Environment, growth, and optimal policy design

Hamid Mohtadi

Department of Economics, University of Wisconsin at Milwaukee, Milwaukee, WI 53201, USA

Received April 1993; final version received July 1995

Abstract

A simple dynamic representative agent model is presented in which the environment enters into the utility and production functions to analyze long-run economic growth under optimal policy designs. The policies that are considered are production taxes or subsidies and quantitative restrictions. Optimal levels of these instruments are designed by a regulator such that (a) the equilibrium growth path mimics the efficient growth path, and (b) the latter is maximized. One finding is that a combination of quantity controls and optimal tax (subsidy) schemes leads to a higher level of social welfare than an optimal tax (subsidy) scheme alone.

Keywords: Environment; Endogenous growth; Regulation

JEL classification: H2; Q2; O4

1. Introduction

How fast should economies grow when faced with environmental concerns? What policies should be adopted to assure economic growth while addressing environmental concerns? These questions have gained increasing importance among policy makers and are likely to continue their rise to prominence in the coming years. Economic models that have been developed to address these concerns often focus on the socially efficient growth path, but not the equilibrium (market-driven) growth path that diverges
from the efficient path due to environmental externalities. As such, they emphasize the potential but not the actual growth path of an environmentally or resource-constrained economy. A focus exclusively on the efficient path cannot satisfactorily address how actual policies should be designed to reach the first-best outcomes on the normative policy issues, and also cannot ‘explain’ actual or observed outcomes on the positive issues.

Focusing primarily on the first set of issues, this paper studies economic growth and optimal policy design when the quality of the environment is considered. Optimal policies here refer to policy instruments that are chosen by an actual government, rather than a central planner, and that induce the market-driven path to mimic the socially efficient path to achieve first-best results. Such an approach requires analyzing both the socially efficient and market-driven paths of growth. Optimal policies result from ‘aligning’ the two paths. Pigouvian taxes, subsidies or quantity regulations on the environment are thus analysed within a dynamic growth context. One interesting finding is that a combination of quantity controls and optimal tax or subsidy schemes leads to a higher level of social welfare than optimal tax or subsidy schemes alone.

The model is based on a representative agent economy consisting of many infinitely lived agents. Evidence suggests that environment affects the economy through amenity, health and productivity channels (e.g. World Bank, 1992, p. 44). The first two channels will be indicated in this paper by an overall ‘consumption’ effect in which environmental quality enters the utility function of agents directly. The third channel is represented by allowing environmental quality to enter the production function.

The paper’s focus on dynamics and externalities resembles the theories of endogenous growth such as the works of Lucas (1988), Romer (1990), Barro (1990), and Rebelo (1991). However, for the most part, the technology here is simply defined as constant returns to scale in capital stock (i.e. in some part of the economy machines produce machines), which is more akin to Rebelo than to other contributors of endogenous growth.

The paper is organized as follows. Section 2 develops the general model, Section 3 specifies a parametric version of the model, and then derives the optimal policy instruments under a steady state. Here, a steady state always exists if the environment affects society’s welfare (utility) but not the economy’s productivity. Thus, Section 4 generalizes the analysis of Section 3

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1 On the exhaustible resource and growth literature see, for example, Solow (1974), Solow and Wan (1976), Hartwick (1978), and Ayers (1988). On the environment and growth literature see Foster (1973), D’arge and Kogiku (1973), Krautkraemer (1985), and Smith (1977). Smith’s is the only one that contains some decentralized interpretations of optimal control theory.

2 For a separate treatment of the effects of environmental quality on health, see Mohtadi and Roe (1992).
by incorporating a productivity effect that yields a non-steady-state outcome and then derives the optimal policy instruments in this case.\textsuperscript{3} Section 5 summarizes and concludes.

2. The basic model

Each agent maximizes his (her) present value of utility streams:

\[ W = \int_0^\infty U(C(t), E(t)) e^{-\rho t} \, dt, \quad U_C, U_E > 0, \quad (1) \]

where \( C \) is a composite consumption bundle and \( E \) represents the 'stock' of the environment. The bar on \( E \) indicates that each agent views this stock as given, although \( E(t) \) varies in the aggregate.

2.1. The decentralized (equilibrium) path

In the absence of any government intervention (e.g. taxes, subsidies) the agents' budget constraint will be given by

\[ C(t) = f(K(t), \bar{E}(t)) - \dot{K}(t), \quad (f_K, f_E > 0), \quad (2) \]

where \( \bar{E}(t) \) in the production function captures the productivity effect of the environment and is not a choice variable to individual agents. \( K(t) \) is the stock of capital. Maximizing (1) subject to (2), the Current Value Hamiltonian for an interior solution is

\[ H = U(C, \bar{E}) + \lambda[f(K, \bar{E}) - C], \quad \text{with} \ K(0) = K_0 \text{ and } E(0) = E_0. \quad (3) \]

The first-order conditions for this interior solution are

\[ \begin{align*}
\mathcal{H}_C &= 0 \rightarrow U_C = \lambda, \quad (4) \\
\mathcal{H}_K &= \rho \lambda - \dot{\lambda} \rightarrow \dot{\lambda} = \lambda(\rho - f_K). \quad (5)
\end{align*} \]

Differentiating (4) in time, substituting for \( \lambda \) and \( \dot{\lambda} \) into (5), and rearranging the terms, the optimal path of consumption in the private economy along the equilibrium path is

\textsuperscript{3} Note that because this paper focuses on domestic environmental policies, abstracting from international policies, the model applies to the case of local rather than global pollutants such as greenhouse gases or ozone-depleting gases. For an example of policies that address global environmental effects, see Diwan and Shafik (1992), and Mohtadi (1992).
\[
\left( \frac{\dot{C}}{C} \right)_t = -\frac{U_c}{CU_{cc}} \left[ f_k(E) - \rho + \frac{U_{CE}}{U_c} \dot{E} \right],
\]

where the dependence on \( C, K, \) and \( t \) are suppressed. Observe that although atomistic agents cannot influence \( E \) in their decision, changes in \( E \) do affect their consumption path and must be taken into account by them. This is shown as \( \dot{E} \) in Eq. (6). This effect depends on the sign of the cross-effect, \( U_{CE} \). If \( U_{CE} > 0 \), which is the common assumption in the literature,\(^4\) this would mean that a secular improvement in environmental quality increases the equilibrium growth rate, ceteris paribus, and a secular deterioration in environmental quality reduces the equilibrium growth rate, ceteris paribus. This would suggest the possibility of a win-win outcome, in which both equilibrium growth increases and environmental quality improves. Intuitively, a rise in \( E \) increases the marginal utility of consumption, raising the incentive to consume at all times and thus to accumulate. This increases the growth rate of consumption. Naturally, if \( U_{CE} < 0 \), then an increase in environmental quality reduces the marginal utility of consumption, reducing the incentive to consume and thus to accumulate, reducing the growth rate. We summarize as follows.

(1) If \( U_{CE} > 0 \), then a 'win-win' outcome is possible in which a secular improvement in environmental quality, ceteris paribus, increases the rate of growth at equilibrium. If \( U_{CE} < 0 \), then no such 'win-win' outcome is possible, so that improvements in environmental quality imply lower growth.

In general, however, the quality of the environment is itself endogenous, as will be discussed now.

2.2. The efficient path

In the aggregate, \( E \) is endogenous. \( E \) is assumed to be a 'stock' that is gradually depleted by the flow of pollutants \( X \), as follows:

\[
\dot{E} = -X.
\]

Eq. (7) implies that the stock \( E \) is a decreasing function of time, or \( E'(t) < 0 \), whether or not the flow \( X \) itself rises or falls in time, i.e. in this 'stock' representation, the quality of the environment declines, whether or

\(^4\) An alternative, but equivalent, assumption in the literature is that the cross-effect of pollution and consumption is negative. See, for example, Becker (1982), D'arge and Kogiku (1973), Foster (1973), and others.
not the flow of pollutants $X$ rises or falls in time. Since capital stock appears to accumulate in time, it would be more convenient to re-express $E$ as a decreasing function of the capital stock, i.e.

$$E = E(K), \quad \text{with } E_K(K) < 0 \text{ and } E(K_0) = E_0.$$  

(8)

In reality, there may be regenerative processes that can slow or even stop the deterioration of the stock of $E$. Even ignoring such regenerative processes, Eq. (8) need not lead to catastrophe in finite time if $E$ approaches zero asymptotically, i.e. if:

$$\lim_{K \to \infty} E(K) \to 0.$$  

(9)

The efficient growth path is found by maximizing (1) subject to the budget constraints (3) and also $E$ endogenous, per Eq. (8). The Current Value Hamiltonian for an interior is

$$\mathcal{H} = U[C, E(K)] + \lambda[f(K, E(K)) - C], \quad \text{with } K(0) = K_0,$$  

(10)

and the first-order conditions for an interior solution are:

$$\mathcal{H}_C = 0 \to U_C = \lambda,$$  

(11)

$$\mathcal{H}_K = \rho \lambda - \dot{\lambda} = (\rho - f_K - f_E E_K)\lambda - U_E E_K.$$  

(12)

Again, differentiating (11) in $t$, substituting for $\lambda$ and $\dot{\lambda}$ in (12), and rearranging the terms, consumption along the efficient path grows at the rate

$$\left(\frac{\dot{C}}{C}\right)_t = -\frac{U_C}{CU_{cc}} \left(f_K + f_E E_K - \rho + \frac{U_E}{U_C} \cdot E_K + \frac{U_{CE}}{U_C} \frac{E_K \dot{K}}{E}\right),$$  

(13)

where $E_K \dot{E} = \dot{E}$ as indicated, so that the last term in (13) is the same as (6). However, Eq. (13) now indicates two additional terms; the term $f_E E_K$ represents the productivity effect of the environment; the term $(U_E/U_C)E_K$ involves the utility effect of the environment. In both, environment enters through the marginal impact of capital accumulation on environment $E_K$. Since $E_K < 0$, as discussed, both these terms are negative, reflecting the adverse effect of capital accumulation on the efficient growth path via a loss of productivity and via social preference away from capital accumulation.
3. Model specification and optimal policies

3.1. Model specification

This section begins with a parametric specification of the functions. This serves analytic tractability. It also facilitates the identification of some key concepts, such as the preference structure, as well as some key policy instruments (to be elaborated below). We begin by positing a simple relationship for the environment function:

$$E(t) = E_0 K(t)^{-\beta}, \quad (\beta > 0).$$

(14)

This formulation implies that as $t \to \infty$, and $K(t) \to 0$, $E(t) \to 0$. Thus, the ‘stock’ of the environment approaches zero only asymptotically, consistent with Eq. (9). When a steady-state path exists, $E/E$ declines at a constant rate, but $E(t)$ itself will fall increasingly more slowly and thus approaches zero only asymptotically. It therefore stays positive for all finite time.

The production function will be represented by a technology that is separable in the effects of $E$ and $K$, and is linear in $K$ but not necessarily in $E$. Specifically, let $f(K, E) = A(E)K$, where $A(E)$ captures the effect of environmental quality on productivity.

The utility function is given by the customary form of constant elasticity (CE) of marginal utility with respect to its argument. However, we generalize this to allow for different weights of $C$ and $E$ in the utility function: $U(C, E) = [(C^\sigma E^\mu)^{1-\sigma} - 1]/(1 - \sigma), (\sigma > 0)$. The generalization is important because the parameter $\mu$, which represents preferences towards the environment, will play a key role in later analysis. Here, $\sigma$ is the usual elasticity of the marginal utility with respect its argument, also known as the intertemporal elasticity of substitution. Imposing the condition $U_{CE} > 0$, discussed earlier, implies that $\sigma < 1$. Conditions $U_{CC} < 0, U_{EE} < 0$ imply that $\nu(1-\sigma) < 1$ and $\mu(1-\sigma) < 1$.

With these specifications, the equilibrium growth rate of consumption will be given by

$$\left(\frac{\dot{C}}{C}\right)_e = \frac{1}{1 - \nu(1 - \sigma)} \cdot \left[ A(E) - \rho + \mu(1 - \sigma) \frac{\dot{E}}{E} \right],$$

(15a)

or, using (14), by

$$\left(\frac{\dot{C}}{C}\right)_e = \frac{1}{1 - \nu(1 - \sigma)} \cdot \left[ A(E) - \rho - \beta \mu(1 - \sigma) \frac{\dot{K}}{K} \right].$$

(15b)

These results have the distinctive feature that the level of environment $E$ appears as a determinant of growth, via the productivity effect. This level effect suggests a 'path dependence' of the growth rate, similar to endogen-
ous growth theories, as is seen, for example, in Romer's (1990) treatment of human capital. However, because $E$ here is itself determined by the path, one will not, in general, obtain a steady-state path. A steady-state path will exist, however, in the absence of the productivity effect of the environment, and is discussed shortly. (The non-steady-state case, in the presence of productivity effects, is discussed in Section 4.)

Using the functional forms above, the parameter-specific efficient growth path is found, in analogy to Eqs. (15) as

$$
\left( \frac{\dot{C}}{C} \right) = \frac{1}{1 - \nu(1 - \sigma)} \left( \left[ 1 - \beta \mu (1 - \sigma) \right] A(E) - \rho - \beta E A_E(E) \right. \\
- \frac{\beta \mu}{\nu} \frac{C}{K} + \mu (1 - \sigma) \frac{\dot{E}}{E},
$$

(16a)
or, equivalently, by

$$
\left( \frac{\dot{C}}{C} \right) = \frac{1}{1 - \nu(1 - \sigma)} \left( \left( 1 - \frac{\beta \mu}{\nu} \right) A(E) - \rho - \beta E A_E(E) \right. \\
+ \frac{\beta \mu}{\nu} \left[ 1 - \nu(1 - \sigma) \right] \frac{\dot{K}}{K},
$$

(16b)

where Eq. (14) and the budget constraint $C = AK - \dot{K}$ are used in the second form. In addition to $1 - \nu(1 - \sigma) > 0$, seen earlier, observe that $\nu$ must exceed $\beta \mu$ if productivity is to be positively related to growth. Thus, we impose this constraint from here on. Intuitively, this implies that the benefit (utility) of growth, through an increase in consumption, must dominate (in elasticity terms) the disutility of growth — via a deterioration in the environment — brought about by an increase in capital accumulation.

Expression (16b) may also be written in a simpler form if the impact of the environment on productivity is expressed by a constant elasticity, $\eta$, where $\eta = E A_E(E)/A(E)$. This is

$$
\left( \frac{\dot{C}}{C} \right) = \frac{1}{1 - \nu(1 - \sigma)} \left( \left[ 1 - \frac{\beta \mu}{\nu} - \beta \eta \right] A(E) - \rho \right. \\
+ \frac{\beta \mu}{\nu} \left[ 1 - \nu(1 - \sigma) \right] \frac{\dot{K}}{K}.
$$

(16c)

Next, Eqs. (15) and (16) are analyzed under the assumption that environmental quality only affects the utility, but not the productivity, of the economy (i.e. we consider the case in which polluted air, for example, may cause sickness, but ignore the adverse effect of polluted air on firm productivity; for example, via the need to install additional ventilators, or the increased cost to the firm of worker sickness). The assumption, in part, serves analytic convenience because it will allow us to derive steady-state
paths when the production function is linear in capital stock. These paths will be the basis for deriving optimal policies in a steady state. (Later, in Section 4, the policy derivations will be extended to where environment affects productivity and the paths are not in a steady state.)

3.1.1. The steady state

If the environment does not affect productivity, \( A \) will be exogenous and \( A_E = 0 \). Then along both the equilibrium and the efficient path, the growth rate is constant and is characterized by, \( \dot{C}/C = \dot{K}/K \). Using this in Eqs. (15b) and (16b), the steady-state paths are

\[
\gamma_e = \frac{1}{1 - (\nu - \beta \mu)(1 - \sigma)} (A - \rho) \\
\gamma_e = \frac{1}{1 - \nu(1 - \sigma)} \left( A - \frac{\rho}{1 - \beta \mu / \nu} \right)
\]

Observe that \( \beta \) and \( \mu \) appear jointly in both equations; if \( \beta \mu \) were to increase, either because environmental decay accelerated (\( \beta \)) or because individuals preferred a cleaner environment (\( \mu \)), the growth rate would fall along both paths.\(^5\) In (18) the quantity \( \rho / (1 - \beta \mu / \nu) \), which exceeds \( \rho \), may be thought of as the 'socially effective discount rate' as far as capital accumulation is concerned. In effect, concern for the environment (\( \mu / \nu \)) reduces the tendency to accumulate capital by increasing this effective rate. As a result, the stock of environment now benefits. A drop in physical capital accumulation now corresponds to an increase in the preservation of 'environmental capital'. Regarding the range of parameters, besides \( \beta \mu < \nu \), other constraints also apply. One is the range noted in footnote (5). Another is the transversality condition on the stock of capital yielding \( (\nu - \beta \mu)(1 - \sigma) < \rho / A \), which is the same along both growth paths and which, in turn, guarantees that \( \gamma_e < 1 \).\(^6\)

Although both growth rates \( \gamma_e \) and \( \gamma_e \) fall in \( \beta \), they exhibit different

\(^5\) The denominator of (17) must be positive if \( \gamma_e \) is to be positively related to productivity, and negatively to the discount rate.

\(^6\) The transversality condition is, \( \lim_{t \to \infty} e^{-\nu t} \lambda(t) K(t) = 0 \). For the equilibrium path, Eq. (5) gives \( \lambda / \lambda = (\rho - A) \) or \( \lambda(t) = \lambda_0 e^{(\rho - A) t} \). Substituting for \( \lambda(t) \) and for \( K(t) = K_0 e^{\nu t} \) into the transversality condition, we find this condition is satisfied if \( \gamma_e < A \). From Eq. (17) this inequality implies that

\[
(\nu - \beta \mu)(1 - \sigma) < \rho / A .
\]

For the efficient path, with \( F_e = 0 \) here, we find, similarly, that \( \gamma_e < A \). From (18) this yields (a) again. Finally, \( \gamma_e < 1 \) if and only if \( (\nu - \beta \mu)(1 - \sigma) < 1 - (A - \rho) \). Comparing the RHS of this inequality with the RHS of (a), we find \( \rho / A < 1 - (A - \rho) \) because \( \rho < A \). Thus, condition (a) guarantees that \( \gamma_e < 1 \).
curvatures. It can be easily verified that $\gamma_e$ is convex in $\beta$ ($\partial^2 \gamma_e / \partial (\beta)^2 > 0$), while $\gamma_e$ is concave in $\beta$ ($\partial^2 \gamma_e / \partial (\beta)^2 < 0$). Since $\gamma_e = \gamma_s$ at $\beta = 0$ (equaling $(A - \rho) / [1 - \nu(1 - \sigma)]$), the above curvatures necessitate that $\gamma_s$ lie above $\gamma_e$ for small $\beta$ and below $\gamma_e$ for larger $\beta$. This result, depicted in Fig. 1(a), means that a large enough value of $\beta$ can drive the socially desirable rate of

Fig. 1. (a) Growth as a function of environmental response to capital accumulation. (b) Region of taxes vs. subsidies.
growth to zero, even though atomistic agents behave in such a way that the equilibrium rate of growth remains above zero. Since $\beta$ is the response of environmental decay to capital accumulation and growth, we can summarize the above result as follows.

(2) When environment affects utility (and not production), then if environmental decay is 'slow' then growth may be 'too low' from a social point of view; if environmental decay is 'rapid', then growth may be 'too high' from a social point of view.

Because this result has important consequences for the analysis of policy that we shall turn to shortly, understanding it is important. Especially, the fact that for some range of $\beta$ the equilibrium growth rate can be too slow is interesting enough to warrant some economic interpretation: consider a given small enough value of $\beta$. Then an incremental addition to the capital stock ($\delta K > 0$) worsens environmental quality (via $E = E_KK$). Given the assumption that $U_{C e} > 0$, Eqs. (6) or (13) tell us that a worsening of environmental quality reduces preference for consumption at all future times, showing up as a reduced growth rate of consumption along either path. In turn, this reduces the incentive for capital accumulation, leading to a drop in both $\gamma$ and $\gamma$ pre steady-state Eqs. (17) or (18). But if capital accumulation slows down, environmental deterioration also slows down! Thus, the consumption growth rate need not fall by as much. However, this feedback from the environment to consumption is external to the private economy, which it ignores but which the central planner internalizes (i.e. the term $(U_{C e}/U_{C c})E_K$ in (13) has no counterpart in (6)). In other words, the private economy, not able to influence $E$, will overcompensate for reduced environmental quality by reducing the consumption rate too much. In turn, in a steady state this means that the private economy will accumulate capital too slowly relative to the social planner solution. The inability of the private economy to adjust to this (second-order) external effect also means that when $\beta$ is large enough, the private economy grows too rapidly (accumulating capital too fast), so that the $\gamma_e$ curve lies above the $\gamma$ curve, as depicted in Fig. 1(a). In effect, $\gamma$ always shows greater sensitivity to changes in $\beta$ because of this additional effect of the environment through utility. By subtracting $\gamma$ from $\gamma_e$ we see that $\text{sign}(\gamma_e - \gamma) = \text{sign}(\beta\mu/\nu) \cdot \lbrack \rho - (\nu - \beta\mu)(1 - \sigma)A \rbrack$, so that $\gamma_e < \gamma$, for small enough $\beta$, and $\gamma > \gamma_e$, otherwise.

3.2. Optimal tax/subsidy policy in a steady state

The reason for analyzing the efficient and the equilibrium paths jointly is to design policies and regulations that can take the economy from a suboptimal path prevailing in equilibrium because of market failures, to the socially efficient path. In a sense we can speak of 'aligning' the two paths.
To illustrate how policy instruments may be used for the purpose of such an alignment, consider a proportional scheme in which a tax or subsidy is imposed on output (income) at a flat rate of $\tau$ ($\tau > 0$ if a tax; $\tau < 0$ if a subsidy). The equation for the steady-state equilibrium path (17) is amended as follows:

$$\gamma_e(\tau) = \frac{1}{1 - (v - \beta \mu)(1 - \sigma)} \cdot [(1 - \tau)A - \rho]. \quad (17')$$

Equating this with $\gamma_\epsilon$ in (18) and solving for $\tau$ we find:

$$\tau_0 = \frac{\beta \mu}{1 - \nu(1 - \sigma)} \left( \frac{1}{\nu - \beta \mu} \cdot \frac{\rho}{A} - (1 - \sigma) \right). \quad (19)$$

From Eq. (19) $\tau_0 \geq 0$, if and only if $\rho \geq (1 - \sigma)(\nu - \beta \mu)A$, since the denominator is already positive. By subtracting (17) from (18) we find that $\gamma_e - \gamma_\epsilon \geq 0$, if and only if the same condition on $\rho$ holds. It follows that $\gamma_e \geq \gamma_\epsilon$ if and only if $\tau_0 \geq 0$. Thus, a Pigouvian output tax is necessary if the pre-tax equilibrium growth rate exceeds the efficient growth rate; a Pigouvian subsidy is necessary if the pre-subsidy equilibrium growth rate is below the efficient rate. This result is shown in Fig. 1(b). Given $\beta$, the subsidy raises $\gamma_e(\tau)$ to the level of $\gamma_\epsilon$ and is thus relevant in the lower range of $\beta$, while the tax lowers $\gamma_e$ to the level of $\gamma_\epsilon$ and is thus relevant in the higher range of $\beta$. This result is summarized below.

\textbf{(2')} When environment affects utility (and not production), if environmental decay is 'slow', then optimal policy implies an output subsidy; if environmental decay is 'rapid', then optimal policy implies an output tax. Optimality aligns the equilibrium path with the efficient path.

How is the subsidy paid for or what becomes of the taxes collected? Consider agents maximizing (1) subject to the after-tax (after-subsidy) budget constraint, $C = (1 - \tau)AK - \dot{K} + \hat{T}$, when $\hat{T}$ is a (non-distortionary) lump-sum amount that represents either a charge levied on agents to finance the subsidy, or a credit that agents receive from the taxes collected, i.e. $\hat{T} \leq \sigma$, if $\tau \geq 0$. Since $\hat{T}$ is a lump sum, it is fixed to the agents but variable in the aggregate, such that $T = \tau AK$. With $\hat{T}$ fixed to agents, the equilibrium growth rate remains at $\gamma_e$. Their aggregate average initial level of consumption, given an initial capital stock, $K_0$, is (using the above budget constraint and the fact that $\dot{K}(0) = \gamma_e K_0$): $C_0 = K_0[(1 - \tau)A - \gamma_e] + T(K_0) = K_0[(1 - \tau)A - \gamma_e + \tau A] = K_0(A - \gamma_e)$. The initial level of consumption under the command economy $C_{os}$ is similarly found (using the central planner budget constraint) to be $C_{os} = K_0(A - \gamma_e)$. Thus, when $\tau$ is chosen so that $\gamma_e = \gamma_\epsilon$, we have $C_{os} = C_0$ and the private path coincides with the efficient path. The
key is that the financing of the subsidy or the return of the tax revenue be in a lump-sum form.

It may also be noted that since $\chi$ maximizes $W$ in Eq. (1) (say $W^*$), setting $\tau$ to equalize $\chi$ to $\gamma$ allows the welfare level under prevailing market conditions to equalize $W^*$. Thus, $\tau_0$ is a first-best policy instrument, given $\beta$. But, as we will see in the next subsection, $\beta$ need not be exogenous and allowing it to be a choice variable may increase the value of $W^*$. Here, we shall see how a tax/subsidy policy may be combined with a policy of regulating the rate of environmental decay, $\beta$, to achieve an even higher welfare level.

3.3. Optimal quantity restriction-cum-tax/subsidy in a steady state

The policy maker can influence the effect of the capital accumulation on the environment by means of regulation. If productivity is itself a function of the degree of regulation, then we can choose jointly the optimal degree of regulation and the optimal tax/subsidy on output. Specifically, the parameter $\beta$ may be used as a key policy instrument. For example, it is desirable for the regulator to seek lower $\beta$ values in order to reduce the deterioration of the environment stock per given increase in capital stock. However, mandating a lower $\beta$ may be costly to individual agents (firms) because it involves the adoption of more expensive techniques, even as it may improve productivity in the aggregate (see the example below). Since the ex-anti decision to employ a technique involves capital in 'putty-putty' form, any cost increases from the adoption of more expensive techniques translate into less output per capital outlay (dollar invested) and thus into reduced productivity. In sum, lower values of $\beta$ would reduce productivity $A$ as seen by individual firms, but raise it as an externality in the aggregate. We can write this as

$$
\beta \uparrow \rightarrow A^\dagger \rightarrow E \rightarrow A^* \text{,}
$$

or, in functional form, as

$$
A = A[\beta, E(\beta)], \quad A_\beta > 0, \quad A_E > 0, \quad A_E E_\beta < 0. \quad (20)
$$

To illustrate, consider the case of a polluted lake into which firms discharge their waste. Mandating a lower $\beta$ means mandating a slower rate of decay in the 'stock' of clean water (measured, for example, by the concentration of pollutants in the water). However, this reduces the ease of disposal, raising costs and reducing productivity per firm ($A^\dagger$). Thus, $A_\beta > 0$, given the quality of the water in the lake. However, lower $\beta$ implies higher water quality ($E_\dagger$) and better health, resulting in higher labor productivity and
thus overall productivity \( A^*_t \). Thus, \( A^*_E E_\mu < 0 \) as an external effect. This suggests a trade-off on the production side, even in the absence of consumption effects. Adding consumption effects certainly involves addition trade-offs.

One is then naturally interested in finding the optimal level of the instrument, \( \beta \). Our approach in finding this level will be to first ignore the external productivity effect of the environment (setting \( A^*_E = 0 \)), focusing only on the trade-offs from direct productivity versus the consumption effects. Later (Section 4) we will also include the external productivity effect. In the absence of this effect, the expressions for the steady-state paths will resemble Eqs. (17) and (18) but show an explicit dependence of \( A \) on \( \beta \):

\[
\gamma = \frac{1}{1 - (\nu - \beta\mu)(1 - \sigma)} \cdot [A(\beta) - \rho],
\]

\[
\gamma = \frac{1}{1 - \nu(1 - \sigma)} \left[ A(\beta) - \frac{\rho}{1 - \beta\mu/v} \right].
\]

We observe that both equations show the trade-off in \( \beta \), as discussed. Because of this trade-off, both curves now exhibit a ‘peak’ in \( \beta \) instead of their earlier monotonic decline. Although a \( \beta \) that maximizes \( \gamma \) may be chosen, this value will be suboptimal and not a first-best instrument. Instead, a two-step approach is adopted: first, we choose a \( \beta \) that maximizes \( \gamma \); second, we choose a tax/subsidy rate that equalizes \( \gamma \) with \( \gamma(\tau) \). In analogy with Eq. (17') the equilibrium growth path of (21) incorporates a tax or subsidy term:

\[
\gamma(\tau) = \frac{1}{1 - (\nu - \beta\mu)(1 - \sigma)} \cdot [(1 - \tau)A(\beta) - \rho].
\]

The value of \( \beta \) that maximizes \( \gamma \) in (22) is given by

\[
\frac{\partial \gamma}{\partial \beta} = 0 \rightarrow (1 - \beta^*\mu/v)^2 A'(\beta^*) = \mu\rho/v .
\]

The second-order condition for the optimum \( \beta \) is satisfied if \( A''(\beta) < 0 \). The tax/subsidy rate at which \( \gamma = \gamma_0 \) was earlier found from Eq. (19). Substituting \( \beta^* \) from (23) into this expression, we now find:

\[
\tau_0^* = \frac{\beta^*\mu}{1 - \nu(1 - \sigma)} \left( \frac{1}{\nu - \beta^*\mu} \cdot \frac{\rho}{A(\beta^*)} - (1 - \sigma) \right).
\]

This value of \( \tau \), in effect, forces \( \gamma \) to intersect \( \gamma \) through the maximum of the latter, as shown in Fig. 2. The figure indicates both a tax and a subsidy

\[ \text{Sign} \left[ \delta^2 \gamma / \delta \beta^2 \right] = \text{sign} \left[ A''(\beta) - 2\rho(\mu/v)^2(1 - \beta\mu/v)^{-3} \right] < 0, \text{ if } A''(\beta) < 0. \]
case depending on whether the 'τ-absent' equilibrium growth curve is above or below $\gamma_n$ at $\beta^*$. The combined $[\beta^*, \gamma_0^*(\beta^*)]$ pair will thus induce the market economy to grow along a socially optimal path. We summarize as follows.

(3) In a steady-state (when considering only the utility effects of the environment), there exists an optimal level of environmental regulation
that, when combined with an optimal tax or subsidy, yields a higher social welfare than any optimal tax or subsidy alone.

The following section extends the analysis of the previous section when the effect of the environment on productivity is also considered.

4. Extension: The environment affects productivity

Since, in this case, \( A_E \) is no longer zero, \( A(\beta) \) is replaced by \( A[\beta, E(\beta)] \) in all the relevant equations. The trade-offs now consist of the growth-enhancing effect of higher \( \beta \) via productivity directly, \( A(\beta, \ldots) \), versus the growth-inhibiting effects of a higher \( \beta \) via the environment and thus, indirectly, productivity, \( A[\ldots, E(\beta)] \), as well as via consumption. When the utility function is specified as before, the equilibrium and efficient paths will be similar to Eqs. (15a) and (16a), but also incorporate this productivity effect:

\[
\left( \frac{\dot{C}}{C} \right)_e = \frac{1}{1 - \nu(1 - \sigma)} \left[ A[\beta, E_e(\beta)] - \rho + \mu(1 - \sigma) \left( \frac{E}{E} \right)_e \right], \tag{15a'}
\]

\[
\left( \frac{\dot{C}}{C} \right)_s = \frac{1}{1 - \nu(1 - \sigma)} \left[ (1 - \beta \mu(1 - \sigma))A[\beta, E_s(\beta)] - \rho - \beta E A_E(E_s) \right. \\
\left. - \frac{\beta \mu}{\nu} \left( \frac{C}{K} \right)_s + \mu(1 - \sigma) \left( \frac{E}{E} \right)_s \right], \tag{16a'}
\]

where \( E_e \) and \( E_s \) denote the state of the environment along each path. These two equations serve as the basis for the optimal policy designs derived below.

4.1. Optimal tax/subsidy policy

Deriving analytical forms for optimal policy instruments turns out to be more complex now, because of the absence of a steady state as explained below. However, in the special case of a 'balanced-path' discussed below, some analytic results do emerge.

Assuming the existence of a \( \beta \) value (to be established later), it follows from the environment function of Section 3 that \( \dot{E}/E = -\beta \dot{K}/K \). Upon substituting this into Eqs. (15a') and (16a'), a long-run 'balanced-path' is found along which consumption and capital stock grow at the same rate—and thus the \( C/K \) ratio is constant—without this common rate being a constant. Constancy of \( \dot{C}/C \) and \( \dot{K}/K \) is ruled out, because in (15a') and (16a') environment \( E \) shows up as a state variable. Such a path is then balanced but not in a steady state. Following the above procedure the two paths are (ignoring, for now, regulation's effect on \( A:A_\beta = 0 \))
where $\eta$ is the elasticity response of productivity to the environment, as previously defined. Given $E$, this productivity effect represents a slowing down of socially optimal growth in (25), where there was no such effect before. Intuitively, higher growth and accumulation worsens the state of the environment, reducing the productivity of capital. The socially efficient path takes account of this factor and prescribes slower growth. If we require that this impact cannot plausibly exceed the direct contribution of productivity to growth (the first term in the bracket in (25)), then $\eta$ must be sufficiently small that $\beta(\mu/\nu + \eta) < 1$, for the 'net' coefficient of productivity, $A$, in (25) to be positive. This range will be useful in the later analysis.

It turns out that when we set $\gamma_e = \gamma_e$ along the 'balanced path' (in search of an optimum tax or subsidy), the levels of $E_s$ and $E_e$ in (24) and (25) are also automatically equalized, so long as both paths begin at the same initial level of $E_0$. This occurs because a $\tau$ value that equates $(\dot{C}/C)_s$ with $(\dot{C}/C)_e$ also equates $(\dot{K}/K)_e$ with $(\dot{K}/K)_s$ along the balanced path. But the latter means that $(\dot{E}/E)_e = (\dot{E}/E)_s$. Starting with the same initial $E_0$ and thereafter changing at the same rate, it follows that $E_s(t) = E_e(t)$ for any $t$ along the balanced path. The value of $\tau$ that equates a $\tau$-appended version of (24) with (25) is

$$\tau_0 = \frac{1}{1 - \nu(1 - \sigma)} \left( \frac{1}{\nu - \beta \mu} \frac{\rho}{A(E)} - (1 - \sigma) \right) + \frac{\beta \nu}{\nu - \beta \mu} \eta \left( 1 + \frac{\beta \mu(1 - \sigma)}{1 - \nu(1 - \sigma)} \right).$$

As in Subsection 3.2, $\tau_0$ turns out to represent a tax ($\tau_0 > 0$) if and only if $\gamma_e > \gamma$, and a subsidy ($\tau_0 < 0$) if and only if $\gamma_e < \gamma$, where $\gamma_e$ is the pre-tax/subsidy growth rate along the equilibrium path. Because of the additional productivity effect of the environment, the prescribed socially optimal path now lies below the previous $\gamma_e$ curve of Figs. 1(a) and 1(b), but the curve $\gamma_e$ stays the same. As shown in Fig. 3, this reduces the subsidy rate or increases the tax rate in the range of $\beta$ where each applies.\(^8\)

\(^8\) Also, while the range of $\beta$ for a subsidy shrinks (as shown in the figure) the range of $\beta$ for taxes may be smaller or larger, since, on the one hand, a downward shift of $\gamma_e$ curve increases it, but, on the other, this downward shift reduces the overall range of $\beta$ in which a non-zero socially optimal growth is possible.
Note that if \( \eta = 0 \), Eq. (26) reduces to the previous case (Eq. (19)). Since \( \partial \tau_0 / \partial \eta > 0 \), \( \tau_0 \) here is larger than in the previous case, if it is a tax (>0), and smaller (in absolute value) if it is a subsidy (<0). In short:

\[
\tau_0 |_{\eta > 0, E} > \tau_0 |_{\eta < 0, E}.
\]

Thus, we have a summary as follows.

(4) Considering the adverse productivity effect of environmental degradation, optimal taxes need to be larger (and optimal subsidies smaller) than when the environment does not affect productivity (assuming \( A_\beta = 0 \)).

The intuition is obvious: if the environment affects productivity, then it is necessary to impose a higher tax (or lower subsidy) than before to discourage excessive growth, since the social cost of environmental decay is now higher.

Also, from (26), \( \partial \tau_0 / \partial E < 0 \). Thus, the more pristine is the environment, the less is the tax needed. Intuitively, this occurs because of a smaller divergence between the social and the market-driven paths, in this case.
4.2. Optimal quantity restriction-cum-tax/subsidy

Incorporating the effect of \( \beta \) on \( A \), we first seek a value of \( \beta \) that maximizes \( \gamma \) under the ‘balanced growth path’, to later combine with the tax/subsidy scheme derived above. The first-order condition now shows the added effect of the environment on productivity via \( \eta \):

\[
\frac{\partial \gamma}{\partial \beta} = 0 \Rightarrow (1 - \beta^* \mu / \nu)^2 (A_{\beta} + A_{E} E_{\beta})
= \mu \rho / \nu + \eta \beta (1 - \beta^* \mu / \nu) (A_{\beta} + A_{E} E_{\beta}) + \eta A.
\] (27)

Comparing with (23), note that \( A'(\beta) \) in (23) is now replaced by \( A_{\beta} + A_{E} E_{\beta} \), which shows the direct and indirect impact of \( \beta \) on productivity. To compare the qualitative results of this model with the previous model, and also to establish the existence of \( \beta^* \), one needs to specify an additional functional form; that of \( A(\beta, \ldots) \). We assume a simple constant elasticity form of \( \beta^\theta \), where \( \theta > 0 \), representing this direct effect. With the external environment effect already represented by \( \eta \), the total functional form is

\[
A = A_0 \beta^\theta \cdot E(\beta)^\eta.
\] (28)

With this specification, and with \( E(\beta) = E_0 K^{-\beta} \), the total differential of productivity in Eq. (27) is \( (\theta/\beta - \eta \ln K) A \), so that (27) becomes:

\[
(\theta/\beta^* - \eta \ln K) \left[ 1 - \beta^* \left( \frac{\mu}{\nu} + \eta \right) \right] \left[ 1 - \beta^* \frac{\mu}{\nu} \right] = \frac{\mu}{\nu} \cdot \frac{\rho}{A(\beta^*), \beta^*)} + \eta.
\] (27')

Eq. (27') shows that the optimal \( \beta^* \) in this non-steady-state world, where the environment affects productivity, depends on the size of \( K \). Further, since the RHS of (27') is positive, and also since both brackets on the left are positive (as \( \beta \mu < \nu \) and \( \beta (\mu / \nu + \eta) < 1 \), it follows that \( \theta/\beta^* - \eta \ln K > 0 \), or \( \theta/\beta^* > \eta \ln K \). Now we provide the following cross-sectional interpretation: rank countries by their per-capita capital stock, starting from \( K = 1 \) for the poorest country. Since \( \eta \ln K \) is then positive, \( \beta^* > 0 \), should it exist (see below). The optimal tax/subsidy-cum-optimal quantity regulation is then found from \( \beta^* \) as the implicit solution of (27') and \( \tau_0(\beta^*) \), when \( \beta^* \) is substituted into (26).

4.2.1. The existence of \( \beta^* \)

Given the above cross-sectional interpretation, the existence of an optimum \( \beta^* \) can be addressed numerically. It is interesting to numerically

\* The second-order condition is satisfied when \( \chi \) is concave in \( \beta \). \( \chi \) turns out to be locally concave in \( \beta \), so long as \( d^2 A / d\beta^2 < 0 \).
Fig. 4. Optimal environmental regulation ($\beta$) as a function of capital stock per capita ($K$), \( \mu = 0.2, \theta = 0.3, \rho = 0.05 \) and \( \eta = 0.05 \).

- $\Delta$, $\mu = 0.25, \theta = 0.2, \rho = 0.05$ and $\eta = 0.05$.
- $\Delta$, $\mu = 0.25, \theta = 0.25, \rho = 0.05$ and $\eta = 0.05$.
examine also how $\beta^*$ varies with $K$, since the sign of $d\beta^*/dK$ in (27') is analytically ambiguous. Fig. 4 establishes the existence of $\beta^*$ for a reasonable set of parameter values and shows that $\beta^*$ always declines in $K$, for all parameter values chosen. Thus, we summarize as follows.

(5) On the basis of numerical methods, when the productivity effect of the environment is considered, optimal levels of regulation on environmental pollution ($\beta^*$) are found to exist, and become more strict the 'richer' are the countries, as represented by higher levels of per-capita capital stock.

In the figure, when the environment does not affect productivity (Section 3) the top flat line results (marked $\cdot$), showing that $\beta^*$ is independent of $K$. This corresponds to $\eta = 0$, in (27'), or to solving for $\beta^*$ in (23). Inclusion of a productivity effect ($\eta = 0$ to $\eta = 0.1$, other parameters fixed) makes optimal regulation dramatically more restrictive, as the flat line shifts to the second curve, marked ($\ast$). Also, the greater is the environmental concern ($\mu/v = 0.2$ to $\mu/v = 0.3$, other parameters fixed) the more restrictive is the environmental regulation as the second curve shifts to the third curve, marked by ($\circ$). Finally, a smaller cost of environmental regulation to firms ($\theta = 0.5$ to $\theta = 0.3$, other parameters fixed), also makes optimal regulation more restrictive as the third curve shifts to the fourth curve, marked by ($\triangle$). These results are all as expected.

Results from this study have important implications for cross-country pollution and the sustainability of growth. These implications are discussed elsewhere, and in relation to the recently available evidence.

5. Summary and conclusion

A dynamic representative agent model is presented, in which the environment enters into the utility and production functions, to investigate the prospects of sustained long-run economic growth under optimal policy designs. The procedure to derive optimal policy instruments involves deriving the equilibrium (market-driven) path of growth, followed by the derivation of the socially efficient path. Optimal policy instruments are then found in two steps; in the first, the two paths are 'aligned' to yield optimal levels of Pigouvian taxes or subsidies, and in the second, the efficient path is maximized to yield the optimal level of environmental regulation.

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See an earlier unpublished draft of this paper available from the author.

This evidence, by Shafik and Bandyopadhyay (1992), suggests that development and growth do not affect the environment uniformly. For example, as per-capita incomes rise, some environmental indicators worsen (e.g. municipal waste), others improve (e.g. public sanitation and sewerage), and a third group experience 'inverted U' patterns (e.g. sulphur dioxide and air particulate matter). Two other studies, by Shafik (1994) and by Grossman and Krueger (1991) reveal similar patterns.
Acknowledgements

This research was supported in part by a grant on Environmental and Natural Resources Policy and Training Project (EPAT) (# 144-DF44) by the USAID. I wish to thank two anonymous referees for their invaluable comments. Earlier versions of this paper were presented at the University of Minnesota, and the Universities of Wisconsin at Madison and Milwaukee. I wish to thank seminar participants at these universities especially, Terry Roe, Yakov Tsur, Elamin El Basha, at the University of Minnesota, and, Magda Kandil and Sunny Kim, at the University of Wisconsin, for comments and criticisms on these earlier versions. Thanks are extended to my research assistant, Sumit Agarwal. Views expressed here are solely my own.

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