P1.17A PARAMETERIZING BOUNDARY LAYER CLOUDS USING PDF METHODS

Vincent E. Larson[1] *, Jean-Christophe Golaz[2], William R. Cotton[2]
[1] Atmospheric Science Group, Dept. of Mathematical Sciences, University of Wisconsin — Milwaukee, Milwaukee, Wisconsin.
[2] Dept. of Atmospheric Science, Colorado State University, Fort Collins, Colorado.

1. INTRODUCTION

This paper investigates the joint probability density function (PDF) of vertical velocity w, liquid water potential temperature θ_l, and total specific water content q_t. If a function $P(w, \theta_l, q_t)$ is a joint PDF, then $P(w, \theta_l, q_t)\, dw\, d\theta_l\, dq_t$ is the probability of obtaining a value of (w, q_t, θ_l) within the range $(w - dw/2) < w < (w + dw/2)$, $(\theta_l - d\theta_l/2) < \theta_l < (\theta_l + d\theta_l/2)$, and $(q_t - dq_t/2) < q_t < (q_t + dq_t/2)$. We study joint PDFs derived from aircraft observations of cumulus, stratocumulus, and cumulus-rising-into-stratocumulus layers.

Our motivation is to provide observational support for a key assumption behind a new cloudy boundary layer parameterization (Golaz et al. 2001). Traditionally, cloud parameterization has been viewed as a collection of tasks. Such tasks include the prediction of heat flux, moisture flux, cloud fraction, and liquid water content. In contrast, Golaz et al. (2001) adopt the alternative viewpoint that the goal of parameterization consists largely of a single task: the prediction of the joint PDF, $P(w, \theta_l, q_t)$. The PDF viewpoint is more general, because if $P(w, \theta_l, q_t)$ is given, the fluxes, cloud fraction, and liquid water content can all be diagnosed. Unfortunately, direct calculation of $P(w, \theta_l, q_t)$ is computationally infeasible at present. Therefore, Golaz et al. (2001) adopt the following methodology. They assume that the joint PDF $P(w, \theta_l, q_t)$ in each grid box and timestep is a member of a family of joint PDFs, namely, a double Gaussian family. From this family of PDFs, Golaz et al. (2001) select a particular member for each grid box and time step by prognosing various moments of the PDF, such as $\overline{w}$, $\overline{w'^2}$, and $\overline{w'q_t'}$, using standard moment equations derived from the Navier-Stokes and advection-diffusion equations. The PDF is then used to close various unclosed terms in the prognostic equations, such as the buoyancy flux $(g/\theta_0)\overline{w'\theta_v'}$ and $\overline{w'q_t'^2}$. Finally, the equations are advanced another timestep, and the cycle repeats. This method, the "assumed PDF" method, has a long history [e.g., Manton and Cotton (1977), Sommeria and Deardorff (1977), Bougeault (1981), Chen and Cotton (1987)]. However, only recently have atmospheric models included w in the PDF (Randall et al. 1992; Lappen and Randall 2001a,b,c). The assumed PDF method has several attractive advantages. First, it is grounded on a sound theoretical foundation, namely the Navier-Stokes and advection-diffusion equations, and testable empiricism, namely the family of PDFs. Second, because the fluxes, cloud fraction, and liquid water are all derived from the same joint PDF, they are all calculated in a self-consistent manner (Lappen and Randall 2001a). Third, predicting the PDF allows one to remove certain systematic biases that occur in the microphysics of models that ignore subgrid variability (Rotstayn 2000; Pincus and Klein 2000; Larson et al. 2001a,b).

To develop a PDF-based parameterization of cloudy boundary layers, it is crucial to choose a satisfactory family of PDFs. We seek a family of PDFs that is simple, because the more parameters a PDF involves, the more moments must be prognosed. We also seek a family of PDFs that is sufficiently flexible and general that it can model both stratocumulus and cumulus regimes. If instead a parameterization package includes separate parameterizations for separate regimes, it faces the difficulty of choosing which scheme to trigger.

This paper uses aircraft data to test how well observed PDFs are fit by four families of PDFs. The first and second families are double delta and single Gaussian functions. The third family is based on a double Gaussian functional form (Lewellen and Yoh 1993). The fourth is a new, analytic formulation based on the double Gaussian form. We compare cloud fraction, specific liquid water content, and liquid water flux calculated from observed and parameterized PDFs.

2. FAMILIES OF PDFS TO BE TESTED

This section briefly summarizes the parameterizations of the joint three-dimensional PDF, $P(w, \theta_l, q_t)$.

I. Double delta function (7 parameters). This PDF consists of two Dirac delta functions whose locations and relative amplitude may vary. A double delta function PDF corresponds to a mass-flux scheme consisting of an updraft and downdraft plume, with no subplume variability in either plume. This PDF is perhaps the simplest that permits nonzero skewness and bimodality. Following Randall et al. (1992) and Lappen and Randall (2001a), we fix the relative amplitude of the delta functions and their positions in the w coordinate by the moments $\overline{w}$, $\overline{w'^2}$, and $\overline{w'^3}$. The positions in the θ_l coordinate are determined by $\overline{\theta_l}$ and $\overline{w'\theta_l'}$, and analogously for q_t. Therefore, by construction, this PDF exactly satisfies the fluxes $\overline{w'\theta_l'}$ and $\overline{w'q_t'}$.

II. Single Gaussian (9 parameters). This PDF consists of a single Gaussian that in general has non-zero correlations between the variables. That is, $\overline{w'\theta_l'}, \overline{w'q_t'}, \overline{q_t'\theta_l'} \neq 0$.

*Corresponding author address: Vincent E. Larson, Department of Mathematical Sciences, University of Wisconsin — Milwaukee, P. O. Box 413, Milwaukee, WI 53201-0413; e-mail: vlarson@uwm.edu

The Gaussian PDF does not allow the possibility of skewness or bimodality, but, unlike the double delta PDF, it does exactly satisfy the scalar variances $\overline{\theta_l'^2}$ and $\overline{q_t'^2}$.

III. Lewellen-Yoh (12 parameters). This PDF is based on a double Gaussian function, that is, the sum of two Gaussians. The positions and relative amplitudes of the two Gaussians may vary, permitting skewed and bimodal shapes. On the other hand, if the two Gaussians overlap, the PDF can reduce to a single Gaussian. Lewellen and Yoh (1993) reduce the number of PDF parameters, so that the PDF is fully determined by the means, variances, co-variances, and $\overline{w'^3}$, $\overline{\theta_l'^3}$, and $\overline{q_t'^3}$. Although the Lewellen-Yoh scheme performs the best of those we tested, it contains two complications. First, its PDF parameters cannot be solved analytically. Second, a host model may predict moments that result in unphysical values of the Lewellen-Yoh PDF parameters. Limiting the PDF parameters to acceptable ranges is somewhat involved but can be done.

IV. Analytic Double Gaussian (10 parameters). Like Lewellen-Yoh, this PDF is based on the double Gaussian shape with a reduced number of parameters. However, in this case the parameters can be found analytically. Furthermore, the only third moment required is $\overline{w'^3}$. The analytic double Gaussian is constructed to satisfy $\overline{q_t'\theta_l'}$ exactly. This was found to be important in the simulations of Golaz et al. (2001).

3. OBSERVATIONAL DATA

To test the aforementioned PDFs, we use aircraft observations from ice-free boundary layer clouds.

The aircraft observations were made during the Atlantic Stratocumulus Transition Experiment (ASTEX) and the First ISCCP (International Satellite Cloud and Climatology Project) Regional Experiment (FIRE). ASTEX investigated stratocumulus layers, cumulus-rising-into-stratocumulus layers, and some cumulus layers, whereas FIRE focused more on stratocumulus layers. We fit the PDF parameterizations to the ASTEX and FIRE datasets separately. These datasets contain many cloud-free legs. We keep these because we want to be able to predict the absence of cloud when appropriate. Furthermore, we wish to test the PDF parameterizations independently on cumulus legs. To do so, we isolate all ASTEX legs that contain some cloud and that occurred on flights that, according to observers' notes, sampled cumulus layers with no stratocumulus or at most broken stratocumulus above. This yields a third dataset, denoted "ASTEX cumulus legs," that consists of eight legs.

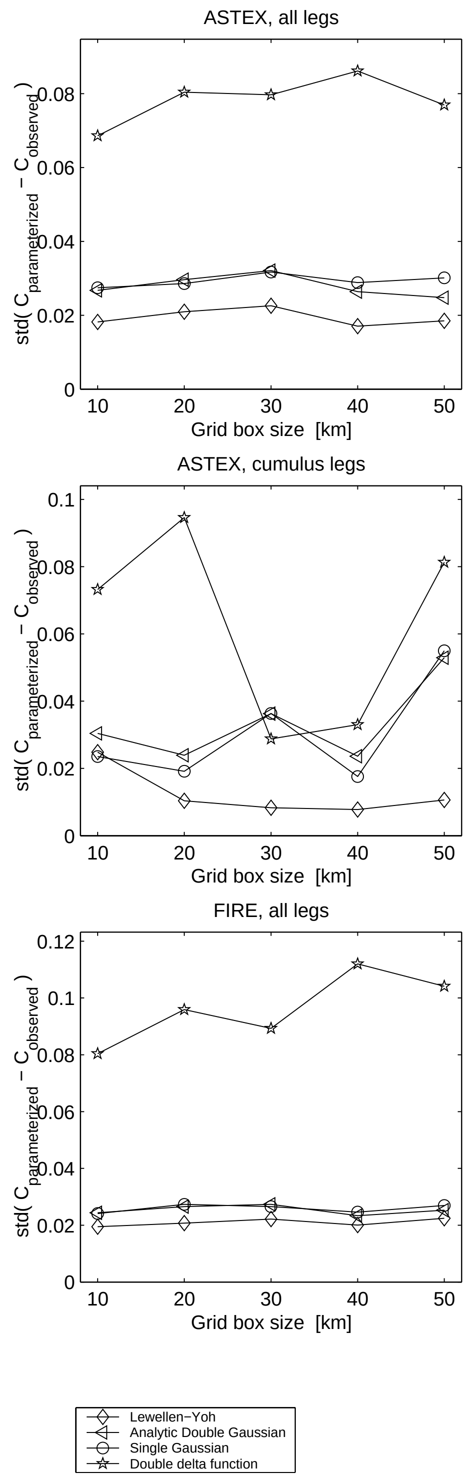

Figure 1: Errors in parameterized cloud fraction. To construct this plot, the three datasets were divided into legs of a given length. Then observed and parameterized cloud fraction of each leg, C_{observed} and $C_{\mathrm{parameterized}}$, were computed for the parameterizations listed in the legend. Next we computed the standard deviation of ($C_{\mathrm{parameterized}}$-$C_{\mathrm{observed}}$) over all the legs. Then we repeated the procedure for different leg lengths.

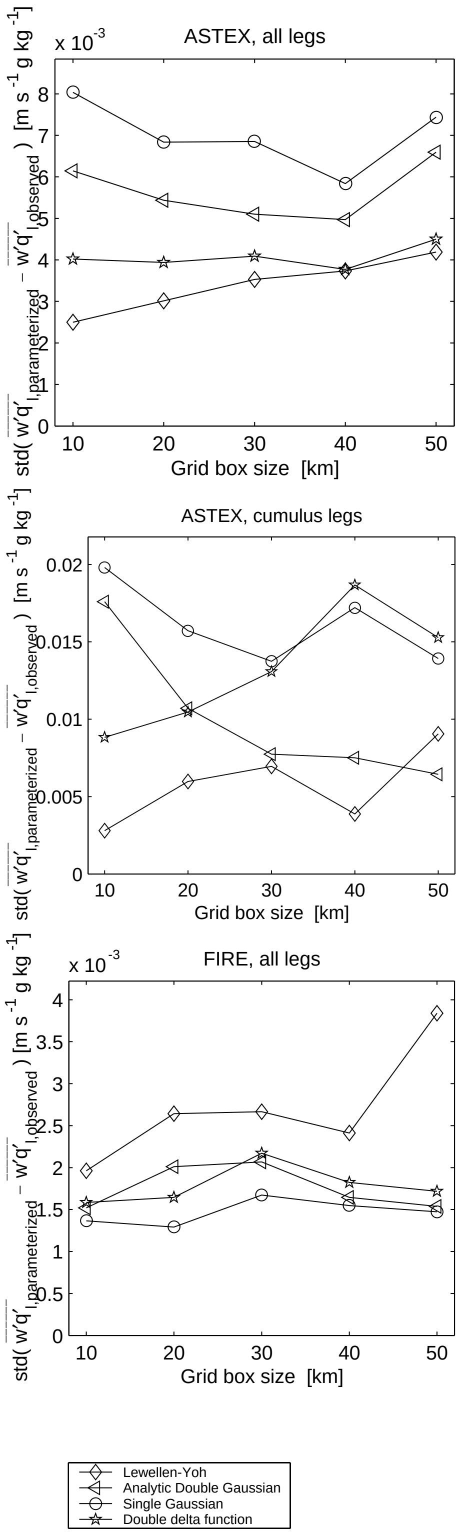

Figure 2: Same as in Fig. 1, but here we compute errors in parameterized turbulent flux of specific liquid water content.

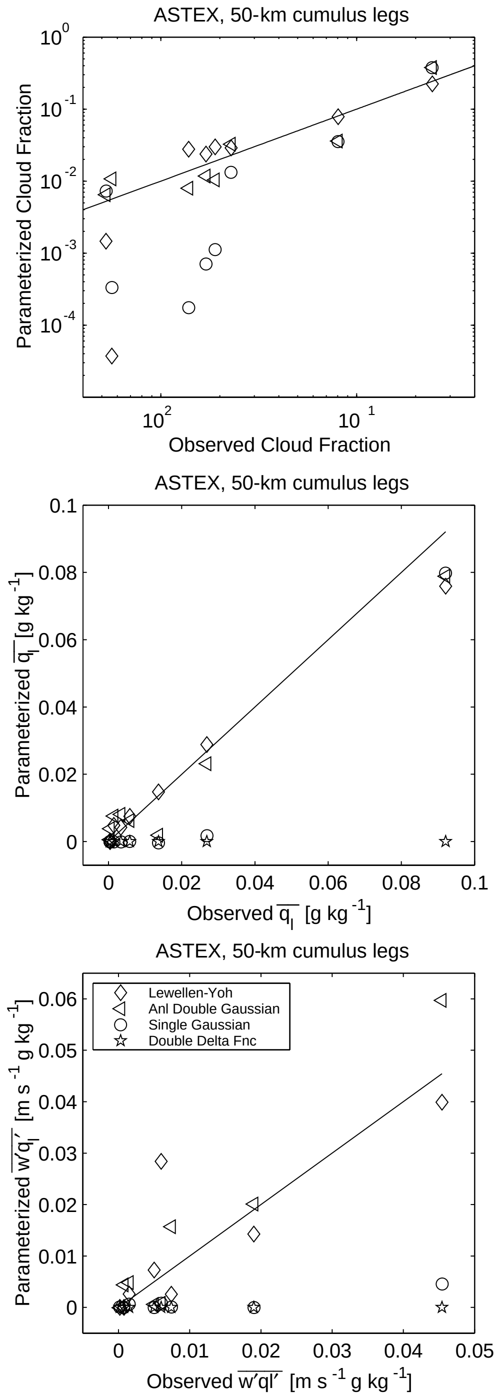

Figure 3: A scatterplot of parameterized vs. observed cloud fraction (C), average specific liquid water content ($\overline{q_l}$), and turbulent flux of specific liquid water content ($\overline{w'q_l'}$). The various parameterizations tested are listed in the legend. Each point corresponds to one long (50-km) ASTEX cumulus leg. A perfect fit would correspond to all points lying along the solid line. The Gaussian PDF (circles) leads to underpredictions. The double delta function (stars) diagnoses no cloud or liquid for any of these legs, and therefore does not appear in the top panel.

4. ERRORS IN PDF PARAMETERIZATIONS

Now we quantify the errors incurred by approximating an observed PDF with a parameterized PDF. We adopt the following procedure. First, we use an aircraft leg to compute observed values of cloud fraction (C) and liquid water flux ($\overline{w'q_l'}$). Second, we use the moments computed from the aircraft leg and the four families of PDFs to parameterize the leg's PDF. Then, using the parameterized PDF, we compute parameterized values of C and $\overline{w'q_l'}$. Finally, we compare these to observed values.

Because we use observed moments to compute parameterized values of C and $\overline{w'q_l'}$, we do not test errors in the host model's prediction of these moments. Instead, we isolate errors due to misrepresentions in the shape of the PDF. Therefore, the errors we plot are the best that a model could hope to achieve if it used these families of PDFs and if it predicted the moments perfectly. The question of how accurately the moments can be predicted is deferred to Golaz et al. (2001).

The errors in C and $\overline{w'q_l'}$ at various leg lengths are shown in Figs. 1 and 2. Consider first the errors in C (Fig. 1). The results from ASTEX cumulus legs are somewhat noisy because there are only 8 legs. Overall, however, the largest errors are associated with the double delta function (7 parameters). Superior to this are the single Gaussian (9 parameters) and analytic double Gaussian (10 parameters) parameterizations, which perform comparably to each other. The best predictions are provided by the Lewellen-Yoh scheme (12 parameters). As expected, we see that parameterizations with more fitting parameters produce better fits. Now consider errors in $\overline{w'q_l'}$, shown in Fig. 2. Overall, this plot reveals no obviously superior scheme. The ASTEX cumulus legs hint, however, that the double Gaussian schemes may perform better for cumulus clouds. The double Gaussian schemes have the capability of producing long tails when skewness is large. This is a decisive advantage when parameterizing cumulus PDFs. However, these tails can occasionally lead to poor predictions of C. Such outliers dominate the errors in the double Gaussian schemes. The double delta function and single Gaussian PDFs are more subdued, but they tend to underpredict $\overline{w'q_l'}$.

A more detailed picture of the errors in long (50-km) ASTEX cumulus legs is shown in Fig. 3. It turns out that for each of these legs, the double delta function diagnoses zero C, q_l, and $\overline{w'q_l'}$. Why is this? The essential difficulty is that the observed distribution contains cloud only within the end of a long tail; however, a double delta function contains no tail. This is related to a problem noted by Wang and Stevens (2000): even if a double delta function represents fluxes well, it tends to underestimate variances.

In Fig. 3, the single Gaussian underestimates C modestly, $\overline{q_l}$ moderately, and $\overline{w'q_l'}$ severely. Similar underpredictions were seen in the simulations of Bougeault (1981). This is related to the fact that the single Gaussian is unskewed, whereas cumulus layers are positively skewed. The underestimates in C are more severe for cases with less cloud. The double Gaussian parameterizations produce scatter, but do not have general underestimates.

5. A CRITIQUE OF MASS-FLUX SCHEMES

Of the four parameterizations tested, the double delta function tends to produce the worst overall estimates of C. Additionally, the double delta function diagnoses no cloud in the observed and simulated cumulus layers. The fundamental problem is that observed and simulated PDFs are spread out and therefore do not resemble double delta functions. One could adjust the locations of the delta functions to model the cloud properties better, but then the fluxes would be compromised.

Several authors have shown that the assumption that a double delta function can adequately represent observed PDFs is the essential assumption behind updraft-downdraft mass-flux schemes with no subplume variability (Randall et al. 1992; de Roode et al. 2000; Lappen and Randall 2001a). Therefore our critique of the double delta function is also an indirect critique of simple updraft-downdraft mass-flux schemes, when they are used to represent cloudy boundary layers.

To overcome the deficiencies of mass-flux models, prior authors have proposed several remedies, listed below.

1. Some authors have suggested coupling a mass-flux scheme to a cloud model that relies on a PDF that is not a double delta function (Lock et al. 2001). This has the drawback that then an inconsistency is built into the foundation of the model, because the cloud and velocity fields satisfy different PDFs rather than a single joint PDF.

2. Other authors examined the possibility of choosing the updraft and downdraft properties via a sophisticated sampling strategy (Wang and Stevens 2000). For instance, one could define the updraft plume to consist of those parcels that have $w' > 0$, $q_l > 0$, and $\theta_v' > 0$. However, Wang and Stevens (2000) were unable to find a satisfactory sampling strategy. Furthermore, it is not obvious how to numerically implement such a re-definition of sampling strategy.

3. Several authors have added subplume variability to mass-flux schemes in an attempt to improve the fluxes. For instance, Lappen and Randall (2001b) used downgradient diffusion to represent subplume-scale fluxes. Others have increased the fluxes by multiplying them by a constant pre-factor whose value could be derived, for example, under the assumption that the underlying PDF is a single joint Gaussian (Petersen et al. 1999). However, the present paper has demonstrated that cloud fields can be misrepresented even if the fluxes are perfect.

None of the above fixes to mass-flux models seems entirely satisfactory. Instead of attempting to modify a mass-flux scheme, it seems more natural to us to attempt to model the PDF as directly as possible. A possible procedure to do this, the assumed PDF method, was outlined in the introduction and is discussed in more detail in Golaz et al. (2001).

6. CONCLUSIONS

In our survey of four PDFs, we found that the Lewellen-Yoh family of PDFs, which is based on a double Gaussian form, performs best. If an analytic scheme is desired, then an analytic double Gaussian presented here performs satisfactorily. In particular, it fits cumulus layers better than the single Gaussian form.

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