Scaling Laws for Energy-constrained Wireless Networks

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Abstract

Although there has been much interest in scaling laws for wireless networks, most work has focussed on characterizing throughput and delay in power-constrained networks. Energy-constrained networks have not received much attention. Previous work [1] on energy-constrained networks was based on models that ignored interference. In this work energy-constrained networks are studied, taking interferences into account.

The number of bits per unit energy is introduced, a quantity that measures performance of a wireless network in terms of energy consumption. Scaling laws for the number of bits per unit energy are analyzed on a extended random wireless network using the physical model. An upper bound is given and it is shown that it is achievable. The tradeoff between the number of bits per unit energy and the average bit delay is characterized. Finally, it is shown that there is no tension between the throughput and the number of bits per unit energy, i.e. they can be simultaneously optimized.

Contents

Chapter 1

Introduction

The work by Gupta and Kumar [2] has caused a significant amount of follow-up work on scaling laws for wireless networks, e.g. [3, 4, 5, 6]. Most of the work in the field has focussed on performance of a wireless network in terms of throughput and transport capacity. The main question asked in this context is how the throughput or transport capacity scales in relation with the number of nodes in the network. The results are stated as upper and lower bounds on the order of throughput that is achievable in a network.

Another quantity that has been studied [7, 8, 9, 10, 11, 12] in wireless networks is the delay of transmissions. The delay is often studied in the context of mobile networks, i.e. networks were nodes are moving around. Mobile networks will not be the focus of this report. In [11] and [12] the tension between throughput and delay is characterized for static as well as mobile networks. Part of this report discusses the different delay models that are used in the literature.

One of the most important points to consider is the network model that is used. The results obtained in [2] are based on a model that allows only point-to-point communications. Two nodes communicating treat signals from all other nodes in the network as interference. One would like to find upper bounds on the performance of wireless networks in the most general model. Some first upper bounds on the throughput under the information-theoretic model were obtained recently [5, 6]. The delay analysis however has only been performed on point-to-point communication models.

All work cited above considers networks where nodes are limited in power. In this report we will analyze networks were each node is limited in energy. This is a relevant model for sensor networks or other networks where e.g. nodes are battery operated. One of the questions to be asked here is how many bits one can transport over the network given a constraint on the energy available in the batteries. To measure performance of a network in this sense we will introduce a new quantity, *bits per unit energy*. We will derive bounds on the achievable number of bits per unit energy and see the relations and tradeoff with the throughput and delay.

There has been some work on energy-constrained wireless networks [1]. The model that is used in this work is different from the one used in e.g. [2, 3, 4, 12]. The difference is that there is no fixed rate of transmission. This is close to an energy minimizing information-theoretic setting [13]. This model is, however, not a complete information-theoretic setting. The communication model that is used is the relaxed protocol model, which is still point-to-point. We comment on the choice of this model in Section 4.3 and conclude that it is preferable to use a different model. The reason is that the relaxed protocol model does contain an inherent notion of the interference caused by transmitting nodes. As a consequence it might be the case that the relaxed protocol model is too optimistic in the power that is required to achieve successful transmissions. If this is the case it affects the number of bits per unit energy. Using a different model allows for a more detailed analysis of the interference and hence of the energy consumption in a network.

In this report we analyze energy-constrained network using a different model, the physical model [2]. Under this model we derive upper and lower bounds on the number of bits per unit energy. We also analyze the relation with the throughput and the delay.

In Chapter 2 we introduce our model and give precise definitions of our model of communication and the performance quantities we are interested in. We introduce bits per unit energy and exactly formulate a problem statement. Chapter 3 gives an overview of previous work in the field. The relation between our network model and other models considered in the literature are explained and we justify the choices we made. Chapter 4 gives the main results obtained in our work. In order to explain the relation with previous work [2, 4, 12] it will also include some known results. In Chapter 5 we analyze in detail the problems given in previous chapters, the results given in Chapter 4 will be proven. Chapter 6, finally, gives an overview of what has been achieved in this project and gives some open problems that are of interest for future work.

Chapter 2

Model and Definitions

This chapter introduces the model and definitions that are used throughout this report. Section 2.1 introduces the extended random network and the restrictions we put on the way communication takes place. In Section 2.2 we introduce the physical model, the model that defines the requirements for successful communication Section 2.3 gives a precise definition of throughput. The number of bits per unit energy is introduced and formalized in Section 2.4. Our delay model is given in Section 2.5. Section 2.6, finally, gives a precise formulation of the problem we are interested in in this report.

2.1 Random Network

Definition 2.1 (Extended Random Network)**.** *An extended random network consists of the following:*

- *A surface of area n.*
- *n communication nodes randomly distributed on this surface according to a uniform distribution. The position of node i is denoted by X_i.*
- *Half of the nodes act as source nodes. The set of source nodes is denoted by $\mathcal{S}$ and it is chosen randomly from all size $n/2$ subsets of $\{1, 2, \ldots, n\}$.*
- *Each of the source nodes has to get its generated bits delivered to another node in the network. The destination node that corresponds to a source node is given by the bijective mapping $D : \mathcal{S} \mapsto \{1, 2, \ldots, n\} \setminus \mathcal{S}$ that is also randomly chosen. We call $(i, D(i))$ a source-destination (S-D) pair.*

We have called the above model an extended random network, to emphasize that we are working on a surface of area n. This in contrast to a dense random network, where the surface is of unit area. In the sequel we will simply speak of a (random) network, unless specified otherwise we will mean an extended random network.

We call the way communication is performed in the network a *scheme*, but we will interchangeably use the terms scheduling algorithm and policy. A specific schemes is denoted by Π. We restrict the way in which communication in the network takes place as follows:

- A source node can send bits to its destination by relaying through other nodes in the network. Although nodes in the network are either source of destination, all nodes can both receive and transmit information to do relaying for other S-D pairs in the network.

- Source nodes generate bits at a common rate λ. This means that a packet becomes available at a source each $1/\lambda$ seconds. We will see in 4.1 that if the network is stable, that λ is called the throughput rate of the network. The generation of packets can also be modelled as an ergodic stochastic process, but this will not change the analysis made in subsequent parts of this report.

- Time is *slotted*. In each time-slot nodes in the network can be either receiving or transmitting information. Transmissions are *packetized*. In a time-slot a node receives/transmits exactly one packet from/to exactly one node. As a consequence there can be at most $n/2$ simultaneous transmissions. Note, moreover, that since in a specific time-slot, a node can be either receiving or transmitting, packets/bits that are received by a relaying node in one-time slot, can not be transmitted before the start of the next time-slot.

- The length of the time-slots is fixed, constant time τ. The rate of transmissions is fixed to W. This implies that packets are of a fixed size $W\tau$.

- All transmissions are with a common power P. The power is restricted to $P_{\max}$, i.e. $P \leq P_{\max}$.

The schemes given in [2, 3, 4, 14, 15, 12] are all admitted by this model. The scheme given in [16, 11] is *not admitted*. We will make an analysis of this scheme in Section 3.3.1 and show that there is no benefit in loosening the model and allowing these kinds of schemes. The basic argument will be that the model needed in this case is not realistic in the sense that it will not give us schemes that are implementable in the real-world. More importantly, we do not need these construction, since all results [16, 11] given by these schemes can also be obtained in the restricted model[12].[1]

2.2 Models for Successful Transmission

Consider the case that node i is transmitting to node j. We define the signal to interference and noise ratio (SINR) for this transmission as

$$\mathrm{SINR}_{(i\to j)} = \frac{P\min[1, \|X_i - X_j\|^{-\alpha}]}{\sigma^2 + \sum_{k\neq i} P\min(1, \|X_k - X_j\|^{-\alpha})}, \tag{2.1}$$

where P is the transmission power used, $\min[1, \|X_i - X_j\|^{-\alpha}]$ is the attenuation from node i at position X_i to node j at position X_j, σ^2 is the common noise power and the sum in the denominator is over all other nodes transmitting at the same time.

We will use the following model for successful transmission between two nodes.

[1] This is the case for the static networks. In networks with mobile nodes, this remains to be shown.

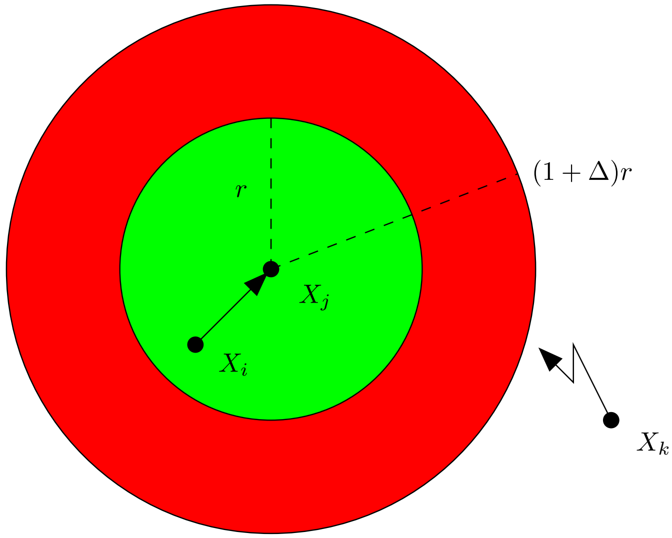

Figure 2.1: The protocol model. Transmission from the node at position X_i to node X_j is successful if and only if the distance from X_i to X_j is at most r and other transmitting nodes are at a distance at least $(1+\Delta)r$.

Definition 2.2 (Physical Model). *Transmission from transmitter i to receiver j is successful if*

$$\mathrm{SINR}_{(i\to j)} \geq \beta. \tag{2.2}$$

Other models of successful transmission have been considered in the literature. We present two common ones here.

Definition 2.3 (Protocol Model). *Let $r > 0$, $\Delta > 0$. Transmission from transmitter i to receiver j is successful if*

$$\|X_i - X_j\| \leq r, \tag{2.3}$$

and moreover,

$$\|X_k - X_j\| \geq (1+\Delta)r, \tag{2.4}$$

for every other transmitting node k.

Definition 2.4 (Relaxed Protocol Model). *Let $r > 0$, $\Delta > 0$ Transmission from transmitter i to receiver j is successful if*

$$\|X_k - X_j\| \geq (1+\Delta)\|X_i - X_j\|, \tag{2.5}$$

for every other transmitting node k.

Figures 2.1 and 2.2 provide some intuition about the protocol model and the relaxed protocol model. Note that in [2], the relaxed protocol model is denoted as the protocol model for arbitrary networks. It was shown in [2] that if a throughput is achievable under the protocol model then it is achievable

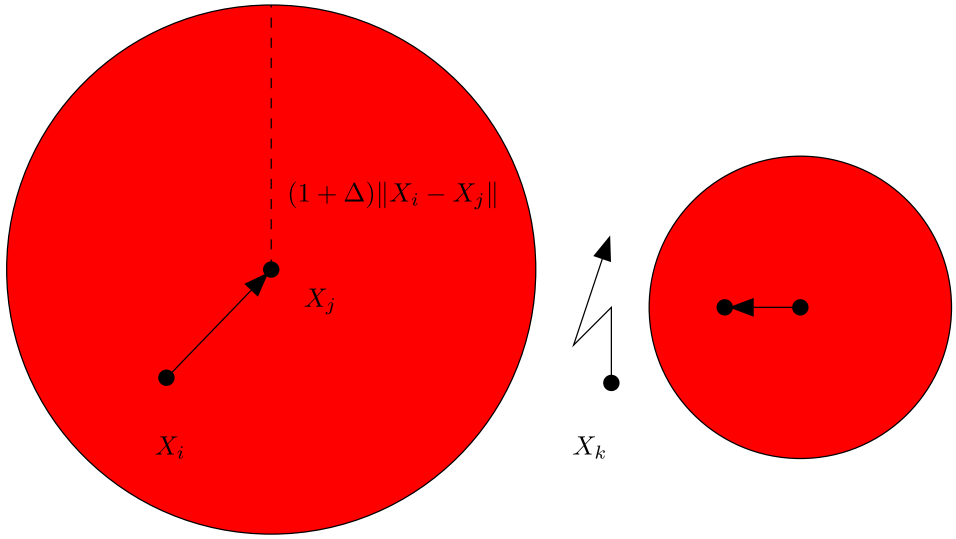

Figure 2.2: The relaxed protocol model. Transmission from the node at position X_i to node X_j is successful if and only if there are no other transmitting nodes within a distance $(1+\Delta)\|X_i - X_j\|$. There is no upper bound on the range of transmissions like in the protocol model.

under the physical model. It was also shown that upper bounds under the relaxed protocol model hold under the physical model. In this sense we can say that the protocol model respectively the relaxed protocol model are restrictions resp. relaxations of the physical model. From [4] we know that the relaxed protocol model and the protocol model do give different behavior in terms of the maximum achievable throughput. The protocol model and relaxed protocol model are included here, because they are used in previous work [2, 3, 12, 11]. We have included these models to simplify our discussion in the sequel and make comparisons between results. Unless explicitly stated otherwise, the model that is used in this report is the physical model.

2.3 Throughput

The most studied property of wireless networks studied is the throughput. It gives the number of bits a source-destination pair can exchange per second, minimized over all pairs. For a scheme to achieve a throughput λ we require each S-D pair in the network to exchange at least λ bits per second.

Definition 2.5 (Throughput of a Scheme)**.** *Given a network realization, using scheme* Π*, let* $B_\Pi(i,t)$ *be the number of source bits that node* $D(i)$ *receives in time slot* t*. Scheme* Π *is said to have throughput rate* λ_Π *on this network realization if*

$$\lambda_\Pi = \max\left\{\lambda' : \min_{i\in\mathcal{S}} \liminf_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} B_\Pi(i,t) \geq \lambda'\right\}. \tag{2.6}$$

We see that for stable network the rate at which bits are generated by the sources equals the throughput.

Definition 2.6 (Feasible Throughput)**.** *Given a network realization, a throughput* λ *is* feasible *if there exists a communication scheme with throughput* λ*.*

Definition 2.7 (Throughput Capacity)**.** *The* throughput capacity *of a random network is of order* $f(n)$ *if there exists constants* c_1 *and* c_2*,* $0 < c_1 < c_2 < \infty$*, such that*

$$\Pr\{\lambda(n) = cf(n) \text{ is feasible}\} \overset{n\to\infty}{\to} 1 \tag{2.7}$$

and

$$\Pr\{\lambda(n) = c'f(n) \text{ is feasible}\} \overset{n\to\infty}{\to} 0. \tag{2.8}$$

The throughput capacity is not explicitly used in our work. The set of achievable throughput rates will be given through Definition 2.14, the set of jointly feasible throughput rates, delays and bits per unit energy. In Chapter 3 we will discuss some of the previous work on the throughput capacity.

2.4 Bits per Unit Energy

One of the contributions of this work is to define a measure for the performance of a wireless network in terms of the amount of energy that is spent to transmit bits. We will introduce a measure for the number of bits that can transported in the network given a constraint on the energy that is spent. We have a choice in how to measure the amount of energy that is spent. The first possibility is to consider the sum of the energies spent by all nodes. The second option is to consider the amount of energy spent by an individual node, maximized over all nodes. In Chapter 1 we have justified our research into energy-constrained networks by giving the example of a sensor network with an energy constraint on each node. We have chosen to define the number of bits per unit energy in terms of an individual energy constraint per node.

The number of *bits per unit energy* for a scheme Π is the number of bits that can be delivered for each of the S-D pair in the network such that every node in the network spends at most one unit energy (Joule). More precisely we have the following:

Definition 2.8 (Bits per Unit Energy of a Scheme)**.** *Under scheme* Π*, let S-D pair* i *exchange bits* $b_i(1), b_i(2), \ldots, b_i(\gamma\mathcal{E})$*. Let* $E^i_\Pi(b)$ *be the energy that node* i *spends to do relaying for bit* b*. The total energy spent by node* i *is*

$$\sum_{j\in\mathcal{T}} \sum_{b=1}^{\gamma\mathcal{E}} E^i_\Pi(b_j(b)). \tag{2.9}$$

The number of bits per unit energy*,* γ*, of scheme* Π *is*

$$\gamma = \max\left\{\gamma' : \left[\max_{i\in\{1,\ldots,n\}} \limsup_{\mathcal{E}\to\infty} \frac{1}{\mathcal{E}} \sum_{j\in\mathcal{S}} \sum_{b=1}^{\gamma'\mathcal{E}} E_i(b_j(b))\right] \le 1\right\}. \tag{2.10}$$

Definition 2.9 (Feasible Bits per Unit Energy)**.** *Given a network realization, bits per unit energy* γ *is* feasible *if there exists a communication scheme that has* γ *bits per unit energy.*

2.5 Delay

Definition 2.10 (Delay of a Bit)**.** *The delay of a bit is the time it takes a bit to reach its destination after it is generated by its source.*

Let $\delta_\Pi^i(j)$ denote the delay of bit j of S-D pair i under scheme Π. The sample mean of delay for S-D pair i is

$$\bar{\delta}_\Pi^i = \limsup_{k\to\infty} \frac{1}{k} \sum_{j=1}^{k} \delta_\Pi^i(j). \tag{2.11}$$

Note that in a specific scheme the delay for a bit can be infinite, i.e. it doesn't reach its destination. Taking the supremum limit in the definition of the sample mean delay for a S-D pair makes sure that this is always properly defined.

Definition 2.11 (Delay of a scheme)**.** *The delay of a scheme* Π *is the average delay over all S-D pairs, i.e.*

$$\delta_\Pi = \frac{1}{|\mathcal{S}|} \sum_{i\in\mathcal{S}} \bar{\delta}_\Pi^i \tag{2.12}$$

This definition of delay is different than the one considered in [16] and [11]. The delay model considered in that work is the packet delay. A second difference between our work and the work in [16] and [11] is that the packet sizes in the model considered there are allowed to scale down. This means that in general the bit delay is a multiple of the packet delay. The two delay models are therefore significantly different. We will discuss the packet delay model in Section 3.3.1. We will comment on other delay models that have not been considered in the literature in Section 6.1.2.

Definition 2.12 (Feasible Delay)**.** *Given a network realization, a delay* δ *is* feasible *if there exists a communication scheme that has delay* δ*.*

2.6 Problem Formulation

The topic of this report is to analyze the set of jointly feasible throughputs, bits per unit energy and delays. More precisely we are interested in the scaling behavior of these quantities. We will consider e.g. a function $\gamma(n)$ and see if a number of bits per unit energy $\gamma(n)$ is feasible on a n node network in the high n limit. We are only interested in the order behavior of the functions we are considering, constants are usually dropped.

Definition 2.13 (Feasible (λ,γ,δ))**.** *Given a realization of an n node network, triple* $(\lambda(n),\gamma(n),\delta(n))$ *is feasible if there exists a scheme that has throughput* $\lambda(n)$*, bits per unit energy* $\gamma(n)$ *and delay* $\delta(n)$*.*

Definition 2.14 (Achievable $(\lambda(n),\gamma(n),\delta(n))$ Region)**.**

$$\mathcal{R} = \left\{ \Big(\lambda(n),\gamma(n),\delta(n)\Big) \; ; \; \Pr\left\{ \Big(\lambda(n),\gamma(n),\delta(n)\Big) \textit{ is feasible}\right\} \stackrel{n\to\infty}{\to} 1 \right\}.$$

In Chapter 4 we will present results of two types. The first type gives conditions on $(\lambda(n), \gamma(n), \delta(n))$ given that is element of $\mathcal{R}$. These are upper bound or converse type results. Examples are upper bounds on the feasible throughputs and the minimum achievable delay given a delay constraint. The upper bound on the throughput will for example be presented as

$$\Big(\lambda(n),\ \cdot\ ,\ \cdot\ \Big) \in \mathcal{R} \Rightarrow \lambda(n) = \mathrm{O}\left(\frac{1}{\sqrt{n}}\right),$$

where the dots on the LHS mean that this holds for any $\gamma(n)$ and $\delta(n)$.

The second type of result gives sets of $(\lambda(n), \gamma(n), \delta(n))$ that are part of $\mathcal{R}$. These are achievability results. An example is that the upper bound on the throughput is achievable, which will be presented as

$$\lambda(n) = \mathrm{O}\left(\frac{1}{\sqrt{n}}\right) \Rightarrow \begin{cases} \exists\ c > 0, \text{ such that} \\ \Big(c\lambda(n),\ \cdot\ ,\ \cdot\ \Big) \in \mathcal{R}, \end{cases}$$

where the dots on the RHS mean that there exist functions $\gamma(n)$ and $\delta(n)$ such that the above holds.

Chapter 3

Previous Work

3.1 A Constructive Scheme

The following scheme was introduced in [2] and later used in [12]. We formulate the scheme in such a way that it can be used on different surfaces, we will elaborate on this after the definition of the scheme.

Scheme 1 (Relaying Along Straight Lines [2, 12]). *Given an n node network. Fix a constant $a(n)$ such that $a(n) = \Omega(\log(n))$.*

- *Create a tessellation on the network surface, such that all cells in the tessellation have area $\Theta(a(n))$ and the maximum distance within a cell is $\Theta(\sqrt{(}n))$*[1]

- *Let the straight line connecting a Source to its Destination be denoted as an S-D line. Packets are transmitted from source to destination by relaying from one cell to a neighbouring cell along the S-D lines.*

- *Activate cells in a TDMA fashion. Only nodes in active cells are allowed to transmit. Let the length of the TDMA schedule be constant. This means that each cell is activated at regular time intervals, the length of the interval does not grow in n.*

- *Each cell is crossed by a certain number of S-D lines, including those belonging to source and destination nodes. Each time a cell is activated it relays a packet for one of the S-D pairs crossing that cell.*

- *The tessellation, the time-slot assignment and the power at which is transmitted are such that all transmissions are successful.*

Given this restriction on communication and the requirements for the scheme given above, we are working exactly in the protocol model [2], Definition 2.3.

The surface that is used in [2] is the two dimensional sphere. On the sphere it is not possible to create a regular tessellation (i.e. where all cells look the

[1] It is possible that requiring a maximum distance of $\Theta(\sqrt{n})$ is too strong. It is not clear if there exists a tessellation of this type on each surface. In the surfaces that have been considered for wireless networks so far, however, the constraint can be met in the sense that suitable tessellations exist.

same). The formulation of the scheme is therefore in terms of a tessellation where the area of the cells does not need to be exactly $a(n)$. The condition on the maximum distance within a cell is necessary to bound the maximum length of a hop. If the surface of the network is a square in the plane or a torus, one can easily create a regular tessellation by considering squares of area $a(n)$.

The reason that we need $a(n) = \Omega(\log(n))$ is as follows. Since we are relaying along straight lines, each cell needs to contain at least one node. If the cell size is constant in the number of nodes, then there will be empty cells with non negligible probability. By taking the cell size of order $\log(n)$ or higher it can be shown [2, 3, 11] that *whp* each cell will contain at least one node.

Lemma 3.1 ([2, 3, 11]). *Each cell in the tessellation contains* $\Theta(a(n))$ *nodes* whp.

One of the main properties needed to analyze throughput, delay and number of bits per unit energy that are achieved by the scheme is stated in the following lemma.

Lemma 3.2 ([2, 3, 11]). *The number of S-D lines passing through a cell is* $\Theta(\sqrt{na(n)})$ whp.

Proof. (*sketch*) We only provide some intuition, but this lemma can be rigorously proven.

Each S-D pair has to cover $\Theta\left(\sqrt{n}\right)$ distance along cells of diameter $\Theta(\sqrt{a(n)})$, and hence passes through $\Theta(\sqrt{n}/\sqrt{a(n)})$ cells. There are $\Theta(n/a(n))$ cells and $\Theta(n)$ S-D lines. Through each cell there are therefore

$$\Theta\left(n\sqrt{\frac{n}{a(n)}}\left(\frac{n}{a(n)}\right)^{-1}\right) = \Theta\left(\sqrt{na(n)}\right) \tag{3.1}$$

S-D lines. □

If the surface of the network is the sphere, the proof of the above lemma is tedious. Since there exists no regular tessellation, the proof relies on VC theory and uniform convergence in the law of large numbers [2]. Working on the rectangle in the plane or on the torus, greatly simplifies the analysis [3, 11].

3.2 Throughput Capacity

One of the contributions of [2] was to show that under the physical model the throughput capacity is $\Omega(1/\sqrt{n\log(n)})$ and $O(1/\sqrt{n})$. The gap in the lower and upper bound was recently closed [4]. The throughput capacity of a random wireless under the physical model is

$$\lambda(n) = \Theta\left(\frac{1}{\sqrt{n}}\right). \tag{3.2}$$

The scheme constructed in [2] to show achievability of $1/\sqrt{n\log(n)}$ is Scheme 1. Note that since Scheme 1 does relaying along straight lines, the area of the cells needs to be of at least order $\log(n)$. It was shown in [4] that by using a constant cell size a throughput of $\Theta(1/\sqrt{n})$ can be obtained. The method is based on percolation theory [17]. The need of full connectivity between cells is exactly what prevents Scheme 1 from achieving the throughput capacity.

Recent work [5, 6] seems to indicate that the upper bound for the throughput capacity given above, does hold in an information-theoretic setting. In this setting, the model of communication is as general as possible. In [6] it is shown that the throughput capacity of a linear or planar network is decreasing in the number nodes. For linear networks the decay is shown to be $O(1/\sqrt{n})$, for planar networks the decay proven in [6] is not as fast. In [5] upper bounds are derived on the transport capacity of a network. These bounds on the network capacity can be used to bound the throughput capacity. What remains to be done is to apply the results of [5] on arbitrary networks, to a random network. A point of attention is that the results in [5] depend on a high attenuation of the signal power over distance ($\alpha > 6$ for the physical model, Definition 2.2.)

In this report only static networks are considered, nodes are at fixed positions. The achievable throughputs of $\Theta(1)$ given in [14, 15] by allowing mobile nodes are not relevant for our model.

3.3 Throughput-Delay Tradeoff

It is shown in [12] that increasing the throughput in a network increases the delay. An exact tradeoff is given that shows what the highest possible throughput is given a certain delay constraint. There are two differences in model that is used with respect to the definition made in Chapter 2:

- The model for successful communications that is used is the relaxed protocol model.
- The definition of delay given is that it is the time a bit spends in the network after it leaves the source. Definition 2.10, on the contrary, states that it is the time between generation of a bit by its source and reception of the bit by its destination.

The results given in [12] do hold for our model, we comment on this in Section 5.1.2. It was already shown in [2] that upper bounds on the throughput under the relaxed protocol model are also upper bounds under the physical model. The upper bound on the throughput-delay tradeoff follows directly from [2] and [12]. It can also be derived from first principles. The upper bound holds for all values of the throughput. Achievability, however, is only proven for throughput rates $O(1/\sqrt{n \log(n)})$. We will comment on achievability of the tradeoff for higher values of the throughput in Section 5.1.3.

The tradeoff result is precisely stated in Theorems 4.3 and 4.4.

3.3.1 The Fluid Model

The fluid model was introduced in [11] to analyze the throughput-delay tradeoff in static and mobile networks without having to worry about queueing of packets at relaying nodes. The fluid model uses the following relaxations of the model given in Section 2.1:

- Packets are not of a fixed size. Different schemes can use different packet sizes, e.g. depending on the number of nodes in the network. Packets that contain only a fraction of a bit are allowed.

- If packet sizes are sufficiently small, nodes are allowed to send more than one packet in one time-slot.
- In one time-slot, one node is allowed to send packets to multiple nodes.

More importantly, the definition of the delay is taken as follows:

- The delay of a *packet* is the average time a packet spends in the network after it leaves the source [11].

Using an appropriately chosen packet size it is shown that there is no queueing of packets at relaying nodes. This allows to conclude that the delay of the packets scales the same way as the number of hops that is taken. It is important to note here that the delay of a bit is not necessarily equal to the packet delay. The constructive scheme used in [11] in fact scales the sizes of packets down proportional to the throughput. The delay of a bit is therefore equal to the packet delay divided by the throughput (which is a number smaller than one).

In follow-up work [12] it is shown that in static networks there is no need to relax the model. The optimal throughput-delay tradeoff can be achieved using the model given in Chapter 2. More importantly, defining delay as the delay of a bit, does not affect the result. The upper bound is the same for the two delay models and it can be achieved.

The scheme given in [16, 11] is *not admitted* by the definitions given in Chapter 2. There is no benefit in loosening the model and allowing these kinds of schemes. The fact is that the model needed in this case is not realistic in the sense that it will not give us schemes that are implementable in the real-world. More importantly, we do not need these construction, since all results [16, 11] given by these schemes can also be obtained in the restricted model[12].[2]

The throughput-delay tradeoff in mobile networks is studied for different models in e.g. [7, 8, 9, 10].

The throughput-delay tradeoff is not explicitly dealt with in [3], but it seems likely that the methods presented there can be extended to analyze the throughput-delay tradeoff. This might give a simplified proof of the results given in [12].

3.4 Energy-Constrained Wireless Networks

There has not been much attention to scaling laws for energy-constrained wireless networks. To the authors best knowledge, the only results on scaling laws for energy-constrained wireless networks are found in [1]. Strongly related work is on scaling laws of wireless networks in the wideband limit [18]. Other work on energy-constrained wireless networks is not considering scaling laws, but is focussing on energy-efficient scheduling [19, 20].

In [1] the tradeoff between energy consumption and delay is analyzed under the following model:

- The network is dense, i.e. the surface is of unit area. If the number of nodes increases, the distance between nodes decreases.

[2] This is the case for the static networks. In networks with mobile nodes, this remains to be shown.

- Transmissions are succesful under the relaxed protocol model of Definition 2.4.

- *Transission rate is not fixed.* Given a power P and a transmission range r, the transmission rate $R(P, r)$ is given by

$$R(P, r) = \frac{1}{2}\left(1 + \frac{Pr^{-\alpha}}{\sigma^2}\right). \tag{3.3}$$

The good point about this model is that not fixing the transmission rate brings it is close to an information-theoretic setting and the capacity per unit cost [13]. There are, however, also some remarks to be made:

1. The power attenuation model that is implied by (3.3) is that at a distance r from the transmitter the received power is $r^{-\alpha}$. Since in a dense network the distance between nodes is smaller than 1, the power that is received is larger than what is actually transmitted.

2. As discussed in Section 2.2 we know that throughputs that are achievable under the protocol model are also achievable under the physical model. It is proven in [2] that this is the case if the transmit power is sufficiently large. It is not shown, however, how power should scale up to get a feasible scheme under the physical model. If the required power scaling is too large then the number of bits per unit energy depicted by the (relaxed) protocol are not representative.

   By using the relaxed protocol model for the derivation of scaling laws for energy consumption, one might loose important features that are present in the physical model and the information-theoretic setting.

In this report we will analyze the tradeoff between energy consumption and delay under the physical model using a fixed transmission rate. We will, moreover, work on an extended network to prevent power amplification at receiving nodes.

Chapter 4

Main Results

In this chapter we give an overview of the results obtained. We will also include some of the results obtained in previous work in order to give a good overview of what is currently known about scaling laws for wireless networks.

The previously known results on the throughput capacity are given in Section 4.1. Section 4.2 contains the results on the throughput-delay tradeoff. In Section 4.3, finally, we present the contributions of our work.

4.1 Throughput Capacity

The following upper bound on the throughput was given in the seminal paper by Gupta and Kumar [2].

Theorem 4.1 ([2])**.** *The throughput is at most order $1/\sqrt{n}$, i.e.*

$$\Big(\lambda(n),\ \cdot\ ,\ \cdot\ \Big) \in \mathcal{R} \Rightarrow \lambda(n) = \mathrm{O}\left(\frac{1}{\sqrt{n}}\right).$$

The upper bound to the throughput that is given in Theorem 4.1 is actually achievable. It was shown in [2] that Scheme 1 on page 11 achieves $\lambda(n) = \Theta(1/\sqrt{n\log(n)}$. The gap was closed in [4].

Theorem 4.2 ([2], [4])**.** *A throughput of up to order $1/\sqrt{n}$ is achievable, i.e.*

$$\lambda(n) = \mathrm{O}\left(\frac{1}{\sqrt{n}}\right) \Rightarrow \begin{cases} \exists\ c > 0, \textit{ such that} \\ \Big(c\lambda(n),\ \cdot\ ,\ \cdot\ \Big) \in \mathcal{R}. \end{cases}$$

Theorems 4.1 and 4.2 establish that the throughput capacity, Defininition 2.7, is $\Theta(1/\sqrt{n})$.

4.2 Throughput-Delay Tradeoff

The following results are given in [11, 12] for the delay model that does not include queueing at the source. We show in Section 5.1.2 that the results hold for our delay model, Definition 2.10, as well. The upper bound on the throughput - delay tradeoff is given by the following theorem.

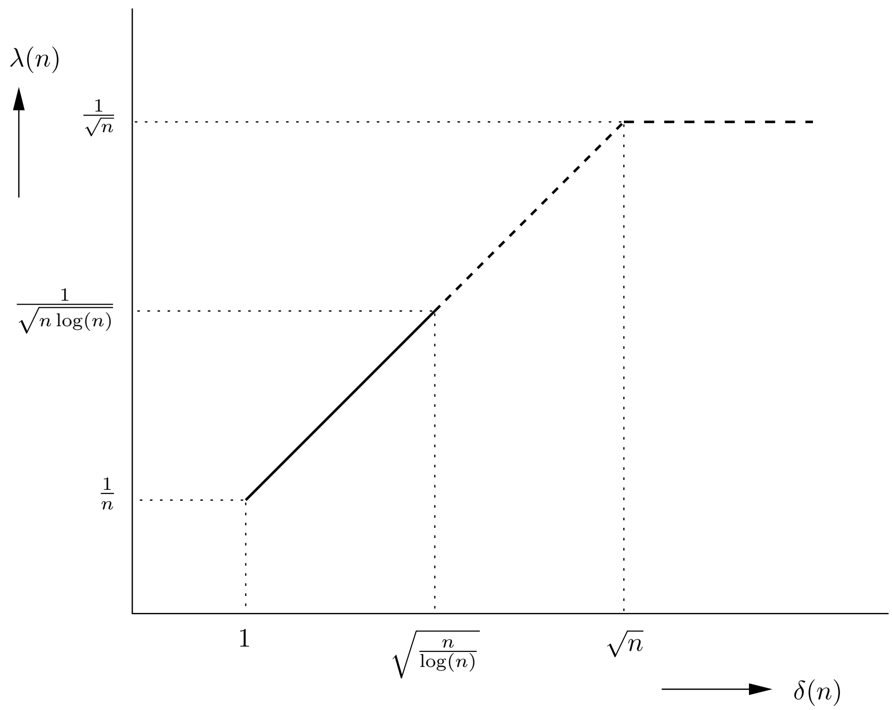

Figure 4.1: Throughput-delay tradeoff. There exist no schemes with delay $\delta(n)$ and throughput $\lambda(n)$ such that the pair $(\delta(n), \lambda(n))$ is above the curve given in the graph. Points on the solid line are achievable, that is given a delay $O(\sqrt{n/\log(n)})$ on the x-axis, there exists a scheme that has throughput equal to the value of the curve on the y-axis. The dashed line gives an upper bound only, achievability remains to be shown.

Theorem 4.3 ([11])**.** *Every scheme that has delay of order at most $\delta(n)$, can have a throughput of at most order $\delta(n)/n$, i.e.*

$$\Big(\lambda(n),\ \cdot\ ,\delta(n)\Big) \in \mathcal{R} \Rightarrow \lambda(n) = \mathrm{O}\left(\frac{\delta(n)}{n}\right),$$

or equivalently

$$\Big(\lambda(n),\ \cdot\ ,\delta(n)\Big) \in \mathcal{R} \Rightarrow \delta(n) = \Omega\left(n\lambda(n)\right).$$

A constructive proof that the above upper bound is achievable is provided [12] based on Scheme 1 on page 11. As a consequence, the result only holds for $\lambda(n) = \mathrm{O}(1/\sqrt{n\log(n)})$.

Theorem 4.4 ([12])**.** *For $\lambda(n) = \mathrm{O}(1/\sqrt{n\log(n)})$, there exists a scheme with $\delta(n) = \Theta(n\lambda(n)$, i.e. it achieves the upper bound given in Theorem 4.3.*

$$\lambda(n) = \mathrm{O}\left(\frac{1}{\sqrt{n\log(n)}}\right) \Rightarrow \begin{cases} \exists\ c_1, c_2 > 0, \textit{ such that} \\ \Big(c_1\lambda(n),\ \cdot\ ,c_2 n\lambda(n)\Big) \in \mathcal{R}, \end{cases}$$

or equivalently

$$\delta(n) = \mathrm{O}\left(\sqrt{\frac{n}{\log(n)}}\right) \Rightarrow \begin{cases} \exists\ c_1, c_2 > 0, \textit{ such that} \\ \Big(c_1\frac{\delta(n)}{n},\ \cdot\ ,c_2\delta(n)\Big) \in \mathcal{R}. \end{cases}$$

We conjecture that the upper bound given in Theorem 4.3 is achievable for the high through regime as well, we will comment on this in Section 5.1.3.

Conjecture 4.5. *For all $\lambda(n)$ below the throughput capacity there exists a scheme that achieves throughput $\Theta(\lambda(n))$ and the lower bound on the delay given in Theorem 4.3, that is*

$$\lambda(n) = \mathrm{O}\left(\frac{1}{\sqrt{n}}\right) \Rightarrow \begin{cases} \exists\ c_1, c_2 > 0, \textit{ such that} \\ \Big(c_1\lambda(n),\ \cdot\ ,c_2 n\lambda(n)\Big) \in \mathcal{R}. \end{cases}$$

The results stated in Theorems 4.1, 4.3 and 4.4 are depicted in Figure 4.1 on the facing page.

4.3 Bits per Unit Energy

There is an upper bound to the number of bits per unit energy that can be achieved. The scaling behavior is equal to that of the throughput capacity.

Theorem 4.6. *The bits per unit energy is at most of order $1/\sqrt{n}$, i.e.*

$$\Big(\ \cdot\ ,\gamma(n),\ \cdot\ \Big) \in \mathcal{R}. \Rightarrow \gamma(n) = \mathrm{O}\left(\frac{1}{\sqrt{n}}\right).$$

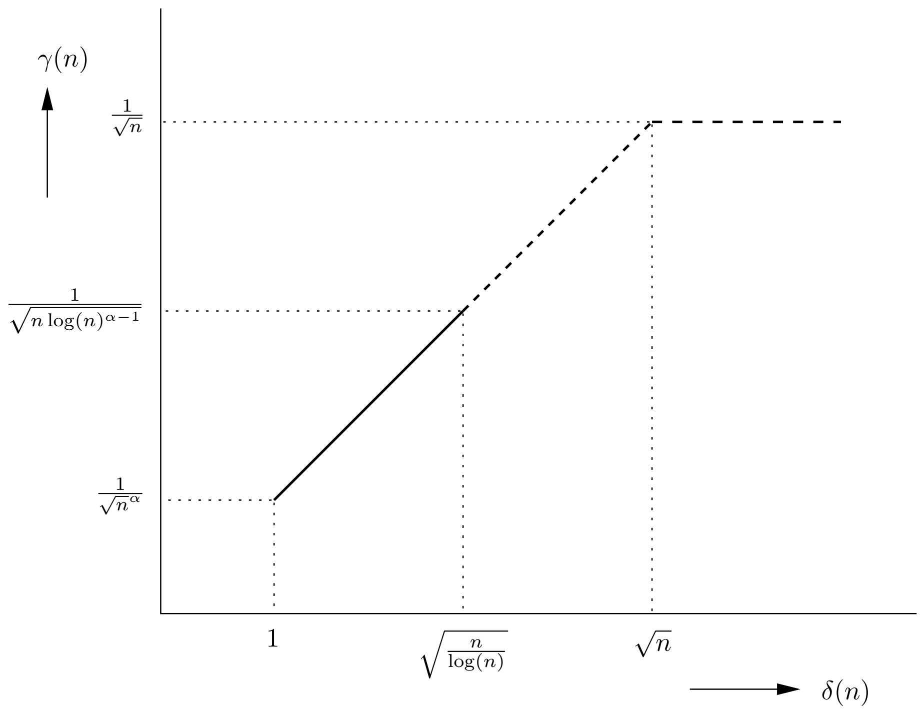

Figure 4.2: Bits per unit energy - delay tradeoff. There exist no schemes with delay $\delta(n)$ and number of bits per unit energy $\gamma(n)$ such that the pair $(\delta(n), \gamma(n))$ is above the curve given in the graph. Points on the solid line are achievable, that is given a delay $O(\sqrt{n/\log(n)})$ on the x-axis, there exists a scheme that has number of bits equal to the value of the curve on the y-axis. The dashed line gives only an upper bound, achievability remains to be shown.

In Section 5.3 we construct Scheme 2 and show that it achieves a number of bits per unit energy of at most order $1/\sqrt{n\log(n)^{\alpha-1}}$. There remains a gap between the upper and the lower bound.

Theorem 4.7. *Let $\gamma(n) = \mathrm{O}(1/\sqrt{n\log(n)^{\alpha-1}})$. There exists a scheme with $\Theta(\gamma(n))$ bits per unit energy, i.e.*

$$\gamma(n) = \mathrm{O}\left(\frac{1}{\sqrt{n\log(n)^{\alpha-1}}}\right) \Rightarrow \begin{cases} \exists\, c>0, \text{ such that} \\ \Big(\,\cdot\,, c\gamma(n),\,\cdot\,\Big) \in \mathcal{R}. \end{cases}$$

Scheme 2 is used to show that a number of bits per unit energy of order $1/\sqrt{n\log(n)^{\alpha-1}}$ is achievable using completely sequential transmissions. This sacrifices throughput and delay. The question is if there exists schemes that have a high number of bits per unit energy that achieve high throughput and low delay at the same time. We first formulate the following upper bound on the number of bits per unit energy given a delay constraint.

Theorem 4.8. *Every scheme that has delay of order at most $\delta(n)$, can have a number of bits per unit energy of at most order $\delta(n)^{\alpha-1}/n^{\alpha/2}$, i.e.*

$$\Big(\,\cdot\,,\gamma(n),\delta(n)\Big) \in \mathcal{R} \Rightarrow \gamma(n) = \mathrm{O}\left(\frac{\delta(n)^{\alpha-1}}{n^{\alpha/2}}\right),$$

or equivalently

$$\Big(\,\cdot\,,\gamma(n),\delta(n)\Big) \in \mathcal{R} \Rightarrow \delta(n) = \Omega\left(\gamma(n)^{1/(\alpha-1)}n^{\alpha/2(\alpha-1)}\right).$$

Similar to the throughput-delay tradeoff, we show achievability of the tradeoff upper bound by using Scheme 1 from Section 3.1. As a consequence the result only holds for the low bits per unit energy regime. The theorem is proven in Section 5.5. The main part of the proof is the show that one can schedule many simultaneous transmissions without needing more power then in the sequentially scheduled Scheme 2. This is shown by proving that the total interference received at a node is bounded by a constant, independent of n the number of nodes. This analysis immediatelly gives us the result stated in Corollary 4.10, which is given below.

Theorem 4.9. *For $\delta(n) = \mathrm{O}(\sqrt{n/\log(n)})$, there exists a scheme with $\gamma(n) = \Theta(\delta(n)^{\alpha-1}/n^{\alpha/2})$, i.e. it achieves the upper bound given in Theorem 4.8. More precisely we have that*

$$\delta(n) = \mathrm{O}\left(\sqrt{\frac{n}{\log(n)}}\right) \Rightarrow \begin{cases} \exists\, c_1, c_2 > 0, \text{ such that} \\ \Big(\,\cdot\,,\ c_1\frac{\delta^{\alpha-1}}{n^{\alpha/2}}\ ,\ c_2\delta(n)\ \Big) \in \mathcal{R}, \end{cases}$$

or equivalently,

$$\gamma(n) = \mathrm{O}\left(\frac{1}{\sqrt{n\log(n)^{\alpha-1}}}\right) \Rightarrow \begin{cases} \exists\, c_1, c_2 > 0, \text{ such that} \\ (\,\cdot\,,\ c_1\gamma(n)\ ,\ c_2\gamma(n)^{1/(\alpha-1)}n^{\alpha/2(\alpha-1)}\ ) \in \mathcal{R}. \end{cases}$$

Note that we used Scheme 1 to prove that a high number of bits per unit energy is available at low delay. Since Scheme 1 achieves the optimal throughput-delay tradeoff as well, we can conclude that a high throughput and a high number of bits per unit energy are jointly achievable.

Corollary 4.10. *There is no tension between number of bits per unit energy and throughput. Given a delay $\delta(n) = \mathrm{O}(\sqrt{n/\log(n)})$, there exists a scheme that achieves both the maximum number of bits per unit energy and the maximum throughput possible given that the delay is of order $\delta(n)$. More precisely we have that*

$$\delta(n) = \mathrm{O}\left(\sqrt{\frac{n}{\log(n)}}\right) \Rightarrow \left\{ \begin{array}{l} \exists\ c_1, c_2, c_3 > 0, \text{ such that} \\ \left(c_1 \frac{\delta(n)}{n}, c_2 \frac{\delta^{\alpha-1}}{\sqrt{n}^{\alpha}}, c_3\delta(n)\right) \in \mathcal{R}. \end{array} \right.$$

Once it has been shown in Section 5.5 that in Scheme 1 interference is bounded by a constant that does not depend on the number of nodes, the result in the above theorem is not hard to see. In our model the transmission rates and time-slot lengths are fixed, the energy needed to transfer a bit is exactly proportional to the power that is used to do this. The power that is needed in turn depends only the length of the hops that are taken. It was shown in [2] that short hop lengths are needed to achieve high throughput. We will see in Section 5.3 that to achieve a high number of bits per unit energy it is also beneficial to take short hops. We conclude that there is no tension between the throughput and the number of bits per unit energy.

In Section 3.4 we have seen that using a different model can give a tension between the number of bits per unit energy and the throughput. The model used in [1] allows for the power to decrease by reducing the transmission rate. This decrease in power is beneficial for the number of bits per unit energy, but it will decrease the througphut of a network. Since we are using a model were the transmission rates are fixed, these effects do not come into play here. One of the topics of future work will be to relax the conditions in our model by allowing the rate to scale down and analyze the tradeoffs that occur in this relaxed model.

The statements of Theorems 4.6, 4.8 and 4.9 are depicted in Figure 4.2 on page 20.

We conjecture that the upper bound to the number of bits per unit energy - delay tradeoff given in Theorem 4.8 is achievable for the entire number of bits per unit energy regime. Theorem 4.9 was proved using Scheme 1. It is shown in [4] that Scheme 1 in general does not achieve high performance, because it is based on full connectivity of cells. The construction used in [4] might be used as a starting point to prove that the bits per unit energy - delay tradeoff is achievable in the high number of bits per unit energy regime. The exact construction can not be used since it is based on the fact that a certain number of nodes perform all the relaying. For a high number of bits per unit energy it needs to be shown that the traffic load can be evenly balanced over all nodes. It will have to be analyzed if this is possible or not.

Conjecture 4.11. *The upper bound given in Theorem 4.8 is achievable for all values of the number of bits per unit energy $\gamma(n) = \mathrm{O}(1/\sqrt{n})$. More precisely*

$$\gamma(n) = \mathrm{O}\left(\frac{1}{\sqrt{n}}\right) \Rightarrow \left\{ \begin{array}{l} \exists\ c_1, c_2 > 0, \text{ such that} \\ \left(\cdot\ ,\ c_1\gamma(n)\ ,\ c_2\gamma(n)^{1/(\alpha-1)}\sqrt{n}^{\alpha/(\alpha-1)} \right) \in \mathcal{R}. \end{array} \right.$$

Chapter 5

Analysis and Proofs

In this chapter we will give proofs of the theorems presented in Chapter 4. Section 5.1 deals with the different delay models used in the literature and our work and extends some of the results given for other delay models to our delay model. Section 5.2 derives upper bounds on the bits per unit energy and the tradeoff thereof with the delay. In Section 5.3 a first achievability proof for the number of bits per unit energy is given. In Sections 5.4 and 5.5 it is shown that there is no tension between throughput and number of bits per unit energy. In those sections it is also shown that part of the delay tradeoff upper bound is achievable. In Section 5.6, finally, we discuss some of the techniques that might be used to show achievability of a high number of bits per unit energy.

5.1 Delay

In this section we collect several results and discussions related to the delay in a network.

5.1.1 Delay is at Least Equal to Number of Hops

Under the physical model we use a constant rate of transmission. The transmission time for a bit is therefore constant.

In the proof of Theorem 4.6 we give in Section 5.2 we will use the fact that the delay of a bit is at least proportional to the number of hops it takes. The next lemma validates this.

Lemma 5.1. *Let a bit b take $h(b)$ hops and have a delay of δ_b. Then*

$$\delta_b = \Omega(h(b)). \tag{5.1}$$

Proof. Note that the communication policy is restricted to work with slotted time, with slots of fixed length (constant in the number of nodes). This means that if a bit is received by a node in one time-slot, this node can only start relaying(/transmitting) this bit in the next time slot.

Let the number hops bit b is taking be denoted by $h(b)$, where the first hop is from the source to the the first relaying node or the destination. In Section 2.1 we defined the time-slot length as a constant τ. Let the bit be transmitted by (relaying) node i at time $q_i\tau + t_i$; that is in time-slot q_i with an offset t_i within the time-slot. The total delay is now given by

$$\delta_b = \sum_{h=1}^{h(b)} \left[\underbrace{q_i\tau + t_h}_{\text{start transmission}} \underbrace{-(q_{h-1}\tau + t_{h-1} + \frac{1}{W})}_{\text{receive bit}} + \underbrace{\frac{1}{W}}_{\text{transmission time}} \right]. \quad (5.2)$$

Now, since a relaying node cannot start transmission of a bit in the same time-slot it receives it, we have $q_h \geq q_{h-1} + 1$. This gives

$$\delta_b \geq \sum_{h=1}^{h(b)} \tau + t_h - t_{h-1} = h(b)\tau + t_{h(b)} - t_0, \quad (5.3)$$

so $\delta_b = \Omega(h(b))$. □

5.1.2 Achievable Throughput-Delay Tradeoff with Queueing at the Source

There is a difference in the delay model used in [12] and Definition 2.10 on page 8. The results given in [12] can, however, be extended to our delay model.

The difference in the two models is that Definition 2.10 includes queueing at the source, whereas the model in [12] does not. If, however, we have a look at the proof technique that is used in [12] we notice that the scheme that is constructed explicitly uses queueing at the source. It follows immediately that the throughput - delay tradeoff achievability does hold for our delay model.

5.1.3 Delay in a Percolation Setting

The achievable throughput-delay tradeoff analysis performed in [12] is based on the fact that relaying is along the straight lines connecting source and destination. As a consequence the results only hold for throughput rates of order smaller than $1/\sqrt{n\log(n)}$. For higher throughput rates it is no longer possible to route along straight lines and it is not known how the delay behaves in this case. In the high throughput case, routing needs to be performed on a percolation cluster [17, 4]. The construction that is used in [4] to proof achievability of throughut $\Theta(1/\sqrt{n})$ does seem to give the right order delay [21]. A quick analysis of the construction given there shows that a bit takes the following hops: i) one hop from the source node to the highway, ii) $\Theta(\sqrt{n})$ hops on the highway in horizontal direction, iii) $\Theta(\sqrt{n})$ hops on the highway in horizontal direction, and iv) one hop to get to the source node. If there is no queuing at the relaying nodes this scheme achieves a delay of order $\sqrt{n}$ at a throughput of order $1/\sqrt{n}$, which is exactly the optimal throughput.

5.2 Upper Bound on Bits per Unit Energy

In this section we will proof Theorems 4.6 and 4.8. The two theorems can be equivalently stated as

Lemma 5.2. *Any scheme that has delay* $\mathrm{O}(\delta(n))$ *must satisfy*

$$\gamma = \mathrm{O}(\min\left[\frac{\delta(n)^{\alpha-1}}{n^{\alpha/2}}, \frac{1}{\sqrt{n}}\right]).$$

Proof. Suppose that we have a sequence of schemes that achieve a bits per unit energy $(\gamma_n)_{n\in\mathbb{N}} = \gamma(n)$ and a delay of $(\delta_n)_{n\in\mathbb{N}} = \delta(n)$. Suppose, moreover, that each node has 1 unit energy available.

In a n node network, to achieve a number of bits per unit energy equal to γ, each source node needs to send γ bits. We make the optimistic assumption that $\gamma n/2$ bits are send through the network, without putting an individual constraint on the number of bits sent by each source node.

Consider bit $b \in \{1, 2, \dots, \gamma\frac{n}{2}\}$. Let $h(b)$ be the number of hops it takes to reach its destination. Moreover let $r(b, h)$ be distance traversed by the hth hop. In order for each bit to reach its destination we need

$$\sum_{b=1}^{\gamma\frac{n}{2}} \sum_{h=1}^{h(b)} r(b, h) \geq \sum_{i\in\mathcal{S}} \gamma \|X_i - X_{\mathcal{D}(i)}\| = \gamma n \bar{L}, \tag{5.4}$$

where $\bar{L} := \frac{1}{n}\sum_{i\in\mathcal{T}} \|X_i - X_{\mathcal{D}(i)}\|$.

Let $E(b, h)$ be the energy used for the hth hop of bit b. Remember that we defined W to be rate of transmission for a hop. Transmitting at a power P results in P/W energy per bit. The minimum energy needed for a hop can be bounded directly from the SINR definition (2.1) by leaving out the interference

$$\begin{aligned} E(b, h) = \frac{P}{W} &\geq \frac{\beta\sigma^2 + \beta\sum_{k\neq i} P\min(1, \|X_k - X_j\|^{-\alpha})}{W\min[1, r(b, h)^{-\alpha}]} && (5.5)\\ &\geq \frac{\beta\sigma^2}{W\min[1, r(b, h)^{-\alpha}]} && (5.6)\\ &= \max[\frac{\beta\sigma^2}{W}, \frac{\beta\sigma^2}{W} r(b, h)^{\alpha}]. && (5.7)\end{aligned}$$

Since each node has 1 J energy, there is n J energy available in the network. We will derive an upper bound on the number of bits per unit energy based on an overall energy constraint. We will see in the sequal that this upper bound can be achieved by a scheme that satisfies an individual energy constraint. The overall energy constraint gives

$$\sum_{b=1}^{\gamma\frac{n}{2}} \sum_{h=1}^{h(b)} E(b, h) \leq n. \tag{5.8}$$

Defining $H := \sum_{b=1}^{\gamma\frac{n}{2}} h(b)$ and using convexity of $x \mapsto x^{\alpha}$, for $\alpha > 2$ we get

$$\left(\sum_{b=1}^{\gamma\frac{n}{2}} \sum_{h=1}^{h(b)} \frac{1}{H}\frac{\beta\sigma^2}{W} r(b, h)\right)^{\alpha} \leq \frac{n}{H}. \tag{5.9}$$

Substituting (5.4) gives

$$\left(\gamma n \bar{L}\right)^{\alpha} \leq \frac{n W^{\alpha} H^{\alpha-1}}{\left(\beta\sigma^2\right)^{\alpha}}. \tag{5.10}$$

At this point we can bound the total number of hops H, in two different ways. The first bound will give an expression in terms of the delay constraint. The second bound will give an expression in terms of the total energy that is available in the network. We will first give the first bound, after which we will proceed with the second one.

Since H is the total number of hops taken for all bits, we can give an upper bound in terms of the energy needed for each hop

$$H = \sum_{b=1}^{\gamma \frac{n}{2}} h(b) \leq \frac{n}{\min_{b,h} E(b,h)} = \frac{nW}{\beta\sigma^2}. \tag{5.11}$$

Substituting the above in (5.10) gives

$$\gamma \leq \frac{W^{\alpha-1}}{(\beta\sigma^2)^{\alpha-1}\,\bar{L}}. \tag{5.12}$$

Finally noting that $\bar{L} := \frac{1}{n}\sum_{i\in\mathcal{S}} \|X_i - X_{\mathcal{D}(i)}\|$, where X_i $(i = 1, 2, \ldots, n)$ are uniformly random points on a area n surface, by the law of large numbers, $\exists c > 0$ such that

$$\frac{\bar{L}}{\sqrt{n}} \overset{n\to\infty}{\to} c \quad whp, \tag{5.13}$$

which concludes the proof of the upper bound that does not include the delay constraint.

To include the delay constraint we proceed as follows. We have seen in Lemma 5.1 that for a bit b, $h(b) \leq \delta/\tau$. Now, since the delay of the scheme is defined as the average delay over all bits, we have

$$H = \sum_{b=1}^{\gamma \frac{n}{2}} h(b) \leq \sum_{b=1}^{\gamma \frac{n}{2}} \frac{\delta}{\tau} = \frac{\gamma \frac{n}{2}}{\tau}\delta. \tag{5.14}$$

Substituting the above in (5.10) and again using (5.13) gives us

$$(\gamma n\sqrt{n})^{\alpha} \leq \frac{nW^{\alpha}\delta^{\alpha-1}(\gamma\frac{n}{2})^{\alpha-1}}{(\beta\sigma^2)^{\alpha}}, \tag{5.15}$$

which, rewritten as

$$\frac{\gamma^{\alpha}}{\gamma^{\alpha-1}} \leq \frac{W^{\alpha}(\frac{1}{2})^{\alpha-1}}{(\beta\sigma^2)^{\alpha}}\frac{\delta^{\alpha-1}}{n^{\alpha/2}}, \tag{5.16}$$

gives the desired result. □

5.3 Achievable Bits per Unit Energy using Sequential Transmissions

In this section we proof Theorem 4.7 and give some intuition on the achievability of a number of bits per unit energy up to order $1/\sqrt{n\log(n)^{\alpha-1}}$. For now we will focus only on the number of bits per unit energy and ignore the throughput and the delay of the constructive scheme we are setting up.

The following scheme is based on Scheme 1 on page 11. The difference is in the scheduling of the cells. We allow only one cell to be active in a time-slot. This means that all transmissions are sequential.

Scheme 2.

- *Create a tessellation on the network surface, such that all cells in the tessellation have area* $\Theta(\log(n))$ *and the maximum distance within a cell is* $\Theta(\sqrt{\log(n)})$.
- *Let the straight line connecting a Source to its Destination be denoted as an S-D line. Packets are transmitted from source to destination by relaying from one cell to a neighbouring cell along the S-D lines.*
- *Activate cells sequentially. Only the active cell is allowed to transmit.*
- *Each cell is crossed by a certain number of S-D lines, including those belonging to source and destination nodes. Each time a cell is activated it relays a packet for one of the S-D pairs crossing that cell.*
- *The tessellation and the power at which is transmitted are such that all transmissions are successful.*

This scheme achieves a number of bits per unit energy of $\gamma = \Theta(1/\sqrt{n \log(n)^{\alpha-1}})$.

Energy per hop is only proportional to length of the hop, because there is no interference. The hops are of length $\Omega(\sqrt{a(n)})$. To satisfy SINR $\geq \beta$, we use a power of order $a(n)^{\alpha/2}$. Since packet sizes and transmission rates are fixed, the energy per hop is directly proportional to the power used. The energy per hop is $\Omega(a(n)^{\alpha/2})$.

Since the distance between source and destination nodes is $\Theta(\sqrt{n})$, the number of hops that each bit has to take is $\Theta(\sqrt{n/a(n)})$. The total energy spent in the network to deliver one bit is therefore $\Theta(\sqrt{na(n)^{\alpha-1}})$.

The energy spent in a n node network to deliver 1 bit for all $n/2$ S-D pairs divided by n the nodes in the network is exactly $\Theta(\sqrt{na(n)^{\alpha-1}})$ J/node. The number of bits per unit energy is $\gamma = \Theta(1/\sqrt{na(n)^{\alpha-1}})$.

From the above argument is follows that if nodes are allowed to pool resources, that is not using an individual energy constraint, a sufficient number of bits per S-D pair can be transmitted. It remains to be shown that the individual energy constraint is not violated. Lemmas 3.1 and 3.2 state that *whp* each cell contains $\Theta(a(n))$ nodes and that *whp* there are $\Theta(\sqrt{na(n)})$ S-D lines crossing a cell. From this it can be concluded that relaying can be evenly balanced over nodes.

Using Scheme 2 results in a number of bits per unit energy of $1/\sqrt{n \log(n)^{\alpha-1}}$, which was to be shown in this section.

5.4 On The Bits per Unit Energy - Throughput Tradeoff

It is clear that by making all transmissions sequential we have a diminished throughput and a large delay. In this section and the next one we will see, however that a high number of bits per unit energy and a high throughput are simultaneously achievable.

One of the key insights from [2] is that in order to achieve high throughput it is necessary to have short hops. It was shown that by scheduling a transmission between two nodes there can be no other transmissions in a disk centered at the transmitter, the disk having a radius proportional to the distance between transmitter and receiver. By taking short hops, there can be many simultaneous transmissions which is improves the throughput.

In the previous section we have seen that using hop length $\sqrt{a(n)}$ the energy spent per bit is $\Theta(\sqrt{na(n)^{\alpha-1}})$ for no interference case. This is obviously minimized by taking the hop length $\sqrt{a(n)}$ as small as possible. We conclude that using short hop lengths is beneficial for both the throughput and the number of bits per unit energy.

The number of simultaneously scheduled transmissions dominates the behavior of the throughput. To achieve a high throughput it is necessary to schedule a high number of simultaneous transmissions. It has to be shown that under the physical model it is possible to have many simultaneous transmissions. An analysis is given in [2], Section IV.D. The concluding Lemma 4.4 [2], however, states that this is possible in the *high power limit*. To achieve high number of bits per unit energy it is necessary to keep the power low. We have seen in the previous section that there exists a scheme where the power grows as $\Theta(a(n)^{a/2})$. If we schedule many simultaneous transmissions we have to analyze how the required power scales with the number of nodes. The conclusion from [2] that there exists a scheme as long the power is large enough is not strong enough for the number of bits per unit energy analysis. We see that there is a possible tradeoff between the number of bits per unit energy and the throughput.

In Section 5.5 we will sharpen the results from [2] and show that it is possible to schedule a high number of simultaneous transmissions (scaling up in the number of nodes) using the same scaling of the power that was used in the previous section using only sequential transmissions. This will prove that a high number of bits per unit energy and high throughput are simultaneously achievable.

5.5 Achievable Bits per Unit Energy using Simultaneous Transmissions

In this section we will show that Scheme 1 on page 11 achieves an optimal number of bits per unit energy. We will show that Scheme 1 achieves

$$\lambda = \Theta\left(\frac{1}{\sqrt{na(n)}}\right), \qquad \gamma = \Theta\left(\frac{1}{\sqrt{na(n)^{\alpha-1}}}\right), \qquad \delta = \Theta\left(\sqrt{\frac{n}{a(n)}}\right).$$

Comparing the above values with Theorems 4.3 and 4.9 shows that at a given delay $\Theta\left(\sqrt{n/a(n)}\right)$ it achieves both the highest possible number of bits per unit energy and the highest possible throughput. The main point to show is that the order of the power that is required is not higher than the power that is neccesary in Scheme 2.

5.5.1 TDMA Cell Scheduling

We need to find a TDMA schedule of the cells that has a cycle of $\Theta(1)$ slots and guarantees successful transmission. We have $\Theta(n/a(n))$ cells, so we need to activate $\Theta(n/a(n))$ cells simultaneously.

In this section we will introduce a constraint for two cells to be active at the same time. We will show that the performance achievable under this condition is optimal.

Given a tessellation on the surface, let $r(n)$ be the maximum distance between points in two neighbouring cells. We now make the following restriction in the way we activate cells. Let $\Delta \geq 0$ be constant.

- Two cells can only be active at the same time if the distance between any two points in these cells is at least $(1 + \Delta)r(n)$.

Combining the restrictions given by Scheme 1 and the above condition, results in the exact requirements given by the protocol model, see Definition 2.3. Since we require successful transmissions, there exists conditions on Δ and the transmit power P. The relation between the two parameters will be analyzed next.

5.5.2 Requirement on P and Δ

There are two parameters to be chosen, the power P and the spacing between active cells Δ. One of the requirements of the scheme we are constructing is that all transmissions are successful. We have the following sufficient condition on P and Δ to satisfy this requirement.

Lemma 5.3. *Transmissions are successful if*

$$\frac{r^\alpha \sigma^2}{P} + \frac{c(\alpha)}{(\frac{\Delta}{2} - 1)^\alpha} \leq \frac{1}{\beta}, \tag{5.17}$$

where $c(\alpha) = \frac{1}{c_a}\left[9 + \frac{3}{\alpha-1} + \frac{6}{\alpha-2}\right]$.

Proof. Given simultaneous transmissions $X_i \to X_j$ and $X_k \to X_l$, we have

$$\|X_i - X_j\| \leq r, \tag{5.18}$$

from the restriction that we only transmit to neighbouring cells and the construction of the tesselation. Moreover

$$\|X_i - X_l\| \geq (1 + \Delta)r. \tag{5.19}$$

This gives

$$\begin{aligned} \|X_l - X_j\| &= \|(X_l - X_i) - (X_j - X_i)\| + \|X_j - X_i\| - \|X_j - X_i\| && (5.20)\\ &\geq \|(X_l - X_i) - (X_j - X_i) + (X_j - X_i)\| - \|X_j - X_i\| && (5.21)\\ &= \|X_l - X_i\| - \|X_j - X_i\| && (5.22)\\ &\geq \Delta r. && (5.23) \end{aligned}$$

Inequality (5.23) states that we need at least disks of radius $\frac{\Delta}{2}r$ around receivers.

We can now lower bound the SINR at a receiving node X_j. Let's first consider the number of receivers in an annulus with inner radius a and outer radius b. Since we need disjoint disks of radius $\frac{\Delta}{2}r$, this number is at most

$$\frac{\pi\left[b + \frac{\Delta}{2}r\right]^2 - \pi\left[a - \frac{\Delta}{2}r\right]^2}{c_A \pi [\frac{\Delta}{2}r]^2}, \tag{5.24}$$

where c_A is a constant depending on the surface (equal to one for the torus, strictly smaller than one for the sphere [2]). The received power at X_j from each of the transmitters sending to these receivers can be at most $P \min[1, \|a - r\|^{-\alpha}]$. There can be no other receivers within a distance Δr of X_j. Bounding the total interference by letting $a = (k+1)\frac{\Delta}{2}r$, $b = (k+2)\frac{\Delta}{2}r$, for $k = 1, 2, \ldots, \infty$, we get the following upper bound for the SINR,

$$\text{SINR} \geq \frac{P \min[1, r^{-\alpha}]}{\sigma^2 + \sum_{k=1}^{\infty} \frac{(k+3)^2 - k^2}{c_A} P \min\left[1, ((k+1)\frac{\Delta}{2}r - r)^{-\alpha}\right]} \tag{5.25}$$

We simplify this by noting that $(k+1)\frac{\Delta}{2}r - r \geq k\frac{\Delta}{2}r + \frac{\Delta}{2}r - kr \geq k(\frac{\Delta}{2} - 1)r$, $\min[1, r^{-\alpha}] = \min[1, (8\log(n))^{-\alpha}] \geq r^{-\alpha}$.

$$\begin{aligned}
\text{SINR} &\geq \frac{Pr^{-\alpha}}{\sigma^2 + \sum_{k=1}^{\infty} \frac{(k+3)^2 - k^2}{c_A} P(k(\frac{\Delta}{2} - 1)r)^{-\alpha}} && (5.26)\\
&= \frac{Pr^{-\alpha}}{\sigma^2 + \frac{P}{c_A}(\frac{\Delta}{2} - 1)^{-\alpha} r^{-\alpha} \sum_{k=1}^{\infty} \frac{6k+3}{k^\alpha}} && (5.27)\\
&\geq \frac{Pr^{-\alpha}}{\sigma^2 + \frac{P}{c_A}(\frac{\Delta}{2} - 1)^{-\alpha} r^{-\alpha}\left[9 + \frac{3}{\alpha - 1} + \frac{6}{\alpha - 2}\right]} && (5.28)\\
&= \frac{P/\sigma^2}{r^\alpha + c(\alpha)P/\sigma^2(\frac{\Delta}{2} - 1)^{-\alpha}}, && (5.29)
\end{aligned}$$

where $c(\alpha) = \frac{1}{c_a}\left[9 + \frac{3}{\alpha-1} + \frac{6}{\alpha-2}\right]$. We will not always explicitly write $c(\alpha)$, but just use c.

For successful communication we require SINR $\geq \beta$ and (5.17) follows. □

We will now illustrate the relation between P and Δ in (5.17). A simple lower bound for P is given by leaving out all interfering terms in the SINR expression, i.e.

$$\frac{Pr^{-\alpha}}{\sigma^2} \geq \beta, \tag{5.30}$$

$$P/\sigma^2 \geq \beta r^\alpha. \tag{5.31}$$

We rewrite (5.17) as

$$\frac{\Delta}{2} - 1 \geq \left(\frac{c\beta}{1 - \frac{\beta r^\alpha}{P/\sigma^2}}\right)^{1/\alpha}. \tag{5.32}$$

Taking $\lim_{P/\sigma^2 \downarrow \beta r^\alpha}$ in the rhs of (5.32), we have a form like

$$\lim_{x \downarrow a} \frac{x}{x - a} = \infty. \tag{5.33}$$

Hence: Letting P/σ^2 tend to the minimum required in the no interference case, requires Δ to go to infinity. Moreover we see from (5.17) that $P \to \infty$ implies

$$\frac{\Delta}{2} - 1 \geq (c\beta)^{1/\alpha}. \tag{5.34}$$

A similar relation was obtained in [2].

We can also give an upper bound to $(\frac{\Delta}{2}-1)$. For that we do need the number of simultaneous transmitter-receiver pairs. In Subsection 5.5.3 we will derive a bound on this number, but for now suppose that it is given by N. From (5.23) it follows that disks of radius $\frac{\Delta}{2}$ around transmitters need to be disjoint. Since the total available area is n

$$N c_A \pi (\frac{\Delta}{2} r)^2 \leq n, \tag{5.35}$$

and hence

$$\frac{\Delta}{2} \leq \frac{\sqrt{n}}{r\sqrt{N c_A \pi}}. \tag{5.36}$$

5.5.3 The Number of Time-slots

Let $G(\mathcal{V})$ be the graph with the cells of the tesselation as its vertices. There exists an edge between vertices V_i and V_j if, by the conditions on the TDMA schedule given in Section 5.5.1, V_i and V_j are not allowed to be active at the same time. Let δ_G be the maximum degree of a graph G.

Lemma 5.4.

$$\delta_{G(\mathcal{V})} \leq \frac{(14+8\Delta)^2}{c_A} - 1.$$

We need the following result from graph theory:

Theorem 5.5 ([22, 23])**.** *A graph G may be colored by $\delta_G + 1$ colors such that for any two colors i and j, where $1 \leq i < j \leq \delta_G + 1$, the number of vertices of color i and color j differ by at most one (i.e. the $\delta_G + 1$ color classes are equal or almost equal in size).*

Lemma 5.4 and Theorem 5.5 give us the following:

- The maximum number of the TDMA schedule does not depend on n and is hence $\Theta(1)$.
- We can create an assignment of cells to time-slots, such that the number of cells in each time-slot is nearly uniform over the time-slots.

Let L be the number of time-slots. From the above arguments it follows that we need

$$L \geq \frac{(14+8\Delta)^2}{c_A}. \tag{5.37}$$

Let N_i, $i = 1, 2, \ldots, L$, be the number of cells in time-slot i. Let, moreover, $N^* = \max_i N_i$. Then

$$\sum_{i=1}^{L} N_i = |\mathcal{V}|, \tag{5.38}$$

which we rewrite as

$$N^* + (L-1)(N^* - 1) \leq |\mathcal{V}|. \tag{5.39}$$

Hence

$$N^* \leq \frac{|\mathcal{V}| + L - 1}{L} \tag{5.40}$$

$$\leq \frac{|\mathcal{V}|}{L} + 1 \tag{5.41}$$

$$\approx \frac{c_A n}{(14 + 8\Delta)^2 \log(n)}, \tag{5.42}$$

where we used (5.37) and $|\mathcal{V}| \leq n/(\pi \log(n))$, and the approximation is tight for large n.

5.5.4 A Lower Bound on P

With the lower bound on the number of simultaneous transmissions in one time-slot, we can now upper bound Δ through (5.36). This will then give a lower bound on P, through (5.17).

Combining (5.42) and (5.34) gives

$$\frac{\Delta}{2} \leq \sqrt{\frac{n}{\log(n)}} \sqrt{\frac{\log(n)}{n} \frac{(14 + 8\Delta)^2}{c_A^2 \pi}} \tag{5.43}$$

$$(\frac{\Delta}{2})^2 \leq \frac{(14 + 8\Delta)^2}{c_A^2 \pi} \tag{5.44}$$

Assuming $c_A = 1$ in the high n limit gives

$$\Delta \leq 11.7 \tag{5.45}$$

Finally, subsituting (5.45) in (5.17) gives

$$P \geq \frac{r^\alpha}{1/\beta - 0.21^\alpha c(\alpha)} \tag{5.46}$$

Note that using $\Theta(n/a(n))$ simultaneous transmissions, requires a power per hop of order $r^\alpha = \sqrt{\log(n)}^\alpha$, which is of the same order as the power per hop used if all transmissions are sequential.

5.5.5 The Number of Bits per Unit Energy

We have seen in the previous sections that, compared to the completely sequential Scheme 2, we do not need higher power per hop to do order $n/\log(n)$ simultaneous transmissions. Hence, we can see that this scheme achieves

$$\gamma = \Omega\left(\frac{1}{\sqrt{n \log(n)^{\alpha-1}}}\right). \tag{5.47}$$

This proofs Theorem 4.9 on page 21.

5.6 Percolation Approach to Bits per Unit Energy

The results from the previous sections suggest that $\gamma = \Theta(1/\sqrt{n})$ is achievable. Achievability for the throughput rate, λ, is shown using a percolation approach [4, 17]. It was discussed in Section 5.1.3 that achievability of the throughput - delay tradeoff can probably be shown using the exact construction given in [4]. It seems that the percolation approach is also needed to prove the achievability of $\Theta(1/\sqrt{n})$ bits per unit energy. It is, however, not clear if the construction used in [4] can be used here. The problem is that this construction is based on the fact that a limited number of nodes carry all the traffic. In order to achieve a high number of bits per unit energy it is neccessary to balance the traffic load over all nodes. If it possible to adopt the techniques from [4] in such a way that traffic load is balanced over nodes, it is possible to prove achievability of the upper bound given in Theorem 4.6.

Chapter 6

Conclusions

In this report we have introduced the number of bits per unit energy as a performance measure for wireless networks with a per node energy constraint. We have also introduced a delay model that is different from previous work in the sense that it does include delay incurred by queueing at the source.

The network model considered is the extended network with the physical model of successful communication. Use of the physical model allows an analysis of the scaling behavior of the required power that is more precise than was possible in [1] using the relaxed protocol model.

We have shown that $1/\sqrt{n}$ is an upper bound to the maximum number of bits per unit energy that can be achieved. Achievability proofs have been given for number of bits per unit energy up to $1/\sqrt{n\log(n)}$ by constructing a scheme based on relaying along straight lines [2]. It was shown that in this regime there is no tradeoff between the throughput and the number of bits per unit energy.

The tradeoff between the number of bits per unit energy and delay has been partly characterized. An upper bound has been given that states that any scheme that has delay smaller or equal to $\delta(n)$ can have a number of bits per unit energy of at most order $\delta(n)^{\alpha-1}/n^{\alpha/2}$. Achievability of this upper bound has been shown for delays up to order $\sqrt{n/\log(n)}$.

6.1 Future Work

We comment on three of the main issues that remain open for future work.

6.1.1 Achievability of the tradeoffs in the high performance regime

The upper and lower bounds that have been given in Theorems 4.8 and 4.9 do not completely characterize the bits per unit energy - delay tradeoff. For the high number of bits per unit energy we have found an upper bound, Theorem 4.8, but we have not shown that this upper bound is achievable. It was pointed out in Section 5.6 that the techniques developed in the work [4] of Franceschetti et al. might be used to find a lower bound.

The throughput - delay tradeoff has been not completely characterized either. For the high throughput an upper bound exists, but achievability has not been shown.

6.1.2 Delay Model

The delay model used in this work is that of the average bit delay. The average is taken over all S-D pairs and over time. This allows for a small number of S-D pairs having relatively large bit delays. A property that might be more relevant in a network is the time-average bit delay, maximized over all S-D pairs. It will be a topic of future research to see if the scaling laws that have been given in this report and [12] still hold under the maximum average bit delay model.

Another natural extension of the delay model is to consider the maximum bit delay, taking the maximum over all S-D pairs and over time. This model will also be the topic of future work.

6.1.3 Information-theoretic Model

The communication model that has been considered in this work is not the most general that can be given. It is desirable to analyze the scaling behavior of number of bits per unit energy and the relation with other properties in an information-theoretic model. Our communication model needs to be generalized in the following ways:

- The signals from other users need not necessarily be treated as noise. It is shown in e.g. [5] that there are situations in which a better performance is achieved if users cooperate and do not treat each other signals as noise.

- The rate of transmission and the packet sizes need not be fixed. It is known that for several problems in information theory [24, 13, 25] the energy consumption is minimized by considering the limiting situation were the transmission rate goes to zero. It has already been shown in [1] that allowing a variable transmission gives rate gives a different behavior for the number of bits per unit energy. This was discussed in Section 4.3

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