1.1 Functions and Function Notation and 1.2 Domain and Range

Notation of Inequalities/Intervals:

$<$ Less Than	$\leq$ Less Than or Equal To
$>$ Greater Than	$\geq$ Greater Than or Equal To
(a, b) # is NOT included	$[a, b]$ # IS included

Note: Infinity (∞) can never be actually "reached," thus it is always associated with parenthesis in interval notation.

Example: State the following solutions as an interval and graph it on a number line.

i) $-5 < x < 1$

<——————————>

ii) $4 \leq y < 9$

<——————————>

iii) $z \geq -3$

<——————————>

iv) $x < 2$, $x > 3$

<——————————>

Properties of Functions: What is a function?

Equation that consists of inputs called __________, the set of outputs called __________,

and a rule that each input determines _______________ output.

Graphically, we can check to see if a graph is a function by applying the _______________.

Example: State whether each rule defines a function, stating the domain and range:

i) $\{(5, 1), (3, 2), (4, 9), (7, 6)\}$

Function: _____

Domain:

Range:

ii) $\{(2, 4), (0, 2), (2, 5)\}$

Function: _____

Domain:

Range:

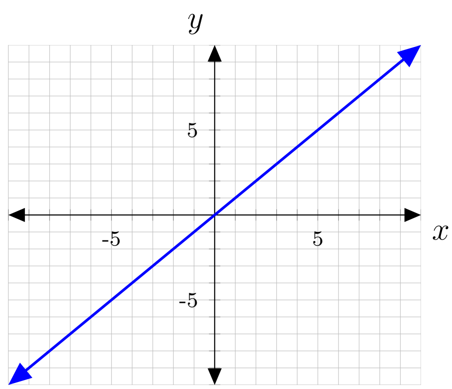
*Note: These questions can also be given as tables or drawings.

Example: Determine if the graph is a function, and state its domain and range:

i) Function: _____

Domain:

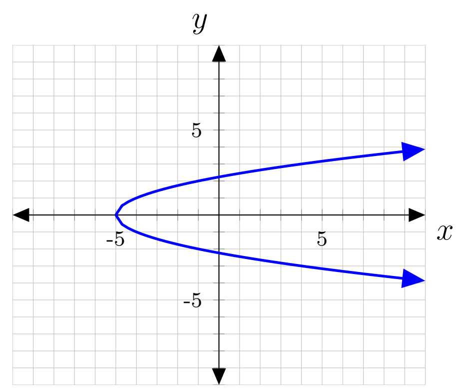
Range:



ii) Function: _____

Domain:

Range:



Function Notation:

$$f(x) = \sqrt{x^2 + 1}$$

Example: Decide whether the relation defines a function of x, and give the domain, range:

i) $y = 2x + 6$

Function: _____

Domain:

ii) $y = \frac{5}{x+2}$

Function: _____

Domain:

iii) $y = \sqrt{2x - 1}$

Function: _____

Domain:

iv) $y^6 = x$

Function: _____

Domain:

Example: Let $f(x) = -3x + 4$ and $g(x) = -x^2 + 4x + 1$. Find the following:

i) $f(0)$

ii) $g(-2)$

iii) Solve $f(x) = -2$

iv) $f(x + 2)$

v) Difference Quotient: $\dfrac{g(x+h) - g(x)}{h}$, where $h \neq 0$

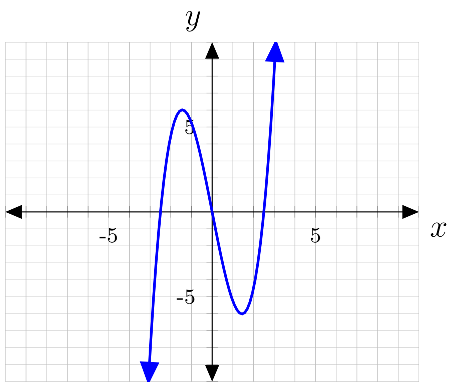
Properties of One-to-One Functions:

Function in which each __________ corresponds to exactly one __________.

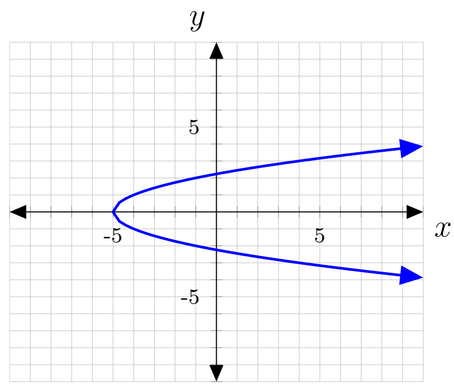
Graphically, we can check to see if a graph is one-to-one by applying the _______________.

Example: Determine if the graph is a one-to-one function:

i) One-to-one: _____



ii) One-to-one: _____

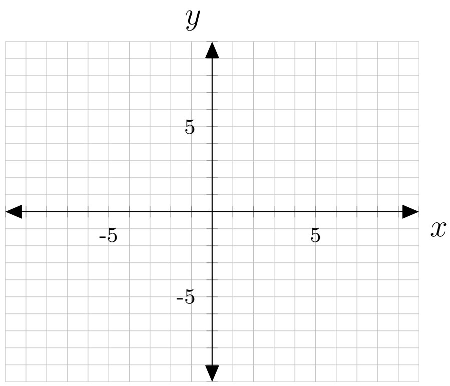


Definition [Piecewise Function]:

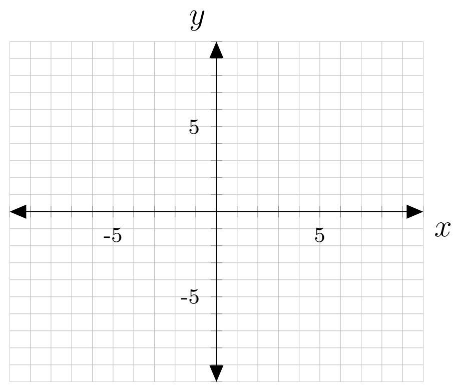
Functions whose whose rules are defined with different equations for different parts of the domain.

Example: Graph each of the following functions:

i) $f(x) = \begin{cases} x+1 & \text{if } x \leq 2 \\ -2x+7 & \text{if } x > 2 \end{cases}$



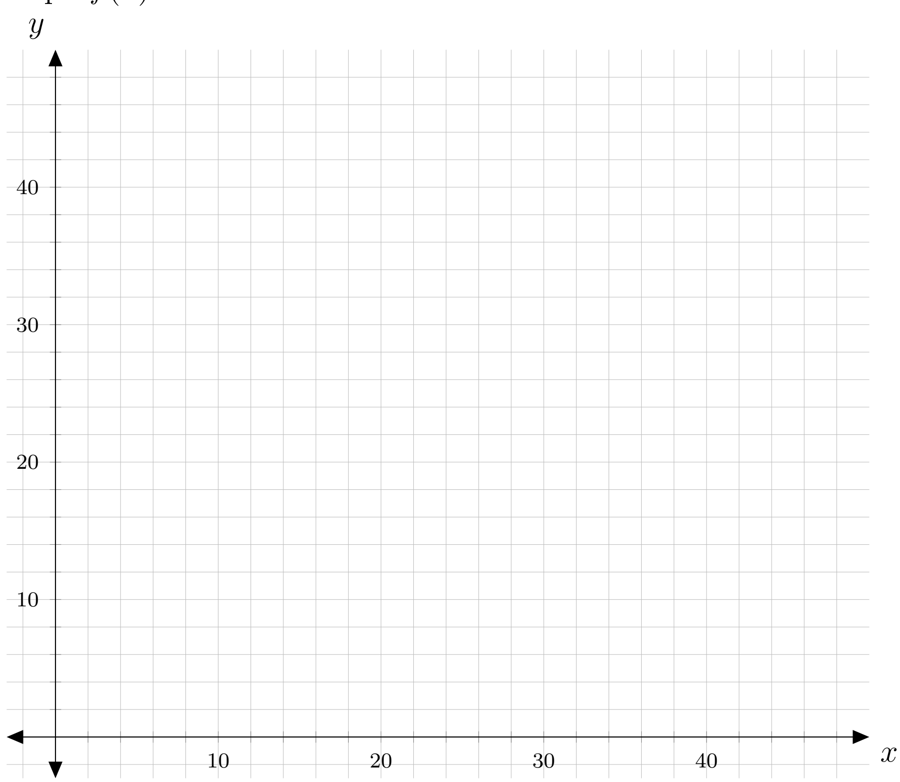
ii) $f(x) = \begin{cases} -3x-2 & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$



Example: The price of Starbucks stock in 2008 can be modeled by the function where x is the number of business days past June 16, 2008:

$$f(x) = \begin{cases} 18.35 - .18x & \text{if } 0 \leq x \leq 22 \\ 12.49 + .10(x-3) & \text{if } 22 < x \leq 44 \end{cases}$$

i) Graph $f(x)$.



ii) What is the lowest stock price during the period given by $f(x)$?

1.1 Problems: 8-10, 14, 17, 18, 28, 30, 32, 33, 37-40, 42, 43, 46, 53, 54, 88, 89, 91

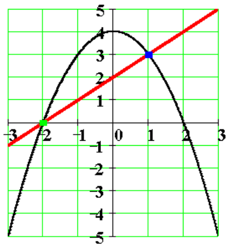
1.2 Problems: 10, 14, 17, 21, 25, 27, 29, 32, 33, 38, 41, 43, 44, 60, 61

1.3 Rates of Change and Behavior of Graphs

Definition [Average Rate of Change of a Function]: $f(x)$ over the interval $[x_1, x_2]$.

$$\text{Average rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example: Find the average rate of $f(x) = -x^2 + 4$ over the interval $[-2, 1]$



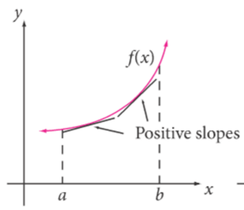
Example: For the following, find the average rate of change on the interval given:

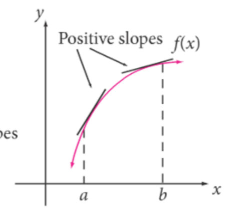
i) $f(x) = x^3 - 1$ on $[1, b]$

ii) $g(x) = 2x - 1$ on $[2, 2 + h]$

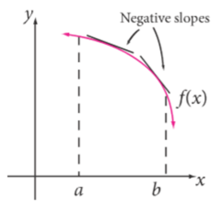
Example: If Dallas is 110 miles away and it took me 90 minutes to get there, what is the average rate of change of my position? What is another name for the rate of change of your position?

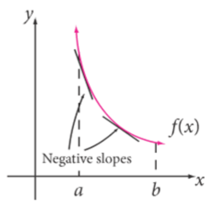
Definition [Increasing/Decreasing]: A function $f(x)$ is **increasing** on an interval (a, b) if its graph rises (from left to right) through (a, b).



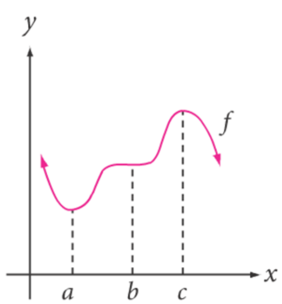


Similarly, $f(x)$ is **decreasing** on an interval (a, b) if its graph falls through (a, b).





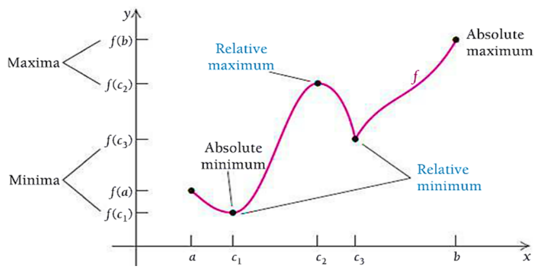
Definition [Relative Extrema]: A point $(c, f(c))$ is called a **relative maximum** if there is an interval I in the domain of $f(x)$ such that $f(c) \geq f(x)$ for all x in I.



Similarly, $(a, f(a))$ is called a **relative minimum** if there is an interval I in the domain of $f(x)$ such that $f(a) \leq f(x)$ for all x in I.

The term **relative extrema** is used to describe points that are either relative maxima or relative minima.

Definition [Absolute Extrema]: A point $(c, f(c))$ is called an **absolute maximum** if $f(c) \geq f(x)$ for all x in the domain of $f(x)$. A point $(a, f(a))$ is called an **absolute minimum** if $f(a) \geq f(x)$ for all x in the domain of $f(x)$. The term **absolute extrema** is used to describe points that are either absolute maxima or absolute minima.



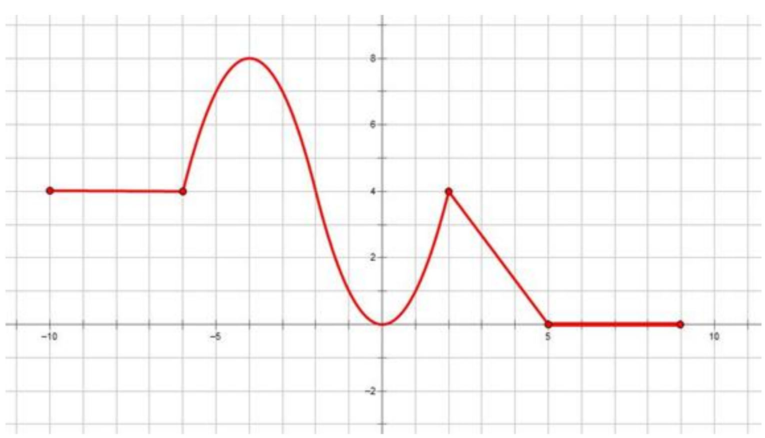
Note: If the interval on which we are finding absolute extrema is closed, like [a,b], then f(a) and f(b) are also candidates for being absolute extrema.

Example: For the function below, label each absolute and relative extrema and determine the intervals on which it is

Increasing:

Decreasing:

Constant:



1.3 Problems: 6, 7, 9, 12, 13, 15-19, 21-25, 27, 29, 32, 43, 44, 47

1.4 Algebra and Composition with Functions

If we are given two functions $f(x)$ and $g(x)$ we can define four new functions as follows.

Properties of Functions:

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Example: Let $f(x) = x + 1$, $g(x) = 2x - 3$, and $h(x) = x^2$. Find the following.

i) $(f - g)(x)$

ii) $(f + g - h)(-2)$

iii) $(gh - f)(1)$

iv) $\left(\frac{f}{g}\right)(3)$

Definition [Composite Function]: Given two functions $f(x)$ and $g(x)$, we define

$$(f \circ g)(x) = f(g(x))$$

Example: Let $f(x) = x - 3$, $g(x) = (x + 2)^2$. Find the following.

i) $(f \circ g)(x)$

ii) $(f \circ g)(1)$

iii) $(g \circ f)(x)$

iv) $(g \circ f)(3)$

Example: For the following, find functions $f(x)$ and $g(x)$ such that the given function can be expressed $h(x) = f(g(x))$.

i) $h(x) = (x+1)^3$

ii) $h(x) = \sqrt{\dfrac{x-1}{x+1}}$

1.4 Problems: 11, 12, 17, 19-21, 27, 29, 32, 41, 42, 45, 47, 57, 59, 61, 77, 79, 80, 85, 86, 94-96

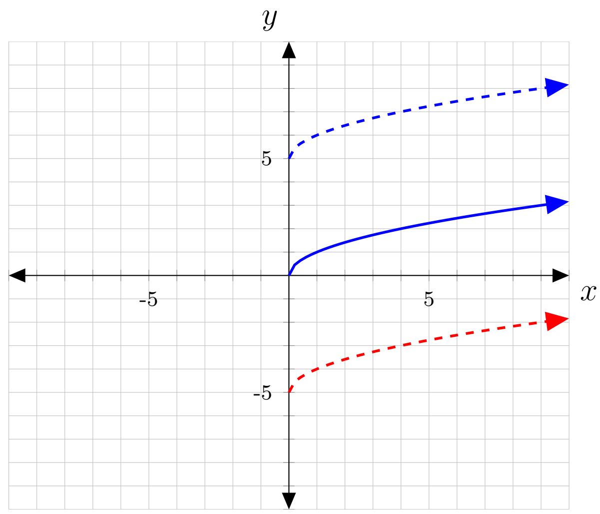
1.5 Transformations of Functions

In this section, we will be looking at vertical and horizontal shifts, reflections, stretches, compressions, and combinations of each.

Definition [Vertical Shift]: Given a function $f(x)$, a new function $g(x) = f(x) + k$, where k is a constant, is a vertical shift of the function $f(x)$.

Note if $k > 0$ the graph is shifted up and if $k < 0$ the graph is shifted down.

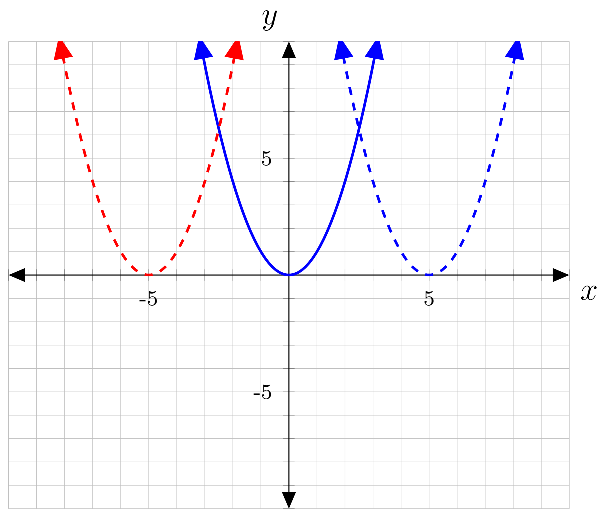
Example:



Definition [Horizontal Shift]: Given a function $f(x)$, a new function $g(x) = f(x-k)$, where k is a constant, is a horizontal shift of the function $f(x)$.

Note if $k > 0$ the graph is shifted right and if $k < 0$ the graph is shifted left.

Example:



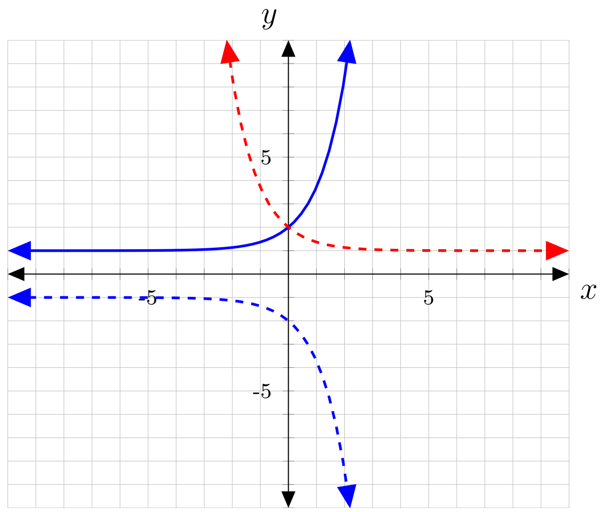
Definition [Reflection]: Given a function $f(x)$, a new function $g(x) = -f(x)$ is a vertical reflection of $f(x)$, or a reflection about the x-axis.

Given a function $f(x)$, a new function $g(x) = f(-x)$ is a horizontal reflection of $f(x)$, or a reflection about the y-axis.

If $f(x) = f(-x)$ for every x, then we say f is an **even function**.

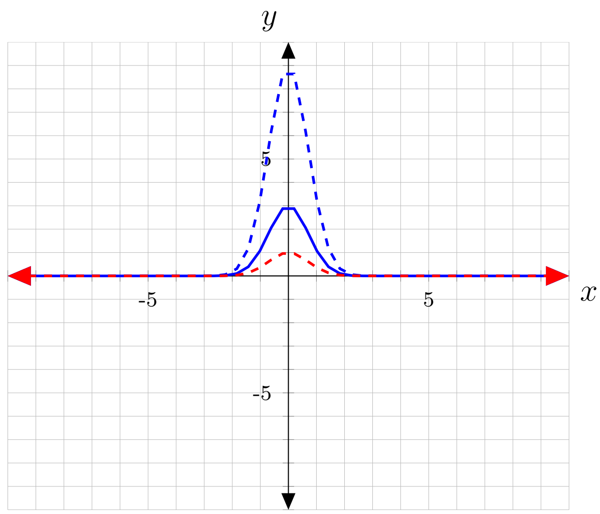
If $f(x) = -f(-x)$ for every x, then we say f is an **odd function**.

Example:



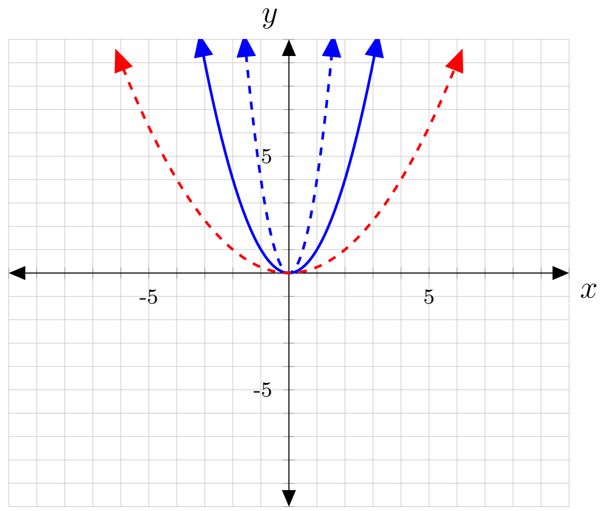
Definition [Vertical Stretch or Compression]: Given a function $f(x)$, a new function $g(x) = af(x)$, where a is a constant, is a vertical stretch or compression of $f(x)$. If $a > 1$ the graph will be stretched and if $0 < a < 1$ the graph will be compressed.

Example:



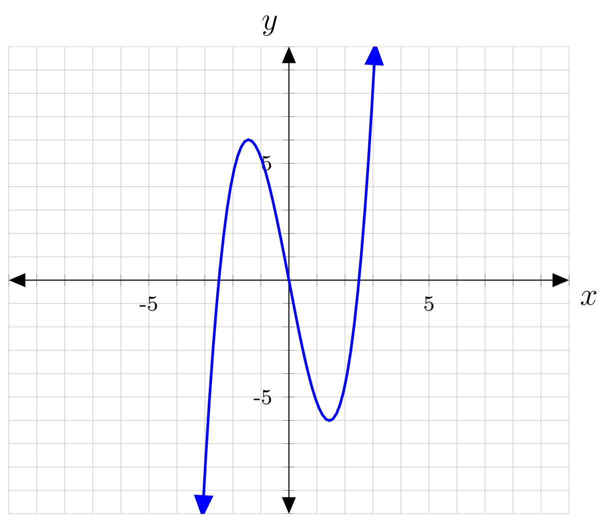
Definition [Horizontal Stretch or Compression]: Given a function $f(x)$, a new function $g(x) = f(bx)$, where b is a constant, is a horizontal stretch or compression of $f(x)$.
If $b > 1$ the graph will be compressed and if $0 < b < 1$ the graph will be stretched by $\frac{1}{b}$.

Example:



Combining Transformations

Example: Given the graph of $f(x) = x^3 - 6.25x$ below, graph $-\frac{1}{2}f(x+1)$ and $f(2x)+1$



1.5 Problems: 6, 9, 13, 15, 19, 33-35, 41-43, 46, 53-55, 58, 67, 68, 78, 79, 81

1.7 Inverse Functions

Properties of Inverse Functions:

Quadratic Function in Standard Form: For any one-to-one functions $f(x) = y$, a function $f^{-1}(x)$ is an inverse function of $f(x)$ if $f^{-1}(y) = x$. This can also be expressed as $f^{-1}(f(x)) = x$ for all x in the domain of f. It also follows that $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.

Note 1- $f^{-1}(x) \neq \dfrac{1}{f(x)}$

Note 1- The range of f is the domain of f^{-1} and the domain of f is the range of f^{-1}

Example: If $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{1}{x} - 2$, is $g = f^{-1}$?

Example: If a is a real number, show $f(x) = \dfrac{x+a}{x-1}$, $x \neq 1$ is its own inverse; that is show $f = f^{-1}$.

Finding Inverse Functions

To find the inverse of a function represented by a graph, we interchange the x and y value of each coordinate of the original function to find the corresponding coordinate of the inverse functions. This is equivalent to reflecting the graph of the original function across the line $x = y$.

To find the inverse of a function represented by a formula,

i) Make sure the function is one-to-one

ii) Interchange x and y

iii) Solve for y.

Example: Find the inverse of the function $f(x) = \dfrac{2}{x-3} + 4$, $x \neq 3$.

Example: Find the inverse of the function $f(x) = h + \sqrt{\dfrac{x-k}{a}}$, $x \geq k$.

1.7 Problems: 6, 11, 12, 15, 16, 23, 25-30, 37, 39, 43, 45, 46