



Higher-order closures and cloud parameterizations

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Outline

- What is higher-order closure?
- Historical perspective
- The Assumed Probability Density Function (PDF) Method
- Some challenges
- To avoid PDFs, should we prognose cloud fraction?
- Sample results
- Conclusions

What is higher-order closure?

The Reynolds-averaged equations (usually 1D) contain turbulent fluxes of momentum, heat, and moisture. Determining these fluxes is a major part of the parameterization problem.

These fluxes may be closed using a downgradient diffusion assumption.

Alternatively, higher-order closure seeks to prognose the fluxes and other moments directly. These prognostic equations introduce new terms that must be closed.

What are the higher-order equations?

In the higher-order model of Golaz et al. (2002), the complete set of prognosed moments is:

$$\begin{aligned} \text{Means :} & \quad \frac{\partial \bar{u}}{\partial t} = \dots \quad \frac{\partial \bar{v}}{\partial t} = \dots \quad \frac{\partial \bar{q}_t}{\partial t} = \dots \quad \frac{\partial \bar{\theta}_l}{\partial t} = \dots \\ \text{2nd - order :} & \quad \frac{\partial \overline{w'q'_t}}{\partial t} = \dots \quad \frac{\partial \overline{w'\theta'_l}}{\partial t} = \dots \quad \frac{\partial \overline{w'^2}}{\partial t} = \dots \\ & \quad \frac{\partial \overline{q_t'^2}}{\partial t} = \dots \quad \frac{\partial \overline{\theta_l'^2}}{\partial t} = \dots \quad \frac{\partial \overline{q'_t\theta'_l}}{\partial t} = \dots \\ \text{3rd - order :} & \quad \frac{\partial \overline{w'^3}}{\partial t} = \dots \end{aligned}$$

w = vertical velocity q_t = total water specific humidity

θ_l = liquid water potential temperature

Details of a subset of higher-order equations: water vapor

Red = prognosed moments

Blue = diagnosed quantities

$$\frac{\partial \bar{q}_t}{\partial t} = -\bar{w} \frac{\partial \bar{q}_t}{\partial z} - \frac{\partial}{\partial z} \overline{w'q'_t} + \left. \frac{\partial \bar{q}_t}{\partial t} \right|_{\text{source}}$$

$$\begin{aligned} \frac{\partial \overline{w'q'_t}}{\partial t} = & -\bar{w} \frac{\partial \overline{w'q'_t}}{\partial z} - \overline{w'^2} \frac{\partial \bar{q}_t}{\partial z} - \frac{\partial \overline{w'^2 q'_t}}{\partial z} \\ & - \overline{w'q'_t} \frac{\partial \bar{w}}{\partial z} + \frac{g}{\theta_0} \overline{q'_t \theta'_v} - \frac{1}{\rho_0} \overline{q'_t} \frac{\partial p'}{\partial z} - \epsilon_{wq_t} \end{aligned}$$

$$\frac{\partial \overline{q_t'^2}}{\partial t} = -\bar{w} \frac{\partial \overline{q_t'^2}}{\partial z} - 2 \overline{w'q'_t} \frac{\partial \bar{q}_t}{\partial z} - \frac{\partial \overline{w'q_t'^2}}{\partial z} - \epsilon_{q_t q_t}$$

A useful distinction: Prognosed moments vs. Diagnostic terms

A useful division is between [prognosed moments] and [diagnostic terms in the prognostic equations]. In this sense, the prognosed higher-order moments can be thought of as part of the “host” model; the diagnosed terms, as the parameterization.

$$\frac{\partial \bar{q}_t}{\partial t} = -\bar{w} \frac{\partial \bar{q}_t}{\partial z} - \frac{\partial}{\partial z} \overline{w'q'_t} + \frac{\partial \bar{q}_t}{\partial t} \Big|_{\text{sources}}$$

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Prognosed variables and memory

Prognosed variables preserve “memory” from timestep to timestep; diagnosed quantities need to be reconstructed at each new each timestep. Extra prognosed variables add information without increasing the number of grid points.

Example: Smoke in an updraft. If a model “forgets” that the smoke is contained in the updraft at the end of a timestep, the model doesn’t know whether to transport smoke up or down at the next timestep.

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Early efforts

- 1D higher-order closure models of the atmospheric boundary layer date back to the 1970s. [e.g. Lewellen and Teske (1973); Wyngaard and Coté (1974); André et al. (1976a,b)]
- These efforts essentially paralleled the development of 3D explicit turbulence models for the boundary layer (later known as LES models) [e.g. Deardorff (1972; 1974a,b; 1980)]

Historical Landmarks

Simulation of dry turbulence

André, J. C., G. de Moor, P. Lacarrère, and R. du Vachat, 1976a: Turbulence approximation for inhomogeneous flows: Part I. The clipping approximation. *J. Atmos. Sci.*, **33**, 476-481.

➤ Third-order modeling using the quasi-normal closure and the clipping approximation

André, J. C., G. de Moor, P. Lacarrère, and R. du Vachat, 1976b: Turbulence approximation for inhomogeneous flows: Part II. The numerical simulation of a penetrative convection experiment. *J. Atmos. Sci.*, **33**, 482-491.

➤ Application to the tank experiment (Willis and Deardorff, 1974).

André, J. C., G. de Moor, P. Lacarrère, G. Therry, and R. du Vachat, 1978: Modeling the 24-hour evolution of the mean and turbulent structures of the planetary boundary layer. *J. Atmos. Sci.*, **35**, 1861-1883.

➤ 24-hour simulation of day 33 of the Wangara experiment.

Historical Landmarks

Turbulence and moist processes

Bougeault, P., 1981a: Modeling the trade-wind cumulus boundary layer. Part I: Testing the ensemble cloud relations against numerical data. *J. Atmos. Sci.*, **38**, 2414-2428.

Bougeault, P., 1981b: Modeling the trade-wind cumulus boundary layer. Part II: A higher-order one-dimensional model. *J. Atmos. Sci.*, **38**, 2429-2439.

- Extension of André 1976 third-order closure model to non-precipitating cloudy convection
- Quasi-normal closure for fourth-order terms
- Skewed distribution for cloud model of θ_l, q_t

Fast forwarding...

- New motivation: higher-order closure modeling as boundary layer cloud *parameterization*.

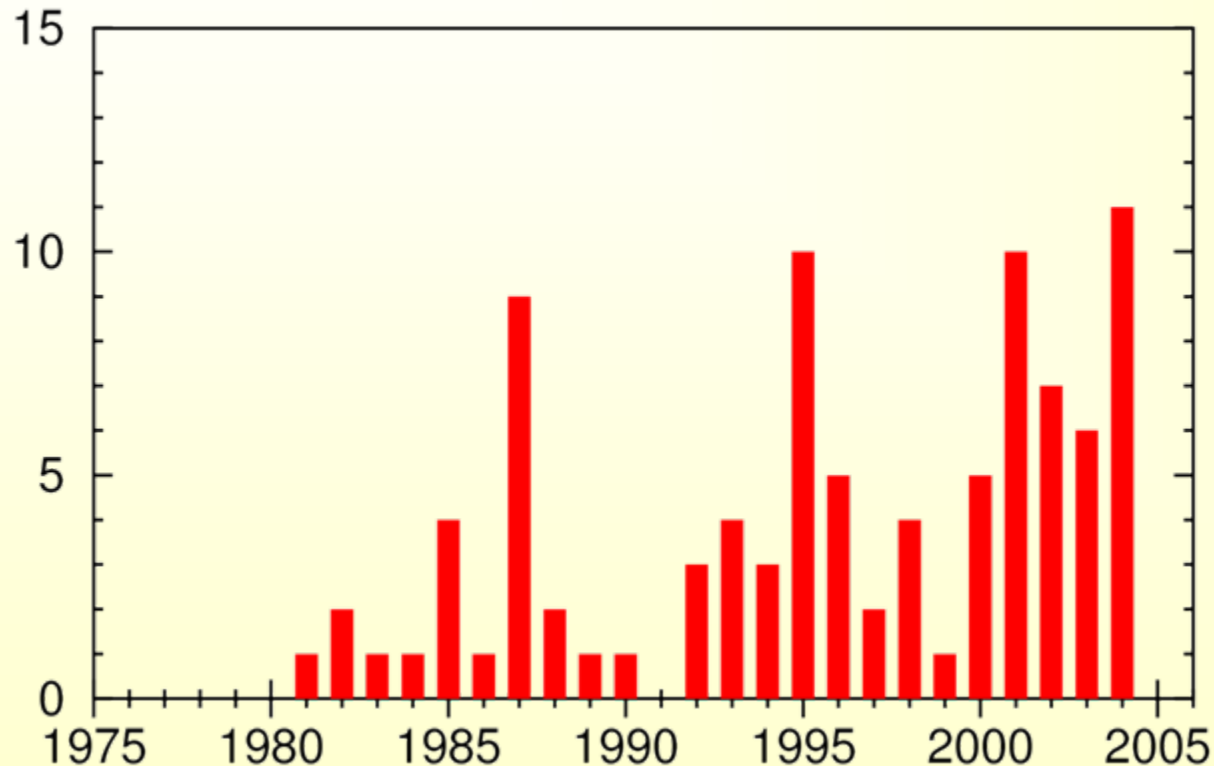
Randall, D. A., Q. Shao, and C.-H. Moeng, 1992: A second-order bulk boundary-layer model. *J. Atmos. Sci.*, **49**, 1903-1923.

- Marriage between higher-order closure and assumed PDF cloud models.
- Lappen and Randall 2001a,b,c built a parameterization based on this approach.

Historical Landmarks

Citation count

Bougeault (1981a,b)



Third-order modeling
of moist convection

**Citation count
(Web of Science):**

Bougeault 1981a: **61**

Bougeault 1981b: **33**

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How do we close the diagnostic terms?

Terms involving pressure or dissipation are closed using standard methods.

Other terms, e.g. **turbulent transport and buoyancy terms**, could be diagnosed if we knew the appropriate PDF. The rest of this talk will focus on these **latter terms**. We close them using a *PDF method*.

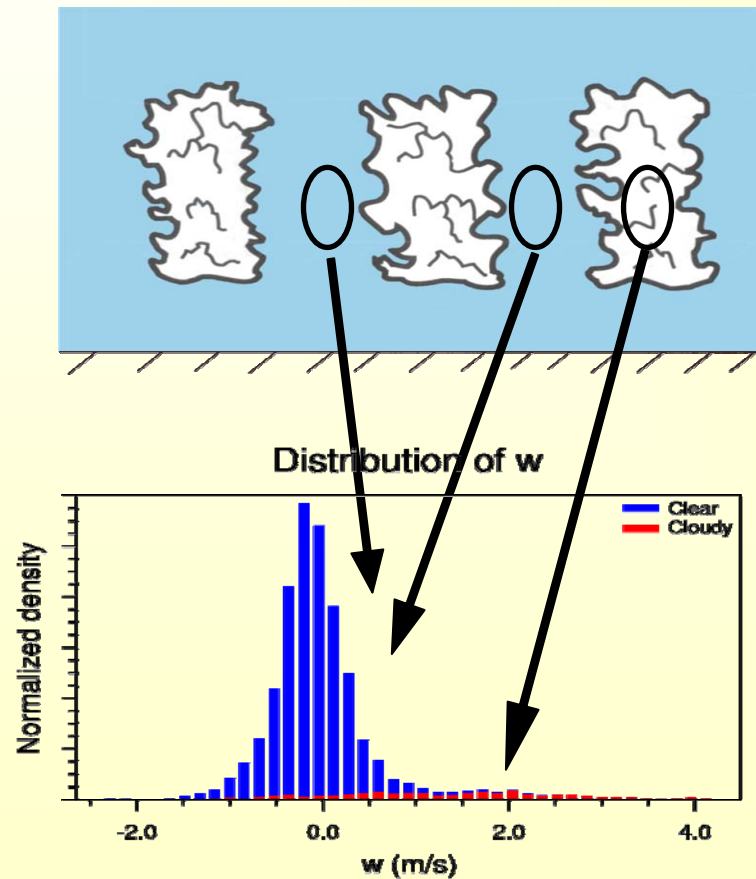
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What is a Probability Density Function?

A PDF is a histogram:



We can generalize the PDF to include several variables

We use a three-dimensional PDF of vertical velocity, w , total water mixing ratio, q_t , and liquid water potential temperature, θ_l :

$$P = P(w, q_t, \theta_l)$$

We close terms by integrating over the PDF. This allows us to couple subgrid interactions of vertical motions and buoyancy. In particular, it lets us compute the buoyancy flux in partly cloudy layers and downdrafts induced by cloud-top radiative cooling in overcast layers.

Randall et al. (1992)

An advantage of using PDFs: Consistency

Using a single, joint (3D) PDF allows us to find many closures that are consistent with one another.

Often cloud parameterization is thought of as the separate closure of many microphysical, thermodynamic, and turbulent terms. In contrast, our focus is on the parameterization of a single, general PDF.

Lappen and Randall (2001)

The Assumed PDF Method

Unfortunately, predicting the PDF directly is too expensive.

Instead we use the Assumed PDF Method. We *assume* a *family* of PDFs, and select a *member* of this family for each grid box and time step. (We assume a double Gaussian PDF family.)

Therefore, the PDF varies in space and evolves in time.

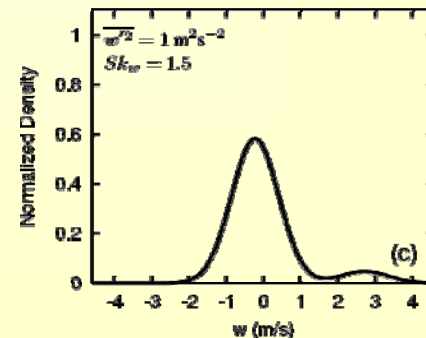
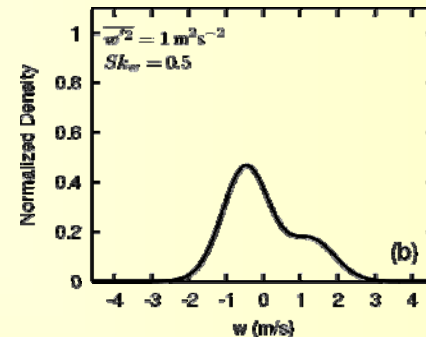
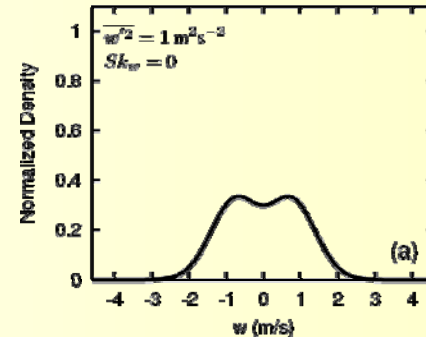
E.g., Manton and Cotton (1977)

The Double Gaussian PDF Family

A double Gaussian PDF is the sum of two Gaussians. It satisfies *three important properties*:

- (1) It allows both negative and positive skewness.
- (2) It has reasonable-looking tails.
- (3) It can be multi-dimensional.

We do not use a completely general double Gaussian, but instead restrict the family in order to reduce the number of parameters.

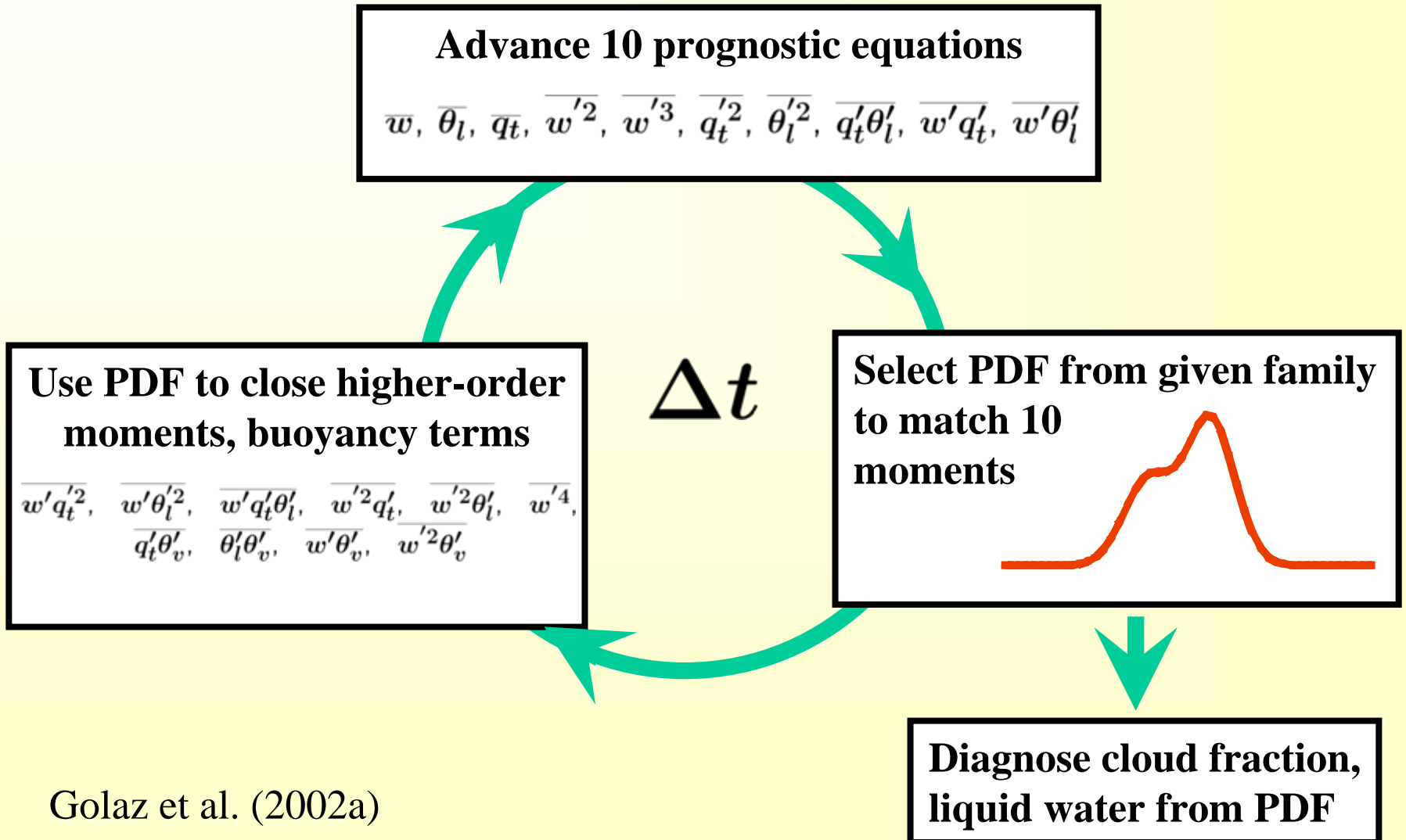


Steps in the Assumed PDF Method

The Assumed PDF Method contains **3** main steps that must be carried out for each grid box and time step:

- (1) Prognose means and various higher-order moments.
- (2) Use these moments to select a particular PDF member from the assumed family.
- (3) Use the selected PDF to compute average of various higher-order terms that need to be closed, e.g. buoyancy flux, cloud fraction, etc.

Schematic of the Assumed PDF method



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One problem: there are many tunable parameters

There is at least one tunable parameter per unclosed term in the prognostic equations. This is a lot.

However, tunable parameters are a necessary part of parameterization. They are not an evil per se.

Rather, the root of the problem is overfitting. The solution is more test cases.

Does tuning deserve its sordid reputation?

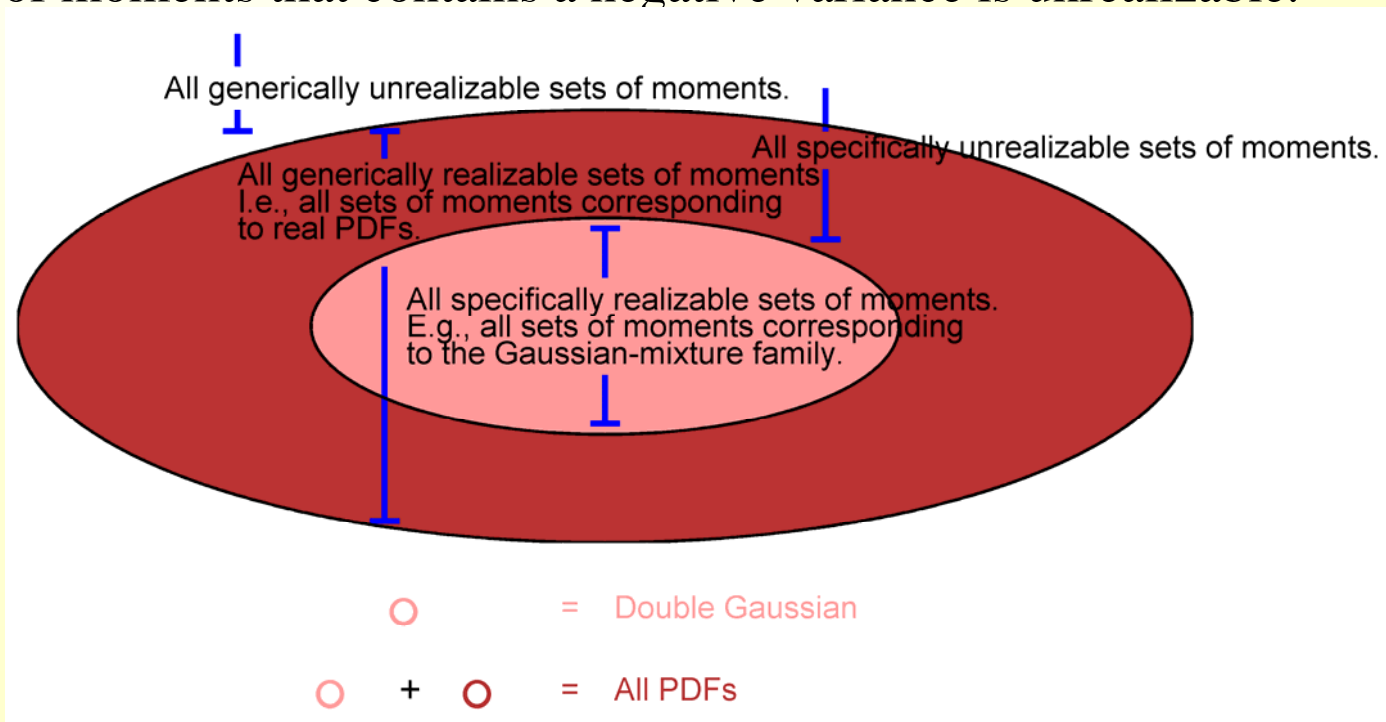
Note an analogy with neural nets. The equations in a neural net contain no physics, only tuning parameters.

Neural net	High-order closure w/ tunable parameters
“Weights”	Tuning parameters
“Learning/Training”	Tuning
“Memorizing”	Overfitting

And yet with enough training data, neural nets work well for some problems. Surely, adding physics, as does higher-order closure, can only improve the performance.

Another problem for higher-order closure: Realizability

The realizability problem is simplified but not removed by PDF methods (Larson and Golaz 2005). Not all sets of prognosed moments lead to a “realizable” PDF, that is, a PDF that is normalized and non-negative everywhere. For instance, any set of moments that contains a negative variance is unrealizable.



Conditions for realizability

If the following conditions on the moments are satisfied, then one can find a corresponding realizable (double Gaussian) PDF (Larson and Golaz 2005):

- (1) Positive variances.
- (2) Correlations between $(-1,1)$.
- (3) A third condition relating the correlations. It disallows impossible cases such as moisture and temperature being perfectly correlated with velocity but perfectly anti-correlated with each other.

Other difficult problems:

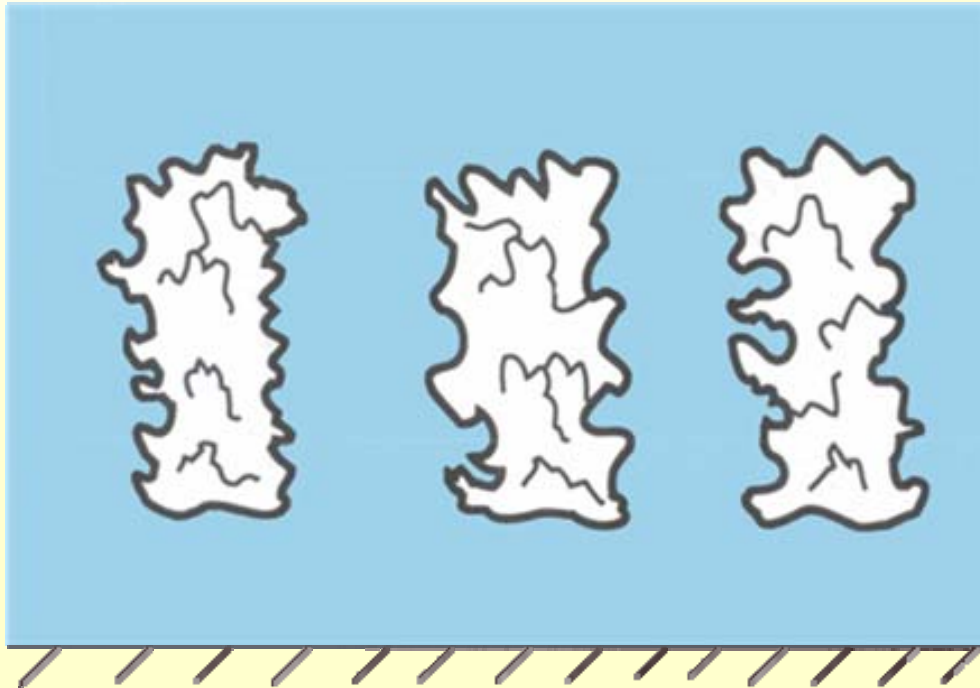
- The need to have fine vertical grid spacing (~ 100 m).
(However, even large-eddy simulations require fine vertical grid spacing.)
- Dissipation length scales. Mellor and Yamada (1982): “The major weakness of all the models probably relates to the turbulent master length scale.”

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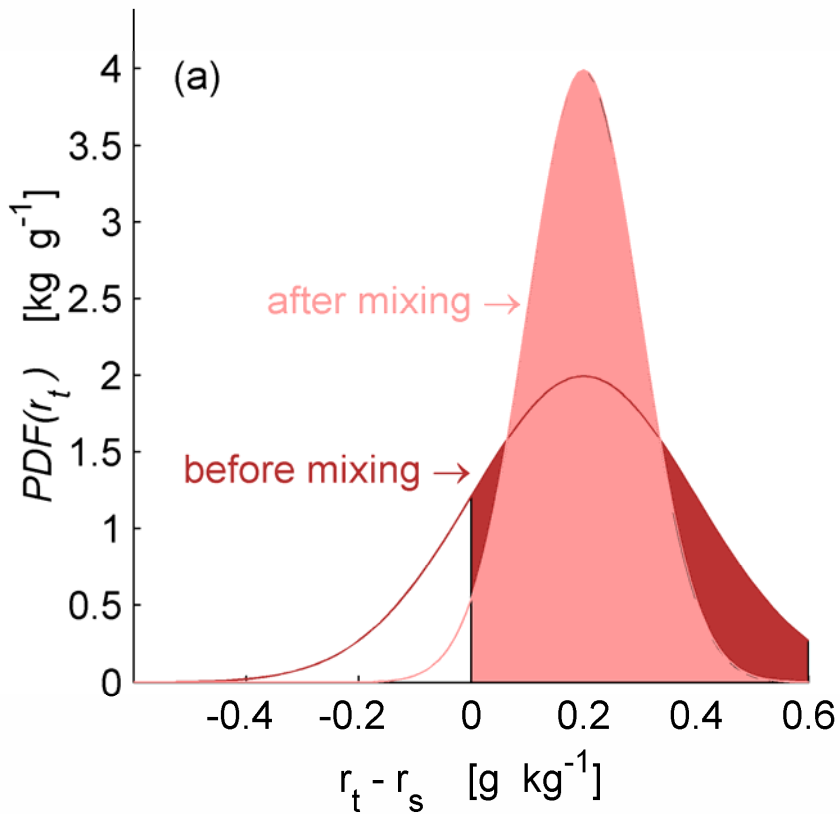
Given the problems confronting the PDF method, should we instead prognose quantities such as cloud fraction?

Clouds can dissipate or grow due to small-scale turbulent mixing. This can happen even in the absence of large-scale gradients.

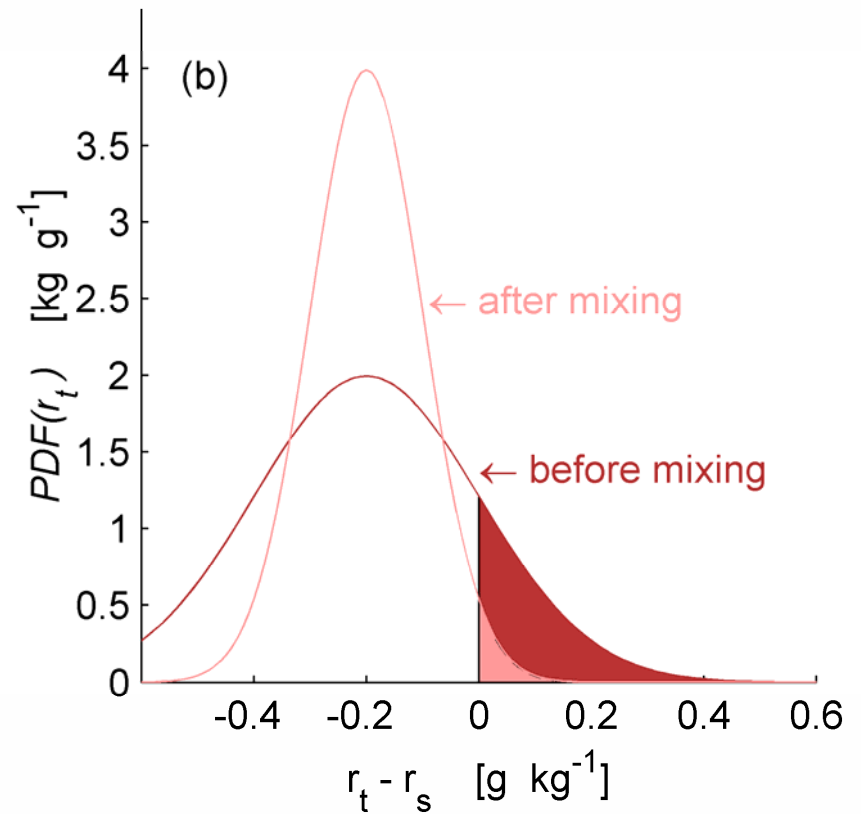


Small-scale mixing may either increase or decrease cloud fraction

Initial High Cloud Fraction \Rightarrow Increase



Initial Low Cloud Fraction \Rightarrow Decrease



Prognosing cloud fraction does not evade the need to know information about the PDF.

Consider the term that governs dissipation of cloud via small-scale mixing.

One standard model yields, roughly speaking, (Larson 2004):

$$\text{Dissipation} \propto [PDF(r_t = r_s)] [\overline{r_t} - r_s]$$

In this model, **dissipation depends on the PDF**, and can either increase or decrease cloud fraction. One can't avoid PDFs!

In contrast, Tiedtke (1993) proposes

$$\text{Dissipation} \propto (r_s - r_v)$$

This *always* decreases cloud fraction, albeit slowly when the grid box is near saturation.

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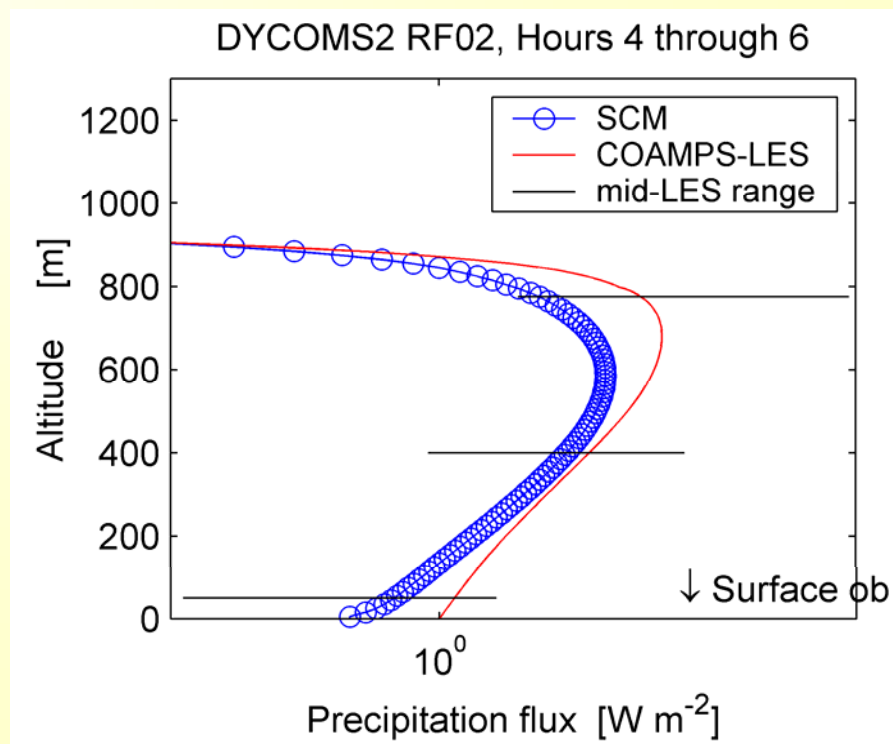
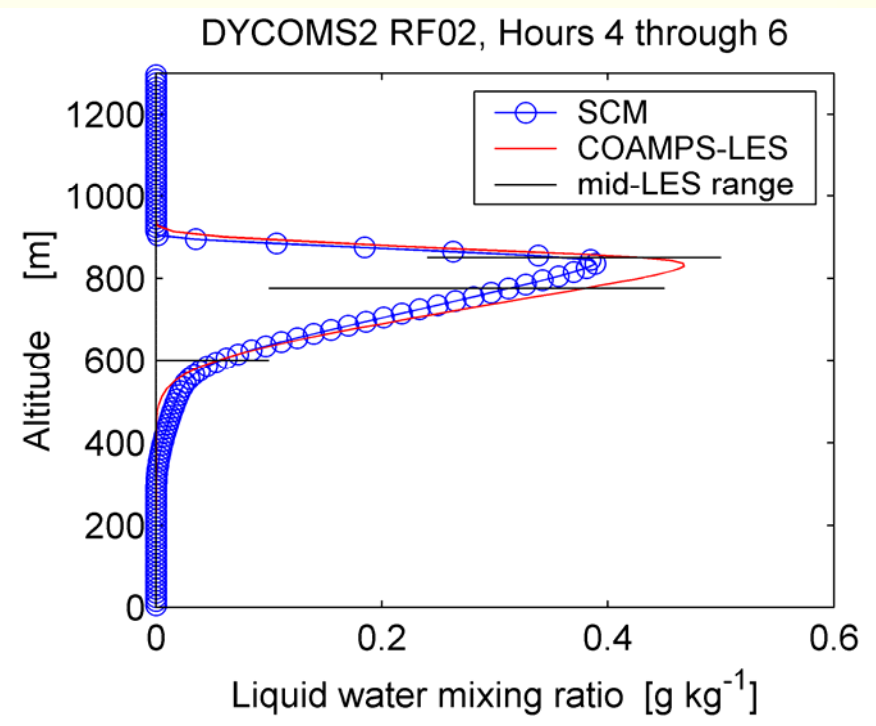
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Sample results

- Parameterization test cases:
 - **FIRE**: nocturnal stratocumulus clouds
 - **BOMEX**: trade-wind cumulus clouds
 - **ARM**: diurnal variation of shallow Cu over land
 - **ATEX**: intermediate regime (Cu under broken Sc)
 - **Wangara**: dry convective layer
 - **DYCOMS-II**: stratocumulus clouds

Drizzle stratocumulus: Results from the recent DYCOMS II RF02 GCSS intercomparison

Our higher-order closure **SCM** is compared with **COAMPS[®]-LES**. Error bars span the middle 50% of LESs in the intercomparison.



Conclusions

- Higher-order closure embeds the physics in the governing equations. That is, it does not attempt to outsmart the Navier-Stokes equations (for the most part).
- The Assumed PDF method allows one to close many terms in an internally consistent way.
- One can sometimes construct simple realizability constraints that state when a set of moments is consistent with an assumed PDF.
- Prognosing cloud fraction does not circumvent the need to know information about the relevant PDF.

Thanks for your attention!