# Generalized Quantifiers

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# Some limitations for Predicate Logic

1. Quantified NPs such as *most* (‘most dogs are black’).

2. Conjoined *NPs* (‘every dog and Pavarotti are hungry’).

3. The semantics of NPs is not fully compositional: ‘every dog’ is a syntactic unit, but is not a semantic unit.

# 1. The problem with *most*

How can we get the meaning of "Most dogs are black"?

- Assume in PC in a quantifier like M(ost). What would be a semantic rule for it?
- (i) $[[_S M\ \beta_i\ S]]^{M,g} = 1$ iff for more than half $u \in U$, $u \in \beta \wedge [[S]]^{M,g[u/ei]} = 1$

  (ii) $[[_S M\ \beta_i\ S]]^{M,g} = 1$ iff for more than half $u \in U$, $u \in \beta \rightarrow [[S]]^{M,g[u/ei]} = 1$

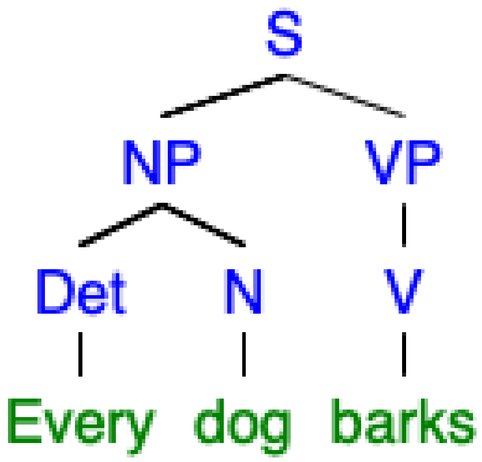
- What would happen in (i) if we have 4 dogs, 4 cats, and 3 dogs are black?
- What would happen in (ii) if we have 4 dogs, 4 cats, and 1 dog is black?

**Does this match our intuitions about the meaning of these sentences?**

# 2. The problem with conjoined NPs

- Conjunction (and disjunction) are cross-categorical operators: they operate on elements of the same syntactic category. However, do we have a general account for all the sentences below (where NPs are conjoined)?
- Every woman and some man are hungry.
- Not every man and some woman is hungry.
- Some woman and some man are hungry.
- Every dog and Pavarotti are hungry.

# 3. The *compositionality* problem




$\forall x[dog(x) \rightarrow barks(x)]$

# A solution: **generalized quantifiers** (Montague 1973, Barwise & Cooper 1981)

- NPs are treated as **sets of sets:**

  - $[[_{NP}$every dog$]^{M,g}$ = the set of sets to which every dog belongs, or the set of properties that every dog has.
  - $[[_{NP}$some dog$]]^{M,g}$ = the set of sets to which at least one dog belongs, or the set of properties that some dog or the other has.
  - $[[_{NP}$the dog$]]^{M,g}$ = the set of sets to which the only individual who is a dog belongs, or the set of properties that a unique dog has.
  - $[[_{NP}$most dogs$]]^{M,g}$ = the set of sets to which more than half the dogs belong, or the set of properties that most dogs have.
  - $[[_{NP}$Pavarotti$]]^{M,g}$ = the set of sets to which Pavarotti belongs, or the set of properties that Pavarotti has.

# In set-theoretic notation:

- $[[_{NP}\text{every dog}]^{M,g} = \{X \subseteq U : [[\text{dog}]] \subseteq X\}$
- $[[_{NP}\text{some dog}]]^{M,g} = \{X \subseteq U : [[\text{dog}]] \cap X \neq \emptyset\}$
- $[[_{NP}\text{the dog}]]^{M,g} = \{X \subseteq U : \text{for some } u \in \text{U}, [[\text{dog}]] = \{u\} \text{ and } u \in X\}$ .
- $[[_{NP}\text{most dogs}]]^{M,g} = \{X \subseteq U : X \cap [[\text{dog}]] \text{ is bigger than } X^{-} \cap [[\text{dog}]]\}$.
- $[[_{NP}\text{no dog}]]^{M,g} = \{X \subseteq U : [[\text{dog}]] \cap X = \emptyset\}$
- $[[_{NP}\text{Pavarotti}]]^{M,g} = \{X \subseteq U : \text{Pavarotti} \in X\}$

# So, how do we evaluate truth conditions?

- If α is an NP and β is a predicate, we say that αβ is true iff [[β]] ∈ [[α]].

- $[[\text{The dog barks}]]^{M,g} = 1$ iff $[[\text{barks}]]^{M,g} \in [[\text{the dog}]]^{M,g}$.
- $[[\text{The dog barks}]]^{M,g} = 1$ iff $[[\text{barks}]]^{M,g} \in \{X \subseteq U : \text{for some } u \in U, [[\text{dog}]] = \{u\} \text{ and } u \in X\}$

Thus, there has to be only one dog, and that dog needs to bark.

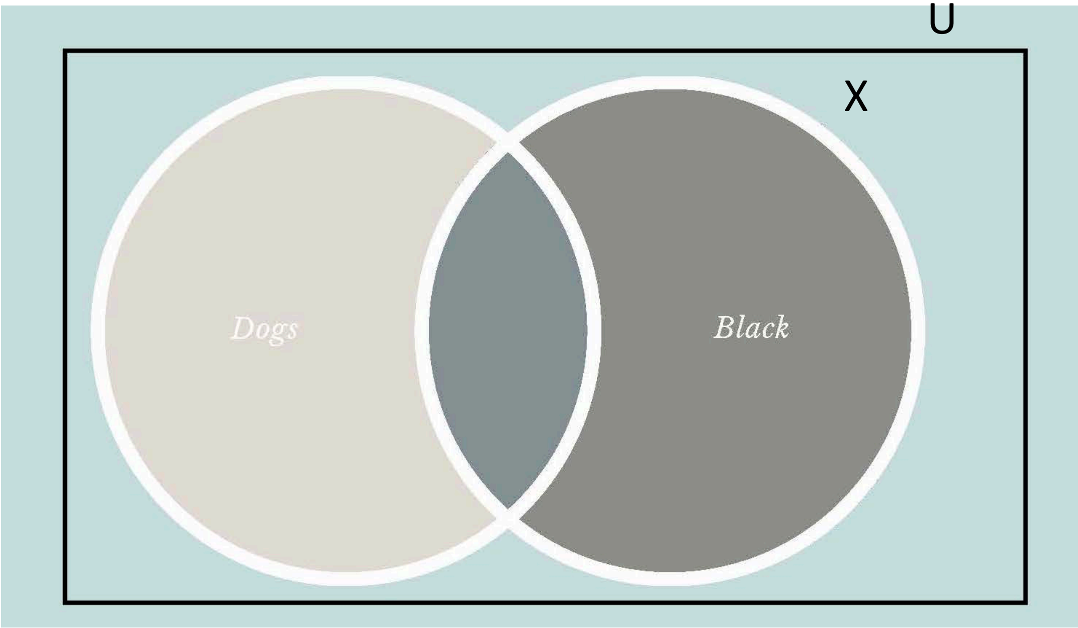
# More generally...

- **For every Y ⊆ *U*** (where Y is the denotation of the common noun):
- [[every]](Y) = {X ⊆ *U* : Y ⊆ X}
- [[some]](Y) = {X ⊆ *U* : X ⋂ Y ≠ Ø}
- [[the]](Y) = {X ⊆ *U* : for some u ∈ U, Y = {u} and u ∈ X} .
- [[most]](Y) = {X ⊆ *U* : X ⋂ Y is bigger than $X^{--}$ ⋂ Y}.
- [[no](Y) = {X ⊆ *U* : Y ⋂ X = Ø}

Chierchia & McConnell-Ginet (2000), Chapter 9, Section 1.

# 1. The problem with *most*

- [most dogs are black]]$^{M,g}$ = 1 iff [[black]]$^{M,g}$ ∈ {X ⊆ *U* : X ∩ [[dogs]] is bigger than X$^{--}$ ∩ [[dogs]]}.

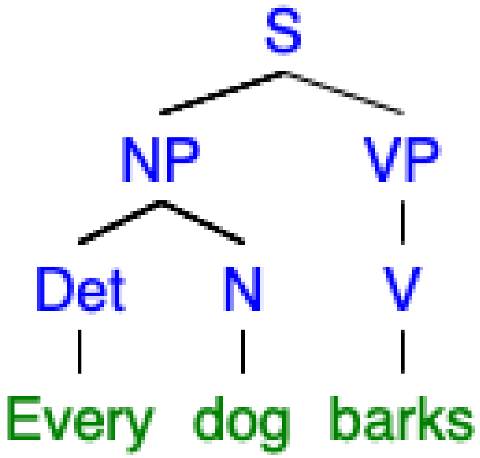


# 2. The problem with conjoined NPs

- Every dog and Pavarotti are hungry
- $[[\text{every dog}]]^{M,g} = \{X \subseteq U : [[\text{dog}]] \subseteq X\}$
- $[[\text{Pavarotti}]]^{M,g} = \{X \subseteq U : \text{Pavarotti} \in X\}$
- $[[\text{every dog and Pavarotti}]]^{M,g} = \{X \subseteq U : [[\text{dog}]] \subseteq X \text{ and Pavarotti} \in X\}$
- $[\text{hungry}]] \in [[\text{every dog}]] \cap [[\text{Pavarotti}]]$ iff
- $[[\text{hungry}]] \in \{X \subseteq U : [[\text{dog}]] \subseteq X\} \cap \{X \subseteq U : \text{Pavarotti} \in X\}$
- $[[\text{hungry}]] \in \{X \subseteq U : [[\text{dog}]] \subseteq X \text{ and Pavarotti} \in X\}$

Conjunction is **SET INTERSECTION** / Disjunction is **SET UNION**.
Negation is **SET COMPLEMENTATION:** if α is an NP: $([[\neg\alpha]]^{M,g} = \mathcal{P}(U) - [[\alpha]]^{M,g})$

# 3. The *compositionality* problem




- $[[\text{Every dog barks}]]^{M,g} = 1$ iff $[[\text{barks}]]^{M,g} \in \{X \subseteq U : [[\text{dog}]] \subseteq \in X\}$

# Let’s work with a model $M^5$

$U^5$ = {John, Bill, Harry, Peter, Andy, Mary}

$V^5$(boy) = {John, Bill, Harry, Peter, Andy}

$V^5$(boring) = {John, Bill, Harry, Mary}

$V^5$(tall) = {John, Bill, Harry, Peter, Andy, Mary}

$V^5$(cute) = {John, Bill, Harry, Peter, Andy, Mary}

$V^5$(fat) = {John}

$V^5$(happy) = {Bill, Peter, Andy, Mary}

$V^5$(blond) = {Harry, Peter, Andy}

$V^5$(rich) = {Harry, Peter, Andy}

$V^5$(smoke) = $\emptyset$

$V^5$(girl) = {m}

# What is the denotation of...?

$[[John]]^{M5,g}$ = {boy, boring, tall, cute, fat}

$[[Bill]]^{M5,g}$ = {boy, boring, tall, happy, cute}

$[[Harry]]^{M5,g}$ = {boy, boring, tall, blond, rich, cute}

$[[Peter]]^{M5,g}$ = {boy, tall, happy, blond, rich, cute}

$[[Andy]]^{M5,g}$ = {boy, tall, happy, blond, rich, cute}

$[[Mary]]^{M5,g}$ = {girl, tall, cute, boring, happy}

# What is the denotation of…?

$[[\text{Every boy}]]^{M5,g}$ = {boy, tall, cute}

$[[\text{Some boy}]]^{M5,g}$ = {boy, boring, tall, cute, fat, happy, blond, rich}

$[[\text{The boy}]]^{M5,g}$ = ∅

$[[\text{Most boys}]]^{M5,g}$ = {boy, boring, tall, happy, blond, rich, cute}

$[[\text{Two boys}]]^{M5,g}$ = {boy, tall, cute, boring, happy, blond, rich}

$[[\text{No boy}]]^{M5,g}$ = {girl, smokes}

# Conjunction and disjunction (example)

$[[\text{every boy and Mary}]]^{M5,g}$ = {tall, cute}

$[[\text{every boy or Mary}]]^{M5,g}$ = {boy, boring, tall, girl, happy, cute}

# Conclusion

Now, instead of the various rules we had for interpreting different quantified sentences **we have one rule**:

$[[[_S NP\ VP]]]^{M,g} = 1$ iff $[[VP]]^{M,g} \in [[NP]]^{M,g}$

This gives the **same results** for the determiners that we could handle as Predicate Logic.  In addition, it **accounts for the determiners that we couldn't**.

# An additional empirical advantage: NPI-licensing

Consider the distribution of *any* N:

They didn't read any book. (grammatical)

* They read any book. (ungrammatical)

Chierchia & McConnell-Ginet (2000), Chapter 9, Section 4.1

# Determiners and grammaticality: Every.

[$_{SUBJ}$Every student who did **anything**] [$_{PRED}$got a prize]
√ (grammatical)

[$_{SUBJ}$Every student] [$_{PRED}$did **anything**]
* (ungrammatical)

| Every : | SUBJ | PRED |
|---|---|---|
| | √ | * |

# Determiners and grammaticality: Some.

$[_{SUBJ}$Some student who did **anything**$]$ $[_{PRED}$got a prize$]$
* (ungrammatical)

$[_{SUBJ}$Some student$]$ $[_{PRED}$did **anything**$]$
* (ungrammatical)

<u>Some</u>: SUBJ PRED
* *

| | SUBJ | PRED |
|---|---|---|
| <u>Some</u>: | * | * |

# Determiners and grammaticality: No.

[$_{SUBJ}$No student who did **anything**] [$_{PRED}$got a prize]
√ (grammatical)

[$_{SUBJ}$No student] [$_{PRED}$did **anything**]
√ (grammatical)

| No: | SUBJ | PRED |
|---|---|---|
| | √ | √ |

# Determiners and grammaticality: Summary

While most Noun Phrases can occur in any position, the distribution of **[any N]** is restricted:

| Determiner | Subject | Predicate |
|---|---|---|
| Every | √ | * |
| Some | * | * |
| No | √ | √ |

# In search for an explanation...

Inferences between sets (supersets and subsets):

That is a dog but it is not an animal. Contradiction!

That is an animal but it is not a dog.

# Determiners and inference: Every.

Every animal is brown


Superset to Subset ↓ = valid

*Every dog is brown – TRUE*

**↓ (subj)**

Every dog is brown


Subset to Superset ↑ = invalid

*Every animal is brown* - FALSE

# Determiners and inference: Some.

Some animal is brown

Superset to Subset ↓ = invalid
Some dog is brown - FALSE

Some dog is brown


Subset to Superset ↑ = valid
Some animal is brown – TRUE

**↑ (subj)**

# Determiners and inference: No.

No animal is brown



Superset to Subset ↓ = valid
*No dog is brown – TRUE*

**↓ (subj)**

No dog is brown



Subset to Superset ↑ = invalid
*No animal is brown - FALSE*

# Inference pattern in SUBJECT position

| | |
|---|---|
| Every | ↓ |
| Some | ↑ |
| No | ↓ |

# Determiners and inference: Every.

Every scarf has dots



Superset to Subset ↓ = invalid
*Every scarf has pink dots - FALSE*

Every scarf has pink dots



Subset to Superset ↑= valid
*Every scarf has dots – True*

**↑(Pred)**

# Determiners and inference: Some.

Some scarf has dots



Superset to Subset ↓ = invalid
Some scarf has pink dots - FALSE

Some scarf has pink dots



Subset to Superset ↑ = valid
Some scarf has dots – TRUE

**↑(Pred)**

# Determiners and inference: No.

No scarf has dots

Superset to Subset ↓ = valid

*No scarf has pink dots – TRUE*

**↓ (Pred)**

No scarf has pink dots

Subset to Superset ↑ = invalid

*No scarf has dots - FALSE*

# Inference pattern in PREDICATE position

| | |
|---|---|
| Every | ↑ |
| Some | ↑ |
| No | ↓ |

# Inference Patterns

| | SUBJECT | PREDICATE |
|---|---|---|
| Every | ↓ | ↑ |
| Some | ↑ | ↑ |
| No | ↓ | ↓ |

Determiners can be **monotone** decreasing (↓ ) or **monotone** increasing (↑) in either position

# Monotonicity. Definition

A generalized quantifier (GQ) is ***monotone increasing*** if for any two sets X and Y, if $X \subseteq Y$, then GQ($X$) entails Q($Y$).

A generalized quantifier (GQ) is ***monotone decreasing*** if for any two sets X and Y, if $X \subseteq Y$, then Q($Y$) entails Q($X$).

Let X = run fast, Y = run.

1. Every boy runs fast.

2. Every boy runs.

3. No boy runs fast.

4. No boy runs.

# Grammaticality and Inference

| | SUBJECT | PREDICATE |
|---|---|---|
| Every | ↓√ | |
| Some | | |
| No | ↓√ | ↓√ |

**The Distribution of Any**

Noun Phrases such as [any N] are grammatical in those structural positions in which inferences from supersets to subsets are valid (in **monotone decreasing** contexts).

# Some conclusions

- Determiners have meanings: they can be thought of as functions from properties to sets of properties.
- These meanings can be described in terms of abstract properties.
- These properties distinguish between structural positions: NP vs. VP.
- There are expressions in natural language that are sensitive to such properties.

# A proposed Universal.
# Determiners and Conservativity

Natural language **only** has **conservative** determiners as basic expressions, where a conservative determiner obeys the following: D(Y)(X) : D(Y)(Y ∩ X)

**Examples:**

- Every man smokes iff every man is a man who smokes
- Some man smokes iff some man is a man who smokes.
- No man smokes iff no man is a man who smokes.

Chierchia & McConnell-Ginet (2000), Chapter 9, Section 4.2

# Some possible non-conservative determiners: $DET_1 (Y) = \{X \subseteq U: Y^{--} \subseteq X\}$ = ALLNON

Consider the following model: U = {a,b,c}

boy = {a,b} smoke = {a,c} cute = {b,c} fat = {a,b,c} rich= {c} happy = {a}

ALLNON (boys) = {smoke, cute, fat, rich}

However, ALLNON is **not conservative**.

To see this compute the following:

"Allnon boys smoke" is true. "Allnon boys are boys who smoke" is false.

smoke $\in$ allnon(boys) smoke $\cap$ boy $\notin$ allnon(boys)

# Is ‘allnon’ a counterexample?

Not really, since both prosodic and meaning cues point to an analysis: [all[non-N]] and not [all-non[N]]

# Some possible non-conservative determiners: $DET_2$ (Y) = {X ⊆ U: X ⊆ Y} = ONLY

Consider the following model: U = {a,b,c}

boy = {a,b}  happy = {b,c}  rich = {b}

ONLY(boys) = {boy, rich}

However, ONLY is **not conservative**.

To see this compute the following:

”Only boys are happy" is false.  ”Only boys are boys who are happy" is true.

happy ∉ only(boys)  happy ∩ boy ∈ only(boys)

# Is ‘only’ a counterexample?

Consider the following evidence:

(1). Determiners cannot occur with pronouns and names but "only" can:

Only John/ Only he came.

(2) "Only" is able to modify a wide variety of syntactic categories (Det cannot):

John only sleeps. (VP)

John sleeps only with his teddy bear. (PP)

John sings only loudly. (Adv.)

Then, only **is not a determiner** and is not subject to the conservativity constraint.