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Operational momentum in large-number addition and subtraction by 9-month-olds

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ABSTRACT

Recent studies on nonsymbolic arithmetic have illustrated that under conditions that prevent exact calculation, adults display a systematic tendency to overestimate the answers to addition problems and underestimate the answers to subtraction problems. It has been suggested that this *operational momentum* results from exposure to a culture-specific practice of representing numbers spatially; alternatively, the mind may represent numbers in spatial terms from early in development. In the current study, we asked whether operational momentum is present during infancy, prior to exposure to culture-specific representations of numbers. Infants (9-month-olds) were shown videos of events involving the addition or subtraction of objects with three different types of outcomes: numerically correct, too large, and too small. Infants looked significantly longer only at those incorrect outcomes that violated the momentum of the arithmetic operation (i.e., at too-large outcomes in subtraction events and too-small outcomes in addition events). The presence of operational momentum during infancy indicates developmental continuity in the underlying mechanisms used when operating over numerical representations.

Introduction

Research on the development of infants' arithmetic abilities has revealed a complex process involving multiple representation systems (Feigenson, Dehaene, & Spelke, 2004; Xu, 2003), perceptual variables (Clearfield & Mix, 2001), and core estimation abilities that become fine-tuned from infancy all

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the way through adolescence (Halberda & Feigenson, 2008; Lipton & Spelke, 2003). There is evidence that infants are able to go beyond simple discrimination of two amounts (e.g., Lipton & Spelke, 2003; Xu & Spelke, 2000) and perform arithmetic operations such as ordering of numerosities (Brannon, 2002), addition and subtraction (McCrink & Wynn, 2004; Wynn, 1992), and even ratio abstraction (McCrink & Wynn, 2007).

Wynn (1992) showed 5-month-olds a puppet display depicting either an addition scenario (1 doll + 1 doll = 1, 2, or 3 dolls) or a subtraction scenario (2 dolls − 1 doll = 1 or 2 dolls). Infants looked longer to the incorrect outcomes than to the correct outcome, leading Wynn to conclude that infants were performing exact addition and subtraction and using an evolutionarily continuous system of number representation similar to that seen in classic animal studies (e.g., Meck & Church, 1983; Platt & Johnson, 1971). However, a growing body of literature supports the idea that, for infants as well as adults, small numbers of objects (<4) and large numbers of objects are subserved by two distinct systems in the brain (Feigenson et al., 2004; Xu, 2003). The small-number system tracks objects spatiotemporally and readily computes spatial extent characteristics such as area and contour length (Clearfield & Mix, 2001; Feigenson, Carey, & Hauser, 2002a; Feigenson, Carey, & Spelke, 2002b; Trick & Pylyshyn, 1994), and the large-number system tallies imprecise and estimated numerical representations (Lipton & Spelke, 2003; Xu & Spelke, 2000). Given that Wynn (1992) used only small numbers of objects and did not control for continuous extent variables such as expected area and contour length, it is possible that infants were relying solely on the precise small-number system. To address the possibility that infants were not using a truly numerical system to add and subtract, McCrink and Wynn (2004) created a modified addition and subtraction paradigm in which continuous extent variables were controlled and the number of objects exceeded the limits of the small-number system. In that study, 9-month-olds were presented with videos of large-number addition (5 + 5 = 10 or 5) or subtraction (10 − 5 = 10 or 5) problems. The infants looked systematically longer to the incorrect outcomes than to the correct outcome in both the addition and subtraction conditions. This finding bolstered the original claim that preverbal infants can indeed perform nonsymbolic addition and subtraction.

This ability has been examined in the adult literature as well (Barth et al., 2006; Cordes, Gallistel, Gelman, & Latham, 2007; McCrink, Dehaene, & Dehaene-Lambertz, 2007). In one recent psychophysical study, McCrink et al. (2007; see also Cordes et al., 2007) showed adults hundreds of short movies depicting nonsymbolic addition or subtraction events. For example, in an addition event, the participants would see an array of objects come onto the screen and become occluded, and then a second array would be added behind the screen. The occluding screen would then disappear to reveal an outcome, and the participants needed to indicate whether the outcome looked correct or incorrect. The resulting outcome arrays ranged from half of the correct outcome to twice the correct outcome, with many intermediate values displayed. Overall, the participants were able to indicate that the correct sum/remainder of the two operands was indeed correct. Importantly, however, the participants' responses also showed *operational momentum,* an effect where the answers to addition problems were systematically overestimated and the answers to subtraction problems were systematically underestimated. This effect was especially pronounced when (a) subtraction problems were being viewed and (b) the magnitude of the operands themselves was large. (In one striking example, the presented subtraction problem of 32 − 16 = 8 was judged to be correct approximately 60% of the time.)

To further examine the results of McCrink et al.'s (2007) study, let us examine what happened when the incorrect outcome presented differed from the correct outcome by a distinct, but not trivial, factor of 1.5. When the outcome presented was greater than the correct outcome (*with-momentum* outcome), adults viewing addition problems indicated that they thought the outcome looked correct 44% of the time. In contrast, when the outcome presented was less than the correct outcome (*against-momentum* outcome), these same adults indicated that they thought the outcome looked correct 18% of the time. When the outcome presented was truly correct, the adults responded that they thought the outcome looked correct approximately 69% of the time. This pattern of response is very different from that found when participants viewed the subtraction problems. When the with-momentum outcome to subtraction problems was presented, participants responded that they thought it was correct 55% of the time. This is identical to the 55% of the time that they responded that the truly correct answer was correct. When the adults viewed an against-momentum outcome, they responded that they thought it was correct 29% of the time. The momentum for subtraction problems was so great that the participants treated the with-momentum and correct outcomes as nearly equivalent.

McCrink et al. (2007) proposed that operational momentum arises from interactions between the spatial and numerical systems and that it is a side effect of attentional updating along the mental number line. Under this hypothesis, attention is shifted to the right (priming greater magnitudes) for addition and to the left (priming lesser magnitudes) for subtraction. Follow-up studies suggest that this over- and underestimation does indeed have a spatial component to it; when given a spatial array of possible answers, adults are biased to respond to the right side of the screen when viewing addition problems and to the left side of the screen when viewing subtraction problems (Knops, Viarouge, & Dehaene, in press; Pinhas & Fischer, 2008).

Given recent evidence showing that infants and adults possess the same sorts of systems for reasoning about numbers (see Feigenson et al., 2004, for a review), and that these systems share similar neural underpinnings in the parietal lobe across development (Izard, Dehaene-Lambertz, & Dehaene, 2008), the question arises as to whether this operational momentum effect is found during infancy as well as adulthood. If infants exhibit a pattern of behavior similar to that of adults, this suggests that operational momentum is not instantiated purely by culture and schooling but rather arises from spatial–numerical mappings that organize our arithmetic calculations from very early on. To examine this question, we provided 9-month-olds with videos depicting addition and subtraction events of 6 + 4 and 14 − 4. After seeing these events, infants then saw a correct outcome of 10 objects, an incorrect outcome of 5 objects (an outcome that is with-momentum for the subtraction operation and against-momentum for the addition operation), and an incorrect outcome of 20 objects (with-momentum for the addition operation and against-momentum for the subtraction operation). The hypothesis that infants' computations are susceptible to operational momentum effects predicts that infants will show longer looking to the incorrect against-momentum outcomes than to the incorrect with-momentum outcomes.

There are two different ways in which this could occur. Infants could exhibit a curvilinear pattern of looking time—looking similarly to the correct outcome and the with-momentum incorrect outcome and looking longer only to the outcome that goes against the momentum of the operation. This pattern would indicate a large degree of operational momentum because the with-momentum outcome is being treated as indistinguishable from the correct outcome. Alternately, infants could show operational momentum by exhibiting a linear pattern of looking time—low levels to the correct outcome, medium levels to the with-momentum outcome, and high levels to the against-momentum outcome. This pattern would indicate a lesser degree of operational momentum because there is still a distinction between the with-momentum outcome and the correct outcome. As noted earlier, the operational momentum effect in adults has consistently been found to be present for both addition and subtraction but is much stronger in subtraction problems (Knops et al., in press; McCrink et al., 2007; Pinhas & Fischer, 2008). If infants share similar mechanisms for adding and subtracting relative to adults, we may find differing degrees of this effect during infancy. Thus, we predict (a) a linear looking time pattern in the *addition* condition (low looking to the correct outcome, medium looking to the with-momentum outcome, and high looking to the against-momentum outcome) and (b) a curvilinear looking time pattern in the *subtraction* condition (low looking to both the correct outcome and the with-momentum outcome with a peak in looking to the against-momentum outcome).

Method

Participants

The final sample consisted of 24 9-month-olds (mean age = 8 months 28 days, range = 8 months 14 days to 9 months 13 days, 13 boys and 11 girls). The participants were recruited from southern Connecticut via commercial mailing lists and hospital flyers. All infants were full-term. An additional 5 infants were tested but excluded due to fussiness that resulted in not seeing all six test trials (2), lack of visibility of gaze (1), experimenter error (1), or parental interference (1).

Design and procedure

The design and procedure closely mirrored that of McCrink and Wynn (2004). The infants were divided into two conditions: addition (12 infants who saw a 6 + 4 event) and subtraction (12 infants who saw a 14 − 4 event).

Infants sat in a car seat positioned 160 cm from a projection screen surrounded by curtains that concealed the experimenters and equipment. Parents sat next to their infants, facing away from the screen so as not to influence the infants' looking behavior. Infants' looking time was surreptitiously coded online through a hole in the curtain by an observer using a computerized key-press program. Infants first saw an occluder familiarization movie and then three outcome familiarization movies. The test movies were then displayed (see "Stimuli" section below). A single continuous look away of 2 s ended the trial, and a curtain was lowered to cover the screen while the next trial was queued. At the beginning of each trial, the curtain was raised and the experimenter said "Watch the movie!" while squeaking a toy. During the test movies, the toy was again squeaked right before the critical subtraction or addition event (as the screen was raised to cover the first set of objects) to ensure that the infants paid attention. Participants were required to watch the initial occlusion of the objects and the addition or subtraction of the next set of objects for the trial to reach the timing stage, which started when the occluder covering the outcome fell. If participants did not see the addition or subtraction of the set of objects, the trial was aborted and started again.

Stimuli

Events were presented on a projection screen (65 cm wide × 50 cm high). All animations were created using Infini-D animating software and rendered into Quicktime movie clips. All movies were rendered against a black background. The infants were shown two types of movies: outcome familiarization movies designed to familiarize them with the sight of the outcomes and the test events themselves.

As noted in the Introduction, there is research suggesting that infants may be sensitive to continuous extent variables that covary with numbers (e.g., area, contour length) and may use these variables in generating expectations about the outcome of an event (Clearfield & Mix, 2001; Feigenson et al., 2002b). Consequently, all test outcomes of 5, 10, and 20 objects were equated for continuous extent values; each had a summed area of 160 cm^2 and a contour length of 280 cm. The objects presented varied in size. The average area of a single object in the 5-object outcome display was 32 cm^2 (contour length of 56 cm), in the 10-object outcome display was 16 cm^2 (contour length of 28 cm), and in the 20-object outcome display was 8 cm^2 (contour length of 14 cm). The objects that underwent the operations shrunk and grew constantly and individually until they moved over to the right side of the screen (where they were covered). When the occluder was raised, the objects were a uniform size. Thus, the outcome arrays contained objects that were very different (e.g., variable in terms of their rectangular dimensions) from the objects that were occluded.

Occluder and outcome familiarization movies

Infants were first shown an occluder familiarization movie in which one rectangle grew and shrank and moved around the screen and was occluded three times. The infants needed to watch at least one of these occlusions to be considered familiarized to our occluder, and they were shown this video until this occurred. The three outcome familiarization movies (one animation per numerical outcome) served to familiarize the infants to the perceptual features of the outcomes. This is important because there is research showing that infants are sensitive to simple repetition of the beginning array of the first operand and should be familiarized to all outcomes so that they are equally interesting to the children (Clearfield & Westfahl, 2006). The movies consisted of a white computerized occluder on the right side of the screen. After getting the infants' attention by squeaking a toy, the occluder moved off-screen to reveal 5, 10, or 20 rectangles. These rectangle arrays were the exact same displays as the testing outcomes.

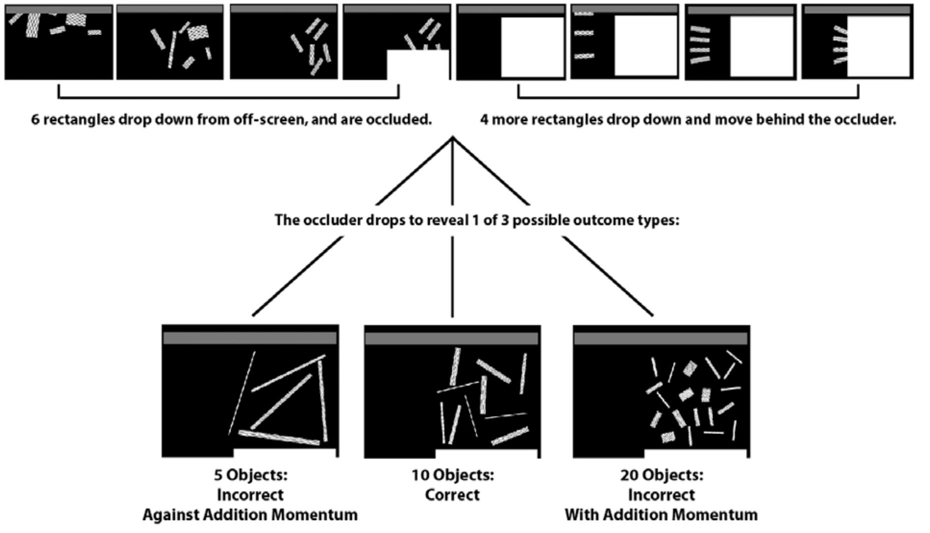

Fig. 1. Schematic of the 6 + 4 addition scenario.

Test movies

In the *addition* condition, a set of 6 rectangles stretched and fell down from off-screen and moved over to the right side of the screen. There they were covered by a white computerized occluder. Then 4 other objects emerged serially from off-screen, lined up on the left side, and moved under the white occluder, which then moved off-screen to reveal either an incorrect outcome of 5 items (the against-momentum outcome), a correct outcome of 10 items, or an incorrect outcome of 20 items (the with-momentum outcome). Each infant saw each outcome (5, 10, and 20), forming a test triplet, and each infant got two test triplets.

In the *subtraction* condition, 14 rectangles stretched down onto the left side of the screen from off-screen. After moving to the right side of the monitor, the white occluder covered them up. Then 4 objects moved out from behind the white occluder and serially off the screen. The white occluder then dropped to reveal either an incorrect outcome of 5 items (the with-momentum outcome), a correct outcome of 10 items, or an incorrect outcome of 20 items (the against-momentum outcome) (see Fig. 1). Again, each infant saw each outcome (5, 10, and 20), forming a test triplet, and each infant got two test triplets. The order of the test triplets was counterbalanced to prevent all infants from always seeing one particular type of outcome (e.g., 5 objects) as the first test trial. The presentation of correct/with-momentum/against-momentum outcome was counterbalanced; of the 24 infants, 7 saw the order 5–10–20, 6 saw the order 10–5–20, 5 saw the order 20–10–5, and 6 saw the order 10–20–5.

Scoring

There was a minimum look criterion of 0.5 s to the outcomes. Participants who did not complete two full triplets of test movies were eliminated from analyses. Looking time was measured by an observer hidden behind a curtain who was unaware of the infants' assigned operation group (addition or subtraction). A second coder reviewed video footage of a subset of participants (25%) and measured the looking times. These times were found to be very highly correlated to the online timing ($r = .98$); thus, all data analyses were performed using results from the primary coder.

Results

Outcome familiarization trials

To ensure that the infants were familiarized equally to each outcome type and possessed no baseline preferences, we performed a repeated-measures 3 (Baseline Type: 5, 10, or 20) × 2 (Gender) × 8 (Order: 5, 10, or 20 outcomes presented first, second, or third after addition or subtraction events) analysis of variance (ANOVA). There was no main effect of outcome familiarization display, $F(2,18) = 2.29$, $p = .129$, and no effects of order, $F(7,9) = 1.82$, $p = .199$, or gender, $F(1,9) = 2.09$, $p = .182$. These looking patterns indicate that the infants looked for similar amounts of time to each outcome type before the testing events occurred (5.9 s to 5 objects, 6.1 s to 10 objects, and 6.6 s to 20 objects).

Test trials

A repeated-measures 3 (Outcome Type: 5, 10, or 20) × 2 (Gender) × 8 (Order: 5, 10, or 20 outcomes presented first, second, or third after addition or subtraction events) ANOVA on infants' looking times revealed a significant main effect of outcome type, $F(2,68) = 4.57$, $p = .014$; no effects of gender, $F(1,34) = 0.075$, $p = .79$, or order, $F(7,34) = 0.352$, $p = .90$; and no significant interactions of outcome type, gender, and/or order.

Planned dependent-samples one-tailed *t* tests were performed to examine the difference between outcome types in the addition and subtraction conditions. Infants in the addition condition looked for 4.37 s to the 5 outcome, 2.77 s to the 10 outcome, and 3.54 s to the 20 outcome. Infants in the subtraction condition looked for 4.42 s to the 5 outcome, 4.51 s to the 10 outcome, and 7.42 s to the 20 outcome. As predicted, infants in the subtraction condition looked significantly longer at the against-momentum incorrect outcome of 20 than at the with-momentum incorrect outcome of 5, $t(23) = -2.77$, $p = .006$. Infants in the addition condition looked marginally significantly longer at the against-momentum outcome of 5 than at the with-momentum outcome of 20, $t(23) = 1.32$, $p = .099$. In both the addition and subtraction conditions, looking to the against-momentum outcome was significantly greater than looking to the correct outcome: addition condition, $t(23) = 2.61$, $p = .008$; subtraction condition, $t(23) = -2.73$, $p = .006$. Finally, looking to with-momentum outcomes was not significantly different from looking to correct outcomes for either the addition or subtraction condition: addition condition, $t(23) = -1.15$, $p = .132$; subtraction condition, $t(23) = -0.12$, $p = .902$ (see Fig. 2).

We performed tests of within-participant contrasts to examine the question of whether there are distinct types of looking time trends in the addition and subtraction conditions. Infants in the addition condition were predicted to exhibit a linear trend with least looking to the correct outcome, intermediate looking to the with-momentum outcome, and most looking to the against-momentum outcome. Infants' responses did in fact follow this pattern; a within-participant contrast analysis revealed a significant linear trend to the data, $F(1,23) = 6.80$, $p = .016$, $MSe = 4.55$, and no significant quadratic trend, $F(1,23) = 0.003$, $p = .955$, $MSe = 5.31$. We also performed a contrast analysis to determine whether there was a quadratic (curvilinear) trend in the subtraction condition, that is, similar looking to the correct and with-momentum outcomes with a "bump" for the against-momentum outcome. Again, infants' responses followed the predicted pattern; there was a significant quadratic trend to the data, $F(1,23) = 6.537$, $p = .018$, $MSe = 8.99$, and no significant linear trend, $F(1,23) = 1.537$, $p = .228$, $MSe = 4.00$).

Discussion

In this experiment, 9-month-olds displayed decreased looking time for correct outcomes to large-number addition and subtraction problems compared with incorrect outcomes. Importantly, this effect was mediated by the nature of the incorrect outcome; infants did *not* look systematically longer

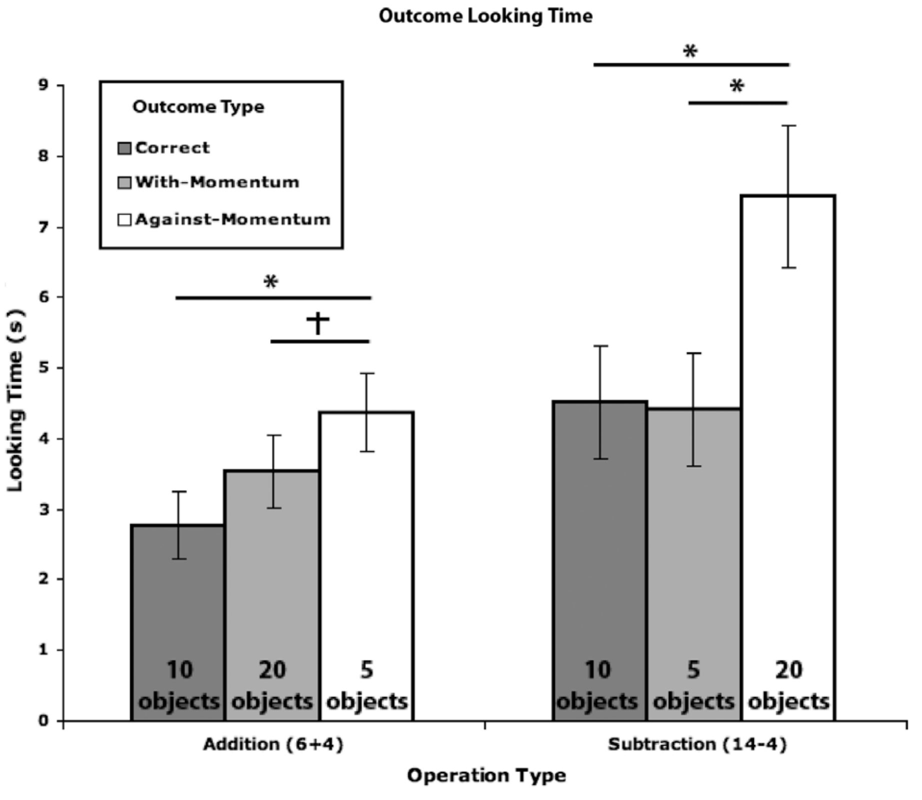

Fig. 2. Looking time to the outcome as a function of operation type (addition or subtraction) and outcome type (correct, with-momentum, or against-momentum). $^{*}p < .01$; $^{\dagger}p < .10$.

to incorrect outcomes of addition problems that exceeded the correct amount or to those subtraction problems that were less than the correct amount.

Moreover, infants who viewed subtraction problems appeared to experience a greater degree of operational momentum than infants who viewed addition problems. Infants in the subtraction condition showed very different looking times to the against- and with-momentum outcomes but failed to distinguish the with-momentum outcome from the correct outcome. In the addition condition, infants exhibited a linear trend to look the most to the against-momentum outcome and the least to the correct outcome, with looking to the with-momentum outcome falling in between. These results are very similar to those found in the adult literature, indicating a developmental continuity not only in the presence of operational momentum but also in the relative strength of this effect in each operation.

There are two distinct interpretations of the findings presented here, and they are not mutually incompatible. Under the first interpretation, the infants in this study experienced true operational momentum. In the initial documentation of this effect in adults, McCrink and colleagues (2007) theorized that it is due to the representation of analog magnitudes as a mental number line and the attentional processes that subtend movement along this number line during arithmetic operations. This spatial extension of attention along the number line is one example of a broader class of anticipatory phenomena known as representational momentum (Freyd & Finke, 1984; Hubbard, 2005); one paradigmatic example is when an observer's memory for the final position of a rotating rectangle is biased toward the momentum of the represented motion (Freyd & Finke, 1984). Subsequent research following up on operational momentum has shown that adding and subtracting lead to response biases to the right and left sides of a screen, respectively (Knops et al., in press; Pinhas & Fischer, 2008). These spatial biases are conceptually related to the Spatial–Numerical Association of Response Codes (SNARC) effect (Dehaene, Bossini, & Giraux, 1993), in which participants are faster to respond to relatively small numbers when they appear on the left side of space and to relatively large numbers when they appear on the right side of space. Both the SNARC effect and the work on left/right spatial shifts in operational momentum suggest that the adults in these studies possess a mental number line that links *less* with *left* and links *more* with *right*.

In many ways, the existence of a mental number line representation during infant development is highly unlikely. Infants have experienced no schooling or education that could have instantiated this organizational schema; thus, there is no reason for them to have a spatial bias for numbers or operating at all. On the other hand, researchers studying developmental dyscalculiacs have proposed that many of their underlying problems come from an inability to spatially represent numerical information (Geary, 1993; Rourke, 1993), suggesting that these two domains are linked from the very beginning and are essential to becoming competent learners of math later in life. Also worth noting is the fact that, despite regional and linguistic differences in how language is read and how words are presented (e.g., left–right in English vs. right–left in Hebrew), the pictorial number line consistently possesses a small–left and large–right mapping.

Under the second interpretation of these data, infants fail to distinguish between certain incorrect and correct outcomes because they are employing the rubric of "if adding, accept more" and "if subtracting, accept less." By this account, they are using an arithmetic principle that does not reference or entail spatial representations of numbers; this account would predict that infants would accept incorrect outcomes even if they were a great distance away from the correct outcome so long as they were more (in the case of addition) or less (in the case of subtraction) than the initial operand. This logical expectation could be part of a "core knowledge" of simple cognitive processes such as numerical logic as proposed by Spelke and colleagues (see Spelke, 2000, for an overview of the core knowledge position), similar to how children possess the logic of arithmetic inversion without formal schooling (Gilmore & Spelke, 2008). However, there is some evidence that the more/less strategy is not the sole way in which infants solve these problems. When shown $10 - 4 = 6$ versus $10 - 4 = 9$ and shown $4 + 5 = 6$ versus $4 + 5 = 9$, 9-month-olds looked longer at the incorrect outcomes even though both outcomes are larger (addition) or smaller (subtraction) than the first operand (McCrink & Wynn, 2005). Other evidence that infants are not using this more/less strategy comes from Wynn (1992); infants in that study looked longer when presented with the problem $1 + 1 = 3$ compared with $1 + 1 = 2$, despite the fact that three objects is still more than the first operand.

Further evidence to distinguish between the two above possibilities can be obtained by careful experimentation with specific magnitudes such as presenting an outlandishly large/small outcome to an addition/subtraction problem. Using different methods to test whether there are spatial biases to infants' arithmetic processing could also be fruitful. For example, one prediction from the account of spatial–numerical operational momentum is that infants will be more likely to orient to an object on the right side after an addition problem and on the left side after a subtraction problem.

These data are only a first step in determining whether the number line is a handy and historically accidental manner of conveying magnitude or an expression of our inherent spatial biases. From them, we can conclude that preverbal infants exhibit operational momentum. The fact that this signature trait is found in both adults and infants indicates a deep continuity across development of the underlying systems used when performing the arithmetic computations of addition and subtraction.

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