Debt Collateralization and Maximal Leverage

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Abstract

We study how allowing agents to use debt as collateral affects asset prices, leverage, and interest rates in a general equilibrium, heterogeneous-agent model with collateralized financial contracts and multiple states of uncertainty. In the absence of debt collateralization, multiple contracts are traded in equilibrium, with some agents borrowing using risky debt and others borrowing with risk-free debt. When agents can use debt contracts as collateral to borrow from other agents, margin requirements decrease, asset prices increase, and the interest rate on risky debt decreases. We characterize equilibrium for N states and L levels of debt collateralization and prove that enough levels of debt collateralization creates an equilibrium featuring maximal leverage on all debt contracts. In the dynamic model, debt collateralization creates larger asset price volatility.

Keywords: Leverage, margins, asset prices, default, securitized markets, asset-backed securities, collateralized debt obligations

JEL classification: D52, D53, G11, G12

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1 Introduction

Prior to the 2007-2008 financial crisis, asset-backed securities (ABS) and collateralized debt obligations (CDOs) significantly contributed to the growth of the market for subprime mortgages. Most prominently, mortgage-backed securities dominated the market and accounted for over half the collateral in CDOs. Although ABS and CDOs played an important role in the housing boom and the financial crisis, we still do not fully understand how these securitized debt markets affect the equilibrium prices of underlying mortgages and leverage on these loans. One of the key features of securitized mortgage markets was the ability to use debt contracts as collateral. This paper asks how debt collateralization, the process by which debt contracts can be used as collateral for new financial contracts, affects equilibrium leverage and asset prices.[2]

To study this issue, we use a general equilibrium model featuring heterogeneous agents and collateralized borrowing following Geanakoplos (1997, 2003). We consider a model with multiple states of uncertainty so that in a leverage economy agents trade multiple contracts in equilibrium. With a few important exceptions, the literature on collateral in equilibrium has focused on binomial models, implying that in equilibrium a single debt contract is traded, which is risk-free.[3] We then allow agents to use debt contracts as collateral to back new financial contracts (i.e., to make new promises) and characterize the equilibrium.

We show that in equilibrium debt collateralization increases the total amount of leverage in the economy, increases asset prices, and decreases interest rates on risky debt. The key insight of our result is that the ability to collateralize debt contracts increases debt prices due to a rise in the "collateral value" of debt. Owing to higher debt prices, leveraged investors

[2]Securitization has many other important features that we abstract away from in our analysis in order to isolate the effect of debt collateralization. Securitization pools loans with similar characteristics to diversify away idiosyncratic risks. Diversification removes less easily quantifiable risks, resulting in securities that are relatively more standardized and thus more liquid. Tranching pools into bonds with different characteristic creates securities with different state-contingencies, improving investors' abilities to hedge and share risks.

[3]Much of the recent financial theory literature has used binomial models to study the effects of fundamentals, tranching, and credit default swaps ("CDS") on prices and volatility. A shortcoming of this approach is that in equilibrium all debt in the economy is risk-free; in other words, binomial models allow only one form of leverage, using risk-free promises. Studying debt collateralization in general equilibrium requires the consideration of a model with risky debt. Multi-state models are richer in the sense that they also allow the study of interactions between different financial innovations such as debt collateralization, tranching and CDS. In current work, Gong and Phelan (2015) study the equilibrium consequences of CDS, and the use of different assets as collateral, in a multi-state model.

are more willing to make large promises to buy risky assets, which in turn increases the price of the risky assets. In fact, every level of debt collateralization further increases collateral values of debt, increasing leverage and the price of the risky asset ultimately backing all debt. With full collateralization, in which all existing debt contracts can be used as collateral, in equilibrium agents use the maximum amount of leverage for the investment they choose. In a dynamic model, debt collateralization also increases volatility in asset prices since leverage and defaults endogenously increase after bad news.

Our modeling environment is a natural way to capture the complexities and innovations in securitized mortgage markets. First, the collateral underlying ABS, whether mortgages or other loans, are themselves debt backed by assets (houses in the case of mortgages).[4] Second, the ABS senior-subordinated tranche structure *implicitly* creates securities in which debt contracts serve as collateral for further debt, and CDOs explicitly use debt (ABS tranches) as collateral to support another level of senior-subordinated tranches. To see this, consider a typical ABS deal, which consists of a pool of mortgages (collateral) supporting senior, mezzanine, and equity/residual tranches. The equity tranche behaves like a leveraged position in the collateral, with the payoff declining "linearly" with the value of the collateral and paying zero when the collateral falls below a certain level. The senior tranche behaves like debt, making a predetermined payoff unless the collateral value falls below a certain threshold, at which point the payoff declines linearly to zero only when the collateral is worth zero. The mezzanine tranches, however, behave like leveraged debt positions. For sufficient values of collateral the tranches get the predetermined payoff (there is not additional upside as with a leveraged position in the collateral), but the subordinated tranches get nothing if the value of the collateral is low (like a leveraged position). In fact, the subordinated tranches are leveraged positions in the debt implicitly "issued" by the subordinated tranches.[5]

Consider a simple, stylized version of an ABS deal with senior, mezzanine, and equity/residual tranches all with face-values of 1, and suppose the value of the collateral could take values of 1, 2, or 3. The senior bonds would get paid 1 for sure; the mezzanine bond would

[4] Nonetheless, there may be good reasons to consider these loans directly as "assets" rather than as loans backed by another asset. This is typically the way mortgage assets are treated in related papers.

[5] In practice the payoffs are complicated by timing of prepayments and how principal payments are allocated to the different tranches.

get paid 1 in only when the collateral is worth 2 or 3, and zero otherwise; and the equity would get paid 1 only in the best state, and zero otherwise. This ABS structure can be equivalently implemented with leveraged investments in the collateral, in the debt backed by the collateral, *and in debt backed by the debt backed by collateral.* The equity investor is effectively buying the collateral with leverage, promising to repay 2 units and defaulting whenever the collateral is worth 2 or less. The mezzanine investor is effectively buying *the promise from the equity investor* and *using this promise as collateral* to borrow 1 from the senior investor. The senior investor buys this promise from the mezzanine investor. This investment scheme exactly replicates the payoffs to the ABS tranches, giving (the mezzanine) investors the ability to use debt as collateral to make new promises.

Wall Street made the concept of using debt as collateral explicit in the creation of CDOs. Because a pool of mortgages can actually take many more values than the number of tranches created in an ABS, the ABS structure does not exactly correspond to the stylized description above: the underlying promises backed by the collateral can be used to make more promises, which is precisely the role of the CDO structure. A CDO did not create pass-through securities backed by mezzanine ABS tranches (in which case the only value of a CDO would have been diversification of idiosyncratic mortgage risk), but rather created leveraged investments in the ABS tranches. Thus, the equity tranche of a CDO created a leveraged investment in ABS tranches, and the senior tranches of a CDO would create investments in debt "issued" by the leveraged (equity) investors. Hence, CDOs (and then CDO-squareds) increased the degree to which debt contracts could be used as collateral to make new promises.[6]

The remainder of this section discusses our relationship to the existing literature. Section 2 introduces a static general equilibrium model with collateral and three states. Section 3 considers the model with leverage, introduces debt collateralization, and characterizes equilibrium. Additionally, Section 3 generalizes to a model with N possible states of the world and with L levels of debt collateralization. Section 4 considers a dynamic model with three time periods to illustrate the increase in volatility due to debt collateralization. Section 5 concludes.

[6] In this process, the supply of safe assets (i.e., perceived to be close to risk-free) increases.

Related Literature

Our paper follows the model of collateral equilibrium developed in Geanakoplos (1997, 2003) and Geanakoplos and Zame (2014), which pioneered the general equilibrium analysis of collateralized lending. Our paper is closely related to the literature on collateral and financial innovation developed in Fostel and Geanakoplos (2008, 2012a,b, 2015a,b) and Fostel et al. (2015). This literature, which uses dynamic, binomial general equilibrium models to explain a variety of economic phenomena, defines financial innovation as the use of new assets as collateral or as the ability to make new promises using existing collateral. Debt collateralization, which expands the set of assets that can be used as collateral, fits directly into this definition of financial innovation.

Several papers study collateral equilibrium with multiple states. Simsek (2013) uses a model with a continuum of states to show how belief disagreements about the future state endogenously create constraints on the amount of leverage that can be used to buy the risky asset. Araujo et al. (2012) examine the effects of default and collateral on risk sharing and prove that with N states, $N-1$ contracts are traded in a collateral equilibrium. Phelan (2015a) studies how changing asset risk and endowment covariances affects asset prices and leverage in equilibrium. Geerolf (2015) studies an economy with a continuum of states and a continuum of agents with differing point-beliefs about the asset payoff. A continuum of contracts are traded in equilibrium, and Geerolf (2015) characterizes the asymptotic distribution of leverage levels in the economy and also shows that interest rates are disconnected from default probabilities, thus explaining the credit-spread puzzle.

Geerolf (2015) uses this setup to study debt collateralization ("pyramiding" in his model), and shows that (1) the asset price increases with each layer of pyramiding, (2) the measure of contracts traded decreases, and (3) the distribution of leverage changes. These results are closely related to ours, however, his approach differs from our model because agents' disagreements are of the form of point-expectations about the asset's value. Thus, all debt contracts in the economy are perceived to be safe by the agents holding them, which has fundamentally different implications for equilibrium. In our model, agents trade different contracts precisely because of perceived risk, and the way that the set of contracts traded in equilibrium decreases is linked intuitively to the debt capacity of each contract.

Our work is related to the literature on securitization. Toda (2014) presents a model in which securitized ABS markets are a way of sharing idiosyncratic risk, and entrepreneurs endogenously borrow from loans with the lowest collateral requirement. ABS reallocate capital to high-risk, high-return technologies, and enhance welfare through improved risk sharing. Others have shown that the additional leverage from securitization is not without risk. Gorton and Metrick (2012) argue that a combination of securitization and repo finance was at the heart of the 2007-2008 financial crisis. Krishnamurthy (2009) discusses how feedback effects in risk capital and risk aversion, repo financing, and counterparty risk decrease liquidity and increase financing cost, causing debt markets to break down as fundamental and market values diverge. Longstaff (2010) shows that malfunctions in debt markets likely occurs through liquidity and risk premium channels, creating contagion for other markets. Geanakoplos and Zame (2011) study how "security pools" affect efficiency of equilibrium.

Our results relate to the literature on how securitized markets create safe and liquid assets. It is well-understood that one of the motivations for the structure of ABS and CDOs was the creation of "safe" (close to risk-free) assets (see for example Gorton and Metrick (2009)). However, in order to create safe tranches, the risky tranches become more risky, which Farhi and Tirole (2014) show can decrease overall liquidity. We provide a possible resolution to the liquidity issue by motivating the demand for *contingent* assets so that the risky tranches are not subject to illiquidity problems. Instead, investors demand contingent promises as a way of increasing leverage, and the creation of risk-free assets naturally follows.

Dang et al. (2011) study how debt collateralization can alleviate asymmetric information problems by creating information-insensitive securities, and they show that the optimal financial instrument is debt backed by debt. Other papers have studied the problems of valuing or rating CDOs (e.g. Bolton et al. (2012), Benmelech and Dlugosz (2010)) or the information problems that arise (e.g. DeMarzo and Duffie (1999), DeMarzo (2005)). In our model there are no informational problems and agents rationally value collateralized debt contracts. Shen et al. (2014) propose a collateral view of financial innovation driven by the cross-netting friction. In our model, debt collateralization is a way of stretching collateral, which is similar to their insight that financial innovation is a response to scarce collateral.

Our work is also related to the corporate finance approach of Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Holmström and Tirole (1997), Acharya and Viswanathan (2011), and Adrian and Shin (2010), in which the endogeneity of leverage relies on asymmetric information and moral hazard problems between lenders and borrowers. These issues are important in loan markets for which the borrower is also a manager who exercises control over the value of the collateral. We consider markets in which the buyer generally has no control or specialized knowledge over the cash flows of the collateral. Our paper also relates to the literature on the effects of credit constraints in Adrian and Boyarchenko (2012), Brunnermeier and Pedersen (2009), Brunnermeier and Sannikov (2014, 2015) and Phelan (2015b).

2 General Equilibrium Model with Collateral

Our analysis focuses on a special class of models with collateral which are the multi-state extension of "C-models," introduced by Geanakoplos (2003). These economies are complex enough to allow for the possibility that financial innovation can have a big effect on prices and equilibrium leverage. But they are simple enough to be tractable and to generate unambiguous (as well as intuitive) results.

2.1 Time and Assets

We begin by considering a two-period, three-state general equilibrium model with time $t = 0, 1$. Uncertainty in the economy is represented by a tree $S = \{0, U, M, D\}$ with a root $s = 0$ at time $t = 0$ and three states of nature $s = U, M, D$ at time 1.

There are two assets in the economy which produce dividends of the consumption good at time 1. As a normalization, the risk-free asset X produces $d_U^X = d_M^X = d_D^X = 1$ unit of the consumption good in every state of the world. The risky asset Y produces $d_U^Y = 1$ unit in state U, $d_M^Y < 1$ units in state M, and $d_D^Y < d_M^Y$ units of the consumption good. To simplify notation, we let $d_M^Y = M$ and $d_D^Y = D$, denoting the state and the payoff by the same variable. Asset payoffs are shown in Figure 2.1.

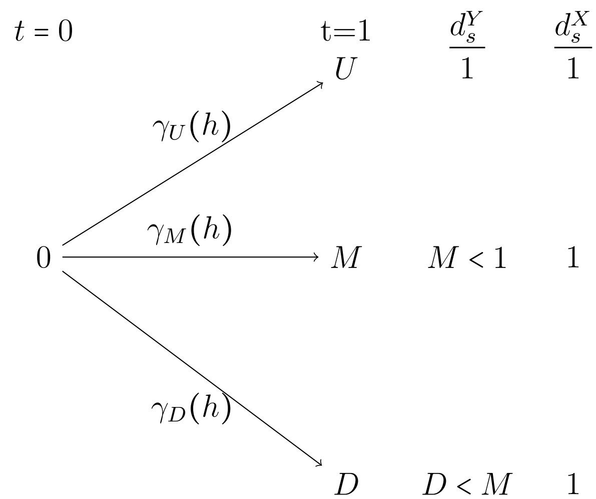

Figure 2.1: Payoff tree of assets X and Y in three-state world

We suppose that agents are uniformly distributed in $(0, 1)$, that is they are described by Lebesgue measure.[7] Investors are risk neutral and have linear utility for the consumption good c at time 1. Each agent h assigns probability $\gamma_U(h)$ to the state U, $\gamma_M(h)$ to the state M, and $\gamma_D(h) = 1 - \gamma_U(h) - \gamma_D(h)$ to the state D. The probabilities $\gamma_U(h)$ and $\gamma_M(h)$ are continuous in h. We further specify that $\gamma_U(h)$ and the ratio $\gamma_M(h)/(\gamma_M(h) + \gamma_D(h))$ are monotonically increasing in h. The second condition implies that the subjective conditional probably of state M, given that U does not occur, is increasing in h. Hence, both conditions imply that a higher h indicates more optimism.

The expected utility of each agent is

$$U^h(c_U, c_M, c_D) = \gamma_U(h)c_U + \gamma_M(h)c_M + \gamma_D(h)c_D,$$

where c_s is consumption in state s. At time 0, each investor is endowed with one unit of each asset.[8]

[7] We will use the terms "agents" and "investors" interchangeably.

[8] None of the results of our analysis depend on risk-neutrality or heterogeneous priors. We could reproduce the distribution of marginal utilities we get from differences in prior probabilities by instead assuming common probabilities, strictly concave utilities, and by allocating endowments of consumption goods so that agents with high h have endowments that are decreasing with the states, and agents with low h have endowments that are increasing with the states. We have chosen to replace the usual marginal analysis of consumers who have interior consumption with a continuum of agents and a marginal buyer. Our view is that the slightly unconventional modeling is a small price to pay for the simple tractability of the analysis.

2.2 Financial Contracts and Collateral

We take all financial innovations exogenously and also suppose that collateral acts as the only enforcement mechanism. At time 0, agents trade financial contracts. A financial contract $j = (A^j, C_j)$, consists of a promise $A^j = (A^j_U, A^j_M, A^j_D)$ of payment in terms of the consumption good, and collateral backing it $C_j = \{X, Y\}$. The lender has the right to seize as much of the collateral as was promised, but no more. Therefore, upon maturity, the financial contract yields $\min\{A^j_U, d^{C_j}_U\}, \min\{A^j_M, d^{C_j}_M\}, \min\{A^j_D, d^{C_j}_D\}$ in states U, M, and D respectively. Note that agents must own collateral before making promises.

To begin our analysis, we suppose that every contract is collateralized by either one unit of X or one unit of Y. When we later allow for debt collateralization, agents are also allowed to borrow against the debt contracts they hold (i.e., the amount owed to them by other agents). Note that even in this scenario, all financial contracts are ultimately collateralized by the assets. We let J^Y and J^X be the set of promises j backed by one unit of Y and X respectively. The set $J = J^X \cup J^Y$ is the set of all possible financial contracts.

We denote the sale of a promise j by $\varphi_j > 0$ and the purchase of the contract by $\varphi_j < 0$. The sale of a contract corresponds to borrowing the sale price and the purchase of a promise is equivalent to lending the price in return for the promise. The sale of $\varphi_j > 0$ units of a contract requires ownership of φ_j units of that asset, whereas the purchase of such contracts does not require ownership.

2.3 Budget Set

Each contract $j \in J$ trades for a price π^j. An investor can borrow π^j by selling contract j in exchange for a promise to pay A^j tomorrow, provided that he owns C_j. We normalize by the price of asset X, taking it to be 1 in all states of the world. Thus, holding X is analogous to holding cash without inflation. We let p denote the price of the risky asset Y. Given asset and contract prices at time 0, each agent decides how much X and Y he holds and trades contracts φ_j to maximize utility, subject to the budget set

$$
\begin{aligned}
B^h(p,\pi) = \{ & (x, y, \varphi, x_U, x_M, x_D) \in R_+ \times R_+ \times R^{J^X} \times R^{J^Y} \times R_+ \times R_+ \times R_+ : \\
& (x-1) + p(y-1) \le \sum_{j \in J} \varphi_j \pi^j & (1) \\
& \sum_{j \in J^X} \max(0, \varphi_j) \le x, \sum_{j \in J^Y} \max(0, \varphi_j) \le y & (2) \\
& c_s = x + y d_s^Y - \sum_{j \in J^X} \varphi_j \min(A_s^j, d_s^X) - \sum_{j \in J^Y} \varphi_j \min(A_s^j, d_s^Y) \}. & (3)
\end{aligned}
$$

Equations (1) and (2) state that expenditures on assets (purchased or sold) cannot be greater than the money borrowed by selling contracts using assets as collateral. Equation (3) states that in the final states, consumption must equal dividends of the assets held minus debt repayment. Recall that a positive φ_j denotes that the agent is selling a contract or borrowing π^j, while a negative φ_j denotes that the agent is buying the contract or lending π^j. Thus there is no sign constraint on φ_j. Additionally, we assume that short selling of assets is not possible.

2.4 Collateral Equilibrium

A collateral equilibrium in this economy is a price of asset Y, contract prices, asset purchases, contract trade and consumption decisions all by agents

$$((p,\pi), (x^h, y^h, \varphi^h, c_U^h, c_M^h, c_D^h)_{h \in (0,1)}) \in (R_+ \times R_+^J) \times (R_+ \times R_+ \times R^{J^X} \times R^{J^Y} \times R_+ \times R_+ \times R_+)^H$$

such that

1. $\int_0^1 x^h dh = 1$
2. $\int_0^1 y^h dh = 1$
3. $\int_0^1 \varphi_j^h dh = 0 \ \forall j \in J$
4. $(x^h, y^h, \varphi^h, c_U^h, c_M^h, c_D^h) \in B^h(p,\pi), \forall h$
5. $(x, y, \varphi, c_U, c_M, c_D) \in B^h(p,\pi) \Rightarrow U^h(c) \le U^h(c^h), \forall h$

Markets for consumption good in all states clear, assets and promises clear in equilibrium at time 0 and agents optimize their utility in their budget sets. Geanakoplos and Zame (2014) show that equilibrium in this model always exists under the assumptions made thus far.

3 Static Model

In this section we introduce the effect of debt collateralization by considering two different versions of the collateral economy, each defined by a different set of feasible contracts J. We describe each variation and the system of equations that characterizes equilibrium. We begin with the 3-state model and then generalize to N states.

3.1 Leverage Economy

We first consider the simplest scenario where agents are allowed to use the risky asset Y as collateral to issue debt contracts. We let them issue non-contingent promises using the asset as collateral. In this case $J = J^Y$, and each $A^j = (j, j, j)$ for all $j \in J = J^Y$.

As shown by Fostel and Geanakoplos (2012b), in equilibrium two contracts are traded: $j_D = D$ and $j_M = M$, with prices π^D and π^M. The interest rate on j_D is zero because it is a safe promise, and thus $\pi^D = D$. However, the delivery of j_M depends on the realization of the state at time 1 and is therefore risky (j_M pays (M, M, D)). This means that any agent making the promise j_M can only borrow $\pi^M < M$. Thus, the interest rate for j_M is strictly positive, defined by $i_M = \frac{M}{\pi^M} - 1$, and is endogenously determined in equilibrium.

In equilibrium there are three marginal investors h_M, h_D, h_π. Agents $h > h_M$ will sell their endowment of X, buy the asset Y, and promise M (issue j_M) for every unit of the asset bought.[9] Agents $h_M > h > h_D$ will sell their endowment of X and buy the risky asset, promising D against every asset bought. Agents $h_D > h > h_\pi$ will sell their endowment of X and Y and buy j_M (effectively lending to agents $h > h_M$). Notice that these agents hold only promises. Agents $h < h_\pi$ will sell their endowment of Y and buy risk-free assets X and contracts j_D backed by the asset. Figure 3.1 illustrates the equilibrium regime.

[9] To simplify notation we will use strict inequalities when referencing the marginal agent, leaving the marginal agent's decision ambiguous. This is without loss of generality since the marginal agent has measure-zero.

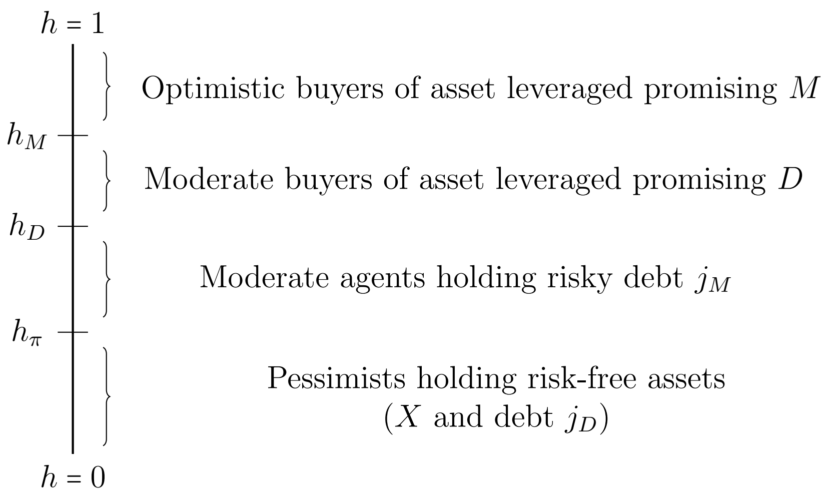

Figure 3.1: Equilibrium regime with leverage in static model.

Marginal investors are indifferent between two different options. Thus, they can be defined by equalizing the expected returns (defined as the expected marginal utility divided by price) on the different investments.

- h_M: indifferent between buying asset with leverage promising M and buying asset with leverage promising D

$$\frac{\gamma_U(h_M)(1-M)}{p-\pi^M} = \frac{\gamma_U(h_M)(1-D)+\gamma_M(h_M)(M-D)}{p-D} \tag{4}$$

- h_D: indifferent between buying asset promising D and holding risky debt j_M

$$\frac{\gamma_U(h_D)(1-D)+\gamma_M(h_D)(M-D)}{p-D} = \frac{(1-\gamma_D(h_D))m+\gamma_D(h_D)d}{\pi^M} \tag{5}$$

- h_π: indifferent between holding risky debt j_M and holding safe assets.

$$\frac{(1-\gamma_D(h_\pi))M+\gamma_D(h_\pi)D}{\pi^M} = 1 \tag{6}$$

In equilibrium, markets must clear. We equate supply and demand for all assets in the economy and obtain the following:

- Supply and demand for risky asset Y:

$$(1 - h_M)\frac{1+p}{p-\pi^M} + (h_M - h_D)\frac{1+p}{p-D} = 1 \tag{7}$$

- Supply and demand of risky debt j_M:

$$(1 - h_M)\frac{1+p}{p-\pi^M} = (h_D - h_\pi)\frac{1+p}{\pi^M} \tag{8}$$

Equation (9) states that the agents buying the risky asset, $h \in (h_D, 1)$, will spend all of their endowment, $(1+p)$, to purchase the risky asset which costs price p and that the demand is equal to the supply of the risky asset, 1. Equation (10) states that the amount of risky debt demanded by agents $h \in (h_M, 1)$ is equal to the amount of risky debt supplied by agents $h \in (h_\pi, h_D)$.

For the beliefs $\gamma_U(h) = 1 - (1-h)^2$, $\gamma_M(h) = h(1-h)^2$, and $\gamma_D(h) = (1-h)^3$, and for payoffs $d_M^Y = .3$ and $d_D^Y = .1$, the equilibrium is:

$$h_M = 0.739, \quad h_D = 0.6636, \quad h_\pi = 0.541, \quad \pi^M = 0.2807, \quad p = 0.8807, \quad i_M = 6.88\%$$

We say that all agents $h > h_M$ are "maximally leveraged" in the sense that making a larger promise would simply result in a transfer of resources to borrowers in the state(s) in which the asset pays its maximum payoff. Agents can choose to promise more to attain more leverage—they can make any promise j—but any promise $j > M$ is in a sense redundant. Notice that any contract $j > M$ has the same deliver as j_M in states M and D (because of default against the asset's payoff) and delivers more only in state U. However, this is exactly the state that investors $h > h_M$ think is comparatively the most likely; thus, agreeing to make a larger promise in U, priced according to more pessimistic agents, would result in raising less than the value of the promise. Agents $h \in (h_D, h_M)$, promising D against each unit of the asset, are not maximally leveraged because promising M changes the delivery to borrowers in both states U and M.

3.2 Economy with Debt Collateralization

We now allow agents to use debt as collateral to make other promises. In the previous economy, agents could use the risky asset Y as collateral; in principal, payments from the risky *debt* are no different from payments from the risky asset. We now suppose that agents can use the payments from risky debt to back further promises. We will refer to the ability to use debt as collateral as "debt collateralization." This follows the definition of financial innovation as the ability to use new assets (financial contracts in this case) as collateral to issue promises.

Specifically, we introduce contracts of the form $j_\ell(j_M) = (\ell, j_M)$. This contract specifies a non-contingent promise (ℓ, ℓ, ℓ) backed by the risky debt j_M acting as collateral. In other words, we allow contracts with $C_j = j_M$ and denote the collateral in the name of the contract. The payoff to $j_\ell(j_M)$ in each state is the minimum of the promise ℓ and the payoff of the debt contract j_M (i.e., $\min\{\ell, d_s^{j_M}\}$). Note that the act of holding j_M and selling the contract $j_D(j_M)$ is equivalent to buying j_M with leverage promising D, yielding a payoff of $(M - D, M - D, 0)$.

We denote equilibrium variables with debt collateralization by a 'hat' ($\hat{\ }$) to distinguish them from their counterparts with leverage. This additional financial innovation leads to the following results.

Lemma 1. *Suppose that in equilibrium agents are able to securitize debt. Then every agent holding risky debt will maximally leverage their purchases of risky debt. That is, all agents holding j_M will sell the promise $j_D(j_M) = (D, j_M)$.*

The full proof is in the appendix, but the intuition is straightforward. Agents holding risky debt are essentially betting on the realization of state U or M at time $t = 1$, as they would lose money if state D is realized. Since these agents are relatively optimistic (compared to agents who choose to hold j_D), they would be willing to sacrifice consumption in state D for the chance to have even more consumption in state U or M. Since j_M pays (M, M, D), promising D maximally leverages the investment in j_M. Put differently, the marginal agent $\hat{h}_\pi$ thinks j_M is priced to exactly compensate for the risk, but every agent $h > \hat{h}_\pi$ thinks the risk is lower than implied by the price and thus would like to leverage their investment in the debt.

This result has four important implications for equilibrium. First, investors in j_M use borrowed money to invest, and second, issuing risk-free debt against j_M increases the supply of safe assets. These two forces imply $\hat{h}_\pi > h_\pi$ and $\hat{\pi}^M > \pi^M$. Third, agents buying the risky asset and promising M can thus borrow more for the same promise, making this investment strategy more attractive. This implies that the marginal investor high-leveraged investor need not be as optimistic, i.e., $\hat{h}_M < h_M$. Fourth, because investing in risky debt is more attractive (owing to the ability to leverage the investment) the marginal investor willing to buy Y and promise D is more optimistic, $\hat{h}_D > h_D$, because the least optimistic agents previously holding the risky asset (in the leverage economy) will now prefer to hold the risky debt with leverage because of its higher return. In fact, in equilibrium no agent chooses this option, which is stated in the following lemma.

Lemma 2. *Let agents be allowed to collateralize debt. Then, every agent holding the risky asset will maximally leverage their purchases of the risky asset. In other words, every agent holding the risky asset will promise M.*

The full proof is in the appendix, but we provide intuition for the result. Any agents who would prefer to buy the risky asset Y and promise D (yielding $(1-D, M-D, 0)$) than to buy the risky asset Y and promise M (yielding $(1-M, 0, 0)$), would do so because they believe state M is relatively more likely than state U. Given these beliefs, and given equilibrium prices, these agents would prefer to get equal payments in M and U rather than a larger payoff in U. Hence, buying the risky debt j_M and promising D, which yields *equal* payments in M and U but still no payment in D ($(M-D, M-D, 0)$), is preferred to buying Y and promising D, which yields a larger payoff in U than in M.

Proposition 1. *In equilibrium, there exist two marginal buyers $\hat{h}_M$ and $\hat{h}_\pi$ such that all $h \in (\hat{h}_M, \hat{h}_\pi)$ will hold risky debt with maximal leverage (promise D); all $h < \hat{h}_\pi$ will hold safe debt and X, and all $h > \hat{h}_M$ will hold the risky asset with maximal leverage (promise M).*

This result follows directly from the previous two lemmas and the fact that optimism is strictly and monotonically increasing in h. Figure 3.2 illustrates the equilibrium regimes with debt collateralization and with leverage.

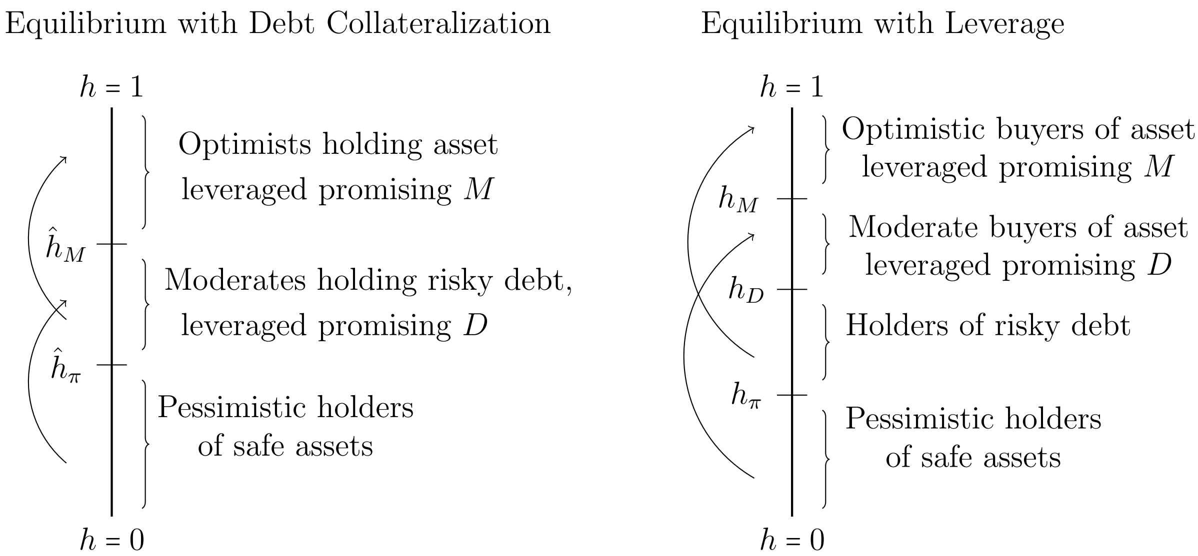

Figure 3.2: Equilibrium with leverage versus equilibrium with debt collateralization.

The key insight for this result is that the price of any asset is a sum of the payoff value and the collateral value. Allowing a debt contract to be used as collateral increases its price—it now has a collateral value—which increases the value to buying the risky asset and issuing that debt contract. Furthermore, because only the risky asset will back risky debt in equilibrium (the risky debt will back safe debt in equilibrium), the collateral value of the risky debt, in effect, gets imparted to the risky asset. Using the risky asset to issue safe debt is "inefficient": the risky asset can be used to back both risky debt and safe debt by issuing risky debt against the asset issuing safe debt against the risky debt. This process creates a new security with collateral value, while using the asset to issue safe debt does not.

Thus, with collateralization we have the following equations defining the marginal investors (again given by equalizing expected returns on two investment options):

- $\hat{h}_M$: indifferent between holding the risky asset with leverage and the risky debt with leverage.

$$\frac{\gamma_U(\hat{h}_M)(1-M)}{p-\hat{\pi}^M} = \frac{\gamma_U(\hat{h}_M)(M-D)+\gamma_M(h_i)(M-D)}{\hat{\pi}^M - D} \tag{9}$$

- $\hat{h}_\pi$: indifferent between holding the risky debt with leverage and the safe asset

$$\frac{\gamma_U(\hat{h}_\pi)(M-D)+\gamma_M(\hat{h}_\pi)(M-D)}{\hat{\pi}^M - D} = 1 \tag{10}$$

The market clearing conditions are given as follows:

- Risky asset Y:

$$\frac{(1-\hat{h}_M)(1+\hat{p})}{\hat{p}-\hat{\pi}^M} = 1 \tag{11}$$

- Risky debt j_M:

$$\frac{(1-\hat{h}_M)(1+\hat{p})}{\hat{p}-\hat{\pi}^M} = \frac{(\hat{h}_M-\hat{h}_\pi)(1+\hat{p})}{\hat{\pi}^M - D} \tag{12}$$

Table 1 presents the equilibrium with debt collateralization and compares to the equilibrium with leverage. Compared to the economy with only leverage, the price of the asset rises and the interest rate on the risky debt decreases because the amount of leverage in the economy increases in equilibrium and there is more risky debt, and more safe debt, being supplied.

Table 1: Equilibrium with Debt Collateralization and with Leverage

	Leverage	Collateralization ($\hat{}$)
p	0.8807	0.888 ↑
π^M	0.2807	0.285 ↑
i_M	6.88%	5.09% ↓
h_M	0.739	0.681 ↓
h_D	0.6636	–
h_π	0.541	0.583 ↑

Notice that the marginal safe-debt investor increases with debt collateralization, reflecting a greater supply of safe debt. Collateralizing risky debt has thus served two purposes: it isolates upside payoffs to agents buying risky debt with leverage, and it creates safe debt for more pessimistic agents, increasing the supply of risk-free securities.[10]

[10] It is worth contrasting this result with Geerolf (2015), in which all debt is perceived to be risk-free because agents have point-mass beliefs. As a result, collateralizing/pyramiding debt contracts does not increase the supply of safe assets. In our model there is a non-trivial demand for safe assets given agents' beliefs.

3.3 Generalization to N States and L Levels of Collateralization

With three states, the economy with leverage features only one risky contract in equilibrium. Once this contract can be used as collateral, no agent invests in risky debt and makes a risky promise. In other words, allowing the first risky contract to be securitized was sufficient so that no further contracts could be securitized. With more than 3 states this result is no longer true. In a leverage economy, multiple *risky* contracts will be traded in equilibrium. If agents can use these initial debt contracts as collateral, in equilibrium some agents will invest in risky debt contracts and make *risky* promises. These second-level debt contracts (backed by debt backed by the asset) could in principal be used as collateral to make further promises. Equilibrium will thus depend on how many "levels of debt" can be used as collateral (i.e., how many stages removed are promises from the risky asset ultimately backing those promises).

This issue is precisely what we analyze in this section. We show that every level of debt collateralization increases the minimum promise made by agents buying the asset, and with "complete collateralization," so that any existing debt contract can be used as collateral, agents make the maximum (natural) promise available for every investment, risky asset or risky debt.

We will now generalize the above lemmas and proposition to a static model with N states and L levels of collateralization. Considering multiple levels of collateralization requires introducing some new notation for debt contracts issued at each level of collateralization. Let the states be given by $S = \{S_1, \dots S_N\}$ with the payout of the asset Y being s_n in the state S_n. For convenience, we well-order the states so that $s_1 < s_2 < \dots < s_N$ and normalize so that $s_N = 1$. Each agent h assigns probability $\gamma_n(h)$ to the state S_n and we have that

$$\sum_{n=1}^{N} \gamma_n(h) = 1, \qquad \gamma_n(h) \geq 0 \quad \forall n, h.$$

We assume that for all $M \in [2, N]$, we have that the ratio

$$\frac{\gamma_M(h)}{\sum_{n=1}^{M} \gamma_n(h)}$$

is monotonically increasing in h. This condition implies that the subjective conditional probability of state S_M given states $\{S_1, \ldots S_M\}$ is increasing in h, denoting that optimism increases uniformly with h.

In equilibrium with leverage, agents can buy the risky asset leveraged with any promise $s_1, \ldots, s_N$ by selling the promise $j_n^0 = (s_n, Y)$ (i.e., we denote debt contracts backed directly by the asset with a superscript 0). That is, the agent promises to pay s_n at time 1 and uses Y as collateral. Note that the payout of the promise can only be realized in states $S_n, \ldots, S_N$. For all states S_l with $l < n$, the seller of j_n^0 defaults and the buyer of j_n^0 seizes the asset as collateral, which is worth $s_l < s_n$. Thus, each j_n^0 pays $(s_n, s_n, \ldots, s_2, s_1)$ in the states $(S_N, S_{N-1}, \ldots, S_2, S_1)$. Additionally, note that $j_1^0 = (s_1, Y)$ is safe debt.

We write Y/j_n^0 to denote the act of holding Y and selling the debt contract j_n^0, and denote the price of the debt contract j_n^0 by π_n^0. In the absence of debt collateralization, we have in equilibrium that agents will do one of the following:

1. hold Y/j_n^0, where $1 \leq n \leq N-1$,
2. hold risky debt j_n^0 with $2 \leq n \leq N-1$,
3. hold safe debt j_1^0 or the safe asset.

Definition 3.1. We say the *first level of debt collateralization* is the creation of promises j_n^1 using j_k^0 as collateral. We write $j_n^1(j_k^0) = (s_n, j_k^0)$ denote the *debt contract* that is traded when an agent holding j_k^0 and sells the promise j_n^1. Note that s_n is the amount promised and we must have $k > n$. Again, an agent holding j_k^0 and selling j_n^1 is denoted by j_k^0/j_n^1.

For a contract j_k^0 to be meaningful collateral for a promise s_n it must be that $s_k > s_n$ because otherwise the payoff to j_k^0 would always be less than the promise (and equality would render the new promise redundant). Thus, in what follows we will only consider when agents use meaningful collateral to make new promises, requiring that $k > n$ for any contract$j_n^1(j_k^0)$. Given this restriction, the payoffs to $j_n^1(j_k^0)$ are the same for every $k > n$, and so we can denote the price of a contract $j_n^1(j_k^0)$ by π_n^1.

Definition 3.2. The L-th level of of debt collateralization is the creation of the promises j_n^L, where $1 \leq i < N-L$ from the existing debt contracts j_k^{L-1}, where $1 < k < N-L+1$. In other words, the buyer of the promise j_k^{L-1} is able to sell the promise j_n^L, using j_k^{L-1} as collateral.

Again, we must have $n < k$. We denote the promise of j_n^L with j_k^{L-1} as collateral by writing $j_n^L(j_k^{L-1}) = (s_n, j_k^{L-1})$. We denote an agent buying j_k^{L-1} and selling j_n^L by j_k^{L-1}/j_n^L.

Thus, each additional level of collateralization involves the creation of new bonds, and allows all previously existing, risky bonds to be purchased with leverage. So long as the backing collateral is meaningful, given the monotonicity of payoffs for debt contracts, the payoff of any contracts is defined by the promise. We use π_n^l to denote the price of any debt security $j_n^L(j_k^{L-1})$ with $k > n$. Note that for all l, $j_1^l(j_j^{l-1}) = (s_1, j_j^{l-1})$. Thus, the price of $j_1^l(j_j^{l-1})$ is s_1 for all l because it is risk-free debt.

We can explicitly characterize equilibrium for any level of collateralization:

Theorem 3.1. *At the L-th level of debt collateralization, the following leveraged positions exist in the economy*

- Y/j_n^0, *where* $L < n < N$
- j_j^l/j_k^{l+1}, *where* $0 \le l < L,\ L - l < j < N - l,\ L - l \le k < j$
- j_ℓ^L, *where* $1 \le \ell < N - L$.

Additionally, more optimistic investors invest in assets with larger face values, and within each asset-class investors are ordered by the amount of leverage they use.

The proof is involved and given in Appendix A, essentially inductively applying the methods used to prove Proposition 1. The intuition for this result is that each level of collateralization increases the collateral value of new promises and of every debt contract that could already be used as collateral. As collateralization increases, more debt contracts have collateral value, which increases the price of the newly securitized debt as well as the values of all the "upstream" debt contracts that can back those promises. As a result, when a security can be used to back promises that serve as collateral L times, making a smaller promise than stimulated by the theorem would not maximize the collateral value of debt contracts. Thus, investors make the largest promise that maximizes the collateral value of "downstream" promises.

We state a few implications of the theorem to provide more meaning. The second and third corollaries follow immediately from Theorem 3.1.

Corollary 3.1. *In an N-state model with $N \geq 3$, there can be at most $N - 2$ levels of debt collateralization.*

Proof. At $N - 2$ levels of debt collateralization, we have that only the agents holding the risky asset Y are holding it leveraged against state S_{N-1}. That is, they hold Y/j^0_{N-1}. Since we must have a Lebesgue-measurable set of agents holding the asset, it must be the case that $N - 2$ is the maximum level of collateralization. □

Corollary 3.2. *With each additional level of debt collateralization, there is one fewer marginal buyer of the risky asset Y.*

Corollary 3.3. *Consider the continuum of agents in the economy. At the maximum $N - 2$ levels of debt collateralization, the interval $(0, 1)$ is broken up into $N+1$ sub-intervals, denoted $(1, \hat{a}_1), (\hat{a}_1, \hat{a}_2), \ldots, (\hat{a}_{n+1}, 0)$. The first interval, $(1, \hat{a}_1)$ consists entirely of agents holding Y/j^0_{N-1}. The second interval, $(\hat{a}_1, \hat{a}_2)$ consists only of agents holding j^0_{N-1}/j^1_{N-2}. In general, the k^{th} interval, where $k > 1$, consists of agents holding $j^{N+2-k}_{N+1-k}/j^{N+1-k}_{N-k}$. In other words, every level of agents in the economy is lending directly to the level above and maximally leveraging the asset or contract in which they invest.*

Debt Collateralization and Tranching

Tranching refers to the process of using collateral to back promises of different types. Using collateral to back contingent promises can increase the collateral value of the asset when agents differentially value state-contingent payoffs. Fostel and Geanakoplos (2012a) show that, with sufficient heterogeneity regarding how states are valued, increasing the contingency of available promises increases the collateral value, and thus the price, of an asset.

Debt collateralization has a similar effect. Each level of debt collateralization increases the collateral value of a larger set of "upstream" debt contracts, which adds to the collateral value of the asset. In the limit, agents isolate payoffs to be above a certain threshold, receiving zero in default states. Equivalently, contracts have a higher collateral value because buying a contract with a larger promise creates a greater degree of state-contingency in the payoff. Debt collateralization is a way of adding a greater degree of state-contingency to non-contingent debt contracts. In this section we formalize this equivalence.

Consider an economy with no borrowing, but the asset Y can be split into the following tranches by a financial intermediary: $T_1, \ldots, T_{N-1}$ where T_1 pays s_1 in all states of the world, and T_k, where $k > 1$, pays $s_k - s_{k-1}$ for all states of the world S_n where $n \geq k$ and 0 otherwise. That is, one unit of the risky asset Y can be used to simultaneously back multiple promises, creating the following tranches:

$$\begin{aligned}
T_N &: \ (s_N - s_{N-1},\ 0,\ 0,\ \ldots\ ,\ 0), \\
T_{N-1} &: \ (s_{N-1} - s_{N-2},\ s_{N-1} - s_{N-2},\ 0,\ \ldots\ ,\ 0), \\
&\vdots \qquad\qquad\qquad \vdots \\
T_2 &: \ (s_2 - s_1,\ s_2 - s_1,\ \ldots\ ,\ s_2 - s_1, 0) \\
T_1 &: \ (s_1,\ s_1,\ \ldots\ ,\ s_1)
\end{aligned}$$

Note that

$$T_1 + T_2 + \cdots + T_N = Y.$$

We refer to the above as *down-tranching* to emphasize the state-contingency applies to "down states" in which the payoff is below the face value.[11]

In this economy, rather than trading the risky asset Y, investors buy and sell the tranches listed above (though they can exactly replicate Y by buying all the tranches). We specify that each investor must hold a non-negative quantity of each tranche and refer to equilibrium as the down-tranching equilibrium. The following result holds:

Corollary 3.4. *The down-tranching equilibrium is equivalent to equilibrium with complete debt collateralization. That is, there exists a bijective mapping of assets and prices from the debt collateralization equilibrium to the down-tranching equilibrium such that the buyers of assets remain the same. Specifically,*

1. *Any agent buying Y/j^0_{N-1} (collateralization) will buy T_N (down-tranching).*

2. *Any agent holding j^l_n/j^{l+1}_{n-1} with $N > n > 1$ (collateralization) will buy T_n (down-tranching).*

[11] In contrast, complete tranching would refer to the creation of Arrow securities for each state so that each tranche was completely state-contingent, not just paying zero in down states.

3. *Any agent holding j_1^L (collateralization) will buy T_1 (down-traching).*

4. *Letting q_N denote the price of T_N, down-tranching equilibrium will have*

   (a) $q_N = \hat{p} - \hat{\pi}_{N-1}^0$.

   (b) $q_n = \hat{\pi}_n^{N-n-1} - \hat{\pi}_{n-1}^{N-n}$

   (c) $q_1 = \hat{\pi}_1^{N-2} = s_1$.

   where $\hat{p}$ and $\hat{\pi}_j^k$, are the equilibrium prices for the asset and debt securities in the complete collateralization equilibrium, respectively.

Proof. This follows because the expected return of holding T_n in the down-tranching equilibrium is the same as holding j_n^l / j_{n-1}^{l+1} in the collateralization equilibrium, when $N > n > 1$. Similarly, the expected return of Y / j_{N-1}^0 is identical to that of T_N; the expected return to holding q_1 is exactly the return of j_1^{N-2}

□

Thus, tranching and debt collateralization have an essential equivalence. Additionally, the down-tranching equilibrium does not require the existence of a separate financial intermediary. To see this, consider an economy where anyone holding the asset can use it to back the promises stated above. Then, every agent will keep/buy the tranche that provides the highest expected return and sell the rest. This provides the equilibrium that we have already stated. One natural candidate for the "intermediary" would be the most optimistic buyers who sell off the remaining tranches to the other investors.

In reality financial innovation included forms of both tranching and debt collateralization. Subprime mortgage pools would be used to create tranches of different seniority. Each tranche of the asset-backed security ("ABS") would pay different amounts depending on the aggregate value of the mortgage pool (i.e., in different states of the world). A typical ABS deal would tranche a pool of mortgages into 4 or 5 rated bonds and a residual, or equity, tranche. These tranches (typically the mezzanine bonds) would then be pooled together to serve as collateral for a CDO, which would issue another 4-5 bonds. And the process would continue as the tranches from the CDO would be collateralized into a CDO-squared. Each stage included both tranching and collateralization of existing debt securities. One can suppose that this process allowed Wall Street to progressively approach the "complete collateralization" or "down-tranching" benchmark using fewer, less complex steps.

4 Price Volatility in a Dynamic Model

The static models illustrates that debt collateralization leads to agents making larger promises, increasing the leverage in the economy as a larger number of agents buy the risky asset against the maximum promise, and even agents investing in risky debt make the maximum promise. In this section we examine how debt collateralization affects volatility and default. We consider a dynamic model following Geanakoplos (2003, 2010). These papers demonstrate how using an asset as collateral creates a "Leverage Cycle" in which asset prices become more volatile because of fluctuations in the asset's collateral value and the distribution of investors' wealth. The main result of this section is that in equilibrium debt collateralization exacerbates and amplifies the leverage cycle, creating more price volatility and more defaults than occur with leverage alone.

We consider a dynamic variation of the model in Section 3 with three periods, $t = 0, 1, 2$. Uncertainty in the payoffs of Y is represented by a tree

$$S = \{0, U, M, D, UU, MU, MD, DU, DD\},$$

illustrated in Figure 4.1. The asset pays only at $t = 2$ with payoffs d_s^Y. To simplify, we will normalize the asset payoffs so that $d_{UU}^Y = d_{MU}^Y = d_{DU}^Y = 1$. Thus the possible "down payoffs" of the asset are d_{MD}^Y and $d_{DD}^Y < d_{MD}^Y$. In other words, the payoff tree is binary at $t = 1$ with a worse possible realization at state M than at D, and at $t = 0$ there is uncertainty about what the minimum possible asset payoff will be.

The risky asset Y has price p_0 at $t = 0$ and prices p_M and p_D in states M and D in $t = 1$. (In state U the price is trivially 1.) Just as before, we first look at an economy where leverage is the only financial innovation and then move on to explore the consequences of debt collateralization.

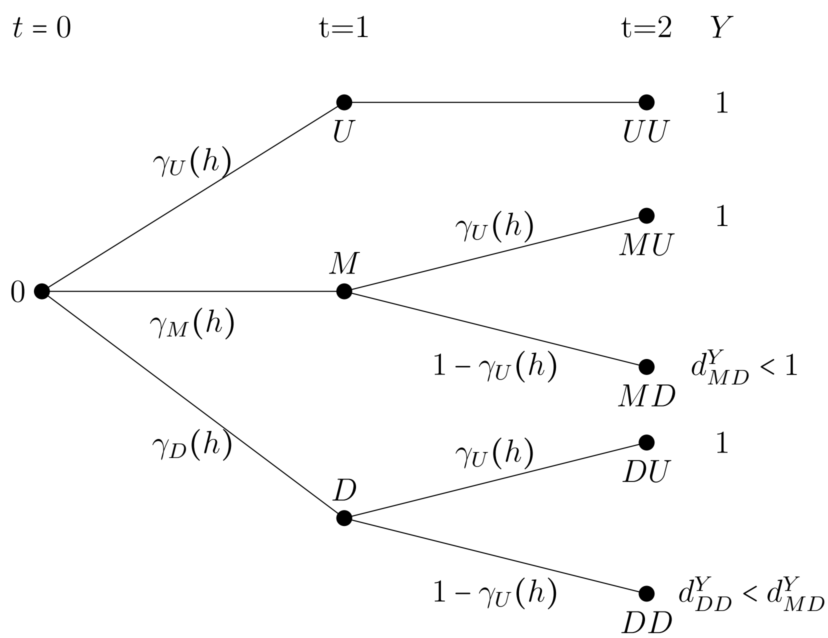

Figure 4.1: Payoff tree for risky asset in dynamic three-state model.

4.1 The Dynamic Economy with Leverage

With leverage, the dynamic equilibrium is essentially different from the static equilibrium because of the dynamic interaction between prices and leverage across time. However, the equilibrium regimes in each state resemble the equilibrium regime in the static economy of Section 3. The dynamic equilibrium with leverage is given by the following.

In equilibrium, at time 0 there are three marginal agents, h_{M0}, h_{D0}, and $h_{\pi 0}$. Agents $h > h_{M0}$ buy the risky asset and promise p_M (i.e., they sell the contract j_{p_M}), which is a risky promise (the contract j_{p_M} delivers $p_D < p_M$ in state D); agents $h \in (h_{D0}, h_{M0})$ buy the risky asset and promise p_D (i.e., they sell the contract j_{p_D}), which is a risk-free promise; agents $h \in (h_{\pi 0}, h_{D0})$ buy the risky debt j_{p_M}; and agents $h < h_{\pi 0}$ buy risk-free asset X and risk-free debt j_{p_D}. Unlike in a binomial economy, there is a possibility of default in the down state D because agents $h \in (h_{M0}, 1)$ cannot pay off the entirety of their debt, having promised p_M when the asset is only worth $p_D < p_M$. We denote the price of the risky debt j_{p_M} by π_0, which has interest rate $i_0 = \frac{p_M}{\pi_0} - 1$.

At time 1, agents receive news about the economy. Margin calls occur and the remaining agents trade assets and make promises. Because the economy is binomial at time 1, in equilibrium agents trade only risk-free contracts. In equilibrium there is one marginal investor in each state, with the remaining optimistic investors buying the risky asset against the maximal risk-free promise possible given the state. Thus, in state M there is a marginal investor h_{MM}. Investors $h > h_{M0}$ have zero wealth after repaying their promise. Investors $h \in (h_{MM}, h_M)$ buy the risky asset and promise M, which is the minimum payoff at $t = 2$. Investors $h < h_{MM}$ buy risk-free assets. In state D there is one marginal investor h_{DD}. Investors $h > h_{D0}$ have zero wealth after repaying their promise. Investors $h \in (h_{DD}, h_{D0})$ buy the risky asset and promise D, which is the minimum payoff at $t = 2$. Investors $h < h_{DD}$ buy risk-free assets.[12]

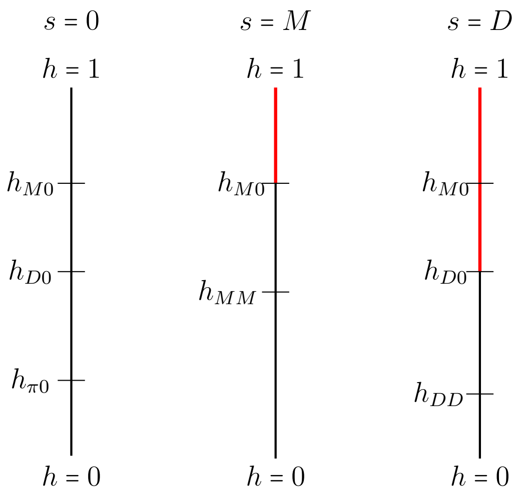

Figure 4.2: State of agents at time 0 and 1 in economy with leverage

Table 2: Wealth of agents at time 1

	State M	State D
$h \in (h_{M0}, 1)$	0	0
$h \in (h_{D0}, h_{M0})$	$\left(\frac{1+p_0}{p-p_D}\right)(p_M - p_D)$	0
$h \in (h_{\pi 0}, h_{D0})$	$\left(\frac{1+p_0}{\pi_0}\right) p_M$	$\left(\frac{1+p_0}{\pi_0}\right) p_D$
$h \in (0, h_{\pi 0})$	$1 + p_0$	$1 + p_0$

Red portions of the unit interval in Figure 4.2 indicate all investors who have lost their wealth and are no longer participating in the market at time 1. Note that we do not know the positions of h_{MM} and h_{DD} relative to the positions of the marginal investors at time $t = 0$. Table 2 gives the wealth of agents at time 1 based on their previous investment choices.

Solving the system numerically with $\gamma_U(h) = h$, $\gamma_M(h) = h(1-h)$, $\gamma_D(h) = (1-h)^2$, and payoffs $d^Y_{MD} = .3$ and $d^Y_{DD} = .1$, we have that h_π holds the safe asset in state M and the risky

[12] Since we do not know the positions of h_{MM} and h_{DD} relative to the marginal investors at time 0, there are several possible equilibrium cases. These cases, as well as the equations defining equilibrium, are listed in Appendix B.

asset in state D. The marginal investors and prices in equilibrium are:

$$h_{M0} = 0.937, \quad h_{D0} = 0.869, \quad h_{\pi 0} = 0.631, \quad p_0 = 0.953, \quad \pi_0 = 0.754,$$

$$h_{MM} = 0.665, \quad h_{DD} = 0.563, \quad p_M = 0.766, \quad p_D = 0.607, \quad i_0 = 1.6\%.$$

The percent price drop in each state, given by $1 - \frac{p_i}{p}$, is 19.68% in state M, and 36.33% in state D.

One reason the crash in D is so large is that investors who bought the risky debt are receiving less than the face value, and less than they invested. This "default mechanism" depresses p_D because remaining investors have less wealth. We isolate the impact of the default mechanism in Appendix B.3 by considering a surprise bailout in $s = D$ to replace the wealth lost to default. Appendix B.3 also compares the 3-state dynamic model to corresponding binomial models and shows that in each case the price crash in the down state of the three state world is greater than the price crash in any of the two-state models and that the biggest price crash is obtained in the case of debt collateralization. This phenomenon occurs precisely because the three-state model with collateralization has more bankrupt agents at time 1 in the down state when compared to the two-state models. Appendix B.4 explores comparative statics for how prices and marginal investors change depending on the payout in d^Y_{MD}.

4.2 The Dynamic Economy with Debt Collateralization

We now suppose that agents at time 0 can use debt as collateral to make further promises. Given our results in the previous section, we can easily characterize equilibrium in the dynamic model.

The dynamic equilibrium with debt collateralization is given by the following. In equilibrium there are two marginal agents at time 0, $\hat{h}_{M0}$, and $\hat{h}_{\pi 0}$. Agents $h > \hat{h}_{M0}$ buy the risky asset and promise p_M; agents $h \in (\hat{h}_{\pi 0}, \hat{h}_{M0})$ buy the risky debt with promise p_M and use it as collateral to promise p_D; and agents $h < \hat{h}_{\pi 0}$ buy the risk-free asset X and the risk-free debt (with promise p_D).

In equilibrium, at time 1 there is one marginal investor in each state as discussed previously. Notice that if the economy is in state M at time 1, then agents $h \in (\hat{h}_{M0}, 1)$ will be bankrupt; if the economy is in state D at time 1, agents $h \in (\hat{h}_{\pi 0}, 1)$ will be bankrupt, illustrated in Figure 4.3.[13]

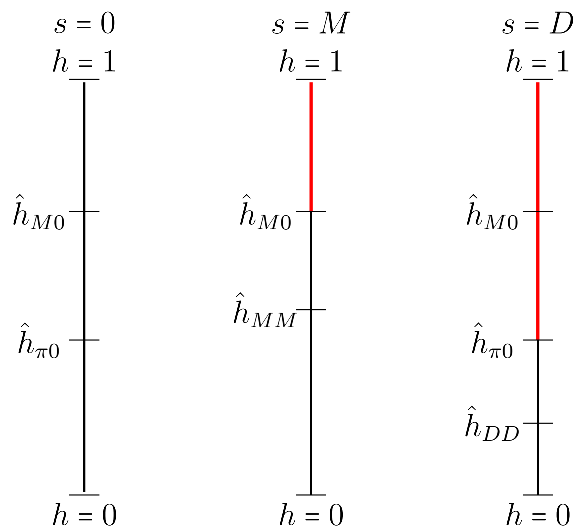

Figure 4.3: State of agents at $t = 0, 1$ in economy with debt collateralization

Table 3 gives the equilibrium with debt collateralization and compares it to the equilibrium with leverage. The price crash is also greater in each state.

Note that the "default mechanism" is effectively much greater with debt collateralization since all debt in the economy is fully collateralized and leveraged. Rather than being poorer in the down state, agents holding risky debt will be completely out of the market. In this case, debt collateralization leads to even more volatility since the agents buying the asset in the down state will be more pessimistic.

Our result that debt collateralization increases volatility is closely related to previous work studying collateral values and volatility. Fostel and Geanakoplos (2012a) show that asset price volatility increases when agents can tranche assets. Tranching increases the collateral value of the risky asset, and in a dynamic setting the "Tranching Cycle" exhibits larger fluctuations in collateral values and in the distribution of wealth. In a two country

[13]Note that we do not know the position of $\hat{h}_{MM}$ relative to the positions of the other marginal investors at time 0, but we do know the relative position of $\hat{h}_{DD}$. The equations defining equilibrium are in Appendix B.

Table 3: Dynamic Equilibrium with Debt Collateralization and with Leverage

	Leverage	Collateralization ($\hat{\ }$)
p_0	0.953	0.970 ↑
p_M	0.766	0.762 ↓
p_D	0.607	0.602 ↓
π_0	0.754	0.762 ↑
i_0	1.6%	0.00985% ↓
h_{M0}	0.937	0.894 ↓
h_{D0}	0.869	–
$h_{\pi 0}$	0.631	0.813 ↑
h_{MM}	0.665	0.660 ↓
h_{DD}	0.563	0.558 ↓
M crash	19.68%	21.48% ↑
D crash	36.33%	37.91% ↑

model, Fostel et al. (2015) show that this result is amplified by international financial flows, which can further increase collateral value because of international demand for collateral-backed financial promises. We have shown that debt collateralization increases the collateral value of debt contracts and of the risky asset. As a result, asset price volatility increases because debt collateralization increases fluctuations in collateral values and in the distribution of wealth.

5 Conclusion

The securitized mortgage markets for ABS and CDOs implicitly and explicitly give investors the ability to use debt contracts as collateral for further promises. We have shown that the ability to collateralize debt backed by a risky asset decreases margins on the risky asset (increases leverage) and decreases the borrowing interest rate for risky debt. When debt serves as collateral, the price of debt increases, giving leveraged investors an incentive make larger promises to use more leverage. As a result, in the dynamic model price crashes are larger because the economy features more leverage, and thus more defaults.

Appendices

A Proofs

Proof of Lemma 1. Suppose for contradiction that there exists an h_i who prefers to hold the risky debt with some amount of leverage L, $0 \leq L < D$, less than the maximum. Since $L < D$ it is risk-free and thus $\hat{\pi}^L = L$. The marginal utilities from investing in j_M against promise L, from investing in j_M against promise D, and from holding safe assets are:

$$\text{debt with leverage L: } \frac{(\gamma_U(h_i) + \gamma_M(h_i))(M - L) + \gamma_D(h_i)(D - L)}{\hat{\pi}^M - L} \tag{13}$$

$$\text{debt with leverage D: } \frac{\gamma_U(h_i)(M - D) + \gamma_M(h_i)(M - D)}{\hat{\pi}^M - D} \tag{14}$$

$$\text{safe asset: } 1. \tag{15}$$

Since by assumption h_i strictly prefers the first option, it must be the case that (13) > (14) and (13) > (15). That is, the investor is optimistic enough to prefer the risky debt to safe debt but not so optimistic as to want zero payoff in D. Hence,

$$\frac{(\gamma_U(h_i) + \gamma_M(h_i))(M - L) + \gamma_D(h_i)(D - L)}{\hat{\pi}^M - L} > \frac{\gamma_U(h_i)(M - D) + \gamma_M(h_i)(M - D)}{\hat{\pi}^M - D}, \tag{16}$$

$$\frac{(\gamma_U(h_i) + \gamma_M(h_i))(M - L) + (1 - \gamma_U(h_i) - \gamma_M(h_i))(D - L)}{\hat{\pi}^M - L} > 1 \tag{17}$$

Simplifying 16, we obtain

$$\hat{\pi}^M - (\gamma_U(h_i) + \gamma_M(h_i))M - \gamma_D(h_i)D > 0 \implies \hat{\pi}^M > (\gamma_U(h_i) + \gamma_M(h_i))M + \gamma_D(h_i)D$$

Simplifying 17, we obtain

$$\hat{\pi}^M - \gamma_D(h_i)D - (\gamma_U(h_i) + \gamma_M(h_i))M < 0 \implies \hat{\pi}^M < \gamma_D(h_i)D + (\gamma_U(h_i) + \gamma_M(h_i))M$$

Note that the above gives us $\hat{\pi}^M > \hat{\pi}^M$. This is a contradiction. Thus, in equilibrium, all agents holding risky debt will do so with maximal leverage. □

Proof of Lemma 2. In equilibrium, each unit of the leveraged risky asset must be backed by one unit of debt, either safe or risky and leveraged. By previous lemma, we have shown that all agents holding risky debt will be maximally leveraged. We therefore know that agents holding the risky asset must either be leveraged against state D or state M and not something in-between.

Suppose for contradiction that there is some agent h_i who prefers to hold the risky asset leveraged against state D and the price of debt is D. That is, the investor is optimistic enough to prefer the risky asset with low leverage to the leveraged risky debt, but not so optimistic as to want to maximally leverage the asset and get zero payoff in M. Note that returns from investment strategies are:

$$\text{Marginal utility from risky asset with debt } D: \quad \frac{\gamma_U(h_i)(1-D)+\gamma_M(h_i)(M-D)}{\hat{p}-D} \tag{18}$$

$$\text{Marginal utility from risky asset with debt } M: \quad \frac{\gamma_U(h_i)(1-M)}{\hat{p}-\hat{\pi}^M} \tag{19}$$

$$\text{Marginal utility from risky debt}: \quad \frac{(\gamma_U(h_i)+\gamma_M(h_i))(M-D)}{\hat{\pi}^M-D} \tag{20}$$

Since by assumption h_i strictly prefers the first option, it must be the case that (18) > (19) and (18) > (20). That is,

$$\frac{\gamma_U(h_i)(1-D)+\gamma_M(h_i)(M-D)}{\hat{p}-D} > \frac{\gamma_U(h_i)(1-M)}{\hat{p}-\hat{\pi}^M} \tag{21}$$

$$\frac{\gamma_U(h_i)(1-D)+\gamma_M(h_i)(M-D)}{\hat{p}-D} > \frac{(\gamma_U(h_i)+\gamma_M(h_i))(M-D)}{\hat{\pi}^M-D} \tag{22}$$

Simplifying 21, we obtain that

$$\gamma_M(h_i)(M\hat{p}+D\hat{\pi}^M-M\hat{\pi}^M-D\hat{p}) > \gamma_U(h_i)(\hat{\pi}^M+MD+D\hat{p}-D-D\hat{\pi}^M-M\hat{p})$$

Simplifying 22, we obtain

$$\gamma_U(h_i)(\hat{\pi}^M+MD+D\hat{p}-D-D\hat{\pi}^M-M\hat{p})+\gamma_M(h_i)(M\hat{\pi}^M+D\hat{p}-D\hat{\pi}^M-M\hat{p}) > 0$$

For convenience, let

$$\alpha := \gamma_M(h_i)(M\hat{p} + D\hat{\pi}^M - M\hat{\pi}^M - D\hat{p}), \quad \beta := \gamma_U(h_i)(\hat{\pi}^M + MD + D\hat{p} - D - d\hat{\pi}^M - M\hat{p})$$

Notice that the above two equations simplify to

$$\alpha > \beta, \qquad \beta - \alpha > 0$$

This is clearly a contradiction since we cannot have both $\alpha > \beta$ and $\beta > \alpha$. Thus, all investors holding the risky asset will be maximally leveraged against state M. □

A.1 Proof of Theorem 3.1

We proceed with the proof by induction, and break the proof into the following two parts.

1. The equilibrium at the first level of collateralization.
2. Equilibrium at the L^{th} level of collateralization.

A.1.1 Equilibrium at the first level of collateralization

To prove the base case, we will show that agents will hold one of the following assets in equilibrium:

- Y/j_i^0, where $2 \le i \le N-1$,
- j_i^0/j_j^1, where $2 \le i \le N-1$, $1 \le j < i$.
- j_j^1, where $1 \le j \le N-2$.

That is, agents will hold the risky asset, Y, leveraged against states $S_2, \ldots, S_{N-1}$; the risky debt contract, j_n^0 (backed by the risky asset), leveraged against some state S_j with $j < n$; or a debt security j_k^1 ($1 \le k \le N-2$), which is backed by risky debt.

Note that the j_k^1 contracts are just securities created in the first round of debt collateralization. Apart from this, the only difference from equilibrium with leverage is that all risky debt contracts backed directly by Y are now bought with leverage, and no agent holds Y, leveraged against S_1. Thus, to prove the base case, it suffices to prove the following two lemmas:

Lemma 3. *In the first level of collateralization, no agent will hold Y/j_1^0.*

Proof of Lemma 3. The intuition for this lemma is nearly identical to the intuition for lemmas 1 and 2. Suppose for contradiction that some agent, h prefers to hold Y/j_1^0. Then, it must be the case that the expected return of holding Y/j_1^0 is greater than holding Y/j_{N-1}^0. This implies that we must have

$$\frac{\sum_{i=1}^{N}\gamma_i(h)(s_i - s_1)}{\hat{p} - \hat{\pi}_1^0} > \frac{\gamma_N(h)(s_N - s_{N-1})}{\hat{p} - \hat{\pi}_{N-1}^0} \tag{23}$$

Rearranging and simplifying, we obtain

$$\sum_{i=1}^{N-1}\gamma_i(h)(s_i - s_1)(\hat{p} - \hat{\pi}_{N-1}^0) + \gamma_N(h)[(s_N - s_1)(\hat{p} - \hat{\pi}_{N-1}^0) - (s_1 - s_{N-1})(\hat{p} - \hat{\pi}_{N-1}^0)] > 0. \tag{24}$$

Furthermore, we know that the expected return of holding Y/j_1^0 is greater than holding j_{N-1}^0/j_1^1, which gives

$$\frac{\sum_{i=1}^{N}\gamma_i(h)(s_i - s_1)}{\hat{p} - \hat{\pi}_1^0} > \frac{\sum_{i=1}^{N-1}\gamma_i(h)(s_i - s_1) + \gamma_N(h)(s_{N-1} - s_1)}{\hat{\pi}_{N-1}^0 - \hat{\pi}_1^1} \tag{25}$$

Rearranging and simplifying, we obtain

$$\sum_{i=1}^{N-1}\gamma_i(h)(s_i - s_1)(\hat{\pi}_{N-1}^0 - \hat{p}) + \gamma_N(h)[(s_N - s_1)(\hat{\pi}_{N-1}^0 - \hat{\pi}_1^1) - (s_{N-1} - s_1)(\hat{p} - \hat{\pi}_1^1)] > 0 \tag{26}$$

A quick check will assure readers that equations (24) and (26) provide a contradiction because the expressions to the left of the > sign are additive inverses and therefore cannot be both strictly greater than 0.

□

Lemma 4. *In the first level of collateralization, any agent buying the promise j_j^0 with $j > 1$ will also sell a promise j_k^1 with $1 \le k < j$.*

Proof of Lemma 4. Now suppose that some agent h prefers to hold a promise j_ℓ^0 with $j > 1$, but not sell a debt security. Then, it must be the case that the expected return of holding j_ℓ^0 is greater than the expected return of holding j_ℓ^0/j_1^1. That is,

$$\frac{\sum_{i=1}^{\ell-1}\gamma_i(h)(s_i) + \sum_{i=\ell}^{N}\gamma_i(h)(s_\ell)}{\hat{\pi}_\ell^0} > \frac{\sum_{i=1}^{\ell-1}\gamma_i(h)(s_i - s_1) + \sum_{i=\ell}^{N}\gamma_i(h)(s_\ell - s_1)}{\hat{\pi}_\ell^0 - \hat{\pi}_1^1} \tag{27}$$

Note that $\hat{\pi}_1^1 = s_1$, since j_1^1 promises s_1 in all states and is therefore safe debt. Thus, rearranging and simplifying 33, we obtain

$$s_1\left(\hat{\pi}_\ell^0 - \sum_{i=1}^{\ell-1}\gamma_i(h)(s_i) - \sum_{i=\ell}^{N}\gamma_i(h)(s_\ell)\right) > 0 \implies \hat{\pi}_\ell^0 - \sum_{i=1}^{\ell-1}\gamma_i(h)(s_i) - \sum_{i=\ell}^{N}\gamma_i(h)(s_\ell) > 0$$

We also know that the expected return of holding j_ℓ^0 must be greater than holding the safe asset. Consequently,

$$\frac{\sum_{i=1}^{\ell-1}\gamma_i(h)(s_i) + \sum_{i=\ell}^{N}\gamma_i(h)(s_\ell)}{\hat{\pi}_\ell^0} > 1 \tag{28}$$

Rearranging and simplifying 34, we obtain

$$\sum_{i=1}^{\ell-1}\gamma_i(h)(s_i) + \sum_{i=\ell}^{N}\gamma_i(h)(s_\ell) > \hat{\pi}_\ell^0 \implies \sum_{i=1}^{\ell-1}\gamma_i(h)(s_i) + \sum_{i=\ell}^{N}\gamma_i(h)(s_\ell) - \hat{\pi}_\ell^0 > 0$$

The above clearly cannot happen because we have that the two equations are additive inverses of each other and therefore cannot both be strictly greater than 0. Thus, no agent holding a risky debt contract will prefer to hold the contract unleveraged.

□

A.1.2 Induction Hypothesis

We now assume that the theorem holds for all levels of collateralization T with $T < L$. Specifically, this means that the theorem holds with $L-1$ levels of collateralization. Looking at this level, we have that agents will hold one of the following assets in equilibrium:

- Y/j_i^0, with $L-1 < i \leq N-1$.

- j_j^l/j_k^{l+1}, with $0 \le l < L-1$, $L-1-l < j < N-l$, and $L-1-l \le k < j$.
- j_ℓ^{L-1}, with $1 \le \ell < N-L+1$.

A.1.3 Equilibrium at the L^{th} level of collateralization

At the L^{th} level of collateralization, we allow all agents holding j_ℓ^{L-1} (with $1 \le \ell < N-L+1$) to sell the promise $j_n^L(j_\ell^{L-1}) = (s_n, j_\ell^{L-1})$ where $1 \le p < \ell$.

We will prove the following:

1. No agent holds Y/j_M^0. This implies that the asset j_M^0/j_{M-1}^1 no longer exists.
2. No agent holding the debt security j_ℓ^{L-1} with $1 < \ell < N-L+1$ will do so without leveraged.
3. No agent will hold j_j^l/j_{L-1-l}^{l+1}, for $0 \le l < L-1$ and $L-1-l < j < N-l$. This implies that all $j_{L-1-l}^{l+1}/A_{L-2-l}^{l+2}$ no longer exist in equilibrium

Note that the above are the changes between the $L-1$ and L^{th} levels of collateralization given by the theorem. We break up the proof into three lemmas, corresponding to the three claims listed above.

Lemma 5. *At the L^{th} level of collateralization, no agent will hold Y/j_L^0.*

Proof of Lemma 5. Suppose for contradiction that some investor h wants to hold Y/j_L^0. Then the leveraged expected return to this asset must be strictly greater than the expected return to holding Y/j_{N-1}^0. This means that

$$\frac{\sum_{i=L}^{N} \gamma_i(h)(s_i - s_L)}{\hat{p} - \hat{\pi}_L^0} > \frac{\gamma_N(h)(s_N - s_{N-1})}{\hat{p} - \hat{\pi}_{N-1}^0}. \tag{29}$$

Rearranging and simplifying the above, we obtain

$$\sum_{i=L}^{N-1} \gamma_i(h)(s_i - s_L)(\hat{p} - \hat{\pi}_{N-1}^0) + \gamma_N[(h)(s_N - s_L)(\hat{p} - \hat{\pi}_{N-1}^0) - (s_N - s_{N-1})(\hat{p} - \hat{\pi}_L^0)] > 0 \tag{30}$$

Additionally, holding Y/j_L^0 must have a higher expected return than holding j_{N-1}^0/j_L^1. Note that at the L^{th} level of collateralization, both j_L^0 and j_L^1 are fully securitized so they have the same price. That is $\hat{\pi}_L^0 = \hat{\pi}_L^1$.

$$\frac{\sum_{i=L}^{N} \gamma_i(h)(s_i - s_L)}{\hat{p} - \hat{\pi}_L^0} > \frac{\sum_{i=L}^{N-1} \gamma_i(h)(s_i - s_L) + \gamma_N(h)(s_{N-1} - s_L)}{\hat{\pi}_{N-1}^0 - \hat{\pi}_L^0} \tag{31}$$

Rearranging and simplifying, we have

$$\sum_{i=L}^{N-1} \gamma_i(h)(s_i - s_L)(\hat{\pi}_{N-1}^0 - \hat{p}) + \gamma_N(h)[(s_N - s_L)(\hat{\pi}_{N-1}^0 - \hat{\pi}_L^0) - (s_{N-1} - s_L)(\hat{p} - \hat{\pi}_L^0)] > 0 \tag{32}$$

The expressions on the left side of the > sign in equations (30) and (32) are additive inverses, and therefore cannot both be strictly greater than 0. Thus, we have a contradiction and no agent will hold Y/j_L^0.

□

Lemma 6. *At the L^{th} level of collateralization, every agent holding j_i^{L-1} with $1 < i < N-L+1$ will sell a promise j_j^L, where $1 \leq j < i$.*

Proof of Lemma 6. Suppose that there exists an agent, h, holding j_i^{L-1} with $1 < \ell < N-L+1$ and prefers not to sell any promises. Then, it must be the case that the expected return of holding j_ℓ^{L-1} is greater than the expected return of holding j_ℓ^{L-1}/j_1^L. That is,

$$\frac{\sum_{i=1}^{\ell-1} \gamma_i(h)(s_i) + \sum_{i=\ell}^{N} \gamma_i(h)(s_\ell)}{\hat{\pi}_\ell^{L-1}} > \frac{\sum_{i=1}^{\ell-1} \gamma_i(h)(s_i - s_1) + \sum_{i=\ell}^{N} \gamma_i(h)(s_\ell - s_1)}{\hat{\pi}_\ell^{L-1} - \hat{\pi}_1^L} \tag{33}$$

Note that $\hat{\pi}_1^L = s_1$, since j_1^L promises s_1 in all states and is therefore safe debt. Thus, rearranging and simplifying 33, we obtain

$$s_1\left(\hat{\pi}_\ell^{L-1} - \sum_{i=1}^{\ell-1} \gamma_i(h)(s_i) - \sum_{i=\ell}^{N} \gamma_i(h)(s_\ell)\right) > 0 \implies \hat{\pi}_\ell^{L-1} - \sum_{i=1}^{\ell-1} \gamma_i(h)(s_i) - \sum_{i=\ell}^{N} \gamma_i(h)(s_\ell) > 0$$

We also know that the expected return of holding j_ℓ^{L-1} must be greater than holding the safe asset. Consequently,

$$\frac{\sum_{i=1}^{\ell-1}\gamma_i(h)(s_i)+\sum_{i=\ell}^{N}\gamma_i(h)(s_\ell)}{\hat{\pi}_\ell^{L-1}}>1 \tag{34}$$

Rearranging and simplifying 34, we obtain

$$\sum_{i=1}^{\ell-1}\gamma_i(h)(s_i)+\sum_{i=\ell}^{N}\gamma_i(h)(s_\ell)>\hat{\pi}_\ell^{L-1} \implies \sum_{i=1}^{\ell-1}\gamma_i(h)(s_i)+\sum_{i=\ell}^{N}\gamma_i(h)(s_\ell)-\hat{\pi}_\ell^{L-1}>0$$

The above clearly cannot happen because we have that the two equations are additive inverses of each other and therefore cannot both be strictly greater than 0. Thus, no agent holding a risky debt contract will prefer to hold the contract unleveraged at the L^{th} level of collateralization.

□

Lemma 7. *At the L^{th} level of collateralization, for all $0 \le l < L-1$, no agent will hold j_k^l/j_{L-1-l}^{l+1}, where $L-l \le k < N-l$.*

Proof of Lemma 7. Suppose for contradiction that there exist some agent h who prefers to be in the position stated above. Then, it must be the case that the is greater than the expected return of holding j_k^l/j_{L-l}^{l+1}. Thus,

$$\frac{\sum_{i=L-l}^{k-1}\gamma_i(h)(s_i-s_{L-1-l})+\sum_{i=k}^{N}\gamma_i(h)(s_k-s_{L-1-l})}{\hat{\pi}_k^l-\hat{\pi}_{L-1-l}^{l+1}}>\frac{\sum_{i=L-l}^{k-1}\gamma_i(h)(s_i-s_{L-l})+\sum_{i=k}^{N}\gamma_i(h)(s_k-s_{L-l})}{\hat{\pi}_k^l-\hat{\pi}_{L-l}^{l+1}}. \tag{35}$$

We rearrange and simplify the above to obtain

$$\sum_{i=L-l}^{k-1}\gamma_i(h)\Omega+\sum_{i=k}^{N}\gamma_i(h)\Psi>0 \tag{36}$$

where

$$\Omega := (s_i-s_{L-1-l})(\hat{\pi}_k^l-\hat{\pi}_{L-l}^{l+1})-(s_i-s_{L-l})(\hat{\pi}_k^l-\hat{\pi}_{L-1-l}^{l+1}) \tag{37}$$

and

$$\Psi := (s_k - s_{L-1-l})(\hat{\pi}_k^l - \hat{\pi}_{L-l}^{l+1}) - (s_k - s_{L-l})(\hat{\pi}_k^l - \hat{\pi}_{L-1-l}^{l+1}) \tag{38}$$

Furthermore, it must also be the case that the expected return of holding j_k^l / j_{L-1-l}^{l+1} is greater than the expected return from holding $j_{L-l}^{l+1} / j_{L-1-l}^{l+2}$. It is important to note here that the price of j_{L-1-l}^{l+2} is the same as the price of j_{L-1-l}^{l+1} because at the L^{th} level of collateralization, both have been securitized to the exact same degree, so the two have the same value. Thus, abusing notation, we can write $\hat{\pi}_{L-1-l}^{l+2} = \hat{\pi}_{L-1-l}^{l+1}$. This give us

$$\frac{\sum_{i=L-l}^{k-1} \gamma_i(h)(s_i - s_{L-1-l}) + \sum_{i=k}^{N} \gamma_i(h)(s_k - s_{L-1-l})}{\hat{\pi}_k^l - \hat{\pi}_{L-1-l}^{l+1}} > \frac{\sum_{i=L-l}^{N} \gamma_i(h)(s_{L-l} - s_{L-1-l})}{\hat{\pi}_{L-l}^{l+1} - \hat{\pi}_{L-1-l}^{l+1}} \tag{39}$$

Rearranging and simplifying the above inequality, we obtain

$$\sum_{L-l}^{k-1} \gamma_i(h)\Upsilon + \sum_{i=k}^{N} \gamma_i(h)\Phi > 0, \tag{40}$$

where

$$\Upsilon := (s_i - s_{L-1-l})(\hat{\pi}_{L-l}^{l+1} - \hat{\pi}_{L-1-l}^{l+1}) - (s_{L-l} - s_{L-1-l})(\hat{\pi}_k^l - \hat{\pi}_{L-1-l}^{l+1}), \tag{41}$$

and

$$\Phi := (s_k - s_{L-1-l})(\hat{\pi}_{L-l}^{l+1} - \hat{\pi}_{L-1-l}^{l+1}) - (s_{L-l} - s_{L-1-l})(\hat{\pi}_k^l - \hat{\pi}_{L-1-l}^{l+1}). \tag{42}$$

A quick check will assure the readers that

$$\Upsilon = -\Omega, \qquad \Phi = -\Psi.$$

This is a contradiction because it means that equations (36) and (40) cannot both be true. Thus, no agent will hold j_k^l / j_{L-1-l}^{l+1}, where $0 \leq l < L-1$ and $L-l \leq k < N-l$.

□

B Equilibrium and Default in the Dynamic Model

B.1 Equilibrium Conditions with Leverage: Section 4.1

For a few marginal investors, we have no doubt about their course of action. The equations defining them are as follows.

Marginal Investors, known

h_{M0}: indifferent between leveraging against p_M and p_D at time $t = 0$. If at time $t = 1$ we are in state D, then h_{M0} is no longer in the market. If at $t = 1$ we are in state M, h_{M0} will choose to hold the risky asset because he is the most optimistic investor in the market. Thus, we have

$$\frac{\gamma_U(h_{M0})(1-p_M)}{p_0-\pi_0} = \frac{\gamma_U(h_{M0})(1-p_D)}{p_0-p_D} + \left(\frac{\gamma_M(h_{M0})(p_M-p_D)}{p_0-p_D}\right)\left(\frac{\gamma_U(h_{M0})(1-d^Y_{MD})}{p_M-d^Y_{MD}}\right).$$

The above equates the marginal utility divided by payoff of holding the risky asset leveraged against p_M and the marginal utility divided by the payoff of holding the asset leveraged against p_D. Note that this differs from previous equations because we need to multiply by the continuation value of the asset at time 2.

h_{MM}: Indifferent between risky asset and riskless asset given the realization of state M at $t = 1$. Since this marginal investor only exists at time $t = 1$ in state M. There is no ambiguity.

$$\frac{\gamma_U(h_{MM})(1-d^Y_{MD})}{p_M-d^Y_{MD}} = 1$$

h_{DD}: Indifferent between risky asset and riskless asset given the realization of state D at $t = 1$. h_{DD} also only exists at time $t = 1$ in state D.

$$\frac{\gamma_U(h_{DD})(1-d^Y_{DD})}{p_D-d^Y_{DD}} = 1$$

Marginal Investors, unknown

h_{D0}: indifferent between leveraging against p_D and holding risky debt at time $t = 0$. If at time $t = 1$ the world is in state D, h_{D0} will choose to hold the risky asset. But, at time $t = 1$ in state M, h_{D0} can either choose to hold the risky asset or the safe asset.

Let h_{D0} hold the risky asset. To simplify notation, let $\gamma^{D0} = \gamma_U(h_{D0})$. Then, equating payoffs (multiplied by continuation values, we have:

$$\begin{aligned}
&\frac{\gamma^{D0}(1-p_D)}{p_0 - p_D} + \left(\frac{\gamma^{D0}(1-\gamma^{D0})(p_M - p_D)}{p_0 - p_D}\right)\left(\frac{\gamma^{D0}(1-d^Y_{MD})}{p_M - d^Y_{MD}}\right) \\
&= \frac{\gamma^{D0}p_M}{\pi_0} + \left(\frac{\gamma^{D0}(1-\gamma^{D0})(p_M)}{\pi_0}\right)\left(\frac{\gamma^{D0}(1-d^Y_{MD})}{p_M - d^Y_{MD}}\right) \\
&+ \left(\frac{(1-\gamma^{D0})^2 p_D}{\pi_0}\right)\left(\frac{\gamma^{D0}(1-d^Y_{DD})}{p_D - d^Y_{DD}}\right)
\end{aligned}$$

When h_{D0} holds the safe asset, we have:

$$\begin{aligned}
&\frac{\gamma^{D0}(1-p_D) + \gamma^{D0}(1-\gamma^{D0})(p_M - p_D)}{p - p_D} \\
&= \frac{(1-(1-\gamma^{D0})^2)p_M}{\pi_0} + \left(\frac{(1-\gamma^{D0})^2 p_D}{\pi_0}\right)\left(\frac{\gamma^{D0}(1-d^Y_{DD})}{p_D - d^Y_{DD}}\right)
\end{aligned}$$

$h_{\pi 0}$: indifferent between holding risky debt and holding riskless asset at time $t = 0$. There are several possibilities for this agent. At time $t = 1$ in state M, the agent can either hold the safe or risky asset. At time $t = 1$ in state D, the agent can either hold the safe or risky asset. To simplify notation, let $\gamma^{\pi 0} = \gamma_U(h_{\pi 0})$. Thus, we have four possible equations defining this agent:

M safe, D safe.

$$\frac{(1-(1-\gamma^{\pi 0})^2)p_M + (1-\gamma^{\pi 0})^2 p_D}{\pi_0} = 1$$

M risky, D risky.

$$\frac{\gamma^{\pi 0}p_M}{\pi_0} + \left(\frac{\gamma^{\pi 0}(1-\gamma^{\pi 0})p_M}{\pi_0}\right)\left(\frac{\gamma^{\pi 0}(1-d^Y_{MD})}{p_M - d^Y_{MD}}\right) + \left(\frac{(1-\gamma^{\pi 0})^2 p_D}{\pi_0}\right)\left(\frac{\gamma^{\pi 0}(1-d^Y_{DD})}{p_D - d^Y_{DD}}\right) = 1$$

M safe, D risky.

$$\frac{(1-(1-\gamma^{\pi 0})^2)p_M}{\pi_0} + \left(\frac{(1-\gamma^{\pi 0})^2 p_D}{\pi_0}\right)\left(\frac{\gamma^{\pi 0}(1-d^Y_{DD})}{p_D - d^Y_{DD}}\right) = 1$$

M risky, D safe.

$$\frac{\gamma^{\pi 0} p_M}{\pi_0} + \left(\frac{\gamma^{\pi 0}(1-\gamma^{\pi 0})p_M}{\pi_0}\right)\left(\frac{\gamma^{\pi 0}(1-d^Y_{MD})}{p_M - d^Y_{MD}}\right) + \frac{(1-\gamma^{\pi 0})^2 p_D}{\pi_0} = 1$$

The known market clearing conditions are for the asset and debt at time 0. Equating supply with demand, we obtain:

Time $t = 0$, Asset:

$$(1-h_{M0})\frac{(1+p_0)}{(p_0-\pi_0)} + (h_{M0}-h_{D0})\frac{(1+p_0)}{(p_0-p_D)} = 1$$

Time $t = 0$, Risky Debt:

$$(1-h_{M0})\frac{(1+p_0)}{(p_0-\pi_0)} = (h_{D0}-h_{\pi 0})\frac{(1+p_0)}{\pi_0}$$

Market Clearing, unknown

Time $t = 1$, State M Asset: We are unsure whether $h_{MM} \in (0, h_{\pi 0})$, $h_{MM} \in (h_{\pi 0}, h_{D0})$, or $h_{MM} \in (h_{D0}, h_{M0})$. This issue can be resolved by considering all cases, solving for equilibrium, and checking that h_{MM} is indeed in the specified interval. $h_{MM} \in (0, h_{\pi 0})$:

$$\frac{(h_{M0}-h_{D0})\left(\frac{1+p_0}{p_0-p_D}\right)(p_M-p_D)}{p_M - d^Y_{MD}} + \frac{(h_{D0}-h_{\pi 0})\left(\frac{1+p_0}{\pi_0}\right)(p_M)}{p_M - d^Y_{MD}} + \frac{(h_{\pi 0}-h_{MM})(1+p_0)}{p_M - d^Y_{MD}} = 1$$

$h_{MM} \in (h_{\pi 0}, h_{D0})$:

$$\frac{(h_{M0}-h_{D0})\left(\frac{1+p_0}{p_0-p_D}\right)(p_M-p_D)}{p_M - d^Y_{MD}} + \frac{(h_{D0}-h_{MM})\left(\frac{1+p_0}{\pi_0}\right)(p_M)}{p_M - d^Y_{MD}} = 1$$

$h_{MM} \in (h_{D0}, h_{M0})$:

$$\frac{(h_{M0}-h_{MM})\left(\frac{1+p_0}{p_0-p_D}\right)(p_M-p_D)}{p_M - d^Y_{MD}} = 1$$

Time $t = 1$, State D Asset: We do not know whether $h_{DD} \in (0, h_{\pi 0})$ or $h_{DD} \in (h_{\pi 0}, h_{D0})$. In the first case, we have:

$$\frac{(h_{D0}-h_{\pi 0})\left(\frac{1+p_0}{\pi_0}\right)p_D}{p_D - d^Y_{DD}} + \frac{(h_{\pi 0}-h_{DD})(1+p_0)}{p_D - d^Y_{DD}} = 1.$$

In the second case, we have

$$\frac{(h_{D0} - h_{DD})\left(\frac{1+p_0}{\pi_0}\right)p_D}{p_D - d^Y_{DD}} = 1.$$

Thus, we obtain the following possible cases in equilibrium:

1. h_{D0} holds risky asset at time 1 in state M.

   (a) $h_{\pi 0}$ holds risky asset in state M and risky asset in state D.

   This implies that $h_{MM}, h_{DD} \in (0, h_{\pi 0})$.

   (b) $h_{\pi 0}$ holds safe asset in state M and safe asset in state D.

   This implies that $h_{MM}, h_{DD} \in (h_{\pi 0}, h_{D0})$.

   (c) $h_{\pi 0}$ holds risky asset in state M and safe asset in state D.

   This implies that $h_{MM} \in (0, h_{\pi 0})$ and $h_{DD} \in (h_{\pi 0}, h_{D0})$.

   (d) $h_{\pi 0}$ holds safe asset in state M and risky asset in state D.

   This implies that $h_{MM} \in (h_{\pi 0}, h_{D0})$ and $h_{DD} \in (0, h_{\pi 0})$.

2. h_{D0} holds safe asset at time 1 in state M.

   This implies that $h_{MM} \in (h_{D0}, h_{M0})$ and $h_{\pi 0}$ holds safe asset in state M.

   (a) $h_{\pi 0}$ holds safe asset in state D.

   This implies that $h_{DD} \in (h_{\pi 0}, h_{D0})$.

   (b) $h_{\pi 0}$ holds risky asset in state D.

   This implies that $h_{DD} \in (0, h_{\pi 0})$.

B.2 Equilibrium Conditions with Collateralization: Section 4.2

The equations defining equilibrium are as follows

Marginal Investors, known

$\hat{h}_{M0}$: indifferent between holding risky asset, leveraged against state M and risky debt leveraged against state D at time 0

$$\frac{\gamma_U(\hat{h}_{M0})(1 - \hat{p}_M)}{\hat{p}_0 - \hat{\pi}_0} = \frac{\gamma_U(\hat{h}_{M0})(\hat{p}_M - \hat{p}_D)}{\hat{\pi}_0 - \hat{p}_D} + \frac{\gamma_M(\hat{h}_i)(\hat{p}_M - \hat{p}_D)}{\hat{\pi}_0 - \hat{p}_D}\left(\frac{\gamma_U(1 - d^Y_{MD})}{\hat{p}_M - d^Y_{MD}}\right)$$

$\hat{h}_{MM}$: Indifferent between holding risky asset and safe asset at time 1, state M.

$$\frac{\gamma_U(\hat{h}_{MM})(1 - d^Y_{MD})}{\hat{p}_M - d^Y_{MD}} = 1$$

$\hat{h}_{DD}$: Indifferent between holding risky asset and safe asset at time 1, state D.

$$\frac{\gamma_U(\hat{h}_{DD})(1 - d^Y_{DD})}{\hat{p}_D - d^Y_{DD}} = 1$$

Marginal Investors, unknown

$\hat{h}_{\pi 0}$: Indifferent between holding risky debt with leverage and holding the safe asset. There are now two possibilities for this agent, rather than 4. Just as before, at time $t = 1$ in state M, the agent can either hold the safe or risky asset. However, at time $t = 1$ in state D, $\hat{h}_{\pi 0}$ will be the most optimistic agent still in the market, forcing him to hold the risky asset. Thus, we have the following two possibilities

M safe.

$$\frac{(\gamma_U(\hat{h}_{\pi 0}) + \gamma_M(\hat{h}_{\pi 0}))(\hat{p}_M - \hat{p}_D)}{\hat{\pi}_0 - \hat{p}_D} = 1$$

M risky: here the payout of the asset in state M at time 1 is multiplied by the continuation value of the asset in time 2 since we have specified that $\hat{h}_{\pi 0}$ will hold the risky asset.

$$\frac{\gamma_U(\hat{h}_{\pi 0})(\hat{p}_M - \hat{p}_D)}{\hat{\pi}_0 - \hat{p}_D} + \left(\frac{\gamma_M(\hat{h}_{\pi 0})(\hat{p}_M - \hat{p}_D)}{\hat{\pi}_0 - \hat{p}_D}\right)\left(\frac{\gamma_U(\hat{h}_{\pi 0})(1 - d^Y_{MD})}{\hat{p}_M - d^Y_{MD}}\right) = 1$$

Market Clearing, known

Time $t = 0$, Risky Asset:

$$(1 - \hat{h}_{M0})\left(\frac{1 + \hat{p}_0}{\hat{p}_0 - \hat{\pi}_0}\right) = 1$$

Time $t = 1$, state D, Risky Asset:

$$(\hat{h}_{\pi 0} - \hat{h}_{DD})\left(\frac{1 + \hat{p}_0}{\hat{p}_D - d^Y_{DD}}\right) = 1$$

Time $t = 0$, Risky Debt:

$$(1 - \hat{h}_{M0}) \left(\frac{1 + \hat{p}_0}{\hat{p}_0 - \hat{\pi}_0} \right) = (\hat{h}_{M0} - \hat{h}_{\pi 0}) \left(\frac{1 + \hat{p}_0}{\hat{\pi}_0 - \hat{p}_D} \right)$$

Market Clearing, unknown

Time $t = 1$, State M Asset: We are unsure whether $\hat{h}_{MM} \in (0, \hat{h}_{\pi 0})$ of $\hat{h}_{MM} \in (\hat{h}_{\pi 0}, \hat{h}_{M0})$. This issue can be resolved by considering both cases, solving for equilibrium, and checking that $\hat{h}_{MM}$ is indeed in the specified interval.

$\hat{h}_{MM} \in (0, \hat{h}_{\pi 0})$:

$$\frac{(\hat{h}_{M0} - \hat{h}_{\pi 0}) \left(\frac{1 + \hat{p}_0}{\pi - \hat{p}_D} \right) (\hat{p}_M - \hat{p}_D)}{\hat{p}_M - d^Y_{MD}} + \frac{(\hat{h}_{\pi 0} - \hat{h}_{MM})(1 + \hat{p}_0)}{\hat{p}_M - d^Y_{MD}} = 1$$

$\hat{h}_{MM} \in (\hat{h}_{\pi 0}, \hat{h}_{M0})$:

$$\frac{(\hat{h}_{M0} - \hat{h}_{MM}) \left(\frac{1 + \hat{p}_0}{\pi - \hat{p}_D} \right) (\hat{p}_M - \hat{p}_D)}{\hat{p}_M - d^Y_{MD}} = 1$$

B.3 Default in the Dynamic 3-state Model

This section isolates the role of default in the dynamic model in two ways. First, it considers a surprise injection of wealth to bailout agents who lost money to default. Second, it maps the 3-state model onto binomial models, in which there is no default, and compares equilibrium in each case.

B.3.1 The Default Mechanism

We demonstrate the impact that default has on asset prices at time 1 by considering the counterfactual scenario. We now suppose that in the down state, holders of risky debt receive an unexpected, exogenous wealth increase at time 1. This can be thought of as a bailout for these agents.

Recall that previously, holders of risky debt had lost some of their wealth in the down state because they were paid back $p_D < \pi_0 < p_M$. Now, the suppose that the holders of risky debt are compensated the difference between p_M and p_D. Notice that because this wealth shock is unexpected, it does not change the equilibrium at time 0 and only at time 1. In the down-state, we now have $h \in (h_{\pi 0}, h_{D0})$ holding p_M units of wealth and $h \in (0, h_{\pi 0})$ holding $1 + p_0$ units of wealth. The equations defining equilibrium are:

Marginal Investor

$$\frac{\gamma_U(h_{DD})(1 - d^Y_{DD})}{p_D - d^Y_{DD}} = 1$$

Market Clearing

$$\frac{(h_{D0} - h_{\pi 0})\left(\frac{1+p}{\pi_0}\right) p_M}{p_D - d^Y_{DD}} + \frac{(h_{\pi 0} - h_{DD})(1 + p_0)}{p_D - d^Y_{DD}} = 1.$$

Using the same specifications as before, as well as the results for $h_{D0}, h_{\pi 0}, \pi_0, p_0$ and p_M from the previous subsection, we find that

$$h_{DD} = 0.5976, \quad p_D = 0.6378, \quad \text{down crash} = 33.08\%.$$

Without this injection of cash, $p_D = 0.606874$ and the crash was 36.33%. The bailouts, offsetting the default mechanism, increases the asset price in D and lowers the volatility. However, it is important to note that this result only occurs if agents do not expect the wealth shock. If agents anticipated the bailout at time $t = 0$, then the increase in the p_D price will lead to higher margins at the initial time period and the expectation that all debt is actually safe.

B.3.2 Comparison With Dynamic Two-State Model

We can compare the volatility in the dynamic three-period model to the dynamic 2-period model in several ways. We can normalize by the expected payoff of the asset, normalize by belief in the up state, as well as normalize by the belief in the downstate.

We first consider normalization by the expected payoff of the asset. We use $\gamma_U(h) = h$ and $\gamma_M(h) = h(1 - h)$ to for the probabilities in the three-state model. We want to set the belief of the upstate, $\varphi(h)$, in the two-state model so that for every agent, the ultimate expected payout of the risky asset is the same in the two models. That is,

$$h+h^2(1-h)+h(1-h)^2+h(1-h)^2\alpha d_{DD}^Y+(1-h)^3 d_{DD}^Y = \varphi(h)+(1-\varphi(h))\varphi(h)+(1-\varphi(h))^2 d_{DD}^Y,$$

where $d_{MD}^Y = \alpha d_{DD}^Y$. Solving the above, we find that

$$\varphi_i = 1 + \frac{\sqrt{(d_{DD}^Y - 1)(1-h)^2(d_{DD}^Y - 1 + (\alpha-1)d_{DD}^Y h)}}{d_{DD}^Y - 1}.$$

Letting $\alpha = 3$, we can solve for equilibrium in the two-state dynamic model.

When we normalize by belief about the up state, we have $\varphi(h) = \gamma_U(h) = h$. Alternatively, if we normalize by belief about the down state, we need $1-\varphi(h) = \gamma_D(h)$.

In summary, we obtain the following differences by normalization.

	Dynamic 3-State		Dynamic 2-State		
	Collateralized debt	No Collateralization	Expected Payout	Belief Up	Belief Down
p_0	.970	0.9531	0.9470	0.9351	0.9876
p_D	.602	0.6069	0.6339	0.6116	0.7314
crash	37.91%	36.33%	33.06%	34.593 %	25.94%

Note that for every normalization, the price crash in the down state of the three state world is greater than the price crash in any of the two-state models and that the biggest price crash is obtained in the case of debt collateralization. Furthermore, the price of the asset in the down state is lowest in the three-state world with debt collateralization.

This phenomenon occurs precisely because the three-state model with collateralization has more bankrupt agents at time 1 in the down state when compared to the two-state models. Thus, the agents who are left to buy the asset are more pessimistic and do not value the asset as highly.

B.4 Comparative statistics

We now briefly explore how price and marginal investors change depending on difference between the payout in the d_{MD}^Y and d_{DD}^Y states. To do this, we fix the payout of the asset in d_{DD}^Y to be 0.1 and solve for equilibrium for a full range of d_{MD}^Y. These results are presented in the table below.

d^Y_{MD}	h_{M0}	h_{D0}	$h_{\pi 0}$	h_{MM}	h_{DD}	p_0	p_M	p_D	π_0	i_0	M crash	D crash
.2	.909	.859	.609	.619	.567	.939	.695	.611	.689	.88%	25.96%	34.95
.3	.937	.869	.631	.665	.563	.953	.766	.607	.754	1.56%	19.7%	36.3%
.4	.958	.875	.649	.712	.562	.963	.827	.604	.811	1.93%	14.2%	37.3%
.5	.974	.876	.668	.759	.558	.971	.880	.602	.862	2.04 %	9.43%	37.98.%
.6	.985	.872	.687	.807	.556	.977	.923	.601	.905	1.95%	5.53%	38.5%
.7	.993	.865	.707	.855	.555	.981	.957	.600	.940	1.77%	2.48%	38.9%
.8	.997	.857	.723	.904	.554	.984	.981	.599	.966	1.59%	.2995%	39.1%
.9	.999	.851	.734	.954	.554	.985	.995	.599	.981	1.47%	-1.01%	39.3%

Table 4: Comparisons of equilibrium for different values of d^Y_{MD}

It is interesting to note that for high enough values of d^Y_{MD}, the price crash in state M is negative (the price increases). For high values of d^Y_{MD} there is so much divergence in the payoffs of the risky asset that at time $t = 1$, once the possibility of state D occurring is eliminated, agents are more optimistic about the payout of the asset and are thus willing to buy more of the risky asset.

References

Acharya, V. V. and S. Viswanathan (2011): "Leverage, moral hazard, and liquidity," *The Journal of Finance*, 66, 99–138.

Adrian, T. and N. Boyarchenko (2012): "Intermediary leverage cycles and financial stability," Tech. Rep. 567, Staff Report, Federal Reserve Bank of New York.

Adrian, T. and H. Shin (2010): "Liquidity and Leverage," *Journal of Financial Intermediation*, 19, 418–437.

Araujo, A., F. Kubler, and S. Schommer (2012): "Regulating Collateral-Requirements When Markets Are Incomplete," *Journal of Economic Theory*, 147, 450–476.

Benmelech, E. and J. Dlugosz (2010): "The Credit Rating Crisis," in *NBER Macroeconomics Annual 2009, Volume 24*, University of Chicago Press, 161–207.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79, pp. 14–31.

Bolton, P., X. Freixas, and J. Shapiro (2012): "The credit ratings game," *The Journal of Finance*, 67, 85–111.

Brunnermeier, M. K. and L. H. Pedersen (2009): "Market Liquidity and Funding Liquidity," *The Review of Financial Studies*, 22, 2201–2238.

Brunnermeier, M. K. and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104, 379–421.

——— (2015): "International Credit Flows and Pecuniary Externalities," *American Economic Journal: Macroeconomics*, 7, 297–338.

Dang, T., G. Gorton, and B. Holmström (2011): "Ignorance and the Optimality of Debt for Liquidity Provision," Tech. rep., MIT and Yale University.

DeMarzo, P. and D. Duffie (1999): "A liquidity-based model of security design," *Econometrica*.

DeMarzo, P. M. (2005): "The pooling and tranching of securities: A model of informed intermediation," *Review of Financial Studies*, 18, 1–35.

Farhi, E. and J. Tirole (2014): "Liquid Bundles," *Journal of Economic Theory*.

Fostel, A. and J. Geanakoplos (2008): "Leverage Cycles and The Anxious Economy," *American Economic Review*, 98, 1211–1244.

——— (2012a): "Tranching, CDS, and Asset prices: How Financial Innovation Can Cause Bubbles and Crashes," *American Economic Journal: Macroeconomics*, 4, 190–225.

——— (2012b): "Why Does Bad News Increase Volatility And Decrease Leverage?" *Journal of Economic Theory*, 147, 501–525.

——— (2015a): "Financial Innovation, Collateral and Investment," Forthcoming.

——— (2015b): "Leverage and default in binomial economies: a complete characterization," Forthcoming.

FOSTEL, A., J. GEANAKOPLOS, AND G. PHELAN (2015): "Global Collateral: How Financial Innovation Drives Capital Flows and Increases Financial Instability," Working Paper.

GEANAKOPLOS, J. (1997): "Promises Promises," in *Santa Fe Institute Studies in the Sciences of Complexity-proceedings volume*, Addison-Wesley Publishing Co, vol. 27, 285–320.

——— (2003): "Liquidity, Default and Crashes: Endogenous Contracts in General Equilibrium," in *Advances in Economics and Econometrics: Theory and Applications, Eight World Conference*, Econometric Society Monographs, vol. 2, 170–205.

——— (2010): "The Leverage Cycle," in *NBER Macroeconomics Annual 2009*, ed. by K. R. D. Acemoglu and M. Woodford, University of Chicago Press, vol. 24, 1–65.

GEANAKOPLOS, J. AND W. R. ZAME (2011): "Security Pools and Efficiency," Mimeo.

——— (2014): "Collateral equilibrium, I: a basic framework," *Economic Theory*, 56, 443–492.

GEEROLF, F. (2015): "Leverage and Disagreement," Working Paper, UCLA.

GONG, F. AND G. PHELAN (2015): "Credit Default Swaps and Debt Collateralization," Working Paper, Williams College.

GORTON, G. AND A. METRICK (2009): "Haircuts," Yale ICF Working Paper No. 09-15, `http://ssrn.com/abstract=1447438`.

——— (2012): "Securitized banking and the run on repo," *Journal of Financial Economics*, 104, 425 – 451.

HOLMSTRÖM, B. AND J. TIROLE (1997): "Financial Intermediation, Loanable Funds, and the Real Sector," *The Quarterly Journal of Economics*, 112, 663–91.

KIYOTAKI, N. AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105, pp. 211–48.

Krishnamurthy, A. (2009): "How debt markets have malfunctioned in the crisis," Tech. rep., National Bureau of Economic Research.

Longstaff, F. A. (2010): "The subprime credit crisis and contagion in financial markets," *Journal of Financial Economics*, 97, 436–450.

Phelan, G. (2015a): "Collateralized Borrowing and Increasing Risk," Mimeo.

——— (2015b): "Financial Intermediation, Leverage, and Macroeconomic Instability." Tech. rep., Williams College.

Shen, J., H. Yan, and J. Zhang (2014): "Collateral-motivated financial innovation," *Review of Financial Studies*, hhu036.

Simsek, A. (2013): "Belief Disagreements and Collateral Constraints," *Econometrica*, 81, 1–53.

Toda, A. A. (2014): "Securitization and Leverage in General Equilibrium," Tech. rep., University of California, San Diego.