Trade Liberalization versus Protectionism: Dynamic Welfare Asymmetries

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Abstract

We investigate whether the losses from protectionism (e.g., Brexit and U.S. China trade war) are equal to the gains from trade liberalization. Standard static trade models imply symmetry: The losses are indeed the same as the gains. In a dynamic setting, we show that the gains exceed the losses. We embed trade between many countries, a la Armington, into a neoclassical growth model. We then examine the transition path from an “observed” steady state to an autarky steady state and also the path in the reverse direction. By construction the two steady states are symmetric with respect to the change in trade costs. We show that the discounted path of changes in consumption can be decomposed into three components: the share of income allocated to consumption (consumption rate), the share of spending on domestically produced goods (measured productivity), and changes in the capital stock. We calibrate the model to 2015 data for 136 countries and assume steady state. The change in welfare due to the change in trade costs is asymmetric: Vietnam, the 95th percentile country in terms of gains from trade, gains almost 60% from liberalization but loses 56% by moving to autarky. Moreover, asymmetry in capital accumulation accounts for the largest share of the asymmetry in dynamic gains and losses, followed by asymmetry in consumption rates. Asymmetry in measured productivity accounts for the smallest share. Under protectionism, the household does not need to reduce investment substantially

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because depreciation reduces the capital stock, which frees up resources for consumption. Under liberalization, however, the household must forego some consumption in order to accumulate capital.

**JEL codes:** E22, F11, O11
1 Introduction

Following decades of extraordinary globalization in the wake of World War II, large portions of the world’s population experienced unprecedented wealth accumulation. In the past few years, however, there has been a reversal in trade integration. Britain’s 2016 Brexit referendum, the U.S.-China trade war beginning in 2018, and export restrictions on essential medical equipment during the COVID-19 pandemic, each represent major events driving regions toward nationalism and to what some regard as a return to protectionism (e.g., Fajgelbaum, Goldberg, Kennedy, and Khandelwal, 2019). These reversals lead to a natural question: Are the losses from current protectionist policies equal to the gains from prior trade integration?

The answer from workhorse static models of trade is no. In such models, welfare gains from trade are computed as the change in income associated with moving from one distribution of trade costs to another distribution (e.g., Arkolakis, Costinot, and Rodríguez-Clare, 2012; Waugh and Ravikumar, 2016). If the increase in trade costs in a protectionism scenario is symmetric to the decreases in a liberalization scenario, then the welfare changes in the two scenarios would be equal.

In this paper, we incorporate dynamics through capital accumulation and argue that the losses from an increase in trade costs (protectionism) are smaller than the gains from a symmetric decrease in trade costs (liberalization), using a consumption-equivalent measure of welfare changes, as in Lucas (1987). The key intuition is that, following trade liberalization, there is an opportunity cost to accumulating capital: The economy has to forego consumption. However, following protectionism, capital depreciates even in the absence of disinvestment. In other words, decades of low trade costs facilitate the accumulation of capital stock, but reversing course and increasing trade costs do not produce large losses immediately since the economy can coast off of previous investments for some time while the capital stock depreciates. The dynamic welfare measure places more weight on consumption dynamics in the short run, so the losses from protectionism are smaller.

The intertemporal conditions of the model play a crucial role in shaping the rate of consumption growth. Therefore, we consider several specifications, each imposing different assumptions on the nature of capital accumulation. In our first specification, we impose an exogenous investment rate as in the Solow growth model (Solow specification, henceforth), so that consumption is proportional to income at every point in time. In our second specification, we allow for the investment rate to be optimally determined in every period along the
transition path, as in the neoclassical growth model (neoclassical specification, henceforth), breaking the proportionality between consumption and investment. Finally, we consider an irreversible-investment specification.

We embed the different specifications of capital accumulation into a model of trade between many countries, a la Armington. We then examine the transition path from an “observed” steady state to an autarky steady state and also the path in the reverse direction. In both cases, we impose a one-time, permanent change in trade costs. By construction, the long run is the same in both cases and the two steady states are symmetric with respect to the change in trade costs. We show that the discounted path of changes in consumption can be decomposed into three components: the share of income allocated to consumption (consumption rate), the share of spending on domestically produced goods (measured productivity), and changes in the capital stock.

We calibrate the model to 2015 data for 136 countries and assume steady state. Our quantitative results are as follows. First, the dynamic gains and losses from the change in trade costs are heterogeneous across countries. Vietnam, for instance, is in the 95th percentile of gains and losses and its welfare change is more than 20 times greater than that for Australia, which is in the 5th percentile. Second, the change in welfare due to the change in trade costs is asymmetric: Vietnam gains almost 60% from liberalization but loses 56% by moving to autarky. Third, asymmetry in capital accumulation accounts for the largest share of the asymmetry in dynamic gains and losses, followed by asymmetry in consumption rates. Asymmetry in measured productivity accounts for the smallest share.

For tractability, bilateral trade is determined by Armington preferences and balanced trade period-by-period. With balanced trade, the steady state can be computed independently from the transition, and hence, is symmetric with respect to the trade cost regime. Each country’s steady-state level of consumption, output, investment, capital stock, and measured productivity can be analytically expressed in terms of its own fundamental productivity, its home trade share, and some elasticities. As in ACR, the long-run steady-state change in income induced by changes in trade costs is a function of only the change in the home trade share and an appropriate trade elasticity. In the long run, the investment rate

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Ravikumar, Santacreu, and Sposi (2019) study the quantitative implications of endogenous capital accumulation in an economy with trade imbalances. In this paper, we expand the type of questions we can address with our methodology and focus on understanding the main channels behind some of the results in Ravikumar, Santacreu, and Sposi (2019). The analysis can equally be carried out using an Eaton and Kortum (2002) structure or a Krugman (1980) structure for trade, and all of the result carry through since the aggregate intratemporal expressions for prices and trade flows are identical (see Arkolakis, Costinot, and Rodríguez-Clare, 2012).
is invariant to the trade regime so changes in consumption are identical to changes in income. In this sense, the steady-state implications are similar to the implications from static models in terms of being symmetric to changes in trade costs, both in the Solow and in the neoclassical specification.

While the Solow and neoclassical specifications deliver identical steady-state implications, the transitional dynamics differ across the two specifications. In the neoclassical specification, in addition to capital and measured productivity, the consumption rate varies along the transition. The de-coupling of consumption from income implies that changes in income, brought about by changes in measured productivity and capital stock, do not fully describe consumption dynamics. In order to evaluate the consumption dynamics, and hence dynamic measures of welfare, we turn to computation. Within our quantitative framework, we develop a tractable accounting method to decompose welfare changes into contributions from (i) changes in capital stock, (ii) changes in consumption share of income, and (iii) changes in home trade share, which reflect changes in measured productivity through trade selection.

In each of our specifications, we provide algorithms that can efficiently compute the exact transitional dynamics for the global economy in a matter of minutes on a basic laptop computer.

These asymmetries have important implications for understanding the effects of reversing trade policy over time—i.e., temporary reductions in trade costs that are subsequently increased, and vice versa. Take the case of a temporary trade liberalization that is followed by a permanent protectionist policy. When there is perfect foresight, the expectation of a reversal of trade policy towards autarky sometime in the future increases consumption today less than if the liberalization were permanent. This consumption smoothing allows for higher investment, resulting into a large capital stock being accumulated prior to the start of the protectionist policy. At that point, households start decumulating capital by letting it depreciate. Hence, the decrease in consumption is lower than without capital accumulation or zero depreciation. Capital depreciation hence plays an important role in generating asymmetric welfare gains, and this has important consequences when there is a sequence of trade policies.

Our paper is related to several strands of literature. First, it is related to recent papers developing algorithms to solve dynamic spatial models (see Ravikumar, Santacreu, and Sposi, 2019; Reyes-Heroles, 2016; Anderson, Larch, and Yotov, 2015). We provide three separate algorithms to efficiently compute the dynamic spatial equilibria. Our first algorithm solves a Solow specification in which the investment rate is exogenous. In particular, our
algorithm solves a sequence of static trade models, where in each period we use the well-known algorithm from Alvarez and Lucas (2007). We then provide a second algorithm to solve a model with endogenous capital accumulation as in the neoclassical growth model. We allow for investment reversibility to study the role of capital vs TFP in delivering welfare gains from trade. we solve an Arrow-Debreu-type economy where income from any period can be allocated toward consumption in any period. Thus the investment rate is determined optimally in every period along the transition under perfect foresight. Finally, we provide a third algorithm to solve a model with endogenous investment that allows for capital irreversibility. In particular, we impose irreversible investment with capital adjustment costs, and employ the algorithm from Ravikumar, Santacreu, and Sposi (2019).

Second, our paper is related to a literature developing sufficient statistics to measure gains from trade. Arkolakis, Costinot, and Rodríguez-Clare (2012) study the losses from moving to autarky, whereas Waugh and Ravikumar (2016) compute sufficient statistics to analyze gains from trade liberalization. All our specifications in the paper admit a sufficient statistics formula to compute steady-state gains from trade. However, this breaks down along the transition because there is capital accumulation with partial depreciation, as in Ravikumar, Santacreu, and Sposi (2019).

Finally, our paper relates to a literature studying the role of a sequencing of economic policies on aggregate outcomes (see Asturias et al., 2016). We design a sequence of trade policy reversals to show that asymmetries through capital accumulation and partial depreciation materialize in the short run and have important welfare effects.

1.1 Empirical Evidence

Using gravity methods, Benasser (2022) identifies asymmetric effects of policy reversals—i.e., entry to and exit from trade agreements—on the volume of trade. We provide suggestive evidence of asymmetries between trade liberalization and protectionist policies on key economic variables for welfare. Specifically, we analyze, empirically, the coincidence of changes in trade openness and changes in the growth rate for each of four variables: investment, capital, consumption and output. We follow Wacziarg and Welch (2008) and use an event study approach and identify periods of trade liberalization and protectionism for a sample of 181 countries. Wacziarg and Welch (2008) use the measure of openness developed by Sachs and Warner (1995), which classifies a country as closed if at least one of the following conditions apply: (a) average tariff rates of 40 percent or more; (b) non-tariff barriers covering 40 percent of trade or more; (c) a black market exchange rate of at least 20 percent lower of
the official one; (d) a state monopoly on major exports; and (e) a socialist economic system.

We divide their sample of countries into two groups. A group for which Wacziarg and Welch (2008) identify one period of permanent liberalization, and a group of countries that move from a regime of liberalization to one of protectionism and then to back to a permanent liberalization.

We first identify countries that had a permanent trade liberalization, and plot the evolution of key economic variables 10 years before and 10 years after the period regime change from being closed to entering liberalization in Figure 12. The graphs plot the average across all countries, with the vertical line at zero being the year of permanent liberalization, each dot in the graph is the average of the variables across all countries in the sample, the dashed horizontal lines represent the 10-year averages prior to and after the liberalization, and the solid continuous line is a three-year moving average. Periods of liberalization are characterized by higher growth rates of consumption, output, investment and capitals stock.

Next, we restrict our analysis to countries that experience liberalization followed by protectionism before moving back to a regime of permanent liberalization. Figure 2 plots the dynamics of investment, capital, consumption and output for countries that underwent a period of temporary liberalization of at least 10 years, followed by a period of protectionism of at least 10 years before a subsequent permanent trade liberalization. The graphs on the left plot the evolution of the variables 10 years before and 10 years after the period when protectionism started. The graphs on the right plot the evolution of key variables 10 years earlier and 10 years after the period of permanent liberalization.

Periods of liberalization exhibit higher growth rates on average. Growth rates go down during protectionism (left panels). Similarly, periods of protectionism exhibit lower growth rates on average. Growth rates increase during periods of liberalization (right panels).

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2The list of countries can be found in... and it excludes those countries had a period of temporary liberalization before the period of permanent liberalization.

3The list of countries is Costa Rica, Guatemala, Honduras, Jamaica, Sri Lanka, Nicaragua, Peru, El Salvador, Syria, Turkey and Venezuela.
Figure 1: Trade Liberalization Dynamics

- **Consumption growth before and after Liberalization**
  - Pre-liberalization mean (mean) consumption growth:
  - Post-liberalization mean (mean) consumption growth:
  - Average consumption growth (mean) consumption growth average

- **Capital growth before and after Liberalization**
  - Pre-liberalization mean (mean) capital growth:
  - Post-liberalization mean (mean) capital growth:
  - Average capital growth (mean) capital growth average

- **Investment Growth before and after Liberalization**
  - Pre-liberalization mean (mean) investment growth:
  - Post-liberalization mean (mean) investment growth:
  - Average investment growth (mean) investment growth average

- **GDP Growth before and after Liberalization**
  - Pre-liberalization mean (mean) GDP growth:
  - Post-liberalization mean (mean) GDP growth:
  - Average GDP growth (mean) GDP growth average
Figure 2: Trade Liberalization and Protectionism Dynamics

**Consumption Growth before and after Protectionism**

**Consumption Growth before and after Liberalization**

**Capital Growth before and after Protectionism**

**Capital Growth before and after Liberalization**

**Investment Growth before and after Protectionism**

**Investment Growth before and after Liberalization**

**GDP Growth before and after Protectionism**

**GDP Growth before and after Liberalization**
2 Model

The world economy consists of $N$ countries, each indexed by $(n, i) = 1, \ldots, N$. Time is discrete and runs from $t = 1, \ldots, \infty$.

**Firms** In each country a retail firm sources goods from all countries. Goods are differentiated based on their country of origin as in [Armington (1969)](https://www.jstor.org/stable/1420888), and the elasticity of substitution between any two origin locations is given by $\theta - 1$. The quantity that country $n$ sources from country $i$ at time $t$ is denoted by $q_{n,i,t}$ and the aggregate basket available in country $n$ is given by

$$ Q_{n,t} = \left[ \sum_{i=1}^{M} (q_{n,i,t})^{\frac{\theta}{\theta - 1}} \right]^{\frac{\theta - 1}{\theta}}, \tag{1} $$

The composite good, $Q_{n,t}$, is allocated to final consumption and final investment.

The production firm in each country produces goods using capital and labor with the following technology

$$ y_{n,t} = A_{n,t} (k_{n,t})^\alpha (\ell_{n,t})^{1-\alpha}. \tag{2} $$

The term $A_{n,t}$ is the fundamental, value-added productivity, which varies across countries. The inputs $k_{n,t}$ and $\ell_{n,t}$ denote the amounts of capital stock and labor used in production and $\alpha$ denotes capital’s share in value added.

**Trade** International trade is subject to iceberg costs whereby destination $n$ must purchase $d_{n,i} \geq 1$ units of the good from origin $i$ in order for one unit to arrive; $d_{n,i} - 1$ units melt away. As a normalization, $d_{n,n} = 1$ for all $n$. Trade is balanced period by period.

**Households** The representative household enters the period with capital stock, $K_{n,t}$, and labor, $L_n$. It supplies these factors inelastically to domestic firms earning a rental rate $r_{n,t}$ on each unit of capital and a wage rate $w_{n,t}$ on each unit of labor. Its income is spent on consumption, $C_{n,t}$ and investment, $X_{n,t}$, both of which have price $p_{n,t}$. The period budget constraint is thus given by

$$ p_{n,t}C_{n,t} + p_{n,t}X_{n,t} = r_{n,t}K_{n,t} + w_{n,t}L_n. \tag{3} $$

Each country admits a representative household of size $L_n$, whose lifetime utility is defined

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4The parameter $\theta$ is the trade elasticity.
as the discounted sum instantaneous utility derived from per-capita consumption over time:

$$\mathcal{U}_n \equiv \sum_{t=1}^{\infty} \beta^{t-1} L_n \ln \left( \frac{C_{n,t}}{L_n} \right),$$

(4)

where $C_{n,t}$ is consumption, $L_n$ population, $\beta \in (0, 1)$ the discount factor. Each period the instantaneous utility depends on per-capita consumption and is weighted by the size of the population.

Investment in physical capital is the only means for saving since trade is balanced. At the end of the period, a fraction $\delta$ of the capital stock depreciates. Capital stock at the beginning of the next period is equal to

$$K_{n,t+1} = (1 - \delta)K_{n,t} + X_{n,t}.$$  

(5)

### 2.1 Equilibrium

A competitive equilibrium satisfies the following conditions: i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technology for accumulating capital, ii) taking prices as given, firms maximize profits subject to the available technologies, and iii) markets clear. At each point in time, we take world GDP as the numéraire: $\sum_n r_{n,t}K_{n,t} + w_{n,t}L_n = 1$ for all $t$.

Prices are determined competitively so that the price faced by country $n$ for a product originating in country $i$ is the country $i$’s marginal cost, scaled by the bilateral trade cost:

$$p_{n,i,t} = \frac{u_{i,t}d_{n,i,t}}{A_{i,t}},$$

(6)

where $u_{i,t} = (r_{i,t}/\alpha)(w_{i,t}/(1 - \alpha))^{(1 - \alpha)}$ is the unit cost for factor inputs in country $i$. Firms earn zero profits so factor payments exhaust total revenue

$$r_{n,t}K_{n,t} = \alpha p_{n,t}y_{n,t} \quad \text{and} \quad w_{n,t} \ell_{n,t} = (1 - \alpha)p_{n,t}y_{n,t}. $$

(7)

Country $n$’s demand for products originating in country $i$ is

$$q_{n,i,t} = (p_{n,i,t}/p_{n,t})^{-(1+\theta)}Q_{n,t},$$

(8)
where $Q_{n,t} = C_{n,t} + X_{n,t}$ is total absorption in country $n$, and

$$p_{n,t} = \left( \sum_{i=1}^{N} (p_{n,i,t})^{-\theta} \right)^{-1/\theta}$$

is the ideal price index for the entire basket of goods absorbed.

Market clearing requires that factor market supply equals demand:

$$K_{n,t} = k_{n,t} \text{ and } L_n = \ell_{n,t}$$

and each country’s gross production is absorbed globally:

$$p_{n,t} y_{n,t} = \sum_{i=1}^{N} p_{n,i,t} q_{n,i,t}.$$  

(11)

To close the model, we impose balance trade period-by-period or, equivalently, that gross production equals gross absorption in each country:

$$p_{n,t} Q_{n,t} = p_{n,t} y_{n,t}.$$  

(12)

Next we describe two alternative specifications regarding household saving decisions that influence transitional dynamics. In the first, the investment rate is exogenously fixed across countries and over time so the household is not forward looking. In the second, we allow for the investment rate to be determined optimally over time by each country.

**Model with Exogenous Investment Rate** In this specification the saving decision is constrained as in the Solow growth model. We refer to this specification as the *Solow specification*. An exogenous fraction, $\varphi$, of current income is spent on current consumption.

$$p_{n,t} C_{n,t} = \varphi \left( r_{n,t} K_{n,t} + w_{n,t} L_n \right).$$

(13)

We solve a sequence of static models recursively starting from period 1 where, in each period, the capital stock is determined by investment made in the previous period. In each period the factors of production are pre-determined so we apply a standard algorithm to compute the static equilibrium in that period as in Alvarez and Lucas (2007). The capital stock available in the subsequent period is the outcome of the investment made in the current
period, which is decided in a non-forward looking fashion. Detailed steps are outlined in Algorithm A.1 with corresponding equilibrium conditions provided in Table A.1.

Neoclassical Model with Endogenous Investment Rate We introduce forward-looking investment decisions that give rise to endogenous capital accumulation. We refer to this specification as the neoclassical specification. All the equations remain the same as before except for the household’s investment condition. Rather than following an exogenous, Solow-Swan-type investment rate to generate intertemporal linkages, the representative household optimally chooses a path for the investment rate. That is, we dispose of equation (13), and derive the following standard intertemporal Euler equation:

\[
\frac{C_{n,t+1}}{C_{n,t}} = \beta \left( 1 + \frac{r_{n,t+1}}{p_{n,t+1}} - \delta \right).
\]

(14)

In terms of finding a solution, we interpret each point in time as an extra dimension in the commodity space. We restrict the time dimension to be bounded by some limit \( T \), which is “sufficiently large” to ensure that the economy has settled into a steady state. Since investment is reversible, we can derive a lifetime budget constraint, and then allocate the net present value of lifetime wealth to consumption at each point in time, similar to an Arrow-Debreu-type economy with only one possible state of nature in each period. Detailed steps are outlined in Algorithm A.2, with corresponding equilibrium conditions provided in Table A.2.

To illustrate this idea, we briefly digress and work through some equations that are explicitly incorporated in the solution method. First, we make use of equations (3) and (5) over all time periods to derive a lifetime budget constrain, from which the net present value of lifetime wealth is given by

\[
W_n \equiv \sum_{t=1}^{T} \frac{w_{n,t}L}{P_{n,t} (1 + \mathcal{R}_{n,t})} + K_{n,1} - \frac{K_{n,T+1}}{1 + \mathcal{R}_{n,T}},
\]

(15)

where \( K_{n,T+1} \equiv K_n^* \) is the new (final) steady state for where the economy is headed. The compounded, real, gross rental rate of capital, net of depreciation (i.e., the rate of return on investment) is defined as

\[
1 + \mathcal{R}_{n,t} \equiv \prod_{t'=1}^{t} \left( \frac{r_{n,t'}}{p_{n,t'}} + 1 - \delta \right).
\]

Since investment is the only channel available for saving, this also equals the compounded
real interest rate up through period \( t \).

Manipulating the Euler equation (14) implies that the net-present value of consumption in any period is some share, \( \xi_{n,t} \), of lifetime wealth, where \( \sum_{t=1}^{T} \xi_{n,t} = 1 \):

\[
\frac{C_{n,t}}{1 + R_{n,t}} = \left( \frac{\beta^t}{\sum_{t'=1}^{T} \beta^{t'}} \right) W_n
\]

### 2.2 A Framework to Account for Welfare Dynamics

We now describe how we evaluate welfare dynamics in each of our two specifications. We compute changes in welfare associated with changes in trade costs using consumption equivalent units, as in Lucas (1987). Specifically, we assume that the world is initially in a steady state at some time \( t^\star \) with consumption level given by \( C_{n,t^\star} \) and compute the proportionate change in that level of consumption that is required to make the household indifferent from moving to new trade costs that yield a consumption path \( \{C_{n,t}\}_{t=t^\star+1}^{\infty} \). The compensating factor, or welfare change, is the value \( \Lambda_n \) such that

\[
\sum_{t=t^\star+1}^{\infty} \beta^{t-t^\star} L_n \ln \left( \frac{\Lambda_n C_{n,t}}{L_n} \right) = \sum_{t=t^\star+1}^{\infty} \beta^{t-t^\star} L_n \ln \left( \frac{C_{n,t}}{L_n} \right).
\]

In other words, \( \Lambda_n \) is the change in steady-state consumption required to make the household indifferent between remaining in the initial steady state or jumping on the new transition path at period \( t^\star + 1 \). We can simplify the expression and explicitly compute the dynamic gains in log points, as

\[
\lambda_n \equiv \ln(\Lambda_n) = (1 - \beta) \sum_{t=t^\star+1}^{t^\star\star} \beta^{t-t^\star} \ln \left( \frac{C_{n,t}}{C_{n,t^\star}} \right) + \beta^{t^\star\star-t^\star} \ln \left( \frac{C_{n,t^\star}}{N_{n,t^\star}} \right),
\]

where \( t^\star\star \) is a point in time such that the economy is sufficiently “close” to the new steady state and the new steady-state consumption level is \( C_{n,t^\star\star} \).

This formulation allows us to separate the dynamic gains from trade into two parts. The first part captures transitional dynamics and the second part captures steady-state gains from trade (the change in consumption between the initial and final steady states). Clearly, as \( \beta \to 1 \), only the second part matters since the first term gets zero weight. Intuitively, if
the future is not discounted, then it does not matter what happens in the first finite set of periods, as the perpetual future dominates in that case. However, as long as $\beta < 1$, then the transitional dynamics start mattering more early on. Nonetheless, it is constructive to study properties of steady-state consumption changes since it determines the total distance travelled during the transition and is important for welfare.

**Steady State**  In each of our specifications we impose balanced trade. In this case, the steady states can be computed independently from the transition paths. The steady-state consumption rate (share of consumption in current income) in the neoclassical model is

$$\varphi^* \equiv \frac{p_n^c C_n^*}{r_n^c K_n^* + w_n^c L_n} = 1 - \frac{\alpha \delta}{\beta} - \frac{(1 - \delta)}{1 - \alpha}.$$ 

Since this rate depends only on the deep parameters that are common across countries and constant over time, it is invariant to the trade regime. Note that the Solow specification admits an identical steady-state equilibrium as the neoclassical specification when the exogenous consumption rate $\varphi$ is chosen appropriately as above.

In both specifications, our model admits an analytical representation of per-capita consumption in the steady state:

$$\frac{C_n^*}{L_n} \propto \left( \frac{A_n}{(\pi_{nn}^*)^\theta} \right) \left( \frac{A_n}{(\pi_{nn}^*)^\theta} \right)^{1-\alpha}.$$  

On the right-hand side, both terms inside of both parentheses are the same. We separate the terms to capture two distinct channels. The first term captures the usual channel whereby trade facilitates specialization and impacts measured, or effective, productivity. The second term reflects the capital accumulation and its dependence on trade costs. Following changes in trade costs, the only terms that change are the home trade shares and so are sufficient to characterize the change in consumption between steady states. From this we can already determine the relative contributions from capital and measured productivity to changes in consumption across steady states. Since the investment rate is constant across steady states, its contribution is zero. The contribution from measured productivity is $1/(1 - \alpha)$ while the contribution from capital is $\alpha/(1 - \alpha)$.

**Definition 2.1.** Two scenarios are steady-state symmetric if the gross change between steady states in one scenario is equal to the inverse of the gross change in the other scenario.  

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In sum, changes in welfare between steady states can be described by changes in measured TFP and changes in the capital stock, and each of these are summarized by the changes in the home trade share. Additionally, changes in the consumption rate do not show up since the consumption rate is invariant to the trade regime.

There is no such convenient analytical representation of consumption along the transition path. Instead, we construct an accounting framework that decomposes the changes in consumption along the transition into the contribution of three components: measured productivity, capital accumulation, and the consumption rate.

**Transitional dynamics**  With $\beta < 1$ and $t^{**}$ chosen sufficiently large so that $\beta^{t^{**}-t^*} \approx 0$, we can measure the welfare change as

$$\lambda_n = (1 - \beta) \sum_{t=t^*+1}^{t^{**}} \beta^{t^*-t^*} \ln \left( \frac{C_{n,t}}{C_{n,t^*}} \right)$$  (17)

In the dynamic general equilibrium of the two specifications we study, consumption can be expressed as a (potentially time varying) share of total output, $Y_{n,t}$, with output being determined by productivity and the capital stock. Specifically, we have two equations that characterize the aggregate dynamics of consumption

$$C_{n,t} = \varphi_{n,t} Y_{n,t}$$
$$Y_{n,t} = Z_{n,t} K_{n,t}^{\alpha} L_{n}^{1-\alpha}$$

The term $\varphi_{n,t}$ is the consumption rate, i.e., the share of consumption in current income. In the Solow specification, this value is treated as an exogenous and constant parameter; in the neoclassical specification, this value is determined endogenously along the transition path. The variable $Z_{n,t}$ denotes *measured* total factor productivity (TFP) and, as is common in workhorse trade models, can be decomposed into an exogenous fundamental productivity and an endogenous component that is described by the home trade share and trade elasticity:

$$Z_{n,t} \propto A_{n,t} (\pi_{n,n,t})^{-1/\theta}$$

We can thus fully characterize the path for consumption, relative to the initial steady state level, in terms of the paths for the consumption rate, the home trade share, and the
capital stock as
\[
\frac{C_{n,t+1}}{C_{n,t^*}} = \left( \frac{\varphi_{n,t+1}}{\varphi_{n,t}} \right) \left( \frac{\pi_{n,n,t+1}}{\pi_{n,n,t^*}} \right)^{-1/\theta} \left( \frac{K_{n,t+1}}{K_{n,t^*}} \right)^{\alpha}
\]

Taking logs and inserting into equation (17), the dynamic welfare change becomes

\[
\lambda_n = (1 - \beta) \sum_{t = t^* + 1}^{\tau^*} \beta^{t-t^*} \left[ \ln \left( \frac{\varphi_{n,t+1}}{\varphi_{n,t}} \right) - \left( \frac{1}{\theta} \right) \ln \left( \frac{\pi_{n,n,t+1}}{\pi_{n,n,t^*}} \right) + \alpha \ln \left( \frac{K_{n,t+1}}{K_{n,t^*}} \right) \right]
\]

\[
= (1 - \beta) \sum_{t = t^* + 1}^{\tau^*} \beta^{t-t^*} \ln \left( \frac{\varphi_{n,t+1}}{\varphi_{n,t}} \right)
\]

\[
- (1 - \beta) \left( \frac{1}{\theta} \right) \sum_{t = t^* + 1}^{\tau^*} \beta^{t-t^*} \ln \left( \frac{\pi_{n,n,t+1}}{\pi_{n,n,t^*}} \right)
\]

\[
+ (1 - \beta)\alpha \sum_{t = t^* + 1}^{\tau^*} \beta^{t-t^*} \ln \left( \frac{K_{n,t+1}}{K_{n,t^*}} \right)
\]

The first term, which we denote by \( \lambda_n^\varphi \), characterizes the direct contribution to welfare from changes in the consumption share. The second term, \( \lambda_n^\pi \), characterizes the contribution from the home trade share. The third term, \( \lambda_n^K \), directly captures the contribution from changes in the capital stock. Each of these terms are dynamically co-dependent as well. Changes in the consumption/investment rate in one period induce changes in the capital stock in subsequent periods; and to the extent that capital grows by more in one country than another, relative country size changes lead to changes in home trade shares. That said, the accounting device is useful to illustrate how each component contributes to dynamics welfare changes along the transition.

We then define \( \zeta_n = \lambda_n^{Lib} + \lambda_n^{Pro} \) as the dynamic welfare asymmetry, or the difference between the gains from liberalization and the absolute value of the losses from protection. In turn, we define \( \zeta_n^\varphi, \zeta_n^\pi, \) and \( \zeta_n^K \) as the contributions to dynamic welfare asymmetries stemming from the three main components: consumption rate, measured productivity (inverse home trade share) and capital stock. By construction, \( \zeta_n = \zeta_n^\varphi + \zeta_n^\pi + \zeta_n^K. \)

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**Definition 2.2.** Two scenarios are dynamically symmetric if, period-by-period, the gross growth rate in one scenario is equal to the inverse of the gross growth rate in the other.

Figure 3 provides a generic illustration of what transition paths for consumption might look like in our neoclassical specification following trade liberalization and following protectionism. The economy stays at the initial steady state through period 0; then, in period 1, the shock hits and the economy begins transitioning to a new terminal steady state. In the left panel, the original paths for consumption \( \{C_t\} \) are shown. The liberalization path (“Lib”) begins in a steady state at a value of 1, then gradually transits to a new steady state of 2. The protectionism path (“Pro”) begins in a steady state at a value of 2, then gradually transits to a new steady state of 1. Thus, the two scenarios are steady state symmetric as per Definition 2.1. Mere visual inspection does not reveal whether or not the two paths are symmetric along the transition. To investigate the presence of asymmetries, the center panel involves two steps. The first step is to compute log of the normalized consumption path under each trade policy (i.e, liberalization or protectionism) by its own initial steady state, \( \{\ln(C_t/C_0)\} \), meaning that both paths begin at the value zero. The second step negates the protectionism path. If the two transition paths are dynamically symmetric, as per Definition 2.2 then the inverse protection path (increasing dashed curve in the middle panel) would overlap with the liberalization path (solid curve in the middle panel).

The figure shows asymmetries in the change of consumption between both trade policies: The rate of consumption growth under liberalization is higher than the drop under protectionism. To map the asymmetries in consumption into asymmetries in welfare, we also need to take into account the role of discounting. The right panel plots the discounted log-normalized paths, \( \{\beta^{t-1}\ln(C_t/C_0)\} \), with the protectionism path still inverted. The area enclosed by the curves and the zero line captures the dynamic welfare change, relative to the initial steady state. The area between these two paths graphically depicts the dynamic welfare asymmetry. Since the liberalization path lies above the inverse protectionism path in this example, the asymmetry would be positive, meaning that the gains from liberalization exceed the losses from protectionism.
3 Quantitative Analysis

Our quantitative approach is to confront the model with data on bilateral trade and production patterns across a large sample of countries. In our model the only parameters that differ across countries are the population size, the productivity level, and bilateral trade costs. We choose the model’s parameters so that they are broadly consistent with those observations, and examine dynamic equilibrium implications of changes in trade costs. In all of our exercises we hold population size and productivity unchanged, relative to the baseline, and vary only the bilateral trade costs.

3.1 Calibration

Our analysis is carried out using a set of 136 countries, spanning the world income distribution, and we assume the world economy is initially in steady state as of 2015. This year is prior to the Brexit vote and the U.S.-China trade war, thus, is unlikely to reflect any anticipatory effects of those shocks. Each period in the model corresponds to one year.

The values for the common parameters are reported in Table 1. We set the discount factor to $\beta = 0.96$ so that the steady-state real interest rate is about 4 percent. We set the share of capital in value added to $\alpha = 0.33$ (Gollin 2002) and the rate of depreciation $\delta = 0.06$. Finally, we use recent estimates of the trade elasticity by Simonovska and Waugh (2014) and set $\theta = 4$.

We calibrate the productivity parameter, $A_t$, and the trade costs $d_{nt}$ using data on bilateral trade flows, geography variables, and income per capita from CEPII.\footnote{http://www.cepii.fr/} We run the following gravity regression with PPML methods as in Silva and Tenreyro (2006).
Table 1: Common parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.96</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital’s share in value added</td>
<td>0.33</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Rate of capital depreciation</td>
<td>0.06</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Trade elasticity</td>
<td>4</td>
</tr>
</tbody>
</table>

$\left( \frac{X_{n,i}}{X_{n,n}} \right) = \exp \left( \sum_{p=1}^{6} \gamma^p \text{dist}_{n,i}^p + \beta B_{n,i} + \ln(S_i) - \ln(S_n) - e_i + u_{n,i} \right),$

where $\frac{X_{n,i}}{X_{n,n}}$ is the ratio country $n$’s import country $i$, relative to its domestic purchases; $\text{dist}_{n,i}^p$ is the contribution to trade costs of the distance between country $n$ and $i$ falling into the $p^{th}$ interval (in miles), defined as $[0,350]$, $[350, 750]$, $[750, 1500]$, $[1500, 3000]$, $[3000, 6000]$, $[6000, \text{maximum}]$; the other control variables are in $B_{n,i}$, and include common border effect, common currency effect, and regional trade agreement, between country $n$ and country $i$. Structurally $S_n = A_n (r_n^{1-\alpha}w_n) - \theta$ is the estimated importer fixed effect for country $n$. Using on rental rates, labor compensation, and $\theta = 4$, we recover $A_n$ from the fixed effect. We obtain $e_i$ as the estimated exporter fixed for country $i$ effect plus $A_i$. Finally, we obtain trade costs from the following expression:

$$-\theta \ln(d_{n,i}) = \sum_{p=1}^{6} \gamma^p \text{dist}_{n,i}^p + \beta B_{n,i} - e_i.$$

Note that we include an exporter fixed effect in the trade cost, $e_i$, which has been shown to fit better the patterns in both country incomes and observed price levels (see Waugh, 2010).

The results are reported in Figure 5. The left panel shows a strong positive correlation between the estimated productivity, relative to the United States, and relative income per capita. The right panel shows countries with higher income per capita have lower export costs, computed as the trade-weighted value of bilateral trade costs of each exporting country.

3.1.1 Model Fit

Figure 5 shows the model fit for the targeted moments—bilateral trade shares (left panel) and home-trade shares (right panel). We find that the correlation between bilateral trade shares in the model and in the data is 0.77; the correlation between home-trade shares in the model and in the data is 0.95.
Figure 4: Calibrated Productivity and Trade Costs

**Fundamental Productivity (left)**

![Fundamental Productivity Graph](image)

**Export Costs (right)**

![Export Costs Graph](image)

Notes: Export cost is computed as the trade-weighted value of bilateral export costs for each country.

Figure 4 shows that the model also does a good job at matching the non-targeted moments—the share in world GDP (left panel) and the share in world trade (right panel). The correlation between the model and the data is 1.00 for the share in world GDP, and is 0.98 for the share in world trade. Finally, the correlation between the model and data for income per capita is 0.82.

Figure 5: Model Fit for Trade Flows

**Bilateral Trade Shares (left)**

![Bilateral Trade Shares Graph](image)

**Home Trade Shares (right)**

![Home Trade Shares Graph](image)

Non-targeted moments. Share in world GDP. Share in world trade. Figure 5.
3.2 Dynamic Welfare Asymmetries

We investigate asymmetries in dynamic welfare gains from trade under two scenarios: Liberalization and protectionism. In the liberalization (Lib) scenario, we begin in a steady state with autarky level trade costs, $d_{n,i} = 10,000$, and then reduce the trade costs to the calibrated levels. In the protection (Pro) scenario, we begin in a steady state with the calibrated level trade costs, and then increase the trade costs to the autarky levels. In both scenarios, we assume that the change in trade cost is a one-time permanent event that happens unexpectedly, and after the sudden change, agents have perfect foresight thereafter. By design, the two changes in trade costs are symmetric, in that the initial distribution of trade costs in one scenario is identical to the final distribution of trade costs in the other. In turn, the two scenarios are steady-state symmetric. Thus, any dynamic asymmetries that emerge purely reflect asymmetric transitional dynamics. Both scenarios are implemented using the two different model specifications described above: the Solow specification and the neoclassical specification.

3.2.1 Dynamic Welfare Gains and Asymmetries

The distribution of welfare changes under each scenario (Lib and Pro) are very similar both in the Solow and in the neoclassical specifications. Among the 136 countries, the distribution of welfare changes from liberalization range from 1.3% to 72.9% in the Solow specification (blue bars, left panel of Figure 8) and from 1.3% to 72.6% in the neoclassical specification (blue bars, right panel of Figure 8). In both cases, the gains are skewed with a mean of
Figure 7: Dynamic Gains and Losses

Notes: Left column depicts results from the Solow version of the model with an exogenous consumption/investment rate. Right column depicts results from the neoclassical version of the model with endogenous consumption/investment rate.

15.1 and a median of 9.2. The ranking of countries in terms of gains and losses under both specifications is identical. The countries at the 5th, 50th, and 95th percentiles are Australia (AUS), Burkina Faso (BFA), and Vietnam (VNM), respectively.

The distribution of losses under protectionism is quite similar to the distribution of gains from liberalization in terms of country ranking (yellow bars, Figure 8). However, when comparing the magnitude of welfare changes under each scenario, we find that the gains from liberalization exceed the losses from protection in both model specifications. That is, we find positive asymmetries in both specifications. The median gain from liberalization is 9.2% and the median loss from protectionism is 9.1%, yielding a 0.1% asymmetry for the median country.

In general, we find that asymmetries are small for most countries. This reflects the fact that the gains from trade are also small for most countries. Given the large skew in the magnitude of the gains from trade, there is also a large skew in the magnitude of the asymmetry. Indeed, the asymmetry is positive in most countries, reaching as high as 4.7% in the Solow model, and as high as 5.5% in the neoclassical model. A small share of countries in our sample have negative asymmetries (i.e., the losses from protectionism exceed the gains from liberalization). In the Solow specification, the asymmetry is negative in 46 countries, whereas there are 11 such countries in the neoclassical specification. However, the negative asymmetries are quite small in magnitude, with the greatest negative asymmetry coming in at −0.01% in the Solow specification, and at −0.02% in the neoclassical specification.
3.2.2 Decomposing the contributions to asymmetries

We now decompose the welfare asymmetries using equation (18). We focus on Vietnam, which falls in the 95 percentile of the dynamic gains from trade. Its total dynamic welfare asymmetry is 3.2% in the Solow model, and is 3.7% in the neoclassical model. Figure 8 illustrates the contribution to dynamic asymmetries from the three components of equation (18): consumption share ($\zeta^c$), measured productivity ($\zeta^\pi$), and the growth in capital stock ($\zeta^K$).

In the Solow specification the capital stock dynamics accounts for more than half (2.2% out of 3.2%) of the dynamic welfare asymmetry. The remaining is explained by measured productivity. In the neoclassical model, the consumption rate accounts for a large component of the dynamic asymmetry (1.4% out of 3.7%), a bit less than the contribution from the capital stock (1.7% out of 3.7%). The remaining 0.6%is accounted for by the measured productivity.

Figure 8: Dynamic welfare asymmetries and decomposition by channel

Solow Specification (left) Neoclassical Specification (right)

Notes: Left column depicts results from the Solow version of the model with an exogenous consumption/investment rate. Right column depicts results from the neoclassical version of the model with endogenous consumption/investment rate.

Figure 9 illustrates how the asymmetries emerge over the transition path for the case of Vietnam. The top row plots the transitional dynamics of the discounted log-change of consumption from its initial steady state and each of its components using equation (18). The left panel shows the results for the Solow specification, whereas the right panel studies the neoclassical specification. In each panel the solid line corresponds to the liberalization scenario, and the dashed line to the protectionism scenario.

In both model specifications, the initial boost in consumption under liberalization exceeds the initial drop in consumption under protection (first row of Figure 9), in absolute value. For a few periods the growth rate of consumption under liberalization continues to exceed the
Figure 9: Transitional Dynamics: Discounted Log-Change from Initial Steady State

**Notes:** Discounted transition paths in log-deviations from initial steady state. "Lib" refers to transition path under trade liberalization following a change from autarky to observed trade costs. "Pro" refers to inverse transition path under protectionism following a change from observed trade costs to autarky. Left column depicts results from the Solow version of the model with an exogenous consumption/investment rate. Right column depicts results from the neoclassical version of the model with endogenous consumption/investment rate.

Rate of decline under protection. Eventually the relationship between the two growth rates reverses. The reason for the eventual reversal is that the two steady states are symmetric, so the total distance covered between steady states is the same in both directions. Since the future is discounted, the growth differences in the beginning dominate the welfare asymmetry,
so that the total dynamic gains from liberalization exceed the losses from protection. This means that the asymmetries mostly materialize in the short-to-medium run.

We now consider the importance of the consumption rate channel in the second row of Figure 9. In the Solow specification (left column), the contribution from the consumption rate is zero by definition, since the consumption rate is exogenous and constant. In the neoclassical model (right column), the consumption rate decreases under liberalization and increases under protectionism as a means to smooth consumption over time. Under liberalization, the consumption rate decreases in magnitude by less than the corresponding increase under protection. The reason being that, under protection, the household can consume the pre-existing capital stock. To the contrary, under liberalization, as capital accumulates and output expands, the share of output allocated to investment needn’t be as large to facilitate further expansion.

The third row of Figure 9 illustrates the transition paths for the measured productivity channel, which is accounted entirely by change in home trade share. This channel is, for the most part, symmetric. However, small asymmetries emerge in the first few periods. The reason is that, as countries accumulate capital at different rates, relative country size changes along the transition so trade shares adjust in response, even in the absence of further changes in trade costs. Under liberalization faster growing countries grow in relative size and undergo an increase in their home trade share after the initial period, and consequently a decrease in measured productivity. On the other hand, under protectionism, there is no under or over shooting for any countries since the home trade share moves to one permanently. This induces asymmetric paths for measured productivity. In the case of Vietnam, being a small country, under liberalization its productivity overshoots under liberalization. For larger countries, the measured productivity under shoots under liberalization.

Finally, in both specifications the primary channel for asymmetries is the capital stock, for which depreciation plays a critical role.

3.2.3 The role of depreciation

The transition path for capital evolves asymmetrically in the short run. Capital depreciation plays a central role in generating asymmetries in the change of consumption during the transition. To illustrate the role of capital depreciation, we can abstract from international trade and assume exogenous, one time, permanent, and symmetric shocks to TFP. This abstraction captures the fact that, through ACR effects, measured TFP increases when trade barriers fall, reflecting decreases in the home trade share. After the shock, the home trade
share only adjusts in response to differential rates of capital accumulation across countries.

We begin the world in steady state in time 0 and consider two scenarios: (i) an increase in TFP and (ii) a symmetric decrease in TFP. Both cases start at the same initial steady state in order to abstract from differences in initial conditions. The long-run steady state changes in capital are symmetric across these two scenarios.

Define symmetric changes in TFP in period 1, relative to their steady state levels:

$$\Gamma^\text{Lib}_z = \frac{Z^\text{Lib}}{Z_0} > 1,$$

$$\Gamma^\text{Pro}_z, 1 = \frac{Z^\text{Pro}}{Z_0} = \frac{1}{\Gamma^\text{Lib}_z} < 1.$$

Since the capital stock does not change on impact, relative to its initial level the change in output in the first period equals the change in TFP. Since the ratios of both consumption and investment are proportional to output, the changes in consumption and investment are also symmetric; that is,

$$\frac{C^\text{Lib}_1}{C_0} \times \frac{C^\text{Pro}_1}{C_0} = \frac{X^\text{Lib}_1}{X_0} \times \frac{X^\text{Pro}_1}{X_0} = \Gamma^\text{Lib}_z \times \frac{1}{\Gamma^\text{Pro}_z} = 1.$$

The asymmetries emerge beginning in period 2. To see this, consider the change in capital from period 1 to period 2:

$$\Gamma_{k,2}^\text{Lib} = \frac{K^2}{K_1} = (1 - \delta) + \frac{X_1}{K_0}$$

$$= (1 - \delta) + \left(\frac{X_1}{X_0}\right) \left(\frac{X_0}{K_0}\right)$$

$$= (1 - \delta) + \left(\frac{X_1}{X_0}\right) \delta$$

In the case of a positive shock, $\frac{X^\text{Lib}_1}{X_0} = \Gamma_z$; analogously, under a negative shock, $\frac{X^\text{Pro}_1}{X_0} = \frac{1}{\Gamma_z}$. Substitute this into the law of motion for capital, and express the asymmetry in the capital stock as of period 2:

$$\Gamma_{k,2}^\text{Lib} \times \Gamma_{k,2}^\text{Pro} = (1 - \delta + \delta \Gamma_z) \left(1 - \delta + \frac{\delta}{\Gamma_z}\right)$$

$$= (1 - \delta)^2 + \delta(1 - \delta) \left(\Gamma_z + \frac{1}{\Gamma_z}\right) + \delta^2$$

First, consider the extreme case with $\delta = 1$. In this case $\Gamma_{k,2}^\text{Lib} \times \Gamma_{k,2}^\text{Pro} = \delta^2 = 1$. That is, with full depreciation, we revert to a static model with no asymmetries.

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Second, consider the other extreme case with \( \delta = 0 \). In this case capital does not depreciate so there is no well-defined steady state, since even small amounts of investment lead to an unbounded sequence of capital. Hence, we resort to a limiting argument. As \( \delta \to 0 \), \( \Gamma^{\text{Lib}}_{k,2} \times \Gamma^{\text{Pro}}_{k,2} \to (1 - \delta)^2 = 1 \).

Finally, consider the more interesting and relevant case with \( \delta \in (0, 1) \). In this case, \( \Gamma^{\text{Lib}}_{k,2} \times \Gamma^{\text{Pro}}_{k,2} = 1 + \delta (1 - \delta) \left( \Gamma_z + \frac{1}{\Gamma_z} \right) > 1 \). This implies that the magnitude of the change in welfare from period 1 to 2 is greater following an increase in TFP than it is following a symmetric decline in TFP. Moreover, the magnitude of the asymmetry is higher for larger TFP shocks (i.e., higher \( \Gamma_z \)).

In this closed economy example TFP does not change after period one, so all of the change in output and, hence, consumption, are driven by changes in capital. Hence, the asymmetries for consumption emerge in the same direction as for the capital stock. Consider the change in consumption between periods 1 and 2.

\[
\frac{C^{\text{Lib}}_2}{C_1} = \left( \frac{K^{\text{Lib}}_2}{K_1} \right)^\alpha = \left( \frac{K^{\text{Lib}}_2}{K^*} \right)^\alpha = (\Gamma^{\text{Lib}}_{k,2})^\alpha, \\
\frac{C^{\text{Pro}}_2}{C_1} = \left( \frac{K^{\text{Pro}}_2}{K_1} \right)^\alpha = \left( \frac{K^{\text{Pro}}_2}{K^*} \right)^\alpha = (\Gamma^{\text{Pro}}_{k,2})^\alpha.
\]

Once we examine an open economy and interpret the productivity shocks as changes in measured productivity resulting from changes in trade costs, then the change in productivity is not a once-and-for all permanent shock. As economies accumulate capital at differential rates, relative sizes change so home trade shares, and hence, measured productivity, change along the transition path. In summary, in a model with a gradual depreciation rate of capital, changes in consumption with respect to the two symmetric trade reforms are asymmetric, and the size of the asymmetry increases with the magnitude of the changes of TFP and depends on the rate of depreciation.

In the Solow specification, it was possible to characterize the growth rate in consumption in the first two periods because both consumption and investment are proportional to output. In a model with an endogenous investment rate, it is not possible to analytically characterize consumption growth in any period in terms of the TFP shock.

Next we carry out a quantitative analysis to evaluate the implications of capital depreciation on welfare asymmetries. To do this we compare the gains from a permanent liberalization to the losses from permanent protectionism under a range of values for the depreciation rate ranging from 0.01 to 0.99. The dynamic asymmetry for Vietnam is plot-
Clearly, the asymmetry declines with the depreciation rate. It is highest when near no depreciation \((\delta = 0.01)\), and is close to zero under when near full depreciation \((\delta = 0.99)\).

**Figure 10: Dynamic Welfare Asymmetry Under Various Depreciation Rates**

3.3 Extension: Neoclassical Model with Irreversible Investment

In the neoclassical specification, we allowed for investment to be reversible, implying that capital can be consumed. This has immediate implications for the transitional dynamics following a protectionist trade policy by mediating the decline in consumption in early period following the shock. In this section we impose irreversible investment by introducing adjustment costs to capital accumulation in a manner that ensures that gross investment is always positive and, thus, capital cannot be consumed. Thus, the only way for the capital stock to decumulate is through depreciation. Specifically, the capital accumulation equation becomes

\[
K_{n,t+1} = (1 - \delta)K_{n,t} + X_{n,t}^\lambda (\delta K_{n,t})^{1-\lambda}.
\]

The additional parameter \(\lambda \in [0, 1]\) governs the adjustment cost or, specifically, the elasticity of capital accumulation with respect to investment. If \(\lambda = 1\) there is no adjustment cost and
this boils down to standard linear depreciation technology. Lower values of \( \lambda \) imply greater adjustment costs.

To simplify notation later on, we invert the capital accumulation equation and express aggregate investment as a function of the capital stock in periods \( t \) and \( t+1 \):

\[
X_{n,t} \equiv \Phi(K_{n,t+1}, K_{n,t}) = (K_{n,t+1} - (1 - \delta)K_{n,t})^{\frac{1}{\lambda}} (\delta K_{n,t})^{\frac{\lambda-1}{\lambda}}.
\]  (19)

The Euler equation governing optimal consumption dynamics is now given by

\[
\frac{C_{n,t+1}}{C_{n,t}} = \left( \frac{\delta^{\lambda-1} (1-\delta)}{\Phi_1(K_{n,t+1}, K_{n,t})} \right) \frac{\Phi_2(K_{n,t+1}, K_{n,t})}{\Phi_1(K_{n,t+1}, K_{n,t})},
\]  (20)

where \( \Phi_1 \) and \( \Phi_2 \) denotes the first derivative of the investment equation (19) with respect to the first and second arguments, respectively.7

The equilibrium conditions of this specification are provided in Table A.3.

Since gross investment must be positive, there are effectively borrowing limits so the formulation of the algorithm from before needs to be changed. In particular, the rate of return to investment in any given period is a function of the investment made in that period as well as the future period. Therefore, we cannot express lifetime income as in equation 15 as a function of only the factor prices since the return to capital depends on the investment decisions made along the transition. As a result, our previous A.2 will not work.

To handle this we introduce a different approach whereby we iterate on consumption rates. First, we guess the entire path for the consumption rates (which may vary across time and space) and solve a sequence of static models as in the Solow specification. we evaluate the resulting transition path to check whether the Euler equation holds in each country and time period. If it does not hold, we adjust the investment rates accordingly to align consumption growth with the real interest rate and go through the same steps. This algorithm is also general enough to handle the neoclassical specification with reversible investment in Section 2 which is a special case with \( \lambda = 1 \).

7Adjustment cost terms: \( \Phi_1(K', K) = \delta^{(\lambda-1)/\lambda} \left( \frac{K'}{K} - (1 - \delta) \right)^{(1-\lambda)/\lambda} \) and \( \Phi_2(K', K) = \Phi_1(K', K) \times \left( (\lambda - 1) \left( \frac{K'}{K} \right) - \lambda(1-\delta) \right) \). When \( \lambda = 1 \), \( \Phi_1 = 1 \) and \( \Phi_2 = \delta - 1 \).
3.3.1 Asymmetries in the Specification with Irreversible Investment

Note The steady state consumption rate is

$$\varphi_n^* \equiv \frac{P_n^* C_n^*}{r_n^* K_n^* + w_n^* L_n} = 1 - \frac{\alpha \delta}{\Phi_1^* + \Phi_2^*},$$

where, in steady state, $\Phi_1^* = 1/\lambda$ and $\Phi_2^* = \delta - 1/\lambda$

The steady state change in consumption is identical to that in the neoclassical specification since the adjustment costs only affect the transitional dynamics between the steady state. That is, there is no cost associated with replacing depreciated capital in the steady state, so the steady state is not impacted by the adjustment cost. For our quantitative analysis, we set $\lambda = 0.76$.

Figure 11: Dynamic Welfare Asymmetries and Decomposition by Channel in Extended Neoclassical Specification
4 Trade policy reversals

Asymmetries in welfare caused by capital accumulation and depreciation materialize in the short-run. As a result, these asymmetries have important implications for welfare changes of trade policy reversals over time. We study two different scenarios using the neoclassical
specification in Section 2. One, which we call *temporary liberalization*, is a scenario where the world begins in a protectionist (autarky) steady state. Then, unexpectedly, in period 1 the trade costs decline to observed levels, but only for a pre-determined period of time, say 5 periods, so that the world is temporarily in a state of liberalization. However, it is know with perfect foresight as of period 1 that the world will revert to autarky after 5 periods. The other scenario, *temporary protectionism* is the opposite. The world begins in the liberalized (observed) steady state in period 0, then unexpectedly moves to protection in period 1 and remains there for 5 periods, then reverts to the observed levels of trade costs thereafter. Again, the eventual reversal is known with perfect foresight as of period 1.

The left column of Figure 13 illustrates the results for temporary trade policies lasting 5 years, and the right column, 10 years. Each of the figures corresponds to Vietnam. The solid lines shows the effects of a temporary liberalization, and the dashed lines, temporary protectionism.

Begin with the temporary liberalization case. When there is perfect foresight, the expectation of a reversal of trade policy towards autarky sometime in the future increases consumption today, but less than if the liberalization were permanent. This consumption smoothing allows for higher investment in the short-run, hence a large capital stock being accumulated prior to the start of the protectionist policy. At that point, households start decumulating capital by letting it depreciate and potentially consuming some of it.

The asymmetric effects and the role of depreciation are stronger when the temporary trade liberalization lasts longer. We observe larger asymmetries in the right column. The reason is that households have more time to smooth consumption more and to gradually accumulate a larger stock of capital when the temporary trade liberalization takes longer to reverse. Indeed, the bottom row of Figure 13 shows that the increase in capital in the right panel is larger than in the left panel.

We observe similar patterns in the case when there is a temporary protectionist policy followed by permanent liberalization. The dashed line of Figure 13 shows the inverse effect of this policy on consumption, capital and productivity. The expectation of a reversal of trade policy towards liberalization in the future decreases consumption today, but the decrease is smoother if the temporary protectionist policy lasts longer. Comparing the solid line with the dashed line, we observe that the initial decrease in consumption during temporary protectionism is lower than during temporary liberalization. This translates into asymmetric welfare gains from trade policy. For Vietnam the welfare gain from a temporary liberalization lasting 5 years is 10.5%, and is 20.3% when it lasts 10 years. The welfare loss from a
temporary protectionist policy lasting 5 years is 7.0% and is 15.2% when it lasts 10 years. In turn, the welfare asymmetries under a 5-year reversal is 3.5% and under 10-year reversals is 5.1%. In the 10-year reversal scenario, the asymmetry exceeds that from a scenario with no reversal, which was 3.7%.

Figure 13: Transitional Dynamics with Trade Policy Reversals

Reverse in period 5 (left) Reverse in period 10 (right)

Notes: Discounted transition paths in log-deviations from initial steady state. "Temp Lib" refers to transition path under trade liberalization following a temporary change from autarky to observed trade costs, and the returning to autarky in period 5 (left) or period 10 (right). "Temp Pro" refers to inverse transition path under protectionism following a temporary change from observed trade costs to autarky, and then returning to observed trade costs in period 5 (left) or period 10 (right).
References


A Dynamic Equilibria and Computational Algorithms

This section of the Appendix provides equilibrium condition and algorithms to compute the exact transitional dynamics for each model specification. The equilibrium conditions for the Solow specification are given in Table A.1 and the corresponding methodology to compute the dynamic equilibrium is in Algorithm A.1. The equilibrium conditions for the neoclassical specification are given in Table A.2 and the corresponding methodology to compute the dynamic equilibrium is in Algorithm A.2. The equilibrium conditions for the neoclassical specification are given in Table A.3 and the corresponding methodology to compute the dynamic equilibrium is in Algorithm A.3.

Table A.1: Equilibrium conditions in one sector model with exogenous investment rate

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
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<tr>
<td>1</td>
<td>( p_{n,i,t} = \frac{u_{i,t}}{A_i} ) ∀(n, t)</td>
</tr>
<tr>
<td>2</td>
<td>( p_{n,t} = \left( \sum_{i=1}^{N} (p_{n,i,t})^{-\theta} \right)^{-\frac{1}{\theta}} ) ∀(n, t)</td>
</tr>
<tr>
<td>3</td>
<td>( p_{n,t}C_{n,t} = \varphi(r_{n,t}K_{n,t} + w_{n,t}L_n) ) ∀(n, t)</td>
</tr>
<tr>
<td>4</td>
<td>( p_{n,t}C_{n,t} + p_{n,t}X_{n,t} = r_{n,t}K_{n,t} + w_{n,t}L_n ) ∀(n, t)</td>
</tr>
<tr>
<td>5</td>
<td>( Q_{n,t} = C_{n,t} + X_{n,t} ) ∀(n, t)</td>
</tr>
<tr>
<td>6</td>
<td>( q_{n,i,t} = (p_{n,i,t}/p_{n,t})^{-1}(1+\theta)Q_{n,t} ) ∀(n, i, t)</td>
</tr>
<tr>
<td>7</td>
<td>( p_{n,t}y_{n,t} = \sum_{i=1}^{N} p_{n,i,t}q_{n,i,t} ) ∀(n, t)</td>
</tr>
<tr>
<td>8</td>
<td>( r_{n,t}k_{n,t} = \alpha p_{n,t}y_{n,t} ) ∀(n, t)</td>
</tr>
<tr>
<td>9</td>
<td>( w_{n,t}\ell_{n,t} = (1-\alpha)p_{n,t}y_{n,t} ) ∀(n, t)</td>
</tr>
<tr>
<td>10</td>
<td>( K_{n,t} = k_{n,t} ) ∀(n, t)</td>
</tr>
<tr>
<td>11</td>
<td>( L_n = \ell_{n,t} ) ∀(n, t)</td>
</tr>
<tr>
<td>12</td>
<td>( p_{n,t}Q_{n,t} = p_{n,t}y_{n,t} ) ∀(n, t)</td>
</tr>
<tr>
<td>13</td>
<td>( K_{n,t+1} = (1-\delta)K_{n,t} + X_{n,t} ) ∀(n, t)</td>
</tr>
<tr>
<td>13s</td>
<td>( X_{n} = \delta K_{n,t} ) ∀(n, t)</td>
</tr>
</tbody>
</table>

Note: Units costs \( u_{n,t} = B (r_{n,t})^\alpha (w_{n,t})^{(1-\alpha)} \) with constant \( B = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \). In steady state, condition 13 collapses to condition 13s.
Algorithm A.1 Model with exogenous investment rate

The following steps reference equilibrium conditions in Table A.1.

1. Start in period 1 with the initial capital stock, $K_{n,1}$, as given. In other periods $t > 1$, capital is pre-determined from the equilibrium investment in period $t - 1$ (described below).

   (a) Make a guess at vector of wages for period $t$: $w_t$. Normalize this guess to the $(N - 1)$-simplex: $\Delta \equiv \{ w \in \mathbb{R}^N : \sum_{n=1}^{N} w_n L_n = 1 \}$.

   (b) Use conditions 8–11 to solve for the vector of rental rates, $r_{n,t}(w_t)$.

   (c) Compute bilateral prices, $p_{n,i,t}(w_t)$, using condition 1. This notation exposes the dependency of the computed prices on the guessed factor prices. Then compute country-level prices, $p_{n,t}(w_t)$, using condition 2.

   (d) Compute investment, $X_{n,t}(w_t)$ and consumption $C_{n,t}(w_t)$, using condition 3 and 4, respectively.

   (e) Solve for total absorption, $Q_{n,t}(w_t)$, using condition 5 and bilateral trade, $q_{n,i,t}(w_t)$, using condition 6.

   (f) Solve for total output, $y_{n,t}(w_t)$, and factor demands—$k_{n,t}(w_t)$ and $\ell_{n,i,t}(w_t)$—using conditions 7–9, respectively.

   (g) Check factor market clearing to see if condition 11 holds. Our preferred metric is the maximum absolute excess demand:

   $$\text{MAED} = \max_{n=1,\ldots,N} \{|\ell_{n,t}(w_t) - L_n|\}.$$  

   If MAED is less than some tolerance (in which case, by Walras’ Law, condition 12 also holds) then proceed to step 2. Otherwise, update the vector of wages as follows:

   $$w_{n,t}^{new} = \frac{w_{n,t} \ell_{n,t}(w_t)}{L_n}$$

   Normalize this updated wage vector to be in the simplex and return to step 1(b).

2. Compute the capital stock in the next period, $K_{n,t+1}$, using condition 13.

3. Return to step 1(a) with the next-period capital stock and continue through period $T$, where $T$ is sufficiently high enough to ensure the economy is “in” the new steady state.
Table A.2: Equilibrium conditions in one sector model with reversible investment

<table>
<thead>
<tr>
<th>Condition</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_{n,t} = \frac{u_{n,t} q_{n,t}}{A_i}$</td>
</tr>
<tr>
<td>2</td>
<td>$p_{n,t} = \left( \sum_{i=1}^{N} (p_{n,i,t})^{-\theta} \right)^{-1/\theta}$</td>
</tr>
<tr>
<td>3</td>
<td>$p_{n,t} C_{n,t} + p_{n,t} X_{n,t} = r_{n,t} K_{n,t} + w_{n,t} L_{n}$</td>
</tr>
<tr>
<td>4</td>
<td>$K_{n,t+1} = (1 - \delta) K_{n,t} + X_{n,t}$</td>
</tr>
<tr>
<td>5</td>
<td>$C_{n,t} = (1 + R_{n,t}) \left( \frac{\beta^t}{\sum_{t'=1}^{T} \beta^{t'}} \right) W_{n}$</td>
</tr>
<tr>
<td>6</td>
<td>$W_{n} = \sum_{t=1}^{T} \frac{w_{n,t} L}{P_{n,t}(1+R_{n,t})} + K_{n,1} - \frac{K_{n,T+1}}{1+R_{n,T}}$</td>
</tr>
<tr>
<td>7</td>
<td>$1 + R_{n,t} \equiv \prod_{t'=1}^{t} (r_{n,t'}^t - \delta)$</td>
</tr>
<tr>
<td>8</td>
<td>$Q_{n,t} = C_{n,t} + X_{n,t}$</td>
</tr>
<tr>
<td>9</td>
<td>$q_{n,i,t} = (p_{n,i,t}/p_{n,t})^{-\theta} Q_{n,t}$</td>
</tr>
<tr>
<td>10</td>
<td>$p_{n,t} y_{n,t} = \sum_{i=1}^{N} p_{n,i,t} q_{n,i,t}$</td>
</tr>
<tr>
<td>11</td>
<td>$r_{n,t} k_{n,t} = \alpha p_{n,t} y_{n,t}$</td>
</tr>
<tr>
<td>12</td>
<td>$w_{n,t} \ell_{n,t} = (1 - \alpha) p_{n,t} y_{n,t}$</td>
</tr>
<tr>
<td>13</td>
<td>$K_{n,t} = k_{n,t}$</td>
</tr>
<tr>
<td>14</td>
<td>$L_{n} = \ell_{n,t}$</td>
</tr>
<tr>
<td>15</td>
<td>$p_{n,t} Q_{n,t} = p_{n,t} y_{n,t}$</td>
</tr>
</tbody>
</table>

Note: Units costs $u_{n,t} = B (r_{n,t})^\alpha (w_{n,t})^{(1-\alpha)}$ with constant $B = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)}$. In steady state, conditions 4 and 5 collapse to conditions 4s and 5s, respectively.
Algorithm A.2 Model with endogenous, reversible investment

The following steps reference equilibrium conditions in Table A.2 given initial and terminal (steady state) capital stocks \((K_1, K_{T+1})\).

1. Make a guess at matrices of factor prices \(\{w_t, r_t\}_{t=1}^T\). Normalize this guess to the \((N-1) \times N \times T\)-simplex: \(\Delta = \left\{ \left( \{w_t, r_t\}_{t=1}^T \right) \in \mathbb{R}^{2NT} : \sum_{n=1}^{N} w_{n,t} L_n = 1, \forall (t) \right\}\).

2. Compute bilateral prices, \(p_{n,i,t} \left( \{w_t, r_t\}_{t=1}^T \right)\), using condition 1. Then compute country-level prices, \(p_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\), using condition 2.

3. First, construct the compounded return to capital \(R_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\) using condition 7, then compute the net-present value of lifetime wealth \(W_n \left( \{w_t, r_t\}_{t=1}^T \right)\) using condition 8. Finally, compute the path for consumption \(C_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\), using condition 5.

4. Compute the paths for investment, \(X_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\), and capital, \(K_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\), using conditions 3 and 4, respectively.

5. Solve for total absorption, \(Q_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\) and bilateral trade, \(q_{n,i,t} \left( \{w_t, r_t\}_{t=1}^T \right)\), using conditions 8 and 9, respectively.

6. Solve for total output, \(y_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\), and factor demands—\(k_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\) and \(\ell_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)\)—using conditions 10–12, respectively.

7. Check factor market clearing to see if conditions 13 and 14 hold:

\[
\text{MAED} = \max_{n=1,\ldots,N, t=1,\ldots,T} \left\{ |k_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right) - K_{n,t}| + |\ell_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right) - L_n| \right\}.
\]

If MAED is higher than some tolerance, then update the matrices of factor prices:

\[
w_{n,t}^{\text{new}} = \frac{w_{n,t} L_t}{L_n} \left( \{w_t, r_t\}_{t=1}^T \right), \quad r_{n,t}^{\text{new}} = \frac{r_{n,t} k_{n,t}}{K_{n,t} \left( \{w_t, r_t\}_{t=1}^T \right)} \left( \{w_t, r_t\}_{t=1}^T \right).
\]

Normalize these updated factor prices to be in the simplex and return to step 2. If MAED is higher than some tolerance, then condition 15 will automatically hold by Walras’ law.
Table A.3: Equilibrium conditions in one sector model with irreversible investment

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( p_{n,t} = \frac{u_{i,t} d_{n,t}}{A_i} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>2</td>
<td>( p_{n,t} = \left( \sum_{i=1}^{N} (p_{n,i,t})^{-\theta} \right)^{-1/\theta} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>3</td>
<td>( p_{n,t} C_{n,t} + p_{n,t} X_{n,t} = r_{n,t} K_{n,t} + w_{n,t} L_n ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>4</td>
<td>( K_{n,t+1} = (1 - \delta) K_{n,t} + X_{n,t}^\lambda (\delta K_{n,t})^{1-\lambda} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>5</td>
<td>( C_{n,t+1}/C_{n,t} = \beta \left( \frac{r_{n,t+1}}{p_{n,t+1}} - \Phi_2 (K_{n,t+2}, K_{n,t+1}) \right) / \Phi_1 (K_{n,t+1}, K_{n,t}) ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>6</td>
<td>( Q_{n,t} = C_{n,t} + X_{n,t} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>7</td>
<td>( q_{n,i,t} = \left( \frac{p_{n,i,t}}{p_{n,t}} \right)^{(1+\theta)} Q_{n,t} ) ( \forall (n, i, t) )</td>
</tr>
<tr>
<td>8</td>
<td>( p_{n,t} y_{n,t} = \sum_{i=1}^{N} p_{n,t} q_{i,n,t} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>9</td>
<td>( r_{n,t} k_{n,t} = \alpha p_{n,t} y_{n,t} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>10</td>
<td>( w_{n,t} \ell_{n,t} = (1 - \alpha) p_{n,t} y_{n,t} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>11</td>
<td>( K_{n,t} = k_{n,t} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>12</td>
<td>( L_n = \ell_{n,t} ) ( \forall (n, t) )</td>
</tr>
<tr>
<td>13</td>
<td>( p_{n,t} Q_{n,t} = p_{n,t} y_{n,t} ) ( \forall (n, t) )</td>
</tr>
</tbody>
</table>

4s | \( X_n^* = \delta K_n^* \) \( \forall (n) \) |

5s | \( r_n^* = \left( \frac{1}{\lambda} \right) \left( \frac{1}{\beta} - (1 - \delta \lambda) \right) p_n^* \) \( \forall (n) \) |

Note: Units costs \( u_{n,t} = B_n (r_{n,t})^\alpha (w_{n,t})^{(1-\alpha)} \) with constant \( B_n = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \). Adjustment costs imply \( \Phi_1 (K', K) = \delta^{(\lambda-1)/\lambda} \left( \frac{1}{\lambda} \right) \left( \frac{K'}{K} - (1 - \delta) \right)^{(1-\lambda)/\lambda} \) and \( \Phi_2 (K', K) = \Phi_1 (K', K) \times \left( (\lambda - 1) \left( \frac{K'}{K} - \lambda (1 - \delta) \right) \right) \). In steady state, conditions 4 and 5 collapse to conditions 4s and 5s, respectively.
Algorithm A.3 Model with endogenous, irreversible investment

The following steps reference equilibrium conditions in Table A.3.

1. Guess a $N \times T$ matrix of nominal investment rates $\{\mathbf{\rho}_t\} \in \mathbb{R}^{NT}$.

2. Given the matrix of investment rates, solve for the “sub-equilibrium” outcomes as in the model with exogenous investment rates using the algorithm in Section A.1. The prices and allocations automatically satisfy all conditions except for, possibly, condition 5.

3. Given sequences of prices and quantities, check condition 5 by defining excess saving as

$$Z_{n,t} \left( \{\mathbf{\rho}_t\}_{t=1}^T \right) \equiv \beta \left( \frac{r_{n,t+1}(\mathbf{\rho}_t)_{t=1}^T - \Phi_2(K_{n,t+1}(\mathbf{\rho}_t)_{t=1}^T),K_{n,t+1}(\mathbf{\rho}_t)_{t=1}^T)}{\Phi_1(K_{n,t+1}(\mathbf{\rho}_t)_{t=1}^T),K_{n,t}(\mathbf{\rho}_t)_{t=1}^T)} - \frac{C_{n,t+1}(\mathbf{\rho}_t)_{t=1}^T}{C_{n,t}(\mathbf{\rho}_t)_{t=1}^T} \right),$$

where our notation makes explicit the dependence of the sub-equilibrium on the guessed matrix of investment rates. Our preferred metric is the \textbf{M}aximum \textbf{A}bsolute \textbf{E}xcess \textbf{D}emand:

$$MAED = \max_{n=1,...,N,t=1,...,T} \left\{ \left| Z_{n,t} \left( \{\mathbf{\rho}_t\}_{t=1}^T \right) \right| \right\}.$$  

If MAED is less than some tolerance, stop. Otherwise, update the investment rates return to step 2:

$$\mathbf{\rho}^{new}_{n,t} = \mathbf{\rho}_{n,t} + Z_{n,t} \left( \{\mathbf{\rho}_t\}_{t=1}^T \right).$$