Stochastic Optimization of Sub-hourly Economic Dispatch with Wind Energy

Harsha Gangammanavar, Suvrajeet Sen and Victor M. Zavala

Abstract— **We present a stochastic programming framework for a multiple timescale economic dispatch problem to address integration of renewable energy resources into power systems. This framework allows certain slow-response energy resources to be controlled at an hourly timescale, while fast-response resources, including renewable resources, and related network decisions can be controlled at a sub-hourly timescale. To this end, we study two models motivated by actual scheduling practices of system operators. Using an external simulator as driver for sub-hourly wind generation, we optimize these economic dispatch models using stochastic decomposition, a sample-based approach for stochastic programming. Computational experiments, conducted on the IEEE-RTS96 system and the Illinois system, reveal that optimization with sub-hourly dispatch not only results in lower expected operational costs, but also predicts these costs with far greater accuracy than with models allowing only hourly dispatch. Our results also demonstrate that when compared with standard approaches using the extensive formulation of stochastic programming, the sequential sampling approach of stochastic decomposition provides better predictions with much less computational time.**

Index Terms—**Economic dispatch, sub-hourly dispatch, stochastic decomposition.**

I. Introduction

THE integration of renewable resources into energy networks, especially in large scale power grids, poses several technological and operational challenges. Among renewable resources, wind and solar energy fall under the class of variable/intermittent energy resources because of their inherent stochasticity. Their variability is not only high in magnitude, but they also exhibit sub-hourly fluctuations. In addition, these resources are known to have a temporal mismatch with the load. These characteristics have generated significant academic interest as well as several new initiatives by power system operators to harness the full benefits of renewable resources.

Renewable integration challenges also necessitate incorporation of faster reserves, storage devices and similar services operating alongside the slow ramping conventional generators in the energy network. To maintain robustness of such a network, their operations should be planned by appropriately modeling the decision processes and using solution approaches that provide verifiable performance.

Currently, power systems planning is mostly based on a two level hierarchy: day-ahead unit commitment (DAUC) and economic dispatch (ED). A substantial number of studies ([1, 2, 3]) have focused on DAUC in the presence of renewable energy resources. By definition, the DAUC models use a 24 hour horizon with one hour resolution and set the initial commitment status for all generation units. Operating reserves and other regulation capacities are also procured in the day-ahead market ([4, 5]). Some power system operators also use short-term unit commitment and real-time unit commitment to update the initial commitments based on new load and renewable generation forecasts, and schedule additional operating reserves. Once the commitments are set and the reserves requirements are established, dispatch schedules (generator set points) are determined by solving the ED model which is the focus of our work.

Due to grid characteristics, the operating practices vary significantly across balancing authorities. For example, the generation scheduling process in Bonneville Power Administration (BPA), a U.S. government electric utility in the Pacific Northwest, is based on hourly bulk energy schedules [6]. BPA can control hydro-generation within an hour, but is unable to access thermal generation within its balancing authority area on a sub-hourly basis. The economic dispatch of conventional generation is based on an hour-ahead forecast and is completed 20 minutes before the hour of delivery. BPA normally has sufficient range of load-following, regulation, and ramping capabilities to handle within-hour imbalances. Advanced ISOs, CAISO for example, use an hour-ahead scheduling process (HASP) and a real-time dispatch (RTD) optimization to handle short term energy imbalances. The RTD process runs every 5 minutes with a horizon of 65 minutes to dispatch internal resources [6]. The intertie resources however, are scheduled non-dynamically for an hour by HASP. Similar operating practices, where certain resources are scheduled at a coarse timescale and others are scheduled at a fine timescale, are used in European TSOs and other balancing authorities in North America. Moreover, deterministic optimization models based on a single scenario are used in this process.

Worldwide, electricity grids are in transition, and recently there are proposals which suggest several reforms to current practices to enable better integration of variable generation (e.g., FERC Order-764 in the U.S. [7]). They require variable generator owners to provide sub-hourly generation and outage data, and power system operators to use sub-hourly scheduling for all generation resources. Balancing authority area operators and ISOs are at different stages to accommodate these changes. For example, within-hour scheduling at 10-minute intervals is being considered for implementation at BPA [6]. Moreover, the need for advanced optimization tools and simulation softwares has been identified as a major requirement for efficient

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implementation of sub-hourly dispatch [8].

In this regard, there have been simulation based investigations of sub-hourly dispatch. For example, the CAISO 2010 report ([9], Appendix C) reveals that sub-hourly analysis tends to overcome relatively small operational issues identified by hourly simulations. However, if the hourly simulations indicate more significant issues, then the sub-hourly simulation shows even larger impact. In another recent work [10], the authors study a power system with renewable resources using a simulation based framework, which integrates system operations at multiple timescales. The authors study the impact of renewable generation variability at different sub-hourly time resolutions and the impact of uncertainty by using different forecasts. Their study shows higher energy imbalance and greater transmission congestion with hourly time resolution, compared to sub-hourly resolutions. Unfortunately, most optimization studies which are based on average processes (e.g., hourly resolution models) tend to choose "optimal" settings with little room for error [11]. Based on the operational issues identified in [9, 10] one can draw similar conclusions for hourly simulation based unit commitment and economic dispatch models. In our study we incorporate optimization as well as simulation at hourly and sub-hourly timescales respectively.

Sub-hourly dispatch has been used for ED in the short-term unit commitment model of [12]. The authors consider 10 minute resolution for dispatch and compute the commitments using only 10 scenarios. These studies ([1, 2, 3, 4, 5]) usually employ approximations which reduce the original model formulation (with a larger set of scenarios) to a much smaller, and computationally manageable model [13]. While such a strategy tends to achieve tractability in optimization, the quality of predictions from such models can be suspect. In Sec. III (discussions related to Tables I,II) we will highlight issues resulting from both small sample approximations as well as hourly wind aggregation in the context of large scale economic dispatch.

To better harvest the benefits of renewable integration one needs to address the following important questions:

- What is the impact of increased variability from renewable generation on economic dispatch? Many jurisdictions in U.S. are mandating significant increases in renewable generation. Such mandates have the potential of introducing far more variability in the electricity generation than has ever been experienced.
- What optimization tools should be used to make choices that will mitigate the impact of increased variability?
- How similar are the decisions obtained from models which have different update intervals for variable generation? More frequent updates demand greater investments in resources for computations and operations. Depending on how similar these decisions are, a system operator has the option to choose computational resources in a cost effective manner.

With the above questions in mind, the main contributions of this work are:

- A stochastic decomposition (2-SD, [14]) framework which can accommodate system operations at multiple timescales as well as exogenous information from state-of-the-art forecast simulators. This framework allows us to plan dispatch operations more effectively than deterministic and/or coarse grained hourly models by making decisions based on a large set of future possibilities. Moreover, the 2-SD framework derives its scalability from its decomposition-based sequential sampling framework which identifies the sample size during the algorithmic process. This is particularly relevant because of sub-hourly wind variability. This algorithmic approach is presented in Sec. II.
- Our computational results are the first of its kind, showing that a) 2-SD provides a viable computational framework for realistic economic dispatch models in the presence of hourly and sub-hourly resource allocation, and b) finer resolution models of wind lead to somewhat higher generation from slow ramping generators to enhance the reliability (i.e., reduce dependence on high cost reserves). These results are presented in Sec. III.

II. Economic Dispatch under Uncertainty

Stochastic programming (SP) has been widely used for power systems applications such as stochastic unit commitment, establishing reserve requirements, and others. Benders decomposition is used in [1] and [12] with a master unit commitment problem, and subproblems to check feasibility of the master solution. In [15] a Lagrangian relaxation algorithm is employed in which a first stage problem schedules slow generators, and a second stage subproblem is used for committing fast generators and dispatch all resources. In [16] robust optimization, with the objective to minimize the worst-case cost associated with commitment solutions, is used in conjunction with stochastic optimization within a unified Benders framework. In [17], the unit commitment horizon is divided into two parts. While stochastic optimization is used for the first part, the second part is solved using interval unit commitment by considering only the central forecast, an upper bound and a lower bound on uncertainty. In all these studies a representative set (with size 10-100) of scenarios is used for optimization. Increasing the number of scenarios often result in very large models, and requires parallelization over a large cluster of computers for optimization ([2]). To the best of our knowledge, our work provides the first real scale application of SP to the economic dispatch problem. In order to relate the economic dispatch problem to SP, we first present a general framework. Following this setup we will identify two alternative operating practices, one motivated by the time lines of BPA and the other by advanced ISOs like CAISO [6].

A. General Framework

We use a two-stage stochastic linear programming framework in which we distinguish between decisions (x) which are made prior to the observation of a random event, and decisions (y) which respond adaptively to the initial choice (x) and the observation (ω). In the SP literature these adaptive decisions are referred to as recourse decisions. The representation of

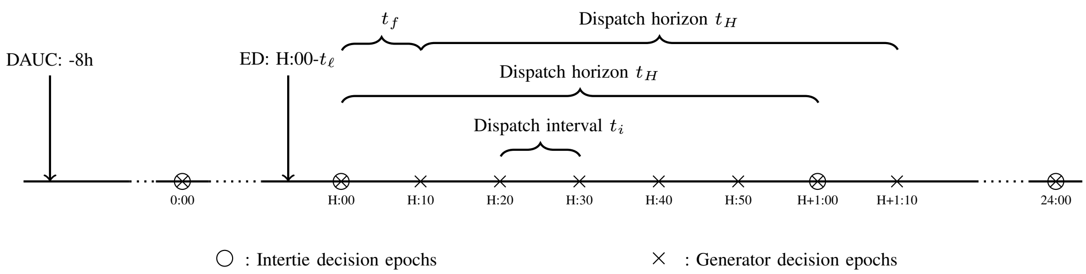


Fig. 1. Dispatch Scheduling Process with $t_f = 10$ minutes

two-stage stochastic linear program with recourse (2SLP) is given by:

$$\min_{x \in \Re^{n_1}} \; f(x) := c^\top x + \mathbb{E}\{h(x, \tilde{\omega})\} \quad \text{s.t. } Ax \le b \tag{1a}$$

where,

$$h(x,\omega) = \min_{y \in \Re^{n_2}} \; d^\top y \quad \text{s.t. } Wy = r(\omega) - Tx, \;\; y \ge 0. \tag{1b}$$

Here, ω is a realization of the random variable $\tilde{\omega}$. The SP literature refers to (1a) as the first stage and (1b) as the second stage.

Ordinarily, the number of possible future scenarios can be so large that the expectation in (1) can only be approximated by either (a) scenario sampling, or (b) selecting a small subset of scenarios [18]. In either case we obtain a prediction of the cost associated with decisions realized from a model. It is customary to verify the quality of these predictions using large sample Monte Carlo simulations.

When the probability space is limited to a finite number (S) of scenarios the problem can be reformulated as one large deterministic problem:

$$\begin{aligned} \min \; & c^\top x + \sum_{s=1}^{S} p^s (d^\top y^s) \\ s.t. \; & Ax \le b \\ & Wy^s = r(\omega^s) - Tx \quad s = 1, \ldots, S \\ & y^s \ge 0 \quad s = 1, \ldots, S \end{aligned} \tag{2}$$

Here p^s denotes the probability of scenario s. For methods using Monte Carlo simulation, the number of scenarios is chosen based on computing resources (the platform and the solver) available, and for such instances one uses $p^s = 1/S$. We will refer to this formulation (2) as the *extensive scenario formulation* (ESF). A very special case of ESF, using a single forecast scenario, is the deterministic dispatch model commonly employed by power system operators. In the following we summarize the time lines and practices as they exist today.

B. Scheduling Time Lines and Practices

The scheduling process for hour H begins at $H - t_\ell$ (Fig. 1), where the leadtime t_ℓ varies from 7.5 minutes (at CAISO) to 20 minutes (at BPA). We assume that the generators are already committed by solving the unit commitment problem (and started) so that sufficient resources are available to supply the expected load. We also assume that the fast-start generators and other load following reserves, as well as regulating reserves have been procured to accommodate fine grain fluctuations. These unit commitments and reserve levels, along with current generation levels, are used as input to our dispatch models. At time $H - t_\ell$ the renewable generation and demand forecasts are locked down and are used to build the model. The dispatch model uses a horizon t_H of about an hour which includes fine grain time intervals at which the variable generation information is updated. This time interval is denoted t_i and typically ranges from 10 minutes to 60 minutes. This process is run every t_f minutes throughout the day, when the generation set points can be revised.

We will study two alternative practices depending on the interval of time between updates of slow-response conventional generation:

1) Hourly coupling models only allow hourly updates of slow-response generators and reflect the scheduling practices at some balancing authority areas, e.g., BPA [6]. In this model the slow-response generators and intertie resource decisions constitute the first stage decision variable x. Both these decisions are updated every hour, that is $t_f = 60$ minutes. The fast-response reserves, intermittent resources, and network decisions are controlled in an adaptive manner at fine timescale every t_i minutes. Note that fast-response reserves refer to those generators and load following reserves which have response times much lower than 10 minutes (the finest resolution considered) [19]. In addition, the second stage will accommodate the revised slow-response conventional generation decisions for the next hour. Since the slow-response generators are fixed for the hour, intra-hour energy imbalance is completely handled through fast-response reserves. All these fine grain decisions are lumped together in our second stage variable y.

2) Sub-hourly coupling models, on the other hand, allow slow-response generator decisions to be updated at sub-hourly intervals. These models are motivated by system operations at advanced ISOs like CAISO [6]. For dispatch beginning at the hour (i.e., time H) one uses the slow-response generators and intertie resource decisions as first stage decision variable x. The generator decisions are fixed only for $t_f = 10$ minutes, at which point in time, these decisions can be revised. On the other hand, the intertie decisions are fixed for the hour.

For dispatch starting at intra-hour epochs (time $H + nt_f$, $n = 1, 2, \ldots$), only slow-response generator decisions constitute the first stage variable x. Revisions at future sub-hourly time periods and next hour intertie decisions, which fall within the model horizon t_H, are included as second stage variables. These, along with fast-response reserves, intermittent resources and network decisions constitute the variable y. Since sub-hourly revisions are considered in this model, hourly coupling decisions are excluded.

C. Alternative Aggregations in Dispatch Models

As mentioned earlier, studies based on pure hourly dispatch models have the potential to hide the complications arising from sub-hourly variability of intermittent resources. To investigate the impact of sub-hourly decisions in the presence of other hourly decisions, we compare dispatch models at different sub-hourly resolutions. The second stage in the 2-SLP model (1a)-(1b) at the finest resolution (10 minutes) includes linear constraints for each sub-hourly interval-n: $\{\mathcal{F}(x, y_n, r(\omega_n)) = 0, y_n \geq 0\}$ for $n = 1, \ldots, 6$. This considerably increases the size of the problem, but identifies the value of fine grain information in deciding the slow-response conventional generation and intertie levels.

The *aggregated models* at 20, 30 and 60 minute resolutions, on the other hand, are created by aggregating the constraints and variables of (1b) by an averaging process. For example, to create the aggregated hourly model we define $\bar{y} = \frac{1}{6}\sum_{n=1}^{6} y_n$, and the quantity $r(\omega_n)$ is replaced by the observed average $\frac{1}{6}\sum_{n=1}^{6} r(\omega_n)$. In this case, we therefore rewrite the sub-hourly constraints as $\mathcal{F}(x, \bar{y}, \frac{1}{6}\sum_{n=1}^{6} r(\omega_n)) = 0$. Clearly, these models are aggregations of the 10-minute model. Nevertheless, these are different instantiations of two-stage stochastic programming models. It is worthwhile to note that the 60-minute aggregated model reflects current operations at BPA, where conventional decisions are revised once every hour based on hourly bulk variable generation information.

D. Algorithms for the Study

All dispatch models were formulated in the 2SLP and ESF forms. As long as the scenarios can be enumerated easily the ESF formulation (2) can be solved as one large LP using state-of-the-art deterministic solvers like CPLEX. Alternatively, one could use some deterministic decomposition scheme for ESF such as Benders' decomposition, Dantzig-Wolfe decomposition or the progressive hedging algorithm [3]. In this paper, the 2SLP problem (1) is solved using two-stage stochastic decomposition algorithm (2-SD) [14].

Conceptually, 2-SD is a stochastic version of deterministic Benders' decomposition [20] or L-shaped method [21]. Like these decomposition methods, 2-SD constructs a piecewise linear approximation to the recourse function and updates the approximation during each iteration of the algorithm. But unlike the deterministic methods, the recourse function is estimated for a small number of outcomes ω of random variable $\tilde{\omega}$. Moreover, because of its sampling based approximations, it is not restricted to only those instances in which the random variables are discrete. This allows 2-SD to be used with external simulators. This is achieved by combining sampling with sequential approximations in such a manner as to reduce the computational effort in generating a new piecewise linear approximation in each iteration.

In 2-SD, a newly sampled outcome vector $\omega^k := (\omega_1^k, \ldots, \omega_N^k)$ is incorporated into a collection of existing outcomes, $\{\omega^1, \ldots, \omega^{k-1}\}$, at iteration k. The algorithm constructs a lower bounding linear approximation of the sample mean function

$$H_k(x) = \frac{1}{k}\sum_{j=1}^{k} h(x, \omega^j). \tag{3}$$

2-SD theory [14] suggests that asymptotic convergence can be achieved by solving just one second-stage LP (1b) in any iteration. Furthermore, previously obtained data (on the optimal dual solutions for (1b)) can be used to define a lower bounding approximation of $h(x, \omega^j)$, for $j < k$.

For the most recent outcome ω^k and first stage decision x^k, we evaluate the recourse function $h(x^k, \omega^k)$ by solving (1b) and obtain the dual optimum solution π_k^k. This dual vector is added to a set V_{k-1} of previously discovered optimal dual vectors. In other words, we recursively update $V_k = V_{k-1} \cup \pi_k^k$. Linear programming duality ensures that for $\pi \in V_k$, $\pi^\top[r(\omega^j) - Tx] \leq h(x, \omega^j)\ \forall x$. Thus, in iteration k, we identify a dual vector in V_k that provides the best lower bounding approximation at $\{h(x^k, \omega^j)\}$, for $j < k$ that is:

$$\pi_j^k \in \text{argmax}\{\pi^\top[r(\omega^j) - Tx^k] \mid \pi \in V_k\}. \tag{4}$$

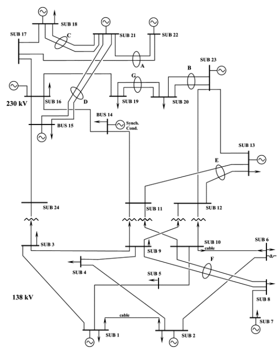
Note that the calculations in (4) are carried out only for previous observations as π_k^k will provide the best lower bound at $h(x^k, \omega^k)$. We use these dual vectors $\{\pi_j^k\}_{j \leq k}$ to generate a lower bounding function for the k^{th} sample mean function:

$$H_k(x) \geq \frac{1}{k}\sum_{j=1}^{k} (\pi_j^k)^\top[r(\omega^j) - Tx] \tag{5}$$

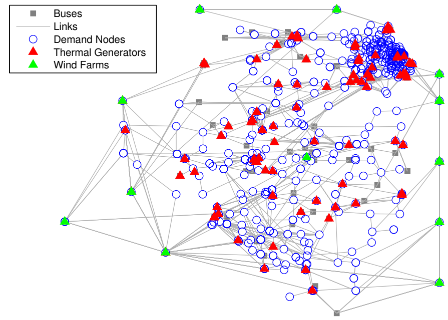
Similar to Benders' decomposition, the lower bound (right-hand-side in (5)) is added as a new linear piece of the piecewise linear approximation of $\mathbb{E}\{h(x, \tilde{\omega})\}$.

One more distinction from deterministic decomposition is the periodic re-adjustment of previous linear pieces in 2-SD [14]. Note that in (5), with increasing iterations we use a larger number of samples to generate the linear approximation. Also, the linear piece at iteration $j < k$ lower bounds the sample mean $H_j(x)$ and not $H_k(x)$. As a result, the earlier linear pieces need to be re-adjusted to ensure that they continue to provide lower bounds on the current sample mean $H_k(x)$. If we assume, without loss of generality, that $h(x, \tilde{\omega}) \geq 0$ (almost surely), then we have $H_k(x) \geq \frac{j}{k} H_j(x)$ $(j = 1, \ldots, k-1)$. Hence, the previously updated subgradient of the function H_j can be used as a lower bounding function of H_k by multiplying it by a factor of (j/k). With this, the approximation for the first-stage objective function at iteration k is given by

$$f_k(x) := c^\top x + \max_{j=1,\ldots,k}\left\{\frac{j}{k} \times \frac{1}{j}\sum_{i=1}^{j} (\pi_i^j)^\top[r(\omega^i) - Tx]\right\} \tag{6}$$



(a) RTS96 System



(b) Illinois System

Fig. 2. Power Systems under Study

Note that these approximations are generated in a recursive manner. The sequence $\{x^k\}$ is generated by 2-SD such that

$$x^{k+1} \in \text{argmin}\{f_k(x) + \frac{1}{2}\|x - \bar{x}^k\|^2 \mid Ax \leq b\} \quad (7)$$

where, $\bar{x}^k$ denotes an incumbent solution (the best solution) at iteration k [14]. This incumbent is updated with the current solution ($\bar{x}^{k+1} = x^{k+1}$) if the (sample mean) point estimate of the objective value at x^{k+1} is better than the estimate at $\bar{x}^k$; else, $\bar{x}^{k+1} = \bar{x}^k$.

Deterministic algorithms which solve convex programs by constructing an outer linearization of the objective function ([20]) are terminated when the difference between the objective function value at a given iterate and a valid lower bound on the objective function values is sufficiently small. The lower bounds obtained by 2-SD are based on sampled information, and hence are stochastic. Therefore, the deterministic termination criterion cannot directly be applied to a sampling based algorithm. The 2-SD approach uses a bootstrapping method to assess the primal-dual gap stability. The algorithm also gauges the impact of new information (new outcome ω^k, new first stage candidate solution x^k, and new dual solutions π_k^k) on the approximation in (6). A measure of this impact and the primal-dual gap stability are used in designing the stopping rules for 2-SD. We refer the reader to [22] and [23] for details about these stopping rules.

The 2-SD algorithm has previously been applied to operations management [24], VLSI design optimization [25], chemical plant expansion [26], water resource engineering [27], among others. This work is the first application of 2-SD algorithm to the stochastic economic dispatch problem. The 2-SD algorithm offers several features which make it amenable for power systems applications:

- *Online sampling:* The 2-SD algorithm does not rely on a-priori selection of scenarios to be used for optimization. Rather, a new sample ω^k is introduced to the observation pool in every iteration, and the approximations are updated based on information collected at (x^k, ω^k). This allows updated forecast scenarios to be included without having to restart the optimization. As a result simulators are easily incorporated as a source of scenarios during optimization. Such simulators are currently used by system operators for diagnosis rather than optimization.
- *Computational edge:* Efficient implementation of (4) and (6), and use of the appropriate data structures [28] allow 2-SD to be used for large scale stochastic optimization problems, often encountered in power systems operations, using minimal computational resources.

III. Computational Results

Our computational studies will address the main questions raised in the introductory section.

Experiments: For our computational study we consider the following experiments:

A. *Comparison of solution methods.* Here we will compare the 2-SD and ESF solution methods.
B. *Sensitivity to Dispatch Interval.* To study the effect of aggregation we solve both the systems with varying dispatch resolution using 2-SD at 10% wind integration level.
C. *Wind Penetration Study:* An increase in penetration level results in greater variability in renewable generation. We investigate the performance of 2-SD and the impact of model choice (hourly v sub-hourly) in systems with different renewable penetration levels.

Our experiments begin by first predicting an optimal or near optimal first stage solution for all instances. In the posterior analysis, the quality of these predicted first stage solutions is verified by fixing them and simulating the dispatch problem (1b) for different wind power realizations. This verification is terminated when a $(1-\alpha) \times 100\%$ confidence interval (CI) of total cost is built. In presenting the verification results we report the estimated mean and the 95% confidence interval of total cost.

Experimental Setup: Test Systems. The study was conducted on two energy networks, the IEEE-RTS96 1-Area system and the Illinois system. The RTS96 system [29], shown in Fig. 2a, is an enhanced test system which has been used in several power system studies [5, 30]. This system is developed based on the data presented in [5, 29, 30] and is modified to include a renewable generator and additional reserves. The system contains 32 thermal generators which produce electricity at generation costs which are functions of the fuel costs and average heat-rates. The load profile used is for a summer weekday. The spatial distribution of demand across the RTS96 system is derived based on Table-2, [29].

The second system (Fig. 2b) is based on realistic data available for the state of Illinois and comprises of 1900 buses, 2538 transmission lines and 870 load nodes. The total system demand for the study horizon is 98639.37 MWh. The system comprises of 237 internal generators and 24 intertie connections. The data consists of a detailed description of network topology, fuel costs, generator ramping constraints and capacities. Twelve wind farm locations at out-of-state buses are included in the network. For both these systems we assume that all the generators are committed over the horizon considered, and are available for dispatch. The demand is assumed to be constant over a one hour time period.

Wind Simulator. The two-stage formulation in (1b), and the ESF formulation (2) allow for randomness in wind generation. The ESF models are built using the Weather Research and Forecasting (WRF) outputs from [31] for 12 wind farm locations in Illinois. The WRF outputs are also used to build a vector autoregression based model. This model uses multiple WRF trial outputs to estimate its parameters, and allows for fast and efficient simulation of wind scenarios for use within stochastic programming algorithms. The model captures the spatio-temporal correlations of wind generation and uses a adaptive sliding window technique to overcome non-stationarity of high resolution, sub-hourly wind. We refer the reader to [32] for more details about the model. The 2-SD optimization and all verification runs for posterior analysis were carried out using scenarios simulated from this model.

For the number of scenarios listed in TABLE I the above data results in ESF formulations with sizes given by ESF Rows and ESF Columns in the table. Note that even for a small network such as RTS96, the size of the linear program easily grows to hundreds of thousands of rows and columns as the number of scenarios increase.

Although our computational experiments restrict the randomness ($\tilde{\omega}$) to wind power generation, the methodology is applicable to cases allowing randomness in demand, operating costs and generator outages.

Platform. The 2-SD algorithm was implemented in C programming language on a 64-bit Intel-core i7-2600 CPU @ 3.4GHzx8 with 8 GB memory. All linear and quadratic programs were solved using CPLEX callable subroutines.

A. Comparison of Solution Methods

For these experiments we use $t_H = 70$ minutes and $t_f = 60$ minutes for hourly coupling model. For the sub-hourly coupling instances t_H is set to 60 minutes and t_f is 10 minutes. Both the models allow variable generation data at $t_i = 10$ minute resolution.

The comparison results of 2-SD algorithm and limited sample ESF method are summarized in TABLE I. The predicted values reported refer to the objective function estimate of each optimization method (LP for ESF, and 2-SD). For 2-SD, one can interpret these values as estimates of lower bounds on the optimal value. The column labeled "Verification CI" (Confidence Interval) refers to the confidence interval calculated by simulating the consequences of first stage solution. These values correspond to estimates of the objective function at the first stage solution, and are therefore estimated upper bounds.

For sample-based stochastic programming models there is no guarantee that the predicted values for any instance will fall within the verification CI. This is observed in the case of ESF in TABLE I where the predicted values are lower than the verification CI for all sample sizes, indicating significant bias in predicting costs. Such bias arises due to the small sample sizes, and hence the quality of solutions from such small sample SP models is hard to discern. On the contrary, values predicted by the 2-SD algorithm use larger sample sizes, and hence have lower bias. This is emphasized by the fact that its prediction values fall within the verification CI which allows us to conclude that the solutions obtained are acceptable. Moreover, the verification column also shows that the estimated mean for 2-SD solutions are uniformly lower than the ones for ESF.

Fig. 3 shows the adjustments of slow-response conventional generation in the sub-hourly coupling models. The recourse computed using a large set of scenarios allows 2-SD to predict solutions which require significantly lower adjustments at sub-hourly intervals when compared to the deterministic ESF approach, which is currently being used by operators. This is indicated by the fact that the adjustment intervals for 2-SD are completely enclosed within those for ESF (see Fig. 3).

Shifting our attention now to solution times for RTS96, note that as the number of scenarios increase for ESF, it loses its computational edge to 2-SD. This is, in fact, highlighted in the

System	Method	# scenarios	ESF Rows	ESF Columns	Predicted Value	Time (s)	Verification CI
RTS96 Hourly coupling	ESF	10	11975	11552	40026.14	0.16	[60074.26, 61697.02]
		20	23885	23072	40795.62	0.41	[41385.75, 41443.03]
		30	35795	34592	40773.41	0.63	[41385.06, 41442.35]
		100	119165	115232	40718.78	2.80	[41385.04, 41442.33]
	2-SD	350	-	-	40945.71	2.35	[40936.34, 40989.01]
Illinois Hourly coupling	ESF	1(mean)	40784	46337	18137161	128.83	[22502915, 22620983]
		5	205538	234351	21886579	2866.56	[22170843, 22272844]
	2-SD	181	-	-	22151783	768.67	[22124133, 22213367]
Illinois Sub-hourly coupling	ESF	1(mean)	36415	40176	12878476	487.73	[13737964, 13830823]
		5	179983	199836	12811076	4080.62	[13722876, 13797483]
	2-SD	138	-	-	13721268	1147.28	[13660228, 13742965]

TABLE I
SOLUTION METHOD COMPARISON (10-MINUTES RESOLUTION, 10% WIND PENETRATION)

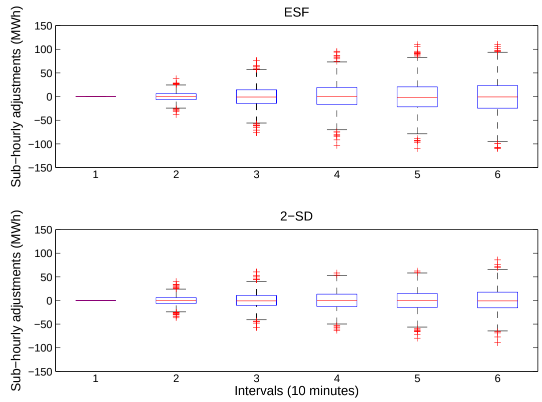


Fig. 3. Adjustments with sub-hourly coupling model in Illinois system

more realistic instance for the state of Illinois where ESF takes almost 50 minutes to manage 5 scenarios. On the other hand, 2-SD offers significant improvements in solution quality and computational times. This makes 2-SD an attractive method for solving large scale stochastic economic dispatch problems. Incidentally, the reader may find it interesting to note that if one were to use an ESF formulation to solve the Illinois instance using hourly coupling model reported in TABLE I, that linear program would have 7.2 million rows and 8.3 million columns.

As one can surmise, the predicted values obtained by 2-SD can also be significantly different from the verification CI. For such cases, we would expect to run the 2-SD algorithm with tighter stopping tolerance, so that the algorithm would use a larger number of samples, leading to improved gap estimates. However, this was not necessary for the instances encountered in this study.

B. Sensitivity to Dispatch Interval

To account for hourly coupling constraints and varying resolutions ($t_i = 10, 20, 30$ and 60 minutes) in hourly coupling model, a common $t_H = 120$ minutes is chosen for comparison in these experiments. In these instances slow-response generation and intertie decisions are held constant for a duration of $t_f = 60$ minutes. The sub-hourly coupling model uses t_H of 60 minutes, over which only intertie decisions are held constant, while the slow-response generation decisions can be updated at $t_i = 10, 20, 30$ and 60 minutes.

The two-stage stochastic programming instances at varying dispatch intervals are solved using 2-SD. The solutions obtained from these instances are verified using the same set of wind scenarios. The prediction and verification results for all the instances are summarized in TABLE II. These results indicate that finer resolution dispatch instances lead to lower costs for both hourly and sub-hourly coupling models. The coarser resolution models underestimate the realistic costs as indicated by the predicted values in TABLE II. This is because, optimization in these instances is carried out with mean ensembles.

Recall that the slow-response conventional generation levels are fixed for $t_f = 60$ minutes in the hourly coupling model, while the variable generation data is available at varying intervals ($t_i = 10, \ldots, 60$ minutes). As the interval is decreased, the fine timescale fluctuations in variable generation are clearly evident during optimization and hence, in this model the first stage reacts cautiously by using more slow-response conventional generation and intertie resources as indicated in TABLE II. The need to avoid infeasibilities due to lack of transmission capacity also contributes to this increase. In the sub-hourly coupling model, although the slow-response conventional generation decisions can be revised every t_i in an adaptive manner, they are limited by their ramping capability. Hence, the first stage uses fine grain data and increases the first stage resources.

TABLE II also lists the p-value, computed from the verification data, associated with the null hypothesis: there is no difference between the 10-minute dispatch solution and solution from lower resolution models. A p-value of less than 0.05, as seen for example in all 60-minute dispatch instances, allows us to reject the null hypothesis at 95% significance level, and we can conclude that their solutions result in statistically different verification results. On the other hand,

System	Resolution (min)	First stage resources (MWh)	Predicted value	Verification			
				Mean	Std. Dev.	Confidence Interval	p-value
RTS Hourly coupling model	10	2076.52	41409.59	41418.99	14.61	[41390.35, 41447.64]	1
	20	2076.96	41213.12	41423.42	14.61	[41394.78, 41452.07]	0.8303
	30	2074.08	41703.08	41765.22	31.70	[41703.08, 41827.35]	< 0.0001
	60	2031.84	40449.79	55454.27	358.05	[54843.49, 56247.04]	< 0.0001
Illinois Hourly coupling model	10	74258.8	29965989	29935161	30631	[29875123, 29995200]	1
	20	74035.9	29839346	29943153	31635	[29881147, 30005160]	0.8559
	30	73712.4	29741583	30027233	33473	[29961624, 30092841]	0.0425
	60	73340.3	29688063	30189580	40474	[30110250, 30268909]	< 0.0001
Illinois Sub-hourly coupling model	10	74371.2	13721268	13701596	21106	[13660228, 13742965]	1
	20	74198.4	13596532	13726059	22744	[13681480, 13770639]	0.4302
	30	74069.4	13543494	13745003	23557	[13702751, 13787255]	0.1501
	60	74082.9	13522440	13907159	24177	[13859771, 13954547]	< 0.0001

TABLE II
COMPARISON OF DISPATCH RESOLUTION (10% WIND PENETRATION)

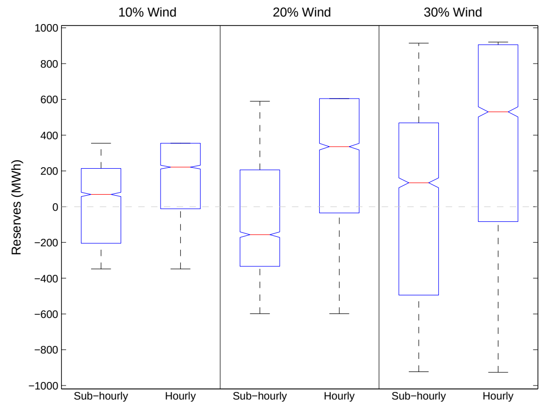


Fig. 4. Reserve utilization in Illinois System with Hourly Coupling Model

Wind%	Original		Modified	
	Mean	Conv.gen.(MWh)	Mean	Conv.gen.(MWh)
10	22168750	74090	9198967	70901
20	38446352	70940	7254406	62973
30	59307934	67905	5285322	54970

TABLE IV
ILLINOIS SYSTEM RESULTS WITH ADDED TRANSMISSION CAPACITY

the null hypothesis cannot be rejected for 20-minute dispatch instances, at 95% significance level.

Fig. 4 shows fast-response reserve utilization in Illinois system when hourly and sub-hourly (10 minute interval) resolution is used with hourly coupling model. The positive and negative values indicate ramp up and down utilization respectively. Recall that this model uses committed reserve levels ($r_{ni}^{min}, r_{ni}^{max}$) as input, this is indicated by the outermost whiskers. The horizontal red lines with notches represent the median. The figure indicates that the reserve requirements increase with an increase in wind penetration levels. Further, the sub-hourly resolution models reduce reserve utilization for all penetration levels when compared to hourly resolution models.

C. *Wind Penetration Study*

Thus far our computational experiments have demonstrated that both sample studies and coarse grain optimization have a potential to be misleading in their predictions (TABLE I, TABLE II). Such observations have also been made in simulations studies conducted by CAISO ([9], see I). Since 2-SD is a simulation-based optimization algorithm, we suspect that it should also be able to predict circumstances that cause congestion even when the dispatch is optimized. In this section we undertake a study to assess the performance of 2-SD at different wind penetration levels on both RTS96 and Illinois networks.

Energy penetration for this study is measured as the ratio of the amount of energy produced from the wind generation to the total energy produced. The results for 10%, 20% and 30% wind integration for both the test networks are provided in TABLE III. These experiments were conducted on the hourly coupling model with $t_H = 70$ minutes, $t_f = 60$ minutes and $t_i = 10$ minutes.

As expected, thermal generation is reduced as the availability of wind is increased. For the RTS96 system congestion was not encountered during 2-SD runs, and hence the net operational cost decreases when the penetration levels are increased (TABLE III). On the other hand, the initial experiments with the Illinois system identifies a small area of the network that needs congestion relief. In this area, increased penetration led to generation curtailment which in turn increased the overall operating cost of the system (TABLE III). Such identification of congestion is due to the combination of optimization and simulation within 2-SD.

Prompted by the specific areas of congestion we introduce additional capacity on a few links which alleviates such congestion. TABLE IV compares the verification results of the original Illinois system with the modified network. With additional transmission capacity the results indicate that the operational cost decreases with increased penetration. The reserve requirements also increase with the penetration levels due to increased volatility as shown in Fig. 4.

The 2-SD framework does not rely on a-priori sampling or knowledge of explicitly provided probability distribution. The algorithm learns the stochastic process in an online manner, and as a result, the number of samples necessary during the runs might vary (see II and TABLE III). TABLE III also highlights the computational performance of 2-SD in instances with higher variability resulting from increased wind penetration. Moreover for congested networks, like the Illinois system at 30% penetration, a deterministic approach will choose a solution which can ensure feasibility only with respect to the sample(s) used to build the model. As a result, the system is much more prone to higher variability of reserves. On the other hand, 2-SD chooses first-stage solution

System	Wind Penetration	Prediction				Verification		
		Samples	First stage resources (MWh)	Value	Time (s)	Mean	Std. Dev.	Confidence Interval
RTS96	10 %	350	2076	40945.71	2.35	40962.68	13.44	[40936.34,40989.01]
	20 %	366	1970	40147.65	2.33	40168.25	24.61	[40120.02, 40216.47]
	30 %	359	1901	39371.52	2.20	39513.22	117.76	[39282.40, 39744.03]
Illinois	10 %	181	74090.5	22151783	768.47	22168750	22763	[22124133, 22213367]
	20 %	189	70940.0	38409069	746.21	38446352	50641	[38347094, 38545610]
	30 %	171	67905.5	59381611	912.74	59307934	98733	[59114417,59501451]

TABLE III
WIND PENETRATION RESULTS WITH HOURLY COUPLING MODEL (10 MINUTE RESOLUTION, 70 MINUTE HORIZON)

which ensures feasibility across all the samples encountered during optimization. As before, the predicted value falls within the verification confidence interval, and hence 2-SD provides good quality solutions even in the presence of high variability.

IV. Conclusion

In this paper, we presented a stochastic economic dispatch framework which allows control of slow-response energy resources and intertie decisions at a coarse timescale, and renewable generation along with other dispatch related decisions at a fine timescale. To the best of our knowledge, this is the first study to incorporate sub-hourly economic dispatch within a stochastic optimization model. We presented two dispatch models which represent alternate operating practices used by power system operators. The results comparing these models at different resolutions illustrated the improvements that can be achieved by sub-hourly dispatch. The improvement in terms of the overall operational cost was due to effective utilization of sub-hourly information in deciding the first stage slow-response generation and intertie levels. The results at various wind integration levels showed reduction in operating reserve usage under sub-hourly dispatch. We also presented a stochastic programming approach, using 2-SD algorithm, to solve these large problems. The results demonstrated the scalability of 2-SD and showed that, when compared with extensive scenario formulation, 2-SD provided verifiably better solutions in far less time. Finally, the 2-SD algorithm was hooked with an external simulator which provided outputs for wind generation. Application of 2-SD algorithm over a rolling horizon, capturing economic dispatch over multiple hours, is currently being studied, and will be reported in the future. We will also investigate the role of storage devices in mitigating the challenges of renewable integration as part of our future research.

Acknowledgment

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Appendix A
Economic Dispatch Formulation

A. Notation

We will use $n = 0$ to denote the first stage decision epoch. With t_H denoting the model horizon and t_i the sub-hourly interval, $N = t_H/t_i$ is the number of sub-hourly decision epochs. $\mathcal{N} = (1, \ldots, N)$ will denote these fine timescale decision epochs. The set of buses, links, demand and intertie nodes are denoted as $\mathcal{B}$, $\mathcal{L}$, $\mathcal{D}$ and $\mathcal{T}$ respectively. The set $\mathcal{G}$ constitutes the slow-response conventional generators, while the set of wind generators and fast-response reserves are denoted as $\mathcal{W}$ and $\mathcal{R}$ respectively. The subscript i represents a subset of the respective set at bus-i.

The first stage variable x consists of intertie decisions $T_i\ \forall i \in \mathcal{T}$ and slow response conventional generation levels $G_{0i}\ \forall i \in \mathcal{G}$. The corresponding production costs are denoted as c_i^{tie} and c_i^{gen} respectively.

Second stage variable y includes ($\forall n \in \mathcal{N}$) the line-$(i,j)$ utilization p_{nij} and bus-i angle θ_{ni}. Additional resources are available through committed fast-response reserves which are used to match energy imbalance resulting from stochastic realizations. These resources are limited by their availability, which is proportional to wind penetration, and is assumed to be known from prior unit/reserve commitment. These resources can provide both ramp-up and ramp-down capabilities and we will use r_{ni} to denote utilization of these resources. Beyond this limit the load can be curtailed by r_{ni}^{ls} and the value of lost load is set at $d_i^{ls}\ \forall i \in \mathcal{L}$ (set to \$2000 in the computational study). Due to network constraints it is possible for generation at any particular node to be left unused. This generation can be ramped down only by a certain amount dictated by the physical ramping constraints. Generation beyond this limit is curtailed, which is denoted as r_{ni}^{gs}, and penalized by including a shedding penalty d_i^{gs} at generation side ($\forall \mathcal{G}$). Finally, we treat wind generation as a must-take resource (Section-3, [33]), provided there are sufficient reserves and no transmission issues in the system. To ensure this we impose a penalty d_i^{ws} on wind curtailment r_{ni}^{ws}. These are also included as second stage decisions. For our computational study we have set these generation shedding penalty to \$500 (a value greater than the highest production cost). Alternately, market based settlement costs/opportunity costs can be used for these curtailment penalties.

The hourly coupling model also includes the intertie decisions and slow-response generation levels for next hour which are denoted as $T_i^+\ \forall i \in \mathcal{T}$ and $G_{0i}^+\ \forall i \in \mathcal{G}$ respectively. The inelastic load is denoted by D_i for the current hour and D_i^+ for the next hour. The sub-hourly coupling model, on the other hand, has conventional generation revisions $G_{ni}\ \forall i \in \mathcal{G}$ and $\forall n \in \mathcal{N}$.

B. Objective

For the hourly coupling model the total cost comprises of the current intertie and conventional generation cost, and the expected value of recourse function. This recourse function includes the cost of generation for next hour and the penalty cost associated with wind, thermal and load curtailment.

$$\sum_{i\in\mathcal{T}} c_i^{tie} T_i + \sum_{i\in\mathcal{G}} c_i^{gen} G_{0i} + \mathbb{E}\Bigg\{ \sum_{i\in\mathcal{T}} c_i^{tie} T_i^+ + \sum_{i\in\mathcal{G}} c_i^{gen} G_{0i}^+ + \sum_{n\in\mathcal{N}} \left(\sum_{i\in\mathcal{G}} d_i^{gs} r_{ni}^{gs} + \sum_{i\in\mathcal{D}} d_i^{ls} r_{ni}^{ls} + \sum_{i\in\mathcal{W}} d_i^{ws} r_{ni}^{ws} \right) \Bigg\}. \tag{8}$$

For sub-hourly coupling model the intertie and generation for next hour are not considered in the above function. However, the conventional generation revisions G_{ni} are included at sub-hourly time intervals. For $n > 1$ the function is given by:

$$\sum_{i\in\mathcal{T}} c_i^{tie} T_i + \frac{1}{N}\sum_{i\in\mathcal{G}} c_i^{gen} G_{0i} + \mathbb{E}\Bigg\{\frac{1}{N}\sum_{i\in\mathcal{G}} c_i^{gen} G_{ni} + \sum_{n\in\mathcal{N}}\left(\sum_{i\in\mathcal{G}} d_i^{gs} r_{ni}^{gs} + \sum_{i\in\mathcal{D}} d_i^{ls} r_{ni}^{ls} + \sum_{i\in\mathcal{W}} d_i^{ws} r_{ni}^{ws}\right)\Bigg\}. \quad (9)$$

When $n = 1$ there are no sub-hourly revisions, and hence the term G_{ni} is not included in (9). The objective is to minimize this cost subject to the constraints presented below.

C. Hourly Constraints

These constraints are associated with slow-response generators.

a. *Generation capacity:*

$$G_i^{min} \le G_{0i} \le G_i^{max}, \quad (10)$$
$$G_i^{min} \le G_{0i}^{+} \le G_i^{max} \quad \forall i \in \mathcal{G}. \quad (11)$$

G_i^{min} and G_i^{max} are the minimum and maximum generation capacity of generator units indexed by i.

b. *Ramping constraints:*

$$\Delta G_i^{min} \le G_{0i} - G_i^{init} \le \Delta G_i^{max}, \quad (12)$$
$$\Delta G_i^{min} \le G_i^{+} - G_{0i} \le \Delta G_i^{max} \quad \forall i \in \mathcal{G}. \quad (13)$$

ΔG_i^{min} and ΔG_i^{max} represent the down and up-ramping limits of generator units. Recall that the initial dispatch levels $\{G_i^{init}\}$ are known inputs to our models.

Constraints (10) and (12) appear as first stage constraints in (1a) for both the hourly and sub-hourly coupling models, while constraints (11) and (13) are bundled into second stage constraints (1b) only for the hourly coupling model. Note that these hourly constraints are not considered for aggregation.

D. Sub-hourly Constraints

The sub-hourly constraints are functions of both first and second stage variables. There will be one set of constraints, $\{\mathcal{F}(x, y_n, \omega_n)\}_{n\in\mathcal{N}}$, associated with each realization ω of the random variable $\tilde{\omega}$.

a. *Power flow equation*: If n belongs to current hour,

$$\sum_{j:(j,i)\in\mathcal{L}} p_{nji} - \sum_{j:(i,j)\in\mathcal{L}} p_{nij} - \sum_{j\in\mathcal{G}_i} r_{nj}^{gs} + \sum_{j\in\mathcal{R}_i} r_{nj} + \sum_{j\in\mathcal{W}_i}(\omega_{nj} - r_{nj}^{ws}) + \sum_{j\in\mathcal{G}_i} G_{0j} = \sum_{j\in\mathcal{D}_i}(D_j - r_{nj}^{ls}) \quad \forall i \in \mathcal{B} \quad (14)$$

The power flow equations ensure that the supply meets the demand at every bus in the network. The next hour power flow equations for the hourly coupling model are obtained by replacing G_{0j} with G_{0j}^{+} and D_j with D_j^{+}. Since sub-hourly coupling model allows for revision of conventional generation decisions at sub-hourly intervals, we will use G_{nj} in the place of the static G_{0j} in the above power flow equation.

b. *Line flow equation:*

$$p_{nij} = \frac{V_i V_j}{X_{ij}}(\theta_{ni} - \theta_{nj}) \quad \forall (i,j) \in \mathcal{L}, n \in \mathcal{N}. \quad (15)$$

Here V_i's are the bus voltages and X_{ij} is line reactance. The real power transmitted on any line and power loss on it are non-linear functions of the difference between the angles at the buses connected by the line. Second-order approximations are used to linearize these functions which make it suitable to be used with standard linear optimization methods. The power flow losses in the network are ignored in this formulation and only the line power flows are considered. [34] provides the details on this linearization of network constraints.

c. *Reserve limits:* Sub-hourly energy imbalance can be addressed using fast-response reserves which are limited by their availability:

$$r_{ni}^{min} \le r_{ni} \le r_{ni}^{max} \quad \forall i \in \mathcal{R}, n \in \mathcal{N}. \quad (16)$$

The limits r_{ni}^{min} and r_{ni}^{max} are available through reserve commitments, and are inputs to our models.

d. *Sub-hourly revisions:* The sub-hourly coupling model allows for sub-hourly revision of conventional generation which are limited by ramp rates of these generators. For $n = 0, \dots, N-1$:

$$\Delta G_i^{min}(N) \le G_{n+1i} - G_{ni} \le \Delta G_i^{max}(N), \forall i \in \mathcal{G} \quad (17)$$

The ramping limits are dependent on t_i, and hence we denote them as functions of N.

e. *Bounds:* The bounds on the second stage variables are enforced due to the physical constraints on the network. $(p_{ij}^{min}, p_{ij}^{max})$ set the limits on the line capacities and $(\theta_i^{min}, \theta_i^{max})$ are the limits on the bus angles. The curtailment variables are limited by the amount of generation and load. For all $n \in \mathcal{N}$:

$$p_{ij}^{min} \le p_{nij} \le p_{ij}^{max} \quad (i,j) \in \mathcal{L}, \quad (18a)$$
$$\theta_i^{min} \le \theta_{ni} \le \theta_i^{max} \quad i \in \mathcal{B}, \quad (18b)$$
$$0 \le r_{ni}^{gs} \le G_{ni} \quad i \in \mathcal{G}, \quad (18c)$$
$$0 \le r_{ni}^{ls} \le D_i \quad i \in \mathcal{D}, \quad (18d)$$
$$0 \le r_{ni}^{ws} \le \omega_{ni} \quad i \in \mathcal{W}. \quad (18e)$$

For the hourly coupling model, the upper bound in (18c) is replaced by G_{0i} for all $n \in \mathcal{N}$.

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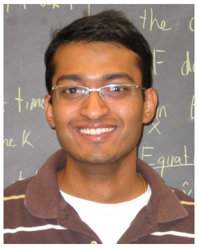

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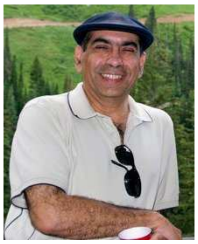

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