

An Investigation of the Average Bid Mechanism for Procurement Auctions*

Wei-Shium Chang[†] Bo Chen[‡]
Humboldt University Berlin Southern Methodist University

Timothy C. Salmon[§]
Southern Methodist University

October 2013

Abstract

In a procurement context, it can be quite costly for a buyer when the winning seller underestimates the cost of a project and then defaults on the project midway through completion. The Average Bid Auction is one mechanism intended to help address this problem. This format involves awarding the contract to the bidder who has bid closest to the average of the bids submitted. We compare the performance of this mechanism with the standard Low Price mechanism to determine how successful the Average Bid format is in preventing bidder losses as well as its impact on the price paid by the buyer. We find the Average Bid mechanism to be more successful than expected due to the surprising fact that bidding behavior remains similar between the Average Bid and Low Price auctions. We provide an explanation for the bidding behavior in the Average Bid auction that is based on subjects having problems processing signals near the extremes of the distribution.

JEL Codes: C91, D44, D82

Key Words: Procurement auctions, average bid, bankruptcy, experiments

1 Introduction

The use of auctions in the procurement process is a standard practice for many organizations who are interested in using the competitive nature of auctions to keep procurement

*The authors thank Francesco Decarolis, Dmitry Ryvkin, Svetlana Pevnitskaya, and Mark Issac for their advice and comments, and Tanapong Potipiti for helpful discussion on our simulations. We also thank various participants in presentations at Texas A&M University and the 2011 Tucson ESA meetings for helpful suggestions. Financial support for this project was provided by IFREE.

[†]Humboldt University of Berlin, Institute for Entrepreneurial Studies and Innovation Management, Unter den Linden 6, 10099 Berlin, Germany. changwei@hu-berlin.de, Phone: +49 30209399013, Fax: +49 30 2093 99030.

[‡]Southern Methodist University, Department of Economics, 3300 Dyer Street, Suite 301 Umphrey Lee Center, Dallas, TX 75275-0496. bochen@smu.edu, Phone: 214-768-2715, Fax: 214-768-1821.

[§]Southern Methodist University, Department of Economics, 3300 Dyer Street, Suite 301 Umphrey Lee Center, Dallas, TX 75275-0496. tsalmon@smu.edu, Phone: 214-768-3547, Fax: 214-768-1821.

costs low. Government agencies and private businesses use a number of different types of auction mechanisms for procuring a wide range of goods and services but the most common mechanism is likely the standard sealed bid first price auction, or low price auction. In this format, potential sellers simultaneously submit asking prices to the buyer. The winning seller is the one who asks for the lowest price and the winner is paid that price.

While the low price mechanism appears to be well suited to the task of forcing sellers to compete to yield a low purchase price for the buyer, this mechanism is not without its drawbacks. In many procurement situations the cost of providing that good or service is unknown to the sellers at the time of the auction. Consider the case of a buyer requesting competing bids from construction companies for a new building. The companies can estimate the construction costs at the time the auction is held but prices of materials will likely change during the construction period, unexpected design complications may emerge, weather may impact scheduling and any number of other unforeseen complications could impact the final total cost in ways not knowable at bid submission. The cost uncertainty is a problem in the low price mechanism because the seller that will tend to win is the one who underestimates the eventual cost by more than the others. If sellers fail to take into account the information they expect to learn contingent upon winning when determining their bids, then they will suffer from what is commonly referred to as the Winner's Curse. This is a well documented phenomenon stemming from the literature on common value auctions and it is typically found that bidders have difficulty learning to avoid it, see Kagel and Levin (2002) for a comprehensive discussion.

While a seller who suffers from the Winner's Curse and loses money should obviously be concerned, in many cases the buyer should also be as well. A construction company who finds that they have agreed to a price less than the construction cost may end up taking longer to complete the project, substituting substandard materials or designs or even walking away from the project. Another way of stating the problem is that once a price is agreed to, there is a principal-agent relationship established between the buyer and the seller and the terms of that contract need to satisfy both the standard incentive compatibility and participation constraints for the seller. If the terms do not satisfy these constraints and the seller exerts an undesired level of effort or walks away from the job, then this will certainly have a negative impact on the buyer.¹ It should therefore be clear that the buyer does have an interest in making certain that a seller is adequately compensated for the work. It is also certainly the case that a buyer does not wish to overpay. While the low price mechanism may be good at generating low prices, prior evidence with first price auctions in a common value environment like this suggests that it is not a very good mechanism for helping bidders avoid losses.

There are a variety of mechanisms that have been considered to solve this problem in a procurement context. An initial issue is whether to use an auction mechanism or to negotiate directly with a small number of possible sellers. Bulow and Klemperer (1996) argues that the additional competition in an auction suggests that it will always perform better but Gil and Oudot (2009) points out that in more complex situations like the ones at issue here negotiations might do better. Chen, Xu, and Whinston (2010) suggests that the

¹The issue of combining incentive contract design with auctions is dealt with in more detail in Fishe and McAfee (1987) and McAfee and McMillan (1986).

way to deal with problems of this sort involve contingent contracting.² Calveras, Ganuza, and Hauk (2004) proposes a solution which consists of a surety bond that the winning seller has to pay for that will compensate the buyer in the event that the seller fails to complete the project. There is also the possibility of renegotiating the terms of a contract after an auction which is analyzed in Bajari, Houghton, and Tadelis (2011) as well as Chang, Salmon, and Saral (2012).

In this study we examine an auction format called an Average Bid Auction. Many organizations have been using this mechanism in an attempt to counter this problem but existing analysis of the mechanism suggests little in the way of support for the practice. The rules of this format involve awarding the contract to the bidder who has submitted a bid closest to the average of all bids with the winning bidder receiving his bid as the purchase price. The naive argument for why this might help in the common value context is that while the lowest bidder may have underestimated the cost of the project, the bidder who submits a bid at about the average may have more accurately estimated the costs. By awarding the project to the bidder closest to the average the buyer may lessen the problem of seller losses. This mirrors the explanation for using the average bid mechanism provided in European Commission (2002) where it is argued that one typical cause of “abnormally low tenders” in the European construction industry is that firms make unintentional estimation errors in preparing tenders and European Commission (2002) recommends mechanisms similar to average bid auctions to eliminate abnormally low tenders.

While this argument might initially seem reasonable, it suffers from a substantial flaw as it does not take into account the possibility that bidding behavior could shift between auction formats. If bidding behavior changes, the average bid mechanism may not solve the problem of seller losses or if it does, it could lead to prices so high as to make that solution not worthwhile. The intention of this current study is to examine this issue both theoretically and empirically to determine whether the average bid mechanism can solve the problem of seller losses without costing the buyer too high of a price premium.

Ours is not the first investigation of the properties of average bid auctions and it is also important to note that there are multiple versions of the mechanism in use. In what might be considered the standard average bid procurement auction that is used by government agencies in Taiwan and by the Florida Department of Transportation (FDOT) in the US, the bid closest to the average is selected and the price is equal to the winning bid. This standard mechanism is generally used when there are a small number of bidders. In situations involving larger numbers of bidders, additional rules may be used such as excluding high and low bids from consideration as an attempt to further eliminate outliers. Engel, Ganuza, Hauk, and Wambach (2006) discusses how this mechanism is implemented in Switzerland while Decarolis (2013) and Conley and Decarolis (2012) provide an extensive description and data on how the mechanism is used in Italy. There are other sorts of exclusions sometimes used such as in Peru and in the State of New York where the elimination criterion is based on the difference between bids and the average bid. Bids that lie 10 percent below or above the average will be thrown out, as stated in the Peruvian regulations for bidding and contracting for public works (Albano, Bianchi, and Spagnolo (2006)). Coviello and

²We note though that the goal in that paper was to solve the problem of a seller underdelivering on quality. This is a different but related problem and so the idea of contingent contracting could be easily adapted to the problem of sellers who make losses due to pre-auction uncertainty.

Mariniello (2014) examine the effects of publicity on participation in these auctions and finds generally positive effects on auction outcomes.

Our analysis will concentrate on the base mechanism in which all bids are considered and the one closest to the average wins. This version has been examined theoretically in Albano, Bianchi, and Spagnolo (2006), though in an environment in which all costs are commonly known, and in Decarolis (2013) under the assumption of private (though uncertain) costs. The basic theoretical prediction is that bidders have the incentive to all submit identical bids leading to potentially a continuum of Nash equilibria in which all bidders submit the same bid at a price high enough to insure profitability for all sellers. The argument behind this is that so long as all bidders are bidding the same price which is above the highest known possible cost, no bidder would prefer to deviate as a deviating bidder will lose the auction for sure (assuming three or more bidders) and forego the chance at a profitable win.

Decarolis (2013) also provides an examination of the underlying empirical questions using a data set of Italian public procurement auctions. This data set involves auctions using the (Italian) average bid auction as well as low price auction augmented by a bidder screening mechanism. In his examination, Decarolis finds that indeed the average bid method results in higher payments to the sellers than in the specialized version of a low price mechanism and that switching to low price auctions from average bid auctions results in weak evidence of quality worsening. While he does not find that sellers are able to collude on the maximally collusive equilibrium of all sellers submitting a common (and very high) bid, he does find that an alternate form of collusion emerges in which bidders bid cooperatively with a subset of collaborators in an attempt to manipulate the bid considered to be average.

The message from these prior papers is a clear recommendation that the average bid mechanism should not be used.³ Albano, Bianchi, and Spagnolo (2006) argues that the prices should be expected to be high and the winner selection random leading to poor efficiency properties. Decarolis (2013) makes a similar point but constructs a model in which the low price auction might perform poorly. In an argument adapted from Zheng (2001), bidders are modeled as having heterogenous costs of default. Bidders with low default costs have limited liability from losses and can therefore bid very aggressively. In such an environment, the low price auction can attract very low speculative bids while the average bid will not. In addition, if the default costs for a buyer are very high it is possible that the high price expected in the average bid auction may be preferable. In the end, Decarolis (2013) still argues against using the average bid mechanism due to poor efficiency properties and the fact that a screening mechanism can be added to a first price auction to reduce the limited liability concerns.

Despite the strength of these arguments against using the average bid auction, one finds

³The average bid mechanism being considered here should not be confused with the median bid CMS Auction examined in Merlob, Plott, and Zhang (2011). There are very substantial differences in these mechanisms. The CMS Auction analyzed there is a very controversial mechanism used for Medicare procurement. Its key characteristics are that it is a multiple unit auction in which bids are non-binding and the lowest bids are accepted up to the point at which supply equals demand. The price received by the sellers is not that market clearing price but a price equal to the median among accepted bids. It is fairly transparent that this is a problematic design as demonstrated in Merlob, Plott, and Zhang (2011). While the name “median bid” auction applied to the CMS auction sounds similar to the “average bid” mechanism we examine, the incentive issues across both mechanisms are unrelated.

that there are many procurement authorities still doing so. Even the Italian agencies conducting the auctions analyzed in Decarolis (2013) chose to continue using it. This suggests something of a puzzle. Either these auctioneers are making a substantial mistake in their mechanism choice or there is some element in the external environments not incorporated into prior analyses that leads to the average bid mechanism not being as problematic as supposed. Our study is designed to determine if there are environments other than those examined in Albano, Bianchi, and Spagnolo (2006), Decarolis (2013) and Conley and Decarolis (2012) which might lead to a different conclusion about the efficacy of the mechanism and provide some basis for why an auctioneer might reasonably choose the mechanism.

A key element in the prior analyses of the average bid auction is that the costs of the sellers have been assumed to be private and independent or commonly known. This is an important point because it is difficult to justify the use of the average bid auction in such an environment. In a common value environment though there exists a problem for both the buyer and seller that the average bid mechanism might solve. This assumption is also important because the common value assumption is a reasonable way to characterize many of the construction contracting examples for which the average bid mechanism is used in practice. Hong and Shum (2002) provides an analysis suggesting what assumptions may be reasonable for different environments.

We present the results of an experimental study which allows for a detailed examination of the impacts of these two mechanisms on bidding behavior in a common value environment in an attempt to understand if the average bid mechanism might be useful. We conduct sessions with bidders participating in only low price auctions and others with bidders participating in only average bid auctions with all other details held constant. We find quite surprisingly that bidding behavior is identical between the two formats. The observed behavior in the low price auction is consistent with what has been previously observed in the literature while the behavior in the average bid auction is inconsistent with prior theoretical predictions. Given that the bids are identical but the pricing rules are different, this leads to prices being higher in the average bid than in the low price. Losses are therefore substantially lower in the average bid, virtually eliminating default problems. We find that the prices in the average bid are roughly in line with the standard theoretical prediction for the low price indicating that the price increase is perhaps not “too high.” Consequently we find evidence suggesting that the average bid auction accomplishes the goals for which it is intended, i.e. eliminating the problems stemming from the Winner’s Curse without leading to too high of a price increase, and in fact even the naive argument for its implementation, i.e. bidding behavior not shifting between formats, may even be accurate.

The success of the average bid auction is based on the fact that the bidding behavior we observe differs from the standard prediction. Instead of bidders coordinating on very high prices they bid according to upward sloping bid functions. In attempting to explain this behavior, we will show that it is not related to the standard types of processing problems usually found in common value auctions. Consequently, the behavior is not explainable by either Cursed Equilibrium, Eyster and Rabin (2005), or Level- k reasoning, Crawford and Iriberri (2007). We show instead that rationalizing the behavior in the average bid auction requires a minimal deviation from standard theory involving only allowing for the fact that the bidders have a difficult time with a complicated inference problem that arises for those who receive signals near the endpoints of the signal distribution.

2 Equilibrium Predictions

The common value environment used here is essentially the same as the one used in the earliest formulations of such an environment dating back to Rothkopf (1969), Wilson (1969) and Wilson (1977). Consider a procurement auction in which n sellers compete to win the right to sell their services or a product to a buyer. Each seller is uncertain about the cost of providing this good or service, but knows that it will be common among all of them. A seller $i \in \{1, \dots, n\}$ receives a private signal of this cost, s_i , that will be conditioned on the actual cost realization of a random variable C , which is only learned after the auction has concluded. We will assume that C is drawn from a commonly known distribution F with pdf f on the range $[C_L, C_H]$ which we will assume to be uniform. The bidders' signals are commonly known to be drawn from an interval around the true cost such that each s_i is independently drawn from a uniform distribution over the range $[c - \theta, c + \theta]$, with c being the realization of C and $\theta > 0$. We assume that the parameters are such that $C_H > C_L > 0$ and $C_H - C_L > 2\theta$.⁴ We will provide analysis of the theoretical bid functions for the low price (LP) mechanism as well as the average bid (AB) mechanism.

2.1 Low Price Auction

In a low price auction, each seller i simultaneously submits a sealed asking price of b_i , and given these asks, the ex post payoff of seller i is

$$\Pi_i(b_i, b_{-i}) = \begin{cases} b_i - c, & \text{if } b_i < b_j \ \forall j \neq i, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

The symmetric risk neutral Nash equilibrium bid function is derived for this environment in Lind and Plott (1991) and Kagel and Richard (2001), which we replicate as follows:

$$b(s_i) = \begin{cases} s_i + \theta - g(s_i), & \text{if } s_i \in [C_L - \theta, C_L + \theta], \\ s_i + \theta - Y(s_i), & \text{if } s_i \in [C_L + \theta, C_H - \theta], \\ C_H - \frac{C_H + \theta - s_i}{(n+1)}, & \text{if } s_i \in [C_H - \theta, C_H + \theta]. \end{cases} \quad (2)$$

where

$$\begin{aligned} g(s_i) &= \left[(C_L + b(C_L + \theta)) P_n(0) + 2n\theta \int_0^{h(s_i)} P_n(u) du \right] P_n^{-1}(h(s_i)), \\ h(s_i) &= \frac{-s_i + C_H + \theta}{2\theta}, \\ P_n(u) &= \exp \left[\ln(1 - u^n) + n \int \frac{du}{1 - u^n} \right], \\ Y(s_i) &= \frac{2\theta}{n+1} \exp \left(\frac{-n}{2\theta} (C_H - \theta - s_i) \right). \end{aligned} \quad (3)$$

As pointed out by Kagel and Levin (1986), the bid function is complicated by the fact that any signal received within θ of the endpoints, C_L and C_H , conveys more information about true cost than a signal in the middle of the distribution, as the true cost cannot lie outside of $[C_L, C_H]$. That must be taken into account in the regions near the bounds. The bid function overall must also take into account the expectation of the negative information conditioned upon winning so that bidders who use this strategy would not suffer from

⁴Such a (natural) specification implies that (1) the true cost is strictly positive and (2) the set of private signals in the 'middle region' $[C_L + \theta, C_H - \theta]$ is non-empty.

the Winner’s Curse. Were bidders to bid according to this strategy, then their bids are a monotonically increasing function of their signals and the bidder with the lowest signal would win. This makes calculating the expected price straightforward, as it requires calculating the relevant order statistic for the lowest expected signal draw.

2.2 Average Bid Auction

We are using the simplest variant of the average bid auction in which each seller simultaneously submits a sealed asking price, b_i . Seller i wins if his ask is closest to the average and the price he receives is equal to his ask. Thus a seller’s payoff is given by

$$\Pi_i(b_i, b_{-i}) = \begin{cases} b_i - c, & \text{if } i \in \arg \min_j |b_j - \frac{1}{n} \sum_{k=1}^n b_k| \\ 0, & \text{otherwise} \end{cases}.$$

If there is a tie, or the set ‘ $\arg \min_j |b_j - \frac{1}{n} \sum_{k=1}^n b_k|$ ’ has cardinality $n^* > 1$, then each bidder identified in the set has equal probability of being declared the winner. Hence the expected value of a seller conditional upon being among the potentially winning set is $(b_i - c)/n^*$. As we shall see, given the nature of the eventual equilibrium we will find it is more important than usual to deal properly with tie bids.

As a first step, it is easy to see that in our common value environment, there exists a continuum of coordination Bayesian Nash equilibria where all sellers ask for the same price (no less than C_H) regardless of their signals. The rationale behind this is essentially identical to the theoretical examination in Albano, Bianchi, and Spagnolo (2006), except that our common value environment additionally requires that, in the interim stage, sellers with all potential signals (including the highest signal ($C_H + \theta$)) find it optimal to choose the coordinated price. This is why the minimum threshold for such a coordinated bid is C_H as this is the lowest bid that guarantees that all types would be willing to submit.

For our analysis we will consider a broader class of equilibria and attempt to determine the existence of a symmetric Bayesian Nash equilibrium where all bidders follow a strictly *increasing* bidding strategy $b(s)$. We can provide an initial general result demonstrating that equilibria with this structure however do not exist:

Proposition 1 *Consider the average bid auction with n bidders where $n \geq 3$. There does not exist a symmetric Bayesian Nash equilibrium in strictly increasing strategies.*

The detailed proof of this proposition is provided in the Appendix. The basic idea is to show that from any proposed strictly increasing equilibrium, there is a set (with positive measure) of types with profitable deviations. While this result is useful, we can also provide more insight into the general incentive issues involved in these auctions by focussing on the special case of auctions of size $n = 3$. The reason to focus on auctions of this size is that the explicit formulation of a bidder’s winning probability for a general n -bidder game is difficult, if not impossible, to obtain for $n \geq 4$. A bidder’s winning probability hinges on the absolute difference between her bid and the average bid, which is in turn determined by the number of bidders and *all* bids submitted. This is in contrast to the low price auction where a bidder’s bid merely serves as a threshold to which the lowest bid of the opponents’ is compared. The $n = 3$ case is a special one in the average bid structure because for

$n = 3$ the bid closest to the average is the same as the median bid and we can formulate an analytically tractable probability statement of a bid being the median bid. This formulation allows us to generate a more specialized and stronger version of Proposition 1 which helps to understand the incentives in the bidding process that we will take advantage of later in attempting to explain the behavior observed in the experiments.

Proposition 2 *In the average bid auction with $n = 3$, bidding $b(s) = k \geq C_H$ for all $s \in [C_L - \theta, C_H + \theta]$ is the **only** type of symmetric increasing Bayesian Nash equilibria of the auction game.*

The proof of Proposition 2 is also in the Appendix. The main idea of the proof is as follows: First, the proof of Proposition 1 indicates that in a symmetric increasing Bayesian Nash equilibrium of the average bid auction, all types in the neighborhoods around the extreme types ($C_L - \theta$ and $C_H + \theta$) have to bid an identical amount.⁵ Once extreme types bid a constant amount, things then unravel so that all types in $[C_L - \theta, C_H + \theta]$ will bid an identical amount in a symmetric equilibrium, i.e., there cannot be a symmetric Bayesian Nash equilibrium that is partially pooling and partially separating. The driving force is that if there were such an equilibrium where types to the right of some type s all bid the same amount (say, z) and all types to the left of type s bid less than z , then all types in a small neighborhood around s would have incentives to lower their bids by an infinitesimal amount. Such a deviation, while lowering the revenue upon winning by a negligible amount, increases the type's expected winning probability significantly, disrupting the equilibrium.⁶ We postpone a detailed discussion on the incentive issues until Section 4.3 where we will attempt to use them in providing an explanation for the bidding behavior we observe in the experiments.

It is important to point out that the breakdown of a strictly increasing Bayesian Nash equilibrium and the unraveling from the end points in Proposition 2 are a result of the mere existence of the end-point regions, **not** their size. Indeed, the proof of Proposition 2 is established without imposing any restriction on the values of extreme cost realizations C_L and C_H (except non-negativity). This implies that extending or modifying the range of the cost realizations does not 'eliminate' the end-point complications and the unraveling effect will always be at work.

While the specific proof of Proposition 2 only holds for the case of $n = 3$, Proposition 1 demonstrates that the key implication of the proposition holds more generally. Further, while we know that strictly increasing bid functions do not arise in equilibrium, we do know that there are multiple equilibria in which all bidders bid an identical amount k with $k \geq C_H$. Coordination on any of these equilibria will lead to prices above those expected in the low price mechanism leading to a clear comparative static prediction that the prices should be higher in the average bid auction than the low price auction.

⁵The proof of Proposition 1 shows that this is the case for a neighborhood around type- $(C_L - \theta)$. A similar argument can be applied to show that this is also the case for a neighborhood around type- $(C_H + \theta)$.

⁶Take type s as an example. By bidding slightly below z , type s increases her winning probability by a non-negligible quantity, as now in the event of winning she does not have to share winning probabilistically with all types (of the other bidders) to the right of s .

3 Experimental Design

The experiments designed to test the theoretical proposals above require two treatments: one using the Low Price Sealed Bid (LP) mechanism and the other using the Average Bid Sealed Bid (AB) mechanism. We chose to conduct them with a between-subject design. Consequently in a session, subjects participated in a series of auctions using either the AB mechanism or the LP mechanism. They did not participate in both and each subject participated in only one session of the experiments.

The parameters used in the experiment are the same as those in Lind and Plott (1991). The true cost, c , in an auction is drawn randomly from a uniform distribution over the range $[150, 1500]$. Sellers receive individual signals, s_i , that are independently drawn from a uniform distribution on the range $[c - \theta, c + \theta]$, where $\theta = 200$. This means that a seller does not know the true cost of winning in the round but he does know that the true cost is at most 200 above or 200 below his signal.

The rules of the auctions are as explained above. Both auction formats are sealed bid institutions. In the LP mechanism, the lowest ask submitted wins. In the AB mechanism, the ask closest to the average wins. In both cases the winner is paid the price they bid and ties are broken randomly. Otherwise the mechanisms are identical and are implemented as such in the laboratory. At the end of each auction round the subjects were told whether they were the winner and the amount of the winning bid. They were not informed about the non-winning bids.

We conducted four sessions of each treatment. Each session had 15 sellers resulting in a total of 122 subjects for the entire experiment.⁷ In each session, sellers participate in 30 auctions and each auction consists of five competitors with competitors randomized across rounds.⁸ Prior to the experiment beginning, subjects were taken through an extensive instruction script that explained to them the rules of the auction mechanisms as well as the complexities of the common value environment.⁹ The instructions included several examples aimed at helping them to understand the environment and how to interpret the signal of the value they received. The structure of the instruction script and the examples in terms of how the common value environment was explained were based on the scripts used in Kagel and Levin (1986). Subjects also bid twice against robot bidders at the end of the instruction phase to help them better understand the environment.

Because it is quite commonly observed that bidders lose money in common value auctions we had multiple bankruptcy rules in place to deal with the possibility of bidders going bankrupt. First, all subjects started with an initial balance of 1000 ECUs. As they made losses, these losses were deducted from this endowment. If their balance became negative,

⁷As will be discussed later, bankruptcy was a possibility in these experiments. There were three subjects who went bankrupt twice, two of whom were replaced by backup subjects. The third went bankrupt for the second time in the last period. The total subjects participating is therefore 122.

⁸Since some of the proofs of our theoretical results are only specified for the $n = 3$ case, one might have expected that the experiments would have been conducted with a similar number of bidders for consistency. We note that these results were proven after seeing the data in our attempt to rationalize the observed behavior. As such, the experiments are not properly seen as a test of this additional theory. The experiments were conceived of as a test of the theory predicting high collusive prices and are a proper test of that prediction. The proofs are limited to the $n = 3$ case due to the fact that it is tractable while the $n = 5$ case is not.

⁹Full instruction scripts are available in the online appendix.

they were declared bankrupt and notified by the software. After this first bankruptcy, the subject was given a new endowment of 500 ECUs. If they went bankrupt a second time, they were asked to leave the experiment with only their show up fee. We recruited an extra subject for every session who went through the instructions at the same time as the other subjects and who replaced any subject who went bankrupt for a second time. In the end, only two subjects had to be replaced and they were both in LP sessions. The conversion rate is \$0.01 per ECU. Overall, subjects earned an average of \$16.70 in LP sessions and \$24.40 in AB sessions including an additional \$10 participation fee. The participants were recruited using ORSEE, Greiner (2004). The software for the experiment was programmed using z-Tree, Fischbacher (2007).

4 Results

Table 1 presents a set of summary statistics concerning the key variables of interest. We have provided the averages by session and by treatment. The first thing to note about the summary statistics is the fact that the average cost and signal numbers are the same across all sessions. This was by design as we used the same cost and signal draws for all sessions. This helps to insure that any differences in prices and behavior are not due to different costs or signal draws between sessions. We have also included a column on benchmark price predictions. In the LP this is the average price that would have been observed had subjects received the signals in the experiment and bid according to the equilibrium strategy defined above. The benchmark for the AB mechanism is the equilibrium that would have had all subjects bidding at the top of the cost distribution.

The summary statistics based on the behavior are the average of the observed bids and of the observed prices. The overall average bid in the LP is 903.5 and 895.2 for the AB. These are obviously quite close. The difference in prices is more substantial as we find average prices to be 746.9 in the LP and 894.7 in the AB. One can conduct simple Wilcoxon tests on the session level averages just to get an idea regarding the differences in the distributions on these measures. As there are only 4 observations in each group the critical value of the test statistic at a significance level of $p = 0.05$ is 10 or $2\Pr(W_{AB} \geq 10) = 0.05$, which requires that all values from one group must be below that of the other to reject the null hypothesis of equal means. This occurs in the case of the auction prices as all LP session averages are below all session averages for the AB. This does not occur for the bid distributions. We will conduct better specified tests regarding these differences in the next section, but it is worth noting that the prices are different between mechanisms even according to this test.

It is also worth noting that the average price in the LP auction is much lower than the theoretically predicted price, 889.43, and lower even than the average cost, 819.7. The average price observed, 895.2, is also definitely lower than the lowest price prediction among the collusive equilibria, 1500. Comparing it to some other benchmarks, we note that it isn't far above the average cost, 819.7, and is approximately the same as the predicted price for the LP, 889.43. While the prices are higher in the AB than the LP, they are not above reasonable benchmarks. It would of course be up to the preferences of a specific buyer whether price increases of this magnitude are worth paying to avoid the potential problems caused by the seller losses in the LP.

Table 1: Summary Statistics

| Treatment | Session | Avg. Cost | Avg. Sig | Pred Price | Avg. Bid | Avg. Price |
|-----------|---------|-----------|----------|------------|----------|------------|
| LP | 3 | 819.7 | 817.3 | 889.43 | 907.9 | 766.9 |
| | 4 | 819.7 | 817.3 | 889.43 | 909.2 | 754.3 |
| | 5 | 819.7 | 817.3 | 889.43 | 926.8 | 761.8 |
| | 6 | 819.7 | 817.3 | 889.43 | 869.9 | 704.8 |
| | All | 819.7 | 817.3 | 889.43 | 903.5 | 746.9 |
| AB | 1 | 819.7 | 817.3 | 1500 | 891.8 | 888.0 |
| | 2 | 819.7 | 817.3 | 1500 | 856.5 | 856.7 |
| | 7 | 819.7 | 817.3 | 1500 | 910.2 | 906.8 |
| | 8 | 819.7 | 817.3 | 1500 | 922.4 | 925.2 |
| | All | 819.7 | 817.3 | 1500 | 895.2 | 894.7 |

4.1 Bidding behavior

Figure 1 provides scatter plots of bids versus signals for both auction formats with non-linear regression lines fitted through both sets of data. There are also reference lines included in the form of the 45 degree line of the signal and then the equilibrium bid function in the LP auction. The visual similarity of the bidding behavior in both mechanisms is striking. The two regression lines through the clouds of observations overlap throughout the entire range of signals. Bids are clustered above the signal and below the RN prediction for the LP mechanism. This indicates the bids are more aggressive than predicted in the LP mechanism which is the common finding for first price auctions. In the AB mechanism we observe no indication that bids are clustering on any particular prices which would have been the case were the subjects are attempting to coordinate on some collusive equilibrium.

Table 2 presents the results of several different specifications of regressions concerning bidding behavior in an attempt to determine the overall structure of the bidding behavior. All of the regressions are conducted using a random effects specification with standard errors clustered at the individual subject level.¹⁰ The variables include the cost signal of the bidders, a dummy for the average bid auction, AB, a dummy for whether or not the round is in the first or last half of the experiment, a dummy variable for if the signal is in $[C_H - \theta, C_H + \theta]$ and then interactions of these variables. The basic specification examines overall differences in behavior between the mechanisms while the additional specifications look at whether we can find any differences in behavior over time and whether that time trend is different between the mechanisms as well as whether there are differences in the three regions of the equilibrium bid function for the LP auction. These regressions provide support for our first result.

Result 1 *There is no statistically significant difference in bidding behavior between the LP and AB mechanisms. While there is a shift in bidding behavior over time, the same shift is observed in both mechanisms.*

¹⁰As noted previously, due to bankruptcies not all initial subjects participated in all rounds. Replacement subjects are identified with their own panel ids. Data on the number of bankruptcies is provided in Table 4.

Table 2: Random Effects Regressions on Bidding Behavior.

| | Bid | | | | |
|---|----------------------|----------------------|----------------------|----------------------|----------------------|
| | (1) | (2) | (3) | (4) | (5) |
| Signal | 0.912*** (0.009) | 0.909*** (0.010) | 0.902*** (0.009) | .951*** (.013) | 0.937*** (0.012) |
| AB | -10.32 (20.947) | -10.30 (20.958) | -4.917 (19.228) | -4.124 (22.059) | -6.992 (15.030) |
| Second 15 | | 32.21** (12.959) | 36.99 (23.476) | | 37.41** (18.820) |
| Region 3 | | | | 609.0*** (109.9) | 455.6*** (113.5) |
| Signal \times AB | 0.006 (0.016) | 0.006 (0.016) | 0.020 (0.016) | -0.006 (0.021) | 0.023 (0.016) |
| Signal \times Second 15 | | 0.010 (0.014) | 0.026* (0.015) | | 0.030** (0.015) |
| Signal \times Region 3 | | | | -0.461*** (0.078) | -0.340*** (0.080) |
| Second 15 \times AB | | | -9.217 (26.389) | | 1.631 (22.297) |
| Second 15 \times Region 3 | | | | | 162.8 (167.9) |
| AB \times Region 3 | | | | 113.8 (130.6) | 248.4 (165.9) |
| Signal \times Second 15 \times AB | | | -0.0314 (0.024) | | -0.051* (0.028) |
| Signal \times Second 15 \times Region 3 | | | | | -0.144 (0.119) |
| Signal \times AB \times Region 3 | | | | -0.063 (0.094) | -0.168 (0.114) |
| Second 15 \times AB \times Region 3 | | | | | -63.90 (236.9) |
| Signal \times Second 15 \times AB \times Region 3 | | | | | 0.065 (0.164) |
| Constant | 155.7*** (15.608) | 137.6*** (12.732) | 134.8*** (12.365) | 132.7*** (15.516) | 113.3*** (11.035) |
| Obs (Clusters) | 3600 (122) | 3600 (122) | 3600 (122) | 3600 (122) | 3600 (122) |

Robust clustered standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

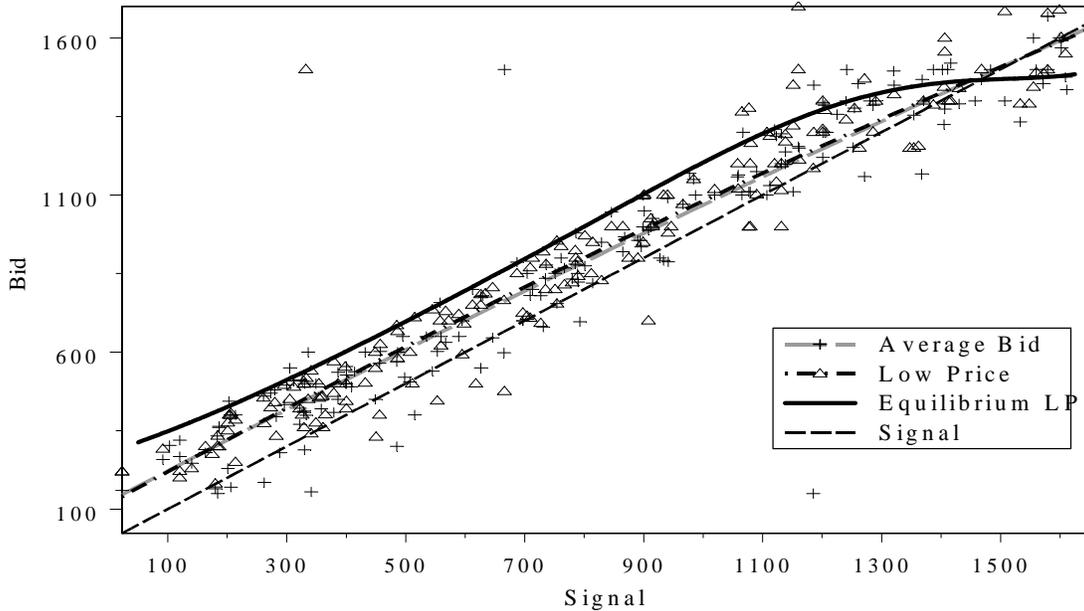


Figure 1: Scatterplot of bids versus signals for both mechanisms. Only 1 out of every 10 observations are plotted.

While the actual bid function for the LP is nonlinear as represented above, it is approximately $s_i + 200$. What we see in the first regression is that bidding behavior on average is $.9s_i + 155$. Both the slope and intercept are different from the theoretical prediction and if one performs more detailed tests on the bidding behavior in the LP auction it is quite clear that bidding is different from the prediction.¹¹ This is of course not surprising and is consistent with prior evidence on bidding in common value auctions. The surprising result is the lack of significant differences in behavior between the mechanisms. We find this lack of difference to be robust across numerous specifications. None of the base effects are significant in any of the regressions and only one of the interactions involving the dummy variable for the AB mechanism is significant. This includes finding no significant effects in specifications 2 and 3 in which we examine whether bidding behavior shifts between the first and second half of the experiment. While we do find that bids increase in the second half of the experiment, the same shift is observed in both mechanisms.

The bidding behavior in the AB mechanism is surprising. The existing equilibrium analysis suggests that the mechanism provides strong incentives for coordination among bidders but due to the fact that bids are strongly influenced by the cost signals there is little indication that bidders engaged in any attempts at coordination. Of course with opponents changing from round to round and no opportunities for conversations about the auctions, coordination would have been difficult in these auctions. We will return later to an attempt to rationalize the bidding behavior in the AB mechanism.

¹¹As a simple version of the test, standard t -tests regarding whether the coefficient on the signal (0.912) is equal to 1 from the first specification generates a p -value < 0.001 indicating that the difference is strongly significantly different. The test for whether the intercept (155.7) is equal to 200 results in a p -value of 0.005 yielding a similar interpretation.

Table 3: Summary of Seller Earnings.

| | Session | Seller Surplus | | | Size Neg Earn | | |
|----|---------|----------------|----------|----------|---------------|----------|----------|
| | | All | 1st Half | 2nd Half | All | 1st Half | 2nd Half |
| LP | 3 | -52.8 | -78.7 | -27.0 | -98.7 | -110.3 | -83.2 |
| | 4 | -65.4 | -104.6 | -26.2 | -98.1 | -115.6 | -70.9 |
| | 5 | -57.9 | -101.5 | -14.3 | -95.8 | -119.5 | -60.74 |
| | 6 | -114.9 | -117.2 | -112.6 | -133.0 | -133.8 | -132.3 |
| | All | -72.8 | -100.5 | -45.0 | -107.8 | -120.1 | -91.7 |
| AB | 1 | 68.3 | 52.6 | 84.0 | -27.9 | -32.8 | -20.9 |
| | 2 | 39.0 | 32.5 | 45.5 | -42.2 | -22.5 | -67.57 |
| | 7 | 87.1 | 68.9 | 105.2 | -41.0 | -37.0 | -46.6 |
| | 8 | 105.5 | 88.3 | 72.8 | -24.7 | -24.6 | -25.5 |
| | All | 75.0 | 62.4 | 87.5 | -36.0 | -27.6 | -49.1 |

Table 4: Summary of Seller Loss Propensity.

| | Session | Loss Propensity | | | # Bankrupt (Twice) |
|----|---------|-----------------|-----------|-----------|-----------------------|
| | | All | 1st Half | 2nd Half | |
| LP | 3 | 63 | 36 | 27 | 1 (0) |
| | 4 | 69 | 42 | 27 | 2 (0) |
| | 5 | 67 | 40 | 27 | 1 (1) |
| | 6 | 81 | 41 | 40 | 4 (2) |
| | All | 280 (.78) | 159 (.88) | 121 (.67) | 8 (3) |
| AB | 1 | 17 | 10 | 7 | 0 (0) |
| | 2 | 32 | 18 | 14 | 0 (0) |
| | 7 | 12 | 7 | 5 | 0 (0) |
| | 8 | 11 | 9 | 2 | 0 (0) |
| | All | 72 (.20) | 44 (.24) | 28 (.16) | 0 (0) |

4.2 Buyer and Seller Earnings

The summary statistics in Table 1 show that the average price in the AB auction, 894.7, was substantially greater than the average price in the LP, 746.9, and this obviously will have consequences on earnings for both sides of the transaction. Table 3 displays the consequences of the price differences on the auction participants. On average, in the LP mechanism, winning sellers lose 72.8 per round while they earn a positive 75.0 per round in the AB. Sellers do occasionally make losses in the AB but what we also see in that table is that conditional on making losses the size of the loss is much greater in the LP (107.8) than in the AB (36). Table 4 contains statistics related to the frequency with which bidders make losses. In the first half of the LP sessions, the winning sellers are losing money in around 40 of the 45 auctions while that only happens in about 10 of the auctions in the AB. In the AB there were no subjects who lost their initial endowment while 8 subjects lost their entire endowment once and 3 of those lost it a second time leading to them being removed from the session.

Table 5: Random Effects Regressions on Auction Price.

| | Auction Price | | |
|-----------------------------------|----------------------|-----------------------|----------------------|
| | (1) | (2) | (3) |
| AB | 160.0*** (19.338) | 177.3*** (19.775) | 166.2*** (33.847) |
| Common Cost | 0.930*** (0.013) | 0.934*** (0.014) | 0.937*** (0.021) |
| CC \times AB | -0.015 (0.016) | -0.017 (0.016) | -0.004 (0.027) |
| Second 15 | | 52.71*** (17.681) | 56.59*** (20.334) |
| CC \times Second 15 | | | -0.005 (0.019) |
| AB \times Second 15 | | -31.18* (18.396) | -11.59 (30.616) |
| CC \times AB \times Second 15 | | | -0.024 (0.033) |
| Constant | -15.68 (15.314) | -44.97*** (16.200) | -47.19** (23.890) |
| Obs (Clusters) | 720 (8) | 720 (8) | 720 (8) |

Robust clustered standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Our next result summarizes the findings on prices and seller surplus.

Result 2 *Auction price is on average higher in the AB than the LP. This means buyer (seller) surplus is lower (higher) in the AB than the LP.*

The statistical support for this result on prices is found in Table 5 which contains a random effects regressions with standard errors clustered at the session level that attempt to disentangle the determinants of auction price. The variables included in the regressions include the dummy variable indicating if the auction is an average bid auction or not, the realized common cost, a dummy variable for whether the round is in the second half of the sessions and interactions between these variables. All three specifications clearly show that the prices in the AB are higher than in the LP. We also see that prices rise over time and they rise approximately the same between both mechanisms. Since costs are common across bidders and across sessions, it should be clear that since prices are significantly higher in the AB, the surplus of the sellers is also significantly higher in the AB than the LP.

Result 3 *Sellers are more likely to lose money in the LP than the AB.*

Table 6 provides the statistical support for this result. It presents a series of random effects probit regressions in columns (1)-(3) where the dependent variable is whether or not

Table 6: Regression of Likelihood of Losses (Probit).

| | Binary for Lost Money | | | | | |
|---------------------------------------|-----------------------|----------------------|----------------------|-----------------------|----------------------|----------------------|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Signal | 0.0006*** (<0.001) | 0.0007** (<0.001) | 0.0008** (<0.001) | 0.0002*** (<0.001) | 0.0002** (<0.001) | 0.0003** (<0.001) |
| AB | -1.770*** (0.266) | -1.795*** (0.272) | -2.058*** (0.392) | -0.595*** (0.066) | -0.603*** (0.067) | -0.679*** (0.088) |
| Signal \times AB | -0.00004 (<0.001) | -0.00007 (<0.001) | -0.00008 (<0.001) | -0.00002 (<0.001) | -0.00003 (<0.001) | -0.00001 (<0.001) |
| Second 15 | | -0.499** (0.249) | -0.646** (0.325) | | -0.180** (0.091) | -0.246** (0.118) |
| Signal \times Second 15 | | -0.00006 (<0.001) | -0.0002 (<0.001) | | -0.00003 (<0.001) | -0.00007 (<0.001) |
| AB \times Second 15 | | | 0.525 (0.513) | | | 0.222 (0.180) |
| Signal \times AB \times Second 15 | | | -0.00002 (<0.001) | | | -0.00004 (<0.001) |
| Constant | 0.348** (0.162) | 0.628*** (0.225) | 0.707*** (0.266) | | | |
| Obs (Clusters) | 720 (119) | 720 (119) | 720 (119) | 720 | 720 | 720 |

Clustered standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

(1)-(3) are panel probit regressions. (4)-(6) are marginal effects from standard probit.

the auction winner lost money in a round as well as a set of marginal effects in columns (4)-(6) calculated from standard probit regressions using the same specifications. The variables used here conform to the same names as those used in prior regressions. Across all six specifications the most significant impact on the probability of losing money is clearly the dummy variable for the AB mechanism indicating that the probability of losing money is much lower in the AB mechanism than in the LP mechanism.

4.3 Explanation of Bidding Behavior

4.3.1 Cursed Equilibrium and Level- k Considerations

Given the surprising finding regarding the bidding behavior in the AB mechanism, we need to determine if there is some way of rationalizing the observed behavior to determine how robust the result might be. As a first step, we note that there are many prior papers that have attempted to provide explanations for the bidding behavior in versions of the LP mechanism. The focus of these explanations is on how and why individuals fail to properly account for the information they should expect to learn conditional upon winning the auction, i.e. that they underestimated the cost by more than anyone else. Eyster and Rabin (2005) propose the notion of Cursed equilibrium. Individuals who are fully-cursed according

to this model believe the actions of others have no correlation with their information and so they make no inference about the signals of others based on their bids. Crawford and Iriberry (2007) propose a model of Level- k behavior. This model presumes that a player of any level- k forms believes that his opponents behave according to the model's predictions for a $(k - 1)$ -individual and best responds to that belief. The reason this model works in a common value setting is that level-1 players are best responding to level-0 players who are typically assumed to bid randomly. A level-1 player can therefore draw no inference about the true cost of the object based upon winning. Both models seem capable of explaining the behavior in first price common value auctions. As our data from this format is consistent with prior experimental results, these models should be expected to provide as good an explanation of the bidding behavior in our data as elsewhere.

Due to the success of Cursed Equilibrium and Level- k in explaining behavior in the LP mechanism, it is sensible to think that they might be similarly useful in explaining the behavior in the AB. Unfortunately this is not the case. Proposition 3 demonstrates that allowing for the possibility of a Cursed equilibrium changes nothing about the analysis behind Propositions 1 and 2 regardless of the degree to which individuals are cursed (i.e., for all $\chi \in [0, 1]$) and consequently it cannot explain the observed behavior.

Proposition 3 *Consider the average bid auction with n bidders where $n \geq 3$. Consider any $\chi \in [0, 1]$. There does not exist a χ -Cursed equilibrium in strictly increasing strategies. In addition, when $n = 3$, bidding $b(s) = k \geq C_H$ for all $s \in [C_L - \theta, C_H + \theta]$ is the **only** type of increasing χ -Cursed equilibria of the auction game.*

The rationale for Proposition 3 is qualitatively identical to that in the previous propositions. Any non-coordinated bidding behavior in neighborhoods around the extreme types unravels, leading to the only possible equilibrium behavior being coordination among all types. A key step here is to show that with fully-cursed bidders ($\chi = 1$) such an unraveling effect results in coordinated bidding behavior for all types. This extreme case, together with the previous result (Proposition 2) for fully rational bidders ($\chi = 0$), establishes Proposition 3. The proof is omitted and can be found in a supplementary Online Appendix.

An initial look at Level- k behavior can demonstrate some fleeting promise as we can show that Level- k does predict behavior different from the coordination equilibria.¹² We can present a closed form analysis for the $n = 3$ case by assuming that level-0 bidders submit a bid that is uniformly distributed on the range $[C_L, M]$ where $M > C_L$ may or may not be the same as C_H . We can explicitly solve for the behavior of a level-1 bidder, $b_1^*(s)$, assuming his two opponents are level-0 type, to be

$$b_1^*(s) = \begin{cases} \frac{2M+3C_L+s+\theta+\sqrt{4M^2-2Ms-2M\theta-6MC_L+(s+\theta)^2+3C_L^2}}{6}, & s \in [C_L - \theta, C_L + \theta], \\ \frac{M+s+C_L+\sqrt{M^2-Ms-MC_L+s^2-sC_L+C_L^2}}{3}, & s \in [C_L + \theta, C_H - \theta], \\ \frac{2M+s-\theta+C_H+2C_L+\sqrt{\Gamma}}{6}, & s \in [C_H - \theta, C_H + \theta], \end{cases} \quad (4)$$

$$\text{where } \Gamma = \frac{(M-s)^2+(M+\theta)^2+(M-C_H)^2+(M-2C_L)^2+2s(C_H-\theta-C_L)}{-2\theta(C_H-C_L)-2C_L C_H}.$$

¹²Omitted calculations and a simulation result on bid functions for Level- k reasoning can be found in Online Appendix.

It can be verified that function $b_1^*(s)$ is continuous and can be upward sloping depending on the values of the parameters. This is the key property necessary to match the observed behavior. Using the parameters from the experiments, and considering the most salient choice of $M = C_H$, the bid function $b_1^*(s)$ for $s \in [C_L - \theta, C_L + \theta]$ becomes

$$b_1^*(s) = \frac{s + 3650 + \sqrt{s^2 - 2600s + 7157500}}{6},$$

which is indeed strictly increasing (and this is true for all $s \in [C_L - \theta, C_H + \theta]$). The problem however is that the bid function predicts bids much higher than observed. A simple benchmark can be seen from the bid that would be placed by someone receiving a signal at the lowest possible cost or $s = C_L$. The prediction is $b_1^*(C_L|M = C_H) = 1068$ while Figure 1 makes it clear that the observed bids for signals in this range are clustered between 300 and 350. Lower values specified for the upper bound M will decrease the predicted bids but it seems difficult to argue why lower values are sensible or are more salient than C_H and even quite low values of M still result in predicted bids higher than observed.¹³ Of course these analytical predictions are for the $n = 3$ case rather than $n = 5$ as in the experiments. While we cannot derive an analytical solution for the $n = 5$ case we can computationally verify that the same property of high predicted bids holds. In particular, the predicted bid assuming $s = C_L$ with $n = 5$ is 975.¹⁴ It is also worth noting that since bids are clearly a function of the signals, we observe no significant behavior that would be consistent with level-0 behavior either.

While we can already see that level-0 and 1 behavior is inconsistent with the data, it remains to investigate the possibility of whether level-2 behavior might conform better to the observed behavior. For level-2 behavior though, the analysis is much more complex and we are not able to provide a full analytic characterization even for $n = 3$. Nevertheless, a numerical analysis (in Online Appendix) shows that a level-2 bidder bids slightly higher than that of a level-1 bidder for low signals, about the same for interior signals and slightly lower than a level-1 bidder at high signals.¹⁵ It does not seem that an appeal to higher level reasoning will allow us to explain the AB behavior using a Level- k structure.

4.3.2 Inference Errors Regarding Extreme Signals

Since these two standard explanations fail here, we must look for other ways to explain the observed behavior in the AB auction. We take a straightforward approach based on assuming that individuals form a belief about the behavior of others that is empirically plausible and best respond to that belief. We will assume that bidders believe that the behavior of others is expected to be equal to the average of the overall bidding behavior observed in the experiments. By combining the relevant coefficients from specification (1) in Table 2, we see that the average behavior of bidders in the AB can be approximated by

¹³We also note that one might propose an alternative version of level-0 bidders by assuming they will not bid below the lowest cost consistent with their signal. While this might be a reasonable specification, it will lead to the prediction of even higher bids.

¹⁴We calculate this bid using the median bid winner determination rule as an approximation for the average bid.

¹⁵The intuition of this behavior is similar to the discussion after Claim 1.

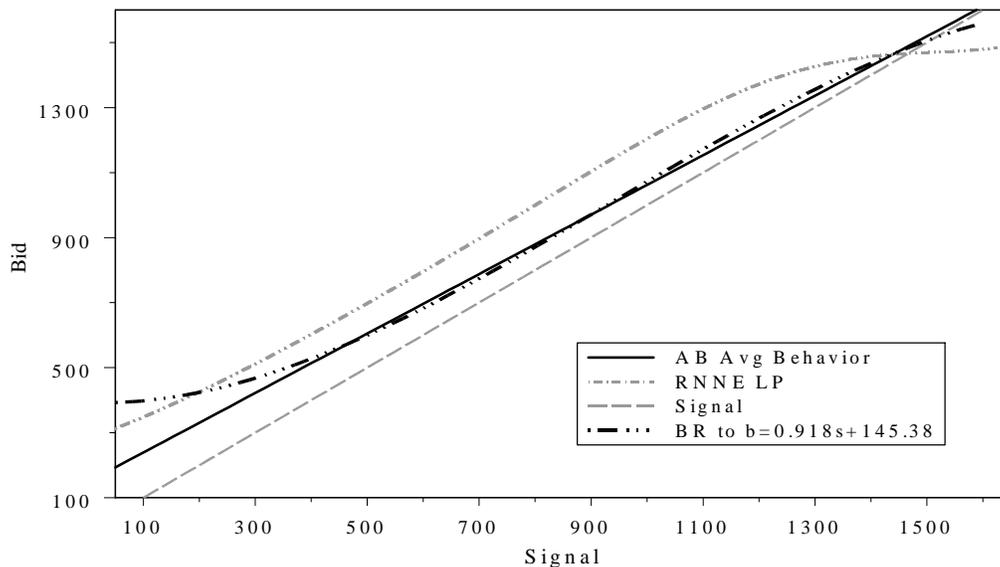


Figure 2: Plot of regression line through observed bids in the AB along with a plot of the approximate best response to that average behavior.

$b_j(s_j) = 0.918s_j + 145.38$.¹⁶ Based upon the assumption that the other 4 bidders are bidding according to $b_j(s_j)$, we can calculate the best response that bidder i would choose for any signal, s_i , he might draw.¹⁷ The result of such a calculation is most easily summarized graphically and can be found in Figure 2 which plots this best response along with the empirical regression line through the bidding data from the AB auction previously shown in Figure 1.

The computationally produced best response lies surprisingly close to the average bid function employed by the subjects. For the interior region, the empirical best response and the observed behavior are practically identical. To begin to explain why the best response bears this shape we first have to understand what inference a winning bidder should make in an AB auction about the true cost of the project conditional upon winning. Under the assumption that all other bidders are using some common bid function that is linear in their signals, if bidder i learns that he wins with bid b_i , then he learns that this bid, properly adjusted based on the bidding strategy of others, is an unbiased estimate of the average of the values of other signals. It is therefore an unbiased estimate of the actual cost. This is a very different piece of information learned than when winning an LP auction. It is worth noting that were bidder i using a bid function close to that of all other bidders, then what bidder i learns is that his own signal, s_i , is an unbiased estimate of the cost of the project. In this local space, a bidder can therefore rationally ignore completely the information learned upon winning, i.e., that his signal is a good estimate of the true cost conditional

¹⁶The constant is constructed as the sum of the overall constant for the regression, 155.7, and adding the AB dummy variable, -10.32 . The coefficient on the signal is the base coefficient, 0.912, plus the interacted coefficient on $\text{Signal} \times \text{AB}$, 0.006.

¹⁷We adopt a brute-force search algorithm to find the numerical best response of a bidder given that the other (four) bidders are bidding according to $b_j(s_j) = 0.918s_j + 145.38$. A detailed illustration of the simulation can be found in the Online Appendix.

upon winning, in favor of a purely naive assumption, i.e., that his signal is unconditionally a good estimate of the true cost, without making a substantial mistake. Of course as bidder i considers bidding behavior farther from that of others then this mistake begins to have consequences. This difference in the information learned upon winning between winning in a LP and AB auction indicates why a direct application of something like Cursed Equilibrium will not help explain the deviations from the equilibrium predictions.

We can also provide an analytic result to better explain the computational one already shown. As before, if we restrict the number of bidders to 3 instead of 5 then the average bid auction is isomorphic to the median bid auction and calculating the probability of a bid being the median is analytically tractable allowing the proof of the following claim:

Claim 1 *Consider the average bid auction with $n = 3$. If the bidders hold the commonly known (but false) conjecture that every other bidder follows a symmetric and strictly increasing bidding strategy, then bidding $\hat{b}(s) = s + \Delta$ with constant $\Delta \geq \theta$ is a best response for every bidder with $s \in [C_L + \theta, C_H - \theta]$.*

The proof of this claim is again relegated to the Appendix. While we cannot extend this proof to the $n = 5$ case as in the experiments, the computational exercise in Figure 2 demonstrates that the principle we prove here for $n = 3$ does hold for $n = 5$ as well. We describe bidders' incentive issues as follows.

In the middle region, if every bidder bids according to $\hat{b}(s)$, then a bidder, upon winning, knows that one of the other two signals is above her signal while the other lies below her signal. So her own signal is indeed a good estimate of the true cost and there is hence no ex post regret. If a bidder deviates to the bid of a lower type, two issues arise: First, her (expected) winning probability drops because of the symmetry of the bidders' signal distributions;¹⁸ Second, if the (underbidding) bidder wins the auction, this additional piece of information reveals that her original signal s is not an accurate estimate of the true cost, and the true cost is likely to be strictly less than s . These issues jointly imply that downward deviations generate strictly smaller expected payoffs. On the other hand, if a bidder deviates to the bid of a higher type, then again her expected winning probability drops, but winning after overbidding brings somewhat good news as the bidder knows her original signal s is likely to be smaller than the true cost. In order to guarantee that a bidder does not want to deviate upward, the profit from following the bid function needs to be high enough to make such a deviation not worthwhile, hence the lower bound requirement (θ) on Δ . For the proof we have specified a lower bound of θ though we note this is not actually the minimum lower bound requirement.

The problem with such a bid function being a valid equilibrium is due to the incentives in the two end regions. These incentive issues depend on logic similar to that behind the proof of Proposition 2. The key issue is that individuals in the endpoint regions possess more information about the possible cost than those in the interior. To be specific, with a signal $s \in [C_H - \theta, C_H + \theta]$ they know that the cost is most likely lower than their signal; with a signal $s \in [C_L - \theta, C_L + \theta]$, they know that the true cost is most likely above their

¹⁸If a bidder bids exactly $s + \Delta$, her expected winning probability is $\frac{1}{3}$, while a downward deviation with magnitude λ (small and positive) resulting in a winning probability $\int_{s-\theta}^{s+\theta-\lambda} \frac{2(s-\lambda-c+\theta)(c+\theta-s+\lambda)}{(2\theta)^3} dc = \frac{(\theta+\lambda)(\lambda-2\theta)^2}{12\theta^3} < \frac{1}{3}$.

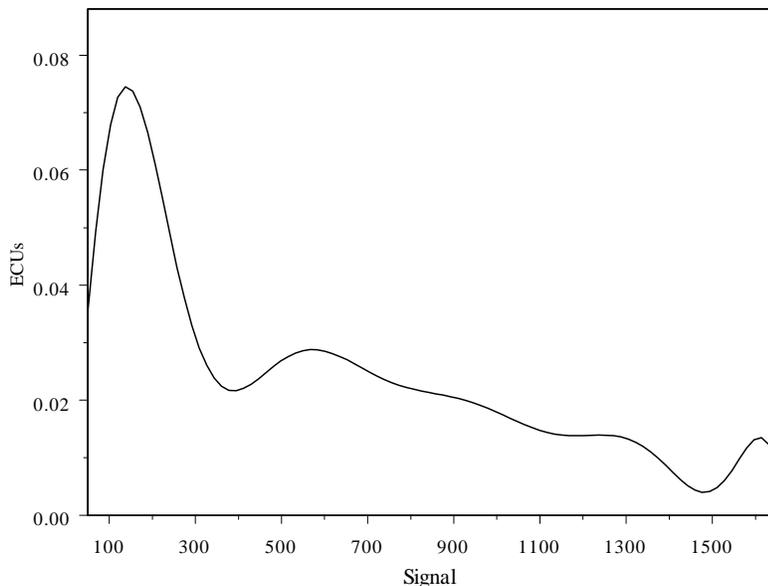


Figure 3: Difference in expected surplus between best response and linearized average behavior weighted by probability of signal.

signal; while with a signal $s \in [C_L + \theta, C_H - \theta]$, they think the true cost is above and below their signal with equal probability. These differences in perception of the true cost relative to signal cause differences in how individuals in the different regions evaluate the profitability of deviations from a linear (or even increasing) bid function. For an individual with a signal in $[C_H - \theta, C_H + \theta]$, a downward deviation increases his probability of winning while an upward deviation decreases it. This is different from someone with a signal in the middle region $[C_L + \theta, C_H - \theta]$ where a deviation in either direction is expected to decrease probability of winning. For any premium large enough to keep bidders in the middle region from deviating upward, it will cause bidders in region $[C_H - \theta, C_H + \theta]$ to want to deviate downward. A similar argument exists in region $[C_L + \theta, C_H - \theta]$, leading them to deviate upward.

Figure 3 provides a useful examination of the difference between the optimal and the average behavior as it shows the difference in expected surplus between bidding according to the best response and bidding according to the linearized strategy, *weighted by the probability of receiving each signal*. One can just integrate the area under this curve to calculate the total difference in expected surplus from following these two strategies; approximately 7.5 ECUs. The figure essentially shows the contribution to the total from all possible signals. Over the interior of the signal distribution an individual loses virtually nothing in expected surplus from following the linearized bidding strategy. The main losses in expected surplus come near the endpoints which is due to the fact that this is where an individual should not be following a linear strategy. As shown above, average earnings by sellers in the AB were

around 75 ECUs. This indicates that by optimally best responding rather than following a linearized strategy, a subject could have increased his earnings by approximately 10%.

Of course the next question is whether subjects do follow strictly linearized strategies. The answer is mixed. By examining the individual plots of bid functions provided in the Online Appendix or even the overall plot shown in Figure 1, we can see that some of the subjects were actually responsive to the fact that they should deviate from the linear strategy near the endpoints. In many of the individual bid function plots we see the predicted flattening of the bidding behavior in the endpoint regions and the aggregate regression plot also possesses flatter portions near the endpoints than in the interior region. This is most noticeable in the upper endpoint region. So for some bidders, the losses from following their behavior will be even lower than what is represented in Figure 3 as they exhibit behavior quite in line with that predicted by assuming they are best responding to the belief that others are following a linearized bid function.

This analysis suggests a straightforward way to rationalize the bidding behavior of the subjects. They appear to be approximately best responding to the behavior of others though there are perhaps two types of “mistakes” regarding behavior near the endpoints. Some subjects fail to understand that when they receive a signal near the endpoint that they should deviate from the linear strategy they use in the interior region. This results in a small loss of expected surplus. Others seem to understand that they should deviate from the linear strategy if they receive a signal in these end regions but perhaps they fail to anticipate that their opponents will do the same thing when their opponents draw a signal in the endpoint regions. Consequently, while behavior does not conform to standard Bayes-Nash predictions, it requires only minimal divergence from the standard model to explain the observed behavior as we need only allow for inference mistakes in the endpoint regions and can otherwise assert a standard best response framework.

5 Conclusion and Discussion

According to prior work, the only way to explain the choice of using an average bid mechanism is that these auctioneers are simply making a mistake. The fact that it is still used actively anyway suggests a puzzle that is worth addressing. Our results indicate that the mechanism can be successfully employed in environments with substantial Winner’s Curse problems that it helps to alleviate thus providing a potential justification for why it is used in practice. In our common value environment, the average bid results in losses to the sellers much less often than the low price auction and when sellers do make losses those losses are much smaller. It does this while leading to prices that should not be considered “too high”, though that is certainly a judgement call for a potential buyer.

As noted in the introduction, the average bid auction has been adopted by many organizations based on an argument that seems initially suspect, i.e. that behavior won’t change much between the low price and average bid mechanisms and so the seller closest to the middle should not be expected to have submitted a bid below the true cost of the project. In the common value environment of our experiments, we find empirically that the bidding behavior in the average bid mechanism does not conform to prior theoretical predictions and instead matches the behavior we observe in the low price format. This result indicates that the naive argument in support of the average bid auction may be roughly accurate

and we can provide an explanation for why this is the case. While the behavior in the low price format is explainable based on prior work on common value auctions, those explanations fail in explaining the behavior in the average bid mechanism. We show that instead the behavior can be explained based on a minimal departure from the standard theory by allowing for bidders to have inference problems for signals received near the endpoint of the distribution. This inference problem is not a new phenomenon and has been observed in prior common value auction studies as described in Kagel and Levin (2002).

Outside of the laboratory environment, we argue that we would actually expect the nature of the behavior to be more similar to what we observed here than the default theoretical prediction. First, the equilibrium we find in Proposition 2 is predicated on the existence of a commonly known and agreed upon upper bound on the cost distribution. While this upper bound is induced in the lab setting, such a (common) upper bound is unlikely to exist in naturally occurring cost distributions. The lack of a commonly agreed upper bound, while possibly introducing new theoretical issues (beyond the scope of the current paper), is likely to decrease the ability to coordinate and to eliminate the inference problems that arise due to the bounds in our analysis. If there are no regions of the signal range where bidders have more information than others, then the logic of Claim 1 should extend through the entire range. Of course a buyer could restore the collusive equilibrium by providing a commonly known maximum possible bid. This will allow for all sellers with costs below to coordinate on this in equilibrium and any sellers with higher costs will choose not to participate. Assuming a buyer does not provide such a handy collusion device to his sellers (though they often do¹⁹), then the equilibria motivating the prior concern about the average bid mechanism will not exist in natural value environments. Further, an auctioneer who finds that all sellers have submitted identical bids will have quite clear evidence that the sellers are colluding. If that auctioneer is a government agency this makes prosecuting collusion simple. Other auctioneers would likely just invalidate the auction. Rational sellers should anticipate these responses and be unlikely to collude in this way. The baseline theoretical prediction therefore seems quite unlikely to emerge in auctions outside of the laboratory.

Given that our conclusions about the average bid mechanism differ from the conclusions drawn in previous work it is important to understand why the different conclusions may have emerged and what this suggests about the use of the average bid mechanism. Whether the data is from the lab or the field, to understand how the results of an empirical study may generalize to different situations one must first have a good understanding of what aspects of the environment were driving the results. It is then crucially important to understand the environmental differences between the environment in which the data were created and the environment to which one wants to apply the results. In our study the environment is one with substantial cost uncertainty due to the common value environment and with fixed populations of bidders who are unable to communicate prior to bidding. The data studied in Decarolis (2013) is from a value environment which is technically unknown though it is argued to be close to a private value environment. The bidder populations were not fixed and the bidders were likely familiar enough with each other to allow for pre-auction

¹⁹Such a collusion device was available in the Italian auctions in Decarolis (2013) where the bidding firms were allowed to submit bids as discounts over a publicly known reserve price. Bidders could collude on the maximum price by bidding a discount of 0.

coordination. Further, the low price mechanism used by the Italian procurers included a costly bidder screening component which likely improved its performance over a standard low price only mechanism.²⁰

As there are multiple differences between the environments it is impossible to know for certain exactly which elements are driving the differences. From the two sets of results there are, however, some indications for how the environmental differences were important to delivering the differences in the results. In the Decarolis data from Italy, he shows as we do that prices rise in the average bid mechanism relative to a low price mechanism. He interprets the price increase as necessarily a problem with the average bid mechanism and in a private value environment, the price rise is unambiguously a problem. In a common value environment in which bidders suffer from the Winner's Curse, the price rise could well be an intended result from adopting the mechanism. It is this rise in prices that helps to alleviate problems from the Winner's Curse by decreasing seller losses. The important empirical question is not whether prices rise, but rather whether the prices rise to a level above what the buyer is willing to pay to lessen the downstream problems that arise from seller losses. In our data, we show that by reasonable metrics the prices are not rising too high though that could change under different parameterizations and based on the preferences of different buyers. The ultimate arbiter of "too high" is certainly the auctioneer and the decision should be made based on the trade-off between these increased costs to contracting compared to the decreased costs in dealing with failed projects.

The indication here is that the minimal environmental condition for considering the average bid auction is the existence a common element to the cost structure of sellers where there is enough uncertainty in the eventual costs to drive a large Winner's Curse problem. Environments that do not meet this criterion are ones for which the average bid auction is likely a poor choice. An additional criterion is that the environment should be one in which participating sellers do not have a history or ability to easily collude with each other. While such collusion can be a problem in any auction format, the average bid structure seems potentially more conducive to such concerns.

It is important to note though that these experiments involved no special elements aimed at fostering or facilitating collusion among the sellers. The environment we chose to investigate here was designed to be a good match for external environments in which either bidders interact infrequently or in which the auctioneer has tight and effective anti-collusion rules in place. In particular we did not allow our sellers access to the sorts of collusive strategies found to be in use by the participants in the Italian auctions in Conley and Decarolis (2012). This was intentional as our goal was to examine the performance of the mechanism in this standard environment. Future studies could certainly add in such things as the ability of sellers to communicate with each other or allow them to use other tools to facilitate collusion. One such tool would be simply to provide sellers with all of the prices submitted in an auction. This perhaps paired with persistent identities of bidders and allowing sellers to compete in the same groups over time could well facilitate subjects learning to engage in more coordinated behavior.

While our experimental results show that the average bid mechanism works as it is

²⁰It is also the case that one study uses lab data generated by student subjects and the other field data generated by professionals. While some see this alone as a substantial difference, empirical evidence suggests this is unlikely to be a concern. See for example Camerer (2011) and Fréchet (2009).

intended to, these results should not be interpreted as a claim that it should be considered an optimal or ideal mechanism. There are likely other mechanisms that might be able to achieve a similar reduction of bankruptcy problems with perhaps less potential for high prices. To know whether an existing mechanism needs to be replaced by an alternative, one first needs to benchmark the performance of the current mechanism. That is what we have done in this study and we find, perhaps surprisingly, that the average bid mechanism performs well. There is still future research to be done to determine if other mechanisms can provide superior solutions to the problems of seller losses but do so in a more cost effective way for the buyers.

Appendix

Proof of Proposition 1. By way of contradiction, suppose on the contrary that there is a Bayesian Nash equilibrium where all the bidders use a strictly increasing bidding strategy $b^*(s)$ with $s \in [C_L - \theta, C_H + \theta]$ and $b^*(C_L - \theta) \geq C_L$.²¹ Suppose all but bidder 1 follow the strategy $b^*(s)$. Consider the case where bidder 1 receives signal $\underline{s} = C_L - \theta$. Bidder 1 hence knows that the true cost is C_L and bidder 1's expected payoff is zero (as the probability of at least one other bidder receiving signal $(C_L - \theta)$ is zero). Suppose such a type of bidder 1 bids instead $b^*(C_L)$, i.e., submits the C_L -type's bid, his expected payoff is

$$\Pi(b^*(C_L); \underline{s}) = P(b^*(C_L) | \underline{s}) (b^*(C_L) - C_L),$$

where $P(b^*(C_L) | \underline{s})$ is the probability of winning for type- \underline{s} of bidder 1 by bidding $b^*(C_L)$. For simplicity, suppose n is odd (the case of n being even can be shown similarly by identifying a scenario with a positive measure such that $b^*(C_L)$ is the winning bid). We then have

$$\begin{aligned} P(b^*(C_L) | \underline{s}) &> (n-1)! \left(\int_{C_L - \theta}^{C_L - \frac{\theta}{2}} \frac{dt}{2\theta} \right)^{\frac{n-1}{2}} \left(\int_{C_L + \frac{\theta}{2}}^{C_L + \theta} \frac{dt}{2\theta} \right)^{\frac{n-1}{2}} \\ &= (n-1)! \left(\frac{1}{4} \right)^{n-1} > 0. \end{aligned}$$

The rationale behind the above inequality is the following: By bidding $b^*(C_L)$, type- \underline{s} of bidder 1 wins the auction if half of the remaining $(n-1)$ bidders receive signals in $[C_L - \theta, C_L - \frac{\theta}{2}]$ and the remaining $\frac{(n-1)}{2}$ bidders receive signals in $[C_L + \frac{\theta}{2}, C_L + \theta]$, which guarantees that $b^*(C_L)$ is the unique bid that is closest to the average of all bids, i.e., a winning bid. The first strict inequality derives from the fact that this is not the only case that $b^*(C_L)$ can be a winning bid.

Since $P(b^*(C_L) | \underline{s}) > 0$ and $b^*(C_L) > C_L$, we have that bidding $b^*(C_L)$ is strictly better (than bidding $b^*(C_L - \theta)$) for type- \underline{s} of bidder 1. By continuity, the same thing holds true for types that are very close to \underline{s} . We hence have found a profitable deviation, a contradiction. ■

²¹A bidding strategy $b^*(s)$ with $b^*(C_L - \theta) < C_L$ can never be a part of an equilibrium as it violates the incentive constraint of the type $s = C_L - \theta$.

Proof of Proposition 2. We proceed with a proof by contradiction. Assume for now that there is an equilibrium where every bidder follows a symmetric and increasing bidding strategy $b(s)$. A bidder's expected payoff given signal s when she bids according to $b(s)$ can hence be written as:

$$\Pi(b(s); s) = \begin{cases} \int_{C_L}^{s+\theta} \frac{2(b(s)-c)}{s+\theta-C_L} \frac{(s-c+\theta)(c+\theta-s)}{(2\theta)^2} dc & \text{if } s \in [C_L - \theta, C_L + \theta], \\ \int_{s-\theta}^{s+\theta} \frac{2(b(s)-c)}{2\theta} \frac{(s-c+\theta)(c+\theta-s)}{(2\theta)^2} dc & \text{if } s \in [C_L + \theta, C_H - \theta], \\ \int_{s-\theta}^{C_H} \frac{2(b(s)-c)}{C_H+\theta-s} \frac{(s-c+\theta)(c+\theta-s)}{(2\theta)^2} dc & \text{if } s \in [C_H - \theta, C_H + \theta]. \end{cases} \quad (\text{A-1})$$

Step 1. First notice that there cannot be an equilibrium where the equilibrium bidding function $b(s)$ has vertical jumps (or discontinuity). If there were such a jump, then the type 'sitting at the jump' would bid at the upper end of the gap. Such a deviation, while not affecting this type's (expected) winning probability, strictly increases the type's payoff in the event of winning. To illustrate, consider a strictly increasing equilibrium where type $s \in [C_H - \theta, C_H + \theta]$ bids z . There is a vertical jump such that all types to the left of s bid values less than z and all types to the right of s bid values above $z + k$ with $k > 0$.²² Type s 's payoff from bidding z is:

$$\Pi(z; s) = \int_{s-\theta}^{C_H} \frac{2(z-c)(s-c+\theta)(c+\theta-s)}{(2\theta)^2(C_H+\theta-s)} dc,$$

By assumption (the vertical jump and $b(s)$ being strictly increasing), no type bids a value in $(z, z + k)$. Hence, type s 's winning probability given cost c remains $2(s-c+\theta)(c+\theta-s)/(2\theta)^2$ when s bids $z + k$ instead. This immediately implies that

$$\begin{aligned} \Pi(z+k; s) - \Pi(z; s) &= \int_{s-\theta}^{C_H} \frac{2k(s-c+\theta)(c+\theta-s)}{(2\theta)^2(C_H+\theta-s)} dc \\ &= \frac{k}{6\theta^2} (C_H + \theta - s)(s + 2\theta - C_H), \end{aligned}$$

or $\Pi(z+k; s) > \Pi(z; s)$. A similar result holds for all types in a small neighborhood with $s' < s$. A similar argument can be applied for cases where $s \in [C_L - \theta, C_L + \theta]$ and $s \in [C_L + \theta, C_H - \theta]$. A contradiction.

Step 2. We next show that the equilibrium bidding function $b(s)$ cannot be partially pooling and partially separating. First, as Proposition 1 demonstrates, there cannot be an equilibrium where the bidding function $b(s)$ is strictly increasing, as all types in a neighborhood around $C_L - \theta$ would deviate upward. Reversing the argument slightly, one can show that all types in a neighborhood around $C_H + \theta$ have incentives to deviate downward. Hence, pooling has to arise in neighborhoods around the extreme types (i.e., $C_H + \theta$ and $C_L - \theta$). We now argue that if types in a neighborhood around $C_H + \theta$ submit an identical bid z (which is at least C_H by the incentive constraint of type $C_H + \theta$), then things unravel so that all types in $[C_L - \theta, C_H + \theta]$ bid the same amount in equilibrium.

Suppose that $b(s)$ is partially separating and partially pooling, i.e., there is a type

²²Other discontinuity cases (upper semicontinuity and discontinuity in a non-strictly increasing equilibrium) can be shown in a similar fashion.

$s \in [C_H - \theta, C_H + \theta]$ such that $b(s')$ is strictly increasing for $s' < s$ and $b(s) = b(s'')$ for all $s'' > s$. As the (pooling) neighborhood around $C_H + \theta$ has positive measure, we let $C_H + \theta - s \geq \Delta > 0$. Type s 's equilibrium payoff by bidding $b(s) = z \geq C_H$ is²³

$$\Pi(b(s); s) = \int_{s-\theta}^{C_H} \frac{(z-c) \left(\frac{(c+\theta-s)^2}{3} + (s-c+\theta)(c+\theta-s) \right)}{(C_H + \theta - s)(2\theta)^2} dc, \quad (\text{A-2})$$

while if type s bids $b'(s) = z - \varepsilon$ for some arbitrarily small $\varepsilon > 0$, we have²⁴

$$\Pi(b'(s); s) \approx \int_{s-\theta}^{C_H} \frac{2(z-c)(s-c+\theta)(c+\theta-s)}{(2\theta)^2(C_H + \theta - s)} dc. \quad (\text{A-3})$$

The difference between (A-2) and (A-3) comes from the fact that when type s deviates downward slightly, he will no longer share winning probabilistically with types to the right of s , who are all bidding $z \geq C_H$. Now since (notice that $s \geq C_H - \theta$ and $C_H + \theta - s \geq \Delta$)

$$\begin{aligned} & \Pi(b'(s); s) - \Pi(b(s); s) \\ & \approx \int_{s-\theta}^{C_H} \frac{(z-c)}{(C_H + \theta - s)(2\theta)^2} \left((s-c+\theta)(c+\theta-s) - \frac{(c+\theta-s)^2}{3} \right) dc \\ & \geq \int_{s-\theta}^{C_H} \frac{(C_H - c)}{(C_H + \theta - s)(2\theta)^2} \left((s-c+\theta)(c+\theta-s) - \frac{(c+\theta-s)^2}{3} \right) dc \\ & = \frac{(s+2\theta - C_H)(C_H + \theta - s)^2}{36\theta^2} \\ & \geq \frac{\Delta^2}{36\theta}, \end{aligned}$$

i.e., type s (and hence all types in a small neighborhood to the right of s by continuity) has strict incentives to deviate downward slightly, a contradiction. A similar argument shows that such a result holds for $s \in [C_L + \theta, C_H - \theta]$ and for $s \in [C_L - \theta, C_L + \theta]$. We conclude that the only type of symmetric increasing Bayesian Nash equilibria is that all types bid an identical amount.

Step 3. Finally, all types bidding an identical amount $z \geq C_H$ (required for C_H type's incentives in equilibrium) for all bidders is indeed a Bayesian Nash equilibrium: Given that the other two bidders are bidding $z \geq C_H$ independently of their types, a bidder always bids z independently of his type, which always generates a positive expected payoff (except possibly for type C_H), while bidding *anything else* gives a payoff of 0, as any bid different from z can never be *closest to the average*, resulting in a zero winning probability. ■

²³In deriving (A-2), notice that given cost c , type s wins in two events: First, the other two bidders both receive signals in $[s, c + \theta]$, in which case all three bidders win with equal probability (i.e., $\frac{(c+\theta-s)^2}{3}$); Second, one of the other two bidders receives a signal in $[s, c + \theta]$ while the remaining bidder receives a signal in $[c - \theta, s]$. In this case, type s shares winning with exactly one other bidder (i.e., $\frac{2(s-c+\theta)(c+\theta-s)}{2}$).

²⁴We have omitted arbitrarily small items ε and $\lambda(\varepsilon)$ in (A-3) — when type s deviates downward slightly, type s is bidding as if he were type $(s - \lambda(\varepsilon))$, where $\lambda(\varepsilon)$ is arbitrarily close to 0 for arbitrarily small ε . Here, $\Pi(b'(s); s) = \int_{s-\theta}^{C_H} \frac{2(z-\varepsilon-c)(s-\lambda(\varepsilon)-c+\theta)(c+\theta-s+\lambda(\varepsilon))}{(2\theta)^2(C_H + \theta - s)} dc$.

Proof of Claim 1. We show that with a conjecture that the other two bidders will bid according to a (symmetric) strictly increasing function, then every bidder finds it a best response to bid $\hat{b}(s) = s + \Delta$, with constant $\Delta \geq \theta$ if $s \in [C_L + \theta, C_H - \theta]$. Given this, a bidder's expected payoff from bidding $\hat{b}(s)$ is

$$\Pi(\hat{b}(s); s) = \int_{s-\theta}^{s+\theta} \frac{2(s + \Delta - c)(s - c + \theta)(c + \theta - s)}{(2\theta)^3} dc = \frac{\Delta}{3}.$$

First, if a bidder bids $\hat{b}'(s) = s + \Delta - \lambda$ with $\lambda > 0$, then

$$\begin{aligned} \Pi(\hat{b}'(s); s) &= \int_{s-\theta}^{s+\theta-\lambda} \frac{2(s + \Delta - \lambda - c)(s - \lambda - c + \theta)(c + \theta - s + \lambda)}{(2\theta)^3} dc \\ &= \frac{\Delta}{3} - \frac{\lambda^2}{48\theta^3} \left(3(2\theta - \lambda)^2 + 4\Delta(3\theta - \lambda) \right) < \frac{\Delta}{3}. \end{aligned}$$

On the other hand, if a bidder bids $\hat{b}''(s) = s + \Delta + \lambda$ with $\lambda > 0$, then

$$\begin{aligned} \Pi(\hat{b}''(s); s) &= \int_{s+\lambda-\theta}^{s+\theta} \frac{2(s + \Delta + \lambda - c)(s + \lambda - c + \theta)(c + \theta - s - \lambda)}{(2\theta)^3} dc \\ &= \frac{(4\Delta\theta + 4\Delta\lambda + 3\lambda^2)(\lambda - 2\theta)^2}{48\theta^3} \\ &= \frac{\Delta}{3} + \frac{\lambda^2}{48\theta^3} \left(3(2\theta - \lambda)^2 - 4\Delta(3\theta - \lambda) \right) \\ &< \frac{\Delta}{3} \text{ if } \Delta \geq \theta. \end{aligned}$$

In deriving the above two inequalities, we have implicitly assumed that $\lambda \leq 2\theta$, as a bigger deviation guarantees a zero winning probability, hence suboptimal. ■

References

- Albano, G. L., M. Bianchi, and G. Spagnolo (2006). Bid average methods in procurement. *Rivista di Politica Economica* 96(1), 41–62.
- Bajari, P., S. Houghton, and S. Tadelis (2011). Bidding for incomplete contracts: An empirical analysis of adaption costs. Working Paper.
- Bulow, J. and P. Klemperer (1996). Auctions versus negotiations. *The American Economic Review* 86(1), 180–194.
- Calveras, A., J.-J. Ganuza, and E. Hauk (2004). Wild bids. gambling for resurrection in procurement contracts. *Journal of Regulatory Economics* 26(1), 41–68.
- Camerer, C. (2011). The promise and success of lab-field generalizability in experimental economics: A critical reply to levitt and list. Working Paper, Caltech.

- Chang, W.-S., T. C. Salmon, and K. J. Saral (2012). Procurement auctions with renegotiation and wealth constraints. Working Paper.
- Chen, J., L. Xu, and A. Whinston (2010). Managing project failure risk through contingent contracts in procurement auctions. *Decision Analysis* 7(1), 23–39.
- Commission, E. (2002). Prevention, detection and elimination of abnormally low tenders in the european construction industry. Directorate Enterprise and Industry.
- Conley, T. G. and F. Decarolis (2012). Detecting bidders groups in collusive auctions. Working Paper, University of Wisconsin.
- Coviello, D. and M. Mariniello (2014). Publicity requirements in public procurement: Evidence from a regression discontinuity design. *Journal of Public Economics* –(–), –.
- Crawford, V. P. and N. Iriberry (2007). Level-k auctions: Can a nonequilibrium model of strategic thinking explain the winner’s curse and overbidding in private-value auctions? *Econometrica* 75(6), 1721–1770.
- Decarolis, F. (2013). Comparing public procurement auctions. Working Paper Boston University.
- Engel, A. R., J.-J. Ganuza, E. Hauk, and A. Wambach (2006). Managing risky bids. In N. Dmitir, G. Piga, and G. Spagnolo (Eds.), *Handbook of Procurement*, pp. 322–344. Cambridge University Press.
- Eyster, E. and M. Rabin (2005). Cursed equilibrium. *Econometrica* 73(5), 1623–1672.
- Fischbacher, U. (2007). z-Tree: Zurich Toolbox For Readymade Economic Experiments. *Experimental Economics* 10(2), 171–178.
- Fishe, R. P. and R. P. McAfee (1987). Nonlinear contracts, zero profits and moral hazard. *Economica* 54(213), 97–101.
- Fréchette, G. (2009). Laboratory experiments: Professionals versus students. NYU Working Paper.
- Gil, R. and J.-M. Oudot (2009). Competitive bidding, renegotiation and relational contracting: Evidence from french defense. Working Paper.
- Greiner, B. (2004). An online recruitment system for experimental economics. In K. Kremer and V. Macho (Eds.), *Forschung und wissenschaftliches Rechnen 2003*, Volume 63, pp. 79–93. Göttingen: Ges. für Wiss. Datenverarbeitung.
- Hong, H. and M. Shum (2002). Increasing competition and the winner’s curse: Evidence from procurement. *Review of Economic Studies* 69(4), 871–898.
- Kagel, J. H. and D. Levin (1986). The winner’s curse and public information in common value auctions. *American Economic Review* 76(5), 894–920.

- Kagel, J. H. and D. Levin (2002). *Common Value Auctions and the Winner's Curse*. Princeton, New Jersey: Princeton University Press.
- Kagel, J. H. and J.-F. Richard (2001). Super-experienced bidders in first-price common-value auctions: Rules of thumb, nash equilibrium bidding, and the winner's curse. *The Review of Economics and Statistics* 83(3), 408–419.
- Lind, B. and C. Plott (1991). The Winner's Curse: Experiments with Buyers and Sellers. *American Economic Review* 81, 335–46.
- McAfee, R. P. and J. McMillan (1986). Bidding for contracts: A principal-agent analysis. *The RAND Journal of Economics* 17(3), 326–338.
- Merlob, B., C. R. Plott, and Y. Zhang (2011). The cms auction: Experimental studies of a median-bid procurement auction with non-binding bids. Caltech Working Paper.
- Rothkopf, M. H. (1969). A model of rational competitive bidding. *Management Science* 15(7), 362–373.
- Wilson, R. B. (1969). Competitive bidding with disparate information. *Management Science* 15(7), 446–448.
- Wilson, R. B. (1977). A bidding model of perfect competition. *Review of Economic Studies* 44(3), 511–518.
- Zheng, C. Z. (2001). High bids and broke winners. *Journal of Economic Theory* 100(1), 129–171.