Tests of Dynamic Extensions to a Family of Trip Table Refinements Methods

Alan J. Horowitz and Layali Dajani

Alan J. Horowitz, Professor, Center for Urban Transportation Studies, University of Wisconsin – Milwaukee, PO Box 784, Milwaukee, WI 53201, voice: 414-229-6685, fax: 414-229-6958, e-mail: horowitz@uwm.edu

Layali Dajani, Project Assistant, Center for Urban Transportation Studies, University of Wisconsin – Milwaukee, PO Box 784, Milwaukee, WI 53201, voice: 414-229-5422, fax: 414-229-6958, e-mail: lhdajani@uwm.edu

Abstract: This paper addresses the problem of using traffic counts to ascertain dynamic origin-destination (OD) tables, when performing dynamic traffic assignments within a traffic window. A family of dynamic OD trip table estimation methods containing two previously unexplored members (single-factor and biproportional) are proposed to solve this problem. The family is tested on a traffic network for the US 45 corridor in Milwaukee, WI. The tests show that dynamic traffic assignments can be made to match ground counts by a dynamic extension to a technique conceptually similar to Fratar factoring of both origins and destinations from a static OD table. The two methods that directly solve for origin or destination factors have computational and statistical advantages over whole-table dynamic OD trip table estimation procedures. Both methods have results that can be readily interpreted.

INTRODUCTION

The statistical estimation of dynamic origin-destination (OD) tables from time-varying ground counts is a logical extension of the static problem (see for examples: 1,2,3,4); however, computational complexity is greatly increased because of the addition of a time dimension. Although there are many potential applications of dynamic OD table estimation, the research for this paper is primarily motivated by these broad traffic engineering or planning contexts:

- Peak hour or peak period urban travel forecasts, either strategic or long-range;
- Multihour or full day forecasts in statewide or other long-distance travel models (5); and
- Short-term traffic forecasts with real-time traffic data.

Historically, the practice of estimating OD tables from ground counts has been highly empirical and burdened with severe computational issues (6). As a partial remedy, this paper explores a method of obtaining row and column factors for full-sized traffic forecasting networks and compares it to a whole-table method and an optimal single expansion factor method. Altogether, these three methods form a family of dynamic OD estimation techniques that allow for flexibility in approach.

- Dynamic Single-Factor. This method obtains an optimal vector of factors, each of which apply to a matrix of origin-destination trips that start within a given time interval. This method implements a philosophy that traffic varies uniformly up or down during a peak period, with the overall structure of the OD table remaining fairly intact.
- Dynamic Biproportional. This method obtains optimal Fratar factors in multiple time periods and would be suitable for any planning situation where dynamic traffic assignment is employed and where better time-of-day factors are needed. This method implements a philosophy that the overall structure of the OD table remains fairly constant, but certain sites within the corridor (such as manufacturers and commercial establishment) might have traffic peaks at slightly different times of day.
- Dynamic Whole-Table. This method finds separate estimates for the number of trips between each origin and each destination that start in each time interval. This method implements a philosophy that there is no regularity in an OD table across time intervals and that each element of the table needs to be independently established. The literature on static and dynamic OD table estimation is dominated by discussions of various whole-table methods.

A method within this family that had been previously tested for static networks (7), uniproportional, has conceptual limitations when applied to dynamic traffic problems and has not been included in this paper.

This paper primarily emphasizes computational experience with these methods. Except for the aforementioned paper on the static problem (7), the literature is silent on development or the use of these techniques. It is possible, of course, to emulate either the dynamic single-factor method or the dynamic biproportional method by estimating a whole matrix by some appropriate technique, then summing the rows and columns. Such an
approach is overly computational and difficult to control. In very large networks with many traffic counts, zones and time intervals, estimating a full dynamic OD table by statistically rigorous methods may be computationally prohibitive.

It should be immediately obvious that dynamic single-factor and biproportional methods cannot possibly provide as good a fit to ground counts as whole-table methods, given the same input data, because there are fewer variables that can be estimated. However, it is far from obvious whether a whole-table method can provide results that are statistically better in any given network, considering that a full static OD matrix might already be available that closely approximates the dynamic matrix.

### FRAMEWORK OF DYNAMIC METHODS OF OD TRIP TABLE ESTIMATION

#### Fourth Level, Whole Trip Table GLS

For the sake of comparison with static methods, this section organizes methods into “levels”, each of which correspond to their respective static method (7). The dynamic single-factor and biproportional methods can both be implemented using generalized least squares (GLS) and are directly related to the conventional GLS method of estimating all the cells of a dynamic OD table, $d_{ij}$, where $i$ is the origin, $j$ is the destination and $d$ is the a trip’s starting time interval. Equation 1 gives a succinct statement of the optimization problem for whole-table GLS,

$$
\min P = \sum_{c=F_a=1}^{L} \sum_{a=1}^{A} w^a \left( V^{ac} - \sum_{d=1}^{L} \sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij}^{acd} d_{ij}^{d^*} \right)^2 + \sum_{d=1}^{L} \sum_{i=1}^{N} \sum_{j=1}^{N} z_{ij} \left( d_{ij}^{d^*} - s_{ij} d_{ij}^d \right)^2
$$

where $V^{ac}$ is a ground count for link direction $a$ as measured over time interval $c$, $d_{ij}^{d^*}$ is the seed trip table for starting time interval $d$, $p_{ij}^{acd}$ is the proportion of trips between zones $i$ and $j$ and start in interval $d$ that use link direction $a$ during interval $c$ (as determined by a dynamic equilibrium traffic assignment) and $F$ and $L$ are the first and last time intervals over which traffic is counted, $N$ is the number of zones and $w^a$ and $z_{ij}$ are weights. Each direction of a two-way link, $a$, may have a separate ground count and may be considered the same as a “link” when only directional links are present in the network. Nonnegativity constraints apply to $d_{ij}^d$.

It should be noted that “trips” in Equation 1 might very well be fragments of a trips between points other than their actual origins and destinations. Although it is possible to have different seed tables for different intervals, the method could also be implemented with just one seed table for all intervals, as is done for the tests in this paper. It should also be noted that there could very well be fewer time intervals with traffic counts than time intervals within the whole simulation, thereby allowing for a spin-up period of time to fully load the network. That is, $F$ does not need to be the first interval of the traffic assignment.

#### Framework Overview

A family of OD table estimation methods can be most compactly described by introducing a set of dynamic “k-factors” to Equation 1. K-factors, often referred to as socioeconomic adjustment factors in the planning literature, are traditionally ad hoc adjustments to OD tables and are sometimes used in attempts to improve the fit of travel forecasting models to observed travel patterns. The notation for a dynamic k-factor is $k_{ij}^d$, where the subscripts and superscripts are as defined previously. In Equation 2, the matrix of $z$’s has been replaced by a single parameter, $z$, in order to simplify the tests in this paper.

$$
\min P = \sum_{c=F_a=1}^{L} \sum_{a=1}^{A} w^a \left( V^{ac} - \sum_{d=1}^{L} \sum_{i=1}^{N} \sum_{j=1}^{N} p_{ij}^{acd} k_{ij}^d d_{ij}^{d^*} \right)^2 + z \sum_{d=1}^{L} \sum_{i=1}^{N} \sum_{j=1}^{N} d_{ij}^{d^*} \sum_{d=1}^{L} \sum_{i=1}^{N} \sum_{j=1}^{N} \left( d_{ij}^d - k_{ij}^d \right)^2
$$

All of the methods are solved in this paper with the gradient projection method (GPM) augmented with PARTAN acceleration steps, as described in the original paper (7). The implementation of this minimization technique will be briefly described later.
Third Level, Fratar Biproportional GLS

The dynamic biproportional method introduces two arrays of factors, one for origins, $x^d_i$, and one for destinations, $y^d_j$, and an overall scale factor, $s$. The origin and destination factors, $x^d_i$ or $y^d_j$, for any starting time interval $d$ may be found by replacing $k^d_{ij}$ with:

$$k^d_{ij} = sx^d_i y^d_j$$  \hspace{1cm} (3)

As before, nonnegativity constraints apply to $x^d_i$ or $y^d_j$. It is also possible to apply bounding constraints on any or all values of $x^d_i$ or $y^d_j$ in order to avoid wild fluctuations from their initial values of 1.0 so that the seed table is not unreasonably distorted for any particular origin or destination. Note that only one value of $s$ is used in the tests for this paper. The scale factor $s$ is optional and might be set prior to the optimization in another attempt to keep $x^d_i$'s and $y^d_j$'s close to 1.0. In order to be consistent with the philosophy of linear regression analysis, $s$ is arbitrarily chosen for the case study described here such that the average of the ground counts is the same as the average estimate of the ground counts when all $x^d_i$'s and $y^d_j$'s are equal to 1 (i.e., the seed OD table is otherwise unmodified). Other means of setting $s$ might be more appropriate in different situations.

It should be observed that the $y^d_j$'s apply to the destinations at the starting interval for the trip, not at the ending interval, in order to remain consistent with the other two methods. The solution to Equations 2 and 3 is not unique if it is unconstrained or if none of the constraints happen to be binding. However, tests reported earlier for the static problem (7) indicated that the possible lack of uniqueness was not an obstacle to obtaining good results.

Second Level, Uniproportional GLS

The uniproportional method is similar to the biproportional method, but all $y^d_j$'s are replaced by $x^d_i$'s. So

$$k^d_{ij} = sx^d_i x^d_j$$  \hspace{1cm} (4)

The uniproportional method is most appropriate to situations where origins equal destinations at each place where trips may begin and end, which would be considered highly unusual in dynamic problems. Therefore, the uniproportional method will not be discussed further.

First Level, Single-Factor GLS

Finally, a single-factor dynamic OD trip table estimation may be achieved by estimating a separate factor for each starting time interval $d$, such that:

$$k^d_{ij} = s^d$$  \hspace{1cm} (5)

Unlike the static method that has just one variable (7), the single-factors for every time period must be obtained using nonlinear optimization techniques and cannot be obtained in closed form.

COMPUTATIONAL ISSUES

Solution Algorithm

Only one solution algorithm was used for these tests: the gradient projection method (GPM) with PARTAN. Faster algorithms or hardware would be needed for real-time applications. Both algorithms are textbook ways of solving nonlinear programming problems. Custom software was written for this research to obtain reasonable execution speeds. The gradient projection method was required so that feasibility could be guaranteed throughout the search process and PARTAN, a well-understood acceleration technique, was required to reduce the number of partial derivatives to be calculated. PARTAN greatly improved execution times over GPM alone. Analytical differentiation was required in all cases to save computation time; however, the partial derivatives for the dynamic biproportional methods are very complex – much more so than for dynamic whole-table GLS – and very time consuming to calculate.
Convergence Criteria
Searches in the gradient projected direction are stopped when the step size, $\eta$, decreases beyond:

$$\eta < \phi \sqrt{H}$$

where $\phi$ is a suitably small number, chosen to be 0.000001 for all tests in this paper, and $H$ is the number of variables. The optimization is terminated when the relative change in the objective function between PARTAN steps is smaller than an arbitrary number, taken to be 0.0001 for the tests presented here.

Discussion
Although there are typically far fewer variables in either the dynamic single-factor or biproportional methods than in a dynamic whole-table GLS problem for any meaningful problem, the input data requirements are identical.

TRAFFIC FORECASTING FRAMEWORK
The OD trip table estimation problem with dynamic equilibrium traffic assignment requires a bilevel (or equivalent) solution procedure, for example (3, 8) among others. In the case study presented here, the minimization problem is embedded within a conventional method of solving for a user-optimal equilibrium traffic assignment using the method of successive averages (MSA). The flow diagram of Figure 1 illustrates the procedure as it would be implemented for an urban traffic forecast, region-wide.

The figure shows three arrows looping back to earlier steps. Two of these loops occur essentially simultaneously at the end of a single equilibrium iteration. The loop on the far right assures that congestion found during traffic assignment is incorporated within a trip distribution step (should a trip distribution step be needed). The loop on the left carries the trip table “refinements”, as determined by the OD table estimation procedure, to the next equilibrium iteration. The refinements are found by subtracting the current equilibrium-averaged seed trip table from the estimated trip table, keeping in mind that the seed trip table could change as the simulation progresses. It is important to note that the minimization problem, a very time consuming process, must be solved once for each MSA average within this framework. An inside loop increments the simulation through time intervals to achieve a dynamic traffic assignment. This third loop emulates the dynamic properties of Dynasmart, a well respected dynamic traffic assignment algorithm developed at the University of Texas, Austin, for FHWA (9). (Note: all software for this project was custom built by the principal author; Dynasmart itself was not used.) A minimization problem is only run once for each MSA average, which incorporates the results for all time intervals.

For the tests of this paper, path building was “dynamic”, that is drivers were assumed to have perfect knowledge of delays throughout their trip and could divert en route to avoid spot congestion. Delays at traffic signals were computed according to the operational analysis procedures of the Highway Capacity Manual, Chapter 16 (10). Delays along uninterrupted sections of freeways and surface arterials are calculated with the BPR curve, using parameters suggested by NCHRP #365 (11).

Procedure for Estimating Dynamic OD Tables
There are at two big questions that might be asked about the estimation methods:

- How well does each method perform given only a seed OD table that was built entirely by rules or assumptions?
- How well does each method perform given a static, rule-based OD table that was already adjusted to match static ground counts?

This paper focuses on the second question, which places emphasis on the dynamic properties of the methods, and lessens the impact that an arbitrary rule might have on the conclusions.

Consequently, the estimation procedure has three parts, as illustrated in Figure 2. The first part consists of finding a serviceable static OD table by making a rough guess as to the traffic pattern through the window. The second part consists of estimating a static OD table from ground counts by a whole-table GLS method, given the guessed OD table from the previous step. Finally, the last part estimates the dynamic OD table from ground counts by any one of the three methods described earlier.
If the set of rules adopted to start the process is poorly chosen, then the second part of the process, estimation of the static table, would appear to be necessary for good performance of either the single-factor and dynamic methods.

**FIGURE 1** A Bilevel Algorithm for Dynamic OD Trip Table Estimation for a Traffic Forecasting Problem
CASE STUDY, US 45 SOUTHBOUND IN MILWAUKEE, WISCONSIN

The case study was derived from data collected while doing a research study for the Wisconsin Department of Transportation to determine the extent of traffic diverting from new freeway ramp meters in the southbound direction of US 45 in and near Milwaukee (12). The corridor for the ramp-meter study consisted of about 16 miles of US 45 and parallel arterial streets. As part of that study many traffic counts were taken over several “incident-free” days. Days with inclement weather or crashes anywhere in the corridor were omitted from the database. Counts were simultaneously collected on the freeway main line, almost all on- and off-ramps in the southbound direction, and at several locations along potential diversion routes. All traffic counts were recorded within 15-minute time intervals. In addition, on-ramp to off-ramp OD matrices for the southbound direction were developed by video logging of license plate numbers, as described by Wu, et al. (12). Data were collected both before and after the installation of new ramp meters in late February 2000. It was originally hoped that the diversion percentages observed from field observations could be validated by traffic assignments, but the traffic assignments were abandoned because of budget limitations and the absence of a reliable dynamic OD table for an area large enough to encompass all diversion effects.

As one might expect in such a large corridor, there are a diversity of land uses. Particularly large employers near the corridor are Harley Davidson and the Milwaukee Regional Medical Center.

For the present study, a network was developed for a window drawn around US 45 and any potential diversion routes. Figure 3 shows the network. The window could be entered or exited from any of 37 external stations (shown as circles), and all traffic was assumed to originate or terminate at one of those external stations. The network had 154 one-way street links and 121 two-way street links. The window was kept deliberately thin in densely developed areas to minimize the amount of strictly internal travel.

The case study concentrates on the afternoon peak hour from 4:30 to 5:30 pm. In order to give a sense of the traffic patterns within the window, the widths of links in Figure 3 are proportional to the assigned traffic volumes from a static traffic equilibrium assignment for a full hour. It should be noted that US 45 is very congested in the southern half of the corridor ahead of the Zoo Interchange where US 45 meets I-94 (near the bottom of Figure 3). Although we are primarily interested in southbound traffic, there is a need to also simulate northbound traffic. Without the northbound traffic being simulated, it would not be possible to accurately estimate delays at signalized approaches according the HCM operational analysis procedures, which are sensitive to the amount of conflicting and opposing traffic.
The case study uses two different lengths of time intervals: 15 minutes and 5 minutes. Because of the size of the traffic window, most vehicles could completely traverse the window within 15 minutes, so a 1-hour dynamic traffic assignment with 15-minute intervals would not be much different from four separate static traffic assignments of 15 minutes each. Assignments with 5-minute intervals would overcome that issue. All dynamic traffic assignments used a spin-up period of 30 minutes, with the first interval beginning at 4:00 pm. Thus, there were six 15-minute interval or eighteen 5-minute intervals. There was no need for a wind-down period.

Altogether 104 link directions had dynamic traffic counts -- those links being strategically selected for the needs of the ramp meter study. This set of links excludes a majority of arterial streets within the network for which only AADT information was available from WisDOT. However, those AADT’s at the edge of the network were interpolated to give hourly origins and destination for the static OD tables used for seeding the dynamic estimation.
process (parts 1 and 2). Five-minute counts were simply interpolated from 15-minute counts, link by link. All static and dynamic traffic assignments were run through 20 MSA averages.

The guessed static OD table was created by simply assuming that OD patterns within the window are governed by a driver’s tendency to go straight through intersections, rather than turn. That is, OD flows without turns are more likely than OD flows with 1 turn, which are more likely than OD flows with 2 turns, etc. This guess leads to the adoption of a doubly-constrained gravity model for which the traditional frictions factors are replaced by ratings of drivers’ propensities to make fewer turns over a short distance. Table 1 shows the adopted propensity factors for OD trips with different numbers of turns, along the minimum-turn path. These propensity values were established by attempting to match the fraction of turns at signalized intersection approaches.

In the case of the dynamic biproportional method, both origin and destination factors were constrained to be within the range of 0.2 to 5 in order to impose reasonable values on the process. While it might be possible for a quitting time for a major employer within or near the corridor, such as Harley-Davidson, to cause a spike in traffic, field observations suggested that such a spike would be well within a five-fold range.

### Table 1 Assumed Turn Propensities for OD Pairs

<table>
<thead>
<tr>
<th>Number of Turns on Minimum Turn Path</th>
<th>Propensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4+</td>
<td>0</td>
</tr>
</tbody>
</table>

### RESULTS

#### Static OD Table Match to Observed Hourly Counts

The results of fitting static OD tables to ground counts was identical for both 15-minute and 5-minute time intervals, using whole-table GLS, as described for parts 1 and 2 of the estimation process. The average ground (directional) count was 1559 vph. The guessed OD table produced an intermediate RMS error to ground counts of 723 vph, which should be considered poor. The final static OD table had an RMS error to link counts of just 138 vph or about 9%, which would normally be considered quite good for planning needs. The RMS deviation between the final and guessed OD table was 48 trips per hour.

#### Static OD Table Match to Observed OD Information

It would be nice to know if the seed table had any level of believability at all. Most attempts at estimating OD tables from ground counts have little or no actual origin-to-destination flow information. However, the ramp meter study produced two afternoon peak hour ramp-to-ramp flow matrices (before and after) for the southbound direction. While incomplete and subject to serious errors from the video-logging technique, these flow matrices would indicate whether the seed table had any credibility. There were 62 ramp-to-ramp flow values. Only the “after” OD table was used. When the guessed static OD table was directly loaded to the network, its ramp-to-ramp OD flows had an RMS error of 250 vph. Given that the average ramp-to-ramp OD flow was just 154 vph, this error should be considered large. However, this RMS error dropped to just 83 vph after the static OD table was fitted to ground counts. While this RMS error might still seem large, its size was strongly influenced by just one particularly large ramp-to-ramp flow that had an error of more than 500 vph (Good Hope Road to Zoo Interchange). This one misfit can be at least partially attributed to a nearly 1000 vph undercounting error in the total destinations at the Zoo interchange in the field-collected ramp-to-ramp flow matrix. Omitting just this one ramp-to-ramp flow reduced the RMS error to 54 vph, a much more acceptable value.

It might have been possible to further improve the fit to the ramp-to-ramp counts by modifying the turn propensity factors or by introducing other logic or by adjusting the network; however, such improvements might have undercut a fair test of the dynamic estimation methods as they would be applied by real-world engineers or planners.
Estimation of Dynamic OD Tables

The results of estimating dynamic OD tables, given an estimated static table, are shown in Tables 2 and 3 for 15-minute intervals and 5-minute intervals, respectively. It is immediately evident that Table 3 is almost exactly one-third of Table 2, indicating that the performance qualities of the three methods are nearly identical between the 15-minute and 5-minute intervals. These tables also reveal the expected result that the whole-table method performs about the same (in terms of percent RMS error) dynamically as statically.

| TABLE 2  Results from 15-Minute Dynamic OD Table Estimation |
|-----------------------------------------------|----------------|-----------------|-------------------|
| Average Count | Link RMS Error | OD Table RMS Change | Number of Variables |
| One Fourth of Static Table | 390 | 93.1 | 0.0 | 0 |
| Single-Factor | 390 | 86.3 | 1.3 | 6 |
| Biproportional | 390 | 52.8 | 5.4 | 444 |
| Whole-Table | 390 | 30.3 | 6.1 | 8214 |

| TABLE 3  Results from 5-Minute Dynamic OD Table Estimation |
|-----------------------------------------------|----------------|-----------------|-------------------|
| Average Count | Link RMS Error | OD Table RMS Change | Number of Variables |
| One Twelfth of Static Table | 130 | 30.7 | 0.0 | 0 |
| Single-Factor | 130 | 27.5 | 0.5 | 18 |
| Biproportional | 130 | 17.2 | 1.7 | 1332 |
| Whole-Table | 130 | 10.2 | 2.0 | 24642 |

The single-factor method provides only a marginal improvement over the optimal static table (only 3.2 vph for the 5-minute interval), indicating the absence of even a moderate peak in traffic within the hour across the whole corridor. The biproportional and whole-table methods fit the ground counts much better, which suggests that peaking is mainly occurring locally at individual origins and destinations or both. Importantly, the largest improvements between methods in Tables 2 and 3 are between the single-factor and biproportional methods.

The fit to ground counts must be weighed against the change to the seed table. The average OD flow over a 15-minute interval was 7.1 vph, and the average OD flow over a 5-minute interval was 2.4 vph. Clearly, the whole-table method is most severely changing the seed table.

F-Test of Significance

Traditionally, the significance of variables in a nonlinear regression is judged by the F-test. However, OD table estimation is not a pure nonlinear regression even if GLS techniques are used, because it is difficult to assess the effective number of degrees of freedom due to the existence and constraining effect of the second (trip table) term of the objective function.

When doing an F-test, the number of variables are zero for no estimation, E (number of time intervals after spin-up) variables for the dynamic single-factor method, 2EN for the dynamic biproportional method, and $EN^2$ variables for the dynamic whole-table GLS method. It should be immediately apparent that there are more variables than data points for both the biproportional and whole-table methods for this case study. Therefore, it is not possible to use the F-test on comparisons involving these methods. The F values between the seed tables (1/4 and 1/12 of the static table) and the single-factor tables are 9.4 and 10.1 for the 15-minute and 5-minute test, respectively.

It should be observed that any method (except those without any degrees of freedom) would have performed better with a lower weight ($z$) on the second term of the objective function. A better way of testing significance is needed.

Intuitive and Spatial Results

The results from the biproportional method can be readily interpreted from a map. For example, Figure 4 shows the origin factors, $x_i^d$, for the last eight 5-minute intervals. The image has been cropped to emphasize those areas of the network of greatest relevance to the ramp meter study. Each set of circles progresses from the lower left to the upper-right and becomes somewhat darker in the later time intervals. The radius (and circumference) of each circle is proportional to the size of the origin factor for that time interval and that node. Medium-sized circles represent
locations where the estimation did not appreciably modify the static origins. Small circles represent fewer trips than
the static table, while large circles represent more trips. With only a few exceptions, within a single node there was
a smooth transition of sizes across the intervals; however, some origins peaked early and other origins peaked late.
Adjacent nodes seem to exhibit similar patterns, which could either be attributed to the times in which trips actually
originate or to artifacts of the estimation process, since adjacent origins would tend to send their trips onto some of
the same links. Destination factors, although considerably different from origin factors, show similar patterns of
variations. The pattern of factors at some nodes are sufficiently resolved to discern the peaking effects of specific
major traffic generators.

The computed origin or destination factors for the dynamic biproportional method did not show any wild
fluctuations. Indeed, all the estimated factors were well within the bounding constraints (0.2 to 5). That is, the
solutions for both 5-minute and 15-minute biproportional estimations after 20 equilibrium iterations were
completely unconstrained, which is consistent with field observations. However, it is conceivable that the
constraints might have been binding at earlier equilibrium iterations, and the constraints may not have been totally
irrelevant to successful convergence for the problem.
FIGURE 4  Variation of Origin Factors over Eight 5-Minute Time Intervals

Computational Effort

It is entirely possible that considerably faster algorithms could be devised for each of the methods and the convergence criteria could have been less strict, but the relative computation times are indicative of the amount of computation required. In addition, all of the runs were performed on an older laptop computer, so the computation
times in Table 4 would be smaller on new computers. Table 4 shows the computation times for the 20 equilibrium iterations of the dynamic part, only.

**TABLE 4 Computation Times for the Dynamic OD Table Estimation, Only (Minutes)**

<table>
<thead>
<tr>
<th></th>
<th>15-Minute Intervals</th>
<th>5-Minute Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-Factor</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>Biproportional</td>
<td>87</td>
<td>770</td>
</tr>
<tr>
<td>Whole-Table</td>
<td>98</td>
<td>1316</td>
</tr>
</tbody>
</table>

The similar computation times between the biproportional and whole-table methods for the 15-minute estimations are likely due to the combination of a relatively small number of OD pairs and the additional work required to compute each partial derivative for the biproportional method. The computational advantage of the biproportional method is more evident for the 5-minute estimations. Previously (7), it was found on larger networks that the computational advantages of the static biproportional method increased rapidly with the number of OD pairs. It is obvious from Table 4 that computation times also rapidly increase with numbers of intervals. Beyond issues of computation time, there were no observed convergence problems with any of the algorithms.

**Discussion**

The routines used in this paper were custom built for this research. An inspection of Figure 1 suggests that these OD table estimation techniques might be readily implemented within commercial software platforms that are modular in nature and have the necessary dynamic traffic assignment requirements. The ability of the equilibrium traffic assignment module to generate the necessary $p_{ij}^{acd}$ array is critical.

Memory requirements were not an issue for these tests, although the need for memory space increases rapidly with network size. The memory requirements are dictated mainly by the five-dimensional $p_{ij}^{acd}$ array, which can at least be conveniently packed in a way that squeezes out all the zeros. For example, the 5-minute whole-table estimation reached a maximum of 187,468 nonzero elements for the $p_{ij}^{acd}$ array by the last equilibrium iteration. Large networks, such as those used by metropolitan planning organizations, would likely be too big for any of these dynamic methods when implemented on a personal computer.

**CONCLUSIONS**

The results of both the dynamic biproportional method and the whole-table method are deemed to be sufficiently good for the needs of the ramp meter diversion analysis. The single-factor method was unable to correctly show the traffic dynamics within the corridor.

The dynamic biproportional method essentially introduces time-of-day adjustment factors by origin and by destination, which are intuitively reasonable and defensible. The dynamic biproportional method has its roots in Fratar factoring, a venerable technique that remains popular for certain types traffic engineering and transportation forecasts.

Only the execution times for the single-factor method were small enough for real-time applications. Even though the dynamic biproportional method was faster than the dynamic whole-table method, the test network was too small to dramatically show the potential gains in computational efficiency.

Even a poor guess of a seed OD table can still yield acceptable results from the OD estimation. The method adopted here for guessing a seed OD table, using turning movements, was less than ideal. A separate static OD estimation was required before a final seed table could be fed to the dynamic estimations. Combining these two techniques produced acceptable results for subsequent dynamic analyses.

**ACKNOWLEDGMENTS**

This research was partially funded by the Midwest Regional University Transportation Center’s partner grant. Count data were supplied by the Wisconsin Department of Transportation.
REFERENCES