitively valid. This reading provides an example of the logical phenomenon of quantifying-in. It is desirable that all valid cases of quantifying-in should be representable by an ideal logical theory. (This is desideratum 5 from §4.)

Putting desiderata 5 and 16 together, one obtains an important derived desideratum. Consider the following instance of argument (II):

(IV) Whatever x believes is true.
x believes that v believes something.
∴ It is true that v believes something.

In view of desiderata 5 and 16 it is desirable to represent (IV) as follows:

$$
\text{(IV}'\text{)}\qquad
\begin{array}{l}
(\forall y)(xB^2y \supset T^1y) \\
xB^2[(\exists u)vB^2u] \\
\hline
\therefore\ T^1[(\exists u)vB^2u].
\end{array}
$$

I conclude by analogy that it is desirable to represent (III) in the following way:

$$\text{(III}'\text{)}\qquad \frac{xB^2[(\exists u)xB^2u]}{\therefore\ (\exists v)xB^2[(\exists u)vB^2u].}$$

What is important about this is that the occurrence of v in the singular term $[(\exists u)vB^2u]$ is an *externally quantifiable* occurrence of a variable.[4] I am thus led to conclude that 'that'-clauses ought to be treated as singular terms which may contain externally quantifiable occurrences of variables.

It will be convenient to represent in some perspicuous way which variables within $[A]$ are externally quantifiable. Let δ be the sequence of externally quantifiable variables in $[A]$. Then, I will rewrite $[A]$ as $[A]^\delta$. So, for example, I will rewrite (III′) as follows:

$$\frac{xB^2[(\exists u)xB^2u]^x}{\therefore\ (\exists v)xB^2[(\exists u)vB^2u]^v.}$$

This allows the externally quantifiable variables to be spotted at a glance.

8. Informal Interpretation

I have concluded that it is desirable to represent 'that'-clauses with the bracket notation. Up to now I have left open how this bracket notation should be interpreted and, in particular, what sort of entity corresponds semantically to a given singular term $[A]$. In order to answer this question we must consider desideratum 1 from §4, which concerns *prima facie* failures of substitutivity of co-extensive expressions within 'that'-clauses.

Consider the following argument:

(V) x believes that everything runs.
Everything runs if and only if everything walks.
———
$\therefore$ x believes that everything walks.

Argument (V) is *prima facie* an instance of the principle of the substitutivity of materially equivalent formulas. However, (V) is intuitively invalid. Thus, it constitutes a *prima facie* violation of the principle of the substitutivity of materially equivalent formulas.

Now consider the following related argument:

(VI) x wonders whether y is the author of *Waverley*.
y = the author of *Waverley*.
$\therefore$ x wonders whether $y = y$.

(VI) is a *prima facie* instance of the principle of the substitutivity of co-referential singular terms. However, there is a reading of (VI) according to which it too is invalid. Thus, we have a *prima facie* violation of this substitutivity principle. Desideratum 1 is simply that arguments containing *prima facie* violations of these two substitutivity principles ought to be represented as invalid in an adequate logical theory.

In the bracket notation (V) would be represented as follows:

$$\text{(V}'\text{)}\qquad \begin{array}{l} xB^2[(\forall y)Ry] \\ \underline{(\forall y)Ry \equiv (\forall y)Wy} \\ \therefore\ xB^2[(\forall y)Wy]. \end{array}$$

And the invalid reading of (VI) would be represented as follows:

$$\text{(VI}'\text{)}\qquad \begin{array}{l} xW^2[y = (\iota z)(Az)]^y \\ \underline{y = (\iota z)(Az)} \\ \therefore\ xW^2[y = y]^y \end{array}$$

where in the first premise the definite description $(\iota z)(Az)$ has narrow scope. Now in order for (V′) and (VI′) to qualify as invalid arguments, what sort of entities must correspond semantically to the singular terms $[(\forall y)Ry]$, $[(\forall y)Wy]$, $[y = (\iota z)(Az)]^y$, and $[y = y]^y$? Both here and in what follows my intention is to use the notion of semantical correspondence in as neutral a way as possible. By doing so, I wish to avoid committing myself to any particular semantical method. And also I wish to take into account the fact that 'that'-clauses might be contextually defined singular terms and, hence, that they might bear no simple semantical relation (e.g., the naming relation) to anything. Even if 'that'-clauses are contextually defined singular terms, their use nevertheless produces ontological commitments;[5] thus, in asking what sort of entity corresponds semantically to the singular terms $[A]$, we are at the very least asking to what sort of entity the use of 'that'-clauses ontologically commits us.

The nominalistic answer to the above question is that linguistic entities—either formulas or inscriptions of formulas—are what correspond semantically to 'that'-clauses. Generally speaking, there are two methods by which one can develop the nominalistic answer in detail. According to the first method, a formula such as $xB^2[(\forall y)Ry]$ is treated in such a way that it contains—at least when fully analysed—either a name for, or a structural description of, a particular formula or inscription. On the second method, a formula such as $xB^2[(\forall y)Ry]$ is treated in such a way that even when fully analysed it does not contain any such metalinguistic name or structural description. Carnap's approach and Quine's syntactical approach are instances of the first method. Scheffler's approach is an instance of the second method.

A fatal difficulty in the first method is that it leads to violations of desideratum 12, the Langford-Church translation test. The argument that these nominalistic analyses lead to faulty translation is familiar enough that I will not go over it here.[6]

The nominalist's second method, by contrast, does satisfy desideratum 12. However, it evidently must do so at the price of violating desideratum 13, Davidson's learnability requirement. (Davidson's learnability requirement is that an idealized representation of natural language should have a finite number of undefined primitive constants.) To see why this is so, let us consider Scheffler's approach as an example. According to this approach, a singular term $[A]$ would be contextually analysed as follows:

$$\ldots [A] \ldots \textit{iff}_{\mathrm{df}}\ (\exists v_k)\ (v_k \text{ is-an-}A\text{-inscription } \& \ldots v_k \ldots)$$

where 'is-an-A-inscription' is an undefined primitive predicate that is satisfied by all and only inscriptions synonymous to A. However, since there are an infinite number of distinct 'that'-clauses in natural language, there must be an infinite number of distinct singular terms $[A]$. Therefore, Scheffler's approach requires an infinite number of undefined primitive predicates 'is-an-A-inscription'. The fact that Scheffler's approach requires an infinite number of undefined primitive predicates not only blocks learnability but also blocks the systematization of the internal logic of 'that'-clauses.

The above considerations, together with a number of others,[7] lead me to conclude that linguistic entities, whether formulas or inscriptions of formulas, are not the sort of entity that correspond semantically to the singular terms $[A]$. And the same conclusion goes for sequences or sets of linguistic entities, or indeed any other kind of object that is linguistic in character.

So what sort of entities do correspond semantically to the singular terms $[A]$? Whatever they are they must render arguments such as (V′) and (VI′) invalid. Further, they must lead to no violations of the Langford-Church translation test. And finally, they must make possible the kind of finitistic treatment of language called for by Davidson's learnability requirement. Now we shall see that these features are had by propositions, which are one kind of intensional entity. (Intensional entities are ones that need not be identical even if they are identical in extension.) To be sure, these features are also had by certain other entities that are not in themselves intensional. But upon analysis these other entities seem always to involve some sort of appeal to intensional entities. (For example, these alternate objects might be sets—or sequences or mereological sums—of intensional entities.) So of the choices available, propositions taken on their own make for the most natural answer to the question. Therefore, other things being equal one may conclude that propositions should be identified as the semantical correlata of the singular terms $[A]$.

Why do propositions meet our needs? Why, for example, does argument (V′) come out as invalid when propositions are identified as the semantical correlata of the singular terms $[(\forall y)Ry]$ and $[(\forall y)Wy]$? The answer is simply that the propositions semantically correlated with these two singular terms are not the same. And this is so even though these propositions have the same extension, i.e., even though they have the same truth value. And why do propositions make it possible to pass the Langford-Church translation test? The answer is that propositions are extralinguistic entities. And thus, when 'that'-clauses are given the recommended interpretation, they can be translated into other languages independently of problematic names for, or structural descriptions of, linguistic entities. Finally, how do propositions make it possible to meet Davidson's learnability requirement? The answer to this question is by no means obvious. Indeed, all previous theories of propositions have failed on this score. (See desideratum 13 on the chart.) This is one of the outstanding problems that a new theory of PRPs must surmount. But it turns out that the syntactic and semantic construction in §§12–14 solves it.

9. The Origin of Intensionality in Language

Intensional entities are, as I have said, entities that can be different from one another even though they are identical in extension. Propositions are 0-ary intensional entities; properties, 1-ary intensional entities; and relations, n-ary intensional entities, for $n \geq 2$. In view of this, there is a natural generalization of the bracket notation which provides singular terms for intensional entities of any finite degree. Let A be any well-formed formula, and let $v_1, \ldots, v_m$ be distinct variables, where $m \geq 0$. (I permit there to be free variables in A that are not among these variables $v_1, \ldots, v_m$.) Then, $[A]_{v_1 \ldots v_m}$ is a singular term whose semantical correlate is an intensional entity of degree m. If $m = 0$, the semantical correlate of this singular term is the proposition that A; if $m = 1$, the semantical correlate is the property of being something v_1 such that A; if $m > 1$, then the semantical correlate is the relation among $v_1, \ldots, v_m$ such that A.

In §6 and §7 it was argued from the point of view of logical syntax that certain complex nominative expressions in natural languages—namely, 'that'-clauses—are best represented by singular terms of the sort provided by the bracket notation $[A]_{v_1 \ldots v_m}$, where $m = 0$. There are analogous arguments to show that certain other complex nominative expressions in natural language—namely, gerundive and infinitive phrases—are best represented as singular terms of the sort provided by our generalized bracket notation $[A]_{v_1 \ldots v_m}$, where $m \geq 1$. By this route, then, the theory of PRPs is found to be part of the logic for natural language.

What is logically distinctive about these singular terms $[A]_\alpha$ is that expressions occurring within them do not obey the substitutivity principles of extensional logic. Thus, when a formula A is enclosed within square brackets (followed by appropriate subscripts), an intensional context is generated. This bracketing operation may therefore be viewed as a generalized *intensional abstraction operation*.

According to the now dominant tradition of C. I. Lewis, Carnap, Hintikka, Kripke, *et al.*, intensionality in natural language originates with a diverse, open-ended list of primitive operators, including, e.g., a strict-implication operator, modal operators, deontic operators, epistemic operators, an assertion operator, a causal-explanation operator, a would-be-fact operator, probability operators, ... The intensional abstraction approach to intensionality is different. Suppose that a multiple-operator theorist has the need for a primitive n-place intensional operator $\mathcal{O}^n$. In this case, the advocate of intensional abstraction will instead have an associated n-place primitive predicate O^n. Thus, where the operator theorist has a new category of operator sentences $\mathcal{O}^n(A_1, \ldots, A_n)$, I will simply have the atomic sentences $O^n([A_1], \ldots, [A_n])$. (As I will show, it is as easy to state the semantics for O^n and $[A_i]$ as it is to state the semantics for $\mathcal{O}^n$.) The intensional-abstraction approach has two distinct advantages over the multiple-operator approach. The first has already been discussed: since these diverse primitive operators cannot take singular terms as arguments, there are infinitely many intuitively valid arguments that cannot be represented by this approach. (Argument (I) in §6 is one such argument.) The second advantage is that the general theory of intensionality that emerges on the operator approach is eclectic and incomplete at best. On the intensional-abstraction approach, however, there is a simple and general theory of intensionality: all intensionality in natural language (or at least all intensionality treatable by some operator or other) has a single origin, namely, a generalized intensional abstraction operator. Because of the simplicity and generality of this approach, I conclude that it provides the best provisional representation of intensionality in natural language.[8]

10. First-Order Language

Early in §6 I asserted that in the science of logic the currently accepted theory includes quantifier logic. At that time I chose to defer the question of whether we ought to adopt a first-order or a higher-order formulation as our standard quantifier logic. I will now take up this question. In chapter 4 I will defend the position that the first-order approach is the more natural and general of the two. Here, I will simply list my reasons for thinking that, from the point of view of formal logico-linguistic theory, the first-order approach is superior to the higher-order approach.

First, first-order quantifier logic is complete; higher-order quantifier logic is not. At the same time, the consistency of first-order quantifier logic is less open to doubt than that of the higher-order counterpart. Other things being equal, it is desirable to construct a new theory within a theoretical framework that is complete and whose consistency is as little open to doubt as possible. Thus, other things being equal, it is desirable to construct a new theory within the framework of a first-order quantifier logic as opposed to higher-order quantifier logic.[9]

Secondly, if the first-order approach to quantifier logic is taken, then, as I will show, it is possible to construct a sound and complete logic for the intensional abstraction operation and, hence, for modal matters and for intentional matters (see desideratum 11). Such a result is out of reach if the higher-order approach is taken.

Thirdly, when the higher-order approach is taken, linguistic predicates and sentences (open or closed) are treated as linguistic subjects. This would seem to open up the possibility of new instances of Frege's '$a = a$'/'$a = b$' puzzle. (This possibility is what lies behind Church's worry about the adequacy of a Russellian semantics to characterize the semantics for *Principia Mathematica*. See §23 and §38.) Thus, in the case of higher-order languages Russellian semantics is problematic; not so in the case of first-order languages. (See desideratum 17.)

Fourthly, in order to avoid the logical and intentional paradoxes (see desiderata 8 and 9), the higher-order approach usually incorporates infinitely many distinct sorts of variables which carry with them an implicit commitment to a theory of logical types. (See desideratum 14.) Type theory, however, imposes especially implausible existence restrictions on PRPs, restrictions that in most cases play no direct role in the avoidance of the paradoxes. The first-order approach, by contrast, can easily avoid the logical and intentional paradoxes without appealing to type theory.

Fifthly, suppose that, in order to avoid the logical and intentional paradoxes, a given higher-order theory incorporates infinitely many distinct sorts of variables. In this case, it will be forced into a violation of desideratum 13, Davidson's learnability requirement. To see this, consider any "transcendental" predicate in natural language, i.e., any predicate in natural language whose extension cuts freely across presumed type boundaries. The 2-place predicate 'contemplate' is an example of such a predicate. Since the open sentence '$(\exists x)x$ contemplates y' is satisfiable by objects in every logical type, 'contemplate' would have infinitely many primitive counterparts $C^2_{o\alpha}$, one for each sort of variable y_α in the higher-order language.[10] The first-order approach, on the other hand, needs just one primitive predicate C^2 to represent the natural language predicate 'contemplate'. In fact, the first-order approach satisfies Davidson's learnability requirement on all counts.

Finally, it seems inevitable (especially in connection with stubborn desiderata such as 3, 8, 9, 23, and 24) that higher-order theories will be considerably more complex than their first-order counterparts. This unnecessary complexity is another count against the higher-order approach.

For all these reasons, it would seem that from the point of view of formal logico-linguistic theory, one is better off using the first-order approach.

Summing up, then, I have the following conclusion. The best representation of intensionality in natural language is provided by a first-order quantificational language that is fitted out with (defined or undefined) complex singular terms such as $[A]_{v_1 \ldots v_m}$, and depending on the value of m, these complex terms are semantically correlated with properties, relations, or propositions. A corollary of this conclusion is one of the underlying tenets of the book, the tenet that the theory of PRPs is part of the logic for natural language and as such is part of logic *per se* with all the attendant privileges and responsibilities.

With this matter at least provisionally settled I am finally ready for the first substantive task of this work, the formalization of intensional logic. The general strategy will be this. Given the above conclusions about the origin and character of intensionality, it follows that I shall have succeeded in formalizing intensional logic if I am successful in spelling out the logical properties of the special complex intensional terms $[A]_\alpha$. This is to be done in the two standard phases. First, I give a semantical characterization of the logical properties of the complex terms $[A]_\alpha$. Secondly, I attempt as nearly as possible to give a syntactical characterization of the same logical properties; this is done by the formulation of an axiomatic first-order intensional logic. Since these two tasks are independent of the question of whether the complex intensional terms $[A]_\alpha$ are defined or undefined, I am free to consider them as if they were undefined. By doing this, I am able to obtain one of the major results of the book, namely, that this first-order intensional logic is sound and complete and, hence, that the syntactic characterization of intensional logic is perfectly equivalent to the logically prior semantical characterization. It is after obtaining these results that I will look at the question of whether these complex intensional expressions can be defined and, in particular, whether they can be defined in a first-order extensional language.

Before moving on to the substantive tasks of the work, however, I want to make a brief digression on the topic of quantifying-in, which was considered in §7.

11. Quine and Church on Quantifying-in*

In §31 of *Word and Object* Quine proposes a way to represent quantifying-in that is rather different from the one I proposed in §7. At the heart of Quine's treatment is a multiplication of the senses of 'believe'. For example, Quine would provisionally represent the intuitively valid argument

x believes that x believes something.

∴ There is someone v such that x believes that v believes something.

in the following alternative manner:[11]

$$\frac{B^3(x, x, [(\exists u)B^2(x, u)]_x)}{\therefore\ (\exists v)B^3(x, v, [(\exists u)B^2(x, u)]_x).}$$

On analogy, then, Quine would represent the intuitively valid argument

x believes that x believes y.

∴ There is someone v and something u such that x believes that v believes u.

as follows:

$$\frac{B^4(x, x, y, [B^2(x, y)]_{xy})}{\therefore\ (\exists u, v)B^4(x, u, v, [B^2(x, y)]_{xy}).}$$

The important thing to notice is that three separate senses of 'believe'—represented by B^2, B^3, and B^4—have already been posited. Since for arbitrarily high numbers n, there are 'that'-clauses containing n distinct externally quantifiable variables, Quine's approach leads to infinitely many primitive 'belief'-predicates—B^2,

* The reader may skip over this section without losing the basic line of development of the book.

B^3, B^4, ..., B^n, ...—and, hence, to a violation of desideratum 13, Davidson's learnability requirement.

Now perhaps this problem can be remedied simply by getting rid of B^4, B^5, B^6,... and by doing their work with finite sequences plus the 'belief'-predicate B^3. There is, however, a further problem in the Quinean approach, a problem that has no easy remedy. Consider the following two formulas:

(1) For all y, if x believes y, then x believes that someone believes y.
(2) x believes y.

From (1) and (2) one can derive the following infinite list of formulas:

(3) x believes that someone believes y.
(4) x believes that someone believes that someone believes y.
(5) x believes that someone believes that someone believes that someone believes y.

....

In my bracket notation these derivations are represented simply as follows:

$$
\begin{array}{lll}
(1') & (\forall y)(xB^2y \supset xB^2[(\exists u)uB^2y]^y) & \\
(2') & \underline{xB^2y} & \\
(3') & \therefore\ xB^2[(\exists u)uB^2y]^y & \\
 & & \text{By (1'), (2'), UI, and MP} \\
(4') & \therefore\ xB^2[(\exists u)uB^2[(\exists u)uB^2y]^y]^y & \\
 & & \text{By (1'), (3'), UI, and MP} \\
(5') & \therefore\ xB^2[(\exists u)uB^2[(\exists u)uB^2[(\exists u)uB^2y]^y]^y]^y & \\
 & & \text{By (1'), (4'), UI, and MP}
\end{array}
$$

....

On the Quinean approach, by contrast, (1)–(3) would be represented as follows:

$$
\begin{array}{lll}
(1'') & (\forall y)(xB^2y \supset B^3(x, y, [(\exists u)uB^2y]_y) & \\
(2'') & \underline{xB^2y} & \\
(3'') & \therefore\ B^3(x, y, [(\exists u)uB^2y]_y) & \\
 & & \text{By (1''), (2''), UI, and MP.}
\end{array}
$$

So far so good. However, it appears impossible on the present approach to go on to represent the derivation of (4) from (1) and (3). For there can be no instantiation of (1″) whose antecedent is (3″). The reason for this is that the antecedent of (1″) is an atomic sentence with two arguments whereas (3″) is an atomic sentence with three arguments. One might think that this problem can be circumvented by somehow using formulas containing B^3 in place of formulas containing B^2. But just try. All straightforward attempts to use this idea to expedite the above derivations just lead to further difficulties of their own. I leave it to the reader to convince himself of this.

My conclusion is that, if one adopts quantifier logic as his initial theoretical framework, there is no reasonable alternative to treating 'believes' as a univocal 2-place predicate and 'that'-clauses as defined or undefined singular terms in which externally quantifiable variables may occur. Indeed, if as I have recommended we use my bracket notation provisionally to represent 'that'-clauses, then the treatment that I believe Quine was looking for in *Word and Object* can, ironically, be achieved as follows:

$$[A]^{v_1 \ldots v_j} =_{\mathrm{df}} \langle\langle v_1, \ldots, v_j\rangle, [A]_{v_1 \ldots v_j}\rangle.$$

Although painfully unnatural, this treatment avoids all the syntactic difficulties that beset Quine's actual treatment.[12] For example, the derivation of (3), (4), ... from (1) and (2) can be represented as follows:

(1‴) $(\forall y)(xB^2y \supset xB^2\langle\langle y\rangle, [(\exists u)uB^2y]_y\rangle)$

(2‴) xB^2y

(3‴) $\therefore\ xB^2\langle\langle y\rangle, [(\exists u)uB^2y]_y\rangle$ By (1‴), (2‴), UI, and MP

(4‴) $\therefore\ xB^2\langle\langle\langle y\rangle, [(\exists u)uB^2y]_y\rangle, [(\exists u)uB^2y]_y\rangle$ By (1‴), (3‴), UI, and MP

....

The important thing to notice, however, is that this treatment, unlike Quine's official treatment, takes 'believes' to be a 2-place predicate and 'that'-clauses to be singular terms that may contain externally quantifiable variables. Thus, the conclusions reached in §§6–7 are sustained. This is all that I wanted to show here.

I will next make a few remarks about the inability of the Frege-

Church approach to adequately represent quantifying-in. This fact is not widely recognized and, therefore, deserves discussion. Consider the following formula:

(6) x believes that y is a spy.

Is it possible to represent this formula in Church's system as follows:

> There is an individual concept y_{ι_1} such that y_{ι_1} is a concept of the individual y_ι and x_ι believes the proposition that is the value of the *spy* sense-function when applied to the argument y_{ι_1}.

i.e.,

$$(\exists y_{\iota_1})(y_\iota \,\Delta\, y_{\iota_1} \;\&\; B_{oo_1\iota}(x_\iota, S_{o_1\iota_1}(y_{\iota_1})))?$$

The answer is negative. To see why, consider the following related sentence:

(7) Someone is the F and x believes that the F is a spy.

Intuitively, this sentence has two logically independent readings, an "opaque" reading and a "transparent" reading. In my bracket notation these two readings can be represented, respectively, by the following:

(8) $(\exists y)(y = (\imath z)(Fz) \;\&\; xB^2[S(\imath z)(Fz)])$
(9) $(\exists y)(y = (\imath z)(Fz) \;\&\; xB^2[Sy]^y)$

where the definite description $(\imath z)(Fz)$ has narrow scope. The opaque reading of (7) is represented in Church's system as follows:

(10) $(\exists y_\iota)(y_\iota = \iota_{\iota(o\iota)}(F_{o\iota}) \;\&\; B_{oo_1\iota}(x_\iota, S_{o_1\iota_1}(\iota_{\iota_1(o_1\iota_1)}(F_{o_1\iota_1}))))$.

This seems to be acceptable. However, suppose that the method suggested earlier for representing quantifying-in within Church's system were adopted. Then, it should be possible to represent the transparent reading of (7) with something like the following:

(11) $(\exists y_\iota)(\exists y_{\iota_1})(y_\iota = \iota_{\iota(o\iota)}(F_{o\iota}) \;\&\; y_{\iota_1} = \iota_{\iota_1(o_1\iota_1)}(F_{o_1\iota_1})$
$\qquad\qquad \&\; y_\iota \,\Delta\, y_{\iota_1} \;\&\; B_{oo_1\iota}(x_\iota, S_{o_1\iota_1}(y_{\iota_1})))$.

However, given the intended interpretation of Δ, $\iota_{\iota(o\iota)}$, $\iota_{\iota_1(o_1\iota_1)}$, $F_{o\iota}$, and $F_{o_1\iota_1}$, the following is a logical truth:

(12) $(\exists y_\iota)(y = \iota_{\iota(o\iota)}(F_{\iota(o\iota)})) \supset \iota_{\iota(o\iota)}(F_{o\iota}) \,\Delta\, \iota_{\iota_1(o_1\iota_1)}(F_{o_1\iota_1})$.

It follows from this that (10) and (11) are logically equivalent. But the two readings of (7) are logically independent. Therefore, (11) cannot be an adequate representation of the transparent reading of (7).

Some advocates of the Frege-Church approach to intensional language are not at all disturbed by this sort of outcome, for they are basically skeptical about the legitimacy of quantifying-in, at least as it arises in connection with the usual examples. However, there are examples of quantifying-in that should move even hard-line advocates of the Frege-Church approach, examples that ought to be representable by every treatment of intensional language. The existence of this sort of example has not, as far as I know, been discussed in the literature.

The following intuitively valid argument illustrates the sort of example I have in mind:

(13) For all y, if x believes y, then x believes that someone believes y.
(14) x believes that A.
(15) $\therefore$ x believes that someone believes that A.

Unlike some of the examples of quantifying-in, this example requires no special education of one's intuitions. Indeed, it is unlikely that there is a reading of this argument according to which it is not intuitively valid. Using my bracket notation, I can represent the argument simply as follows:

(13′) $(\forall y)(xB^2y \supset xB^2[(\exists z)zB^2y]^y)$
(14′) $xB^2[A]$
(15′) $\therefore\ xB^2[(\exists z)zB^2[A]]$ By UI and MP.

In contrast to this approach, the Frege-Church approach (as it stands) does not appear to provide any adequate representation of this intuitively valid argument.

To see what the problem is, consider the following candidate representations within Church's system. First, one might attempt to represent the argument as follows:

(13″) $(\forall p_{o_1})(\forall p_{o_2})(p_{o_1} \Delta\, p_{o_2} \supset (B_{oo_1\iota}(x_\iota, p_{o_1}) \supset B_{oo_1\iota}(x_\iota, (\exists y_{\iota_1})(B_{o_1o_2\iota_1}(y_{\iota_1}, p_{o_2})))))$
(14″) $B_{oo_1\iota}(x_\iota, A_{o_1})$
(15″) $\therefore\ B_{oo_1\iota}(x_\iota, (\exists y_{\iota_1})(B_{o_1o_2\iota_1}(y_{\iota_1}, A_{o_2}))).$

True, the inference from (13″) and (14″) to (15″) is valid—or at least it is when supplemented with $A_{o_1} \Delta A_{o_2}$ as an additional premise. However, this way of representing the above argument is not adequate; for (13″) is too strong. To see why, let p_{o_1} be some proposition believed by x_ι and let (13″) be true. Then, for every concept p_{o_2} of p_{o_1}, the following would have to be true:

$$B_{oo_1\iota}(x_\iota, (\exists y_{\iota_1})(B_{o_1o_2\iota_1}(y_{\iota_1}, p_{o_2}))).$$

But this is implausible. To dramatize the implausibility, consider an example given by Church in a different connection (p. 22 n., 'A Formulation'). Let p_{o_1} be the proposition that it is necessary that everything has some property or other. This proposition is in fact the proposition mentioned on lines 27–8 of page 272 of Lewis and Langford's *Symbolic Logic*. Consider the following two sentences:

(16) x_ι believes that someone believes that it is necessary that everything has some property or other.

(17) x_ι believes that someone believes the proposition mentioned on lines 27–8 of page 272 of Lewis and Langford's *Symbolic Logic*.

Clearly, there is a reading of (16) and a reading of (17) according to which it is possible for (16) to be true when (17) is false. In my bracket notation these readings of (16) and (17) would be represented as follows:

(16′) $xB^2[(\exists y)yB^2[N^1[(\forall u)(\exists v)(u \Delta v)]]]$

(17′) $xB^2[(\exists y)yB^2(\iota w)(M^1 w)]$

where M^1 represents 'is mentioned on lines 27–8 of page 272 of Lewis and Langford's *Symbolic Logic*' and $(\iota w)(M^1 w)$ is a definite description having narrow scope. Let us keep these readings in mind. Now suppose that x believes our proposition p_{o_1}. In this event, (13) requires that (16) on the indicated reading is true; (13), however, does not require that (17) on the indicated reading is true. By contrast, (13″) requires that on the indicated readings both (16) and (17) are true. Thus, (13″) does not adequately represent (13); it is too strong.

The following is a second attempt to represent the inference from (13) and (14) to (15) within Church's system:

$$
\begin{array}{ll}
(13''') & (\forall p_{o_1})(B_{oo_1\iota}(x_\iota, p_{o_1}) \supset (\exists p_{o_2})(p_{o_1} \,\Delta\, p_{o_2} \\
 & \qquad \& \; B_{oo_1\iota}(x_\iota, (\exists y_{\iota_1})(B_{o_1 o_2 \iota_1}(y_{\iota_1}, p_{o_2}))))) \\
(14''') & \underline{B_{oo_1\iota}(x_\iota, A_{o_1})} \\
(15''') & \therefore\; (\exists p_{o_2})(A_{o_1} \,\Delta\, p_{o_2} \;\&\; B_{oo_1\iota}(x_\iota, (\exists y_{\iota_1})(B_{o_1 o_2 \iota_1}(y_{\iota_1}, p_{o_2})))).
\end{array}
$$

Although this argument is valid, it too fails to adequately represent the original. The reason for this is that the Churchian representation of (13) is now too weak. To see why, suppose that x_ι believes the proposition p_{o_1} discussed earlier, and suppose further that (16) is false on the reading isolated above. It follows that (13) is false as well. But notice that on the readings isolated above it is possible for (17) to be true even when (16) is false. In addition, if (17) on this reading is true, then so is (13'''). Therefore, from the fact that (13) is false it does not follow that (13''') is false. Hence, (13''') fails to adequately represent (13); it is too weak.

There is a third strategy by which one could attempt to represent the inference from (13) and (14) to (15) within Church's system. Namely, one could incorporate the modified Quinean treatment that I described earlier. According to this rather artificial treatment, objects of belief are identified with ordered pairs:

$$[A]^{v_1 \ldots v_j} =_{\mathrm{df}} \langle\langle v_1, \ldots, v_j\rangle, [A]_{v_1 \ldots v_j}\rangle.$$

However, incorporating this treatment within Church's system not only would violate the spirit of the Frege-Church theory but also would generate excessive complications in connection with the matter of type restrictions. It should be noted, moreover, that such a treatment would be inconsistent with the principle of identity underlying Church's Alternative (2), namely, the principle that necessary equivalence is sufficient for identity. To see this, note that the following is intuitively valid:

$$(\forall x)(\forall y)N^1[x = x \equiv y = y]^{xy}.$$

Therefore, given the principle of identity underlying Church's alternative 2, the following should also be valid:

$$(\forall x)(\forall y)[x = x]^x = [y = y]^y.$$

However, on the modified Quinean treatment, this sentence is definitionally equivalent to

$$(\forall x)(\forall y)\langle\langle x\rangle, [x = x]_x\rangle = \langle\langle y\rangle, [y = y]_y\rangle.$$

But the latter sentence is invalid, for if $x \neq y$, then

$$\langle\langle x\rangle, [x = x]_x\rangle \neq \langle\langle y\rangle, [y = y]_y\rangle.$$ [13]

From the foregoing criticisms it does not follow that there is no way to construct a unified representation of quantifying-in within Church's systems. However, no such unified representation suggests itself.[14]

Incidentally, before winding up these comments on quantifying-in, I should note that, if one were to attempt to develop a treatment of quantifying-in within Carnap's framework or Scheffler's framework, problems involving multiple embeddings of 'that'-clauses would arise. However, it appears that, by adapting the artificial modified Quinean treatment that I described above, one could surmount these problems at least formally. On the assumption that this is so, I have given Carnap and Scheffler '+' grades for desideratum 5 on the chart in §4.

I hope that this digression on alternate treatments of quantifying-in has helped to bring out the virtues of my bracket notation for representing quantifying-in. However, it is now time to leave these philosophical issues behind and to commence the study of formal intensional logic.