10.3 Polar Coordinates

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Intro to polar coordinates

**Cartesian Coordinates**

**Polar Coordinates**

Example: Plot the point

\[ r = -\sqrt{2}, \quad \theta = \frac{9\pi}{4} \]

\[(r, \theta) = (-\sqrt{2}, \frac{9\pi}{4}) = (-\sqrt{2}, \frac{\pi}{4})\]
Intro to polar coordinates

Cartesian Gridlines:

\[ x^2 + y^2 = r^2 \]

Polar Gridlines:

\[ \theta = \frac{\pi}{2} \]

\[ \theta = \frac{5\pi}{2} \]

\[ \theta = \frac{9\pi}{2} \]

\[ \theta = -\frac{3\pi}{2} \]
Consider the point defined by the Cartesian coordinates \((x = 1, y = 1)\).

Which polar coordinates, \((r, \theta)\), do NOT identify the same point?

A \((\sqrt{2}, \frac{\pi}{4})\)

B \((\frac{9\pi}{4}, \sqrt{2})\)

C \((\sqrt{2}, -\frac{7\pi}{4})\)

D \((-\sqrt{2}, \frac{5\pi}{4})\)
Example

Sketch the curve \( r = 2 \sin \theta \).

\( \theta = 0 \), \( r = 0 \)

\( \theta \) increasing to \( \frac{\pi}{2} \); \( r \) increase to 2.

\( \theta \) increasing to \( \pi \); \( r \) decrease to 0.

\( \theta \) increasing to \( \frac{3\pi}{2} \); \( r \) decreases to -2.

\( \theta \) increasing to \( 2\pi \); \( r \) increase to 0.
Example

Sketch the curve \( r = \cos(3\theta) \).

Graph \( r \) vs \( \theta \) in Cartesian, just to get an idea of how \( r \) changes as \( \theta \) changes.

“Loop pieces of the graph in polar coordinates”
Matching. \textbf{Homework.}

(i) \( r = 2 \sin(\theta) + 1 \)  
(ii) \( r = 3 \sin(2\theta) \)

(iii) \( r = \sin(2\theta) + 2 \)  
(iv) \( r = 2 \sin(3\theta) \)
Polar coordinates ↔ Cartesian coordinates

\( x = r \cos \theta \), \( y = r \sin \theta \)

\( r = \sqrt{x^2 + y^2} \), \( \theta = \arctan \frac{y}{x} \)
Poll.

\[ x = r \cos \theta \]
\[ y = r \sin \theta \]
\[ r = \sqrt{x^2 + y^2} \]
\[ \theta = \arctan \frac{y}{x} \]

Which parametric equations describe the polar curve below?

\[ r = \cos(\theta) \]

A. \( x = \cos(\theta), \quad y = \sin(\theta) \)

B. \( x = \cos^2(\theta), \quad y = \cos(\theta) \sin(\theta) \)

C. \( x = \theta, \quad y = \cos(\theta) \)

D. \( x = \frac{1}{2} \cos(2\theta) + \frac{1}{2}, \quad y = \frac{1}{2} \sin(2\theta) \)
Poll.

Which polar equation describes the line \( x = 3 \)?

A. \( r = 3 \)

B. \( r = 3 \cos(\theta) \)

C. \( r = \frac{3}{\cos(\theta)} \)

D. \( r = 3 \tan(\theta) \)