

1.1 The Real Numbers

There is a good bit of information in Sec. 1.1 that we will not review.

Order of Operations: P E MD AS

- 1) **Parenthesis:** Within each set of parentheses or brackets, start with the innermost set and work outward. Also, work separately above and below any fraction bars.
- 2) **Exponents:** Find all powers and roots, working from left to right.
- 3) **Multiplication/Division:** Do any multiplications or divisions in the order in which they occur, left to right.
- 4) **Addition/Subtraction:** Do any additions or subtractions in the order in which they occur, left to right.

Example: Simplify the following:

i)
$$\frac{2(3 - 7) + 4(8)}{4(-3) + (-8 + 5)(7 - 9)}$$

ii)
$$\frac{x^2 - 3\sqrt{y + 12}}{\sqrt{x^2 + y}}$$
 where $x = 6$, $y = 13$

Notation of Inequalities/Intervals:

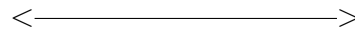
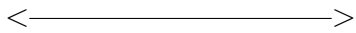
$<$ Less Than	\leq Less Than or Equal To
$>$ Greater Than	\geq Greater Than or Equal To
(a, b) # is NOT included	$[a, b]$ # IS included

Note: Infinity (∞) can never be actually "reached," thus it is always associated with parenthesis in interval notation.

Example: State the following solution as an interval and graph it on a number line.

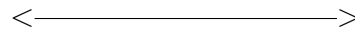
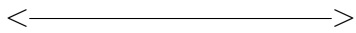
i) $-5 < x < 1$

ii) $4 \leq y < 9$



iii) $z \geq -3$

iv) $x < 2$



1.1 Problems: 13, 14, 16, 22, 25, 26, 28, 45-50

1.2 Polynomials

Definition [Exponents]: Consider a^n : Here a is the base and n is the exponent.

This denotes the product $a \cdot a \cdot a \dots a$ (n times)

i) Consider 4^2 -4^2 $(-4)^2$

ii) Consider 4^3 -4^3 $(-4)^3$

iii) Consider how do -4^2 and $(-4)^2$ differ? How about -4^3 and $(-4)^3$?

Properties of Exponents:

i) $(a^m)(a^n) = a^{m+n}$

ii) $(a^m)^n = a^{mn}$

iii) $(ab)^n = a^n b^n$

iv) $(a)^0 = 1$, for $a \neq 0$ Note: 0^0 is not defined

Properties of Polynomials:

Examples of polynomials: $5x^2 + x + 1$, $-2x^2$ (monomial), 5

i) Here x is a **variable** and the constant before each variable is a **coefficient**

ii) The degree of a polynomial is the largest exponent of x with nonzero coefficient.

Adding and Subtracting Polynomials

i) Remember to ONLY add/sub **like** terms! $5x^2y^9z$, $5x^3y^5z^7$, $-2x^2y^9z$

ii) Be careful when subtracting: you will need to **distribute** the negative if there are parenthesis.

Multiplying Polynomials

i) Monomial \times Polynomial \Rightarrow Distribute the Monomial to each term in the polynomial

ii) Binomial \times Binomial \Rightarrow FOIL (First, Outside, Inside, Last)

iii) Polynomial \times Polynomial \Rightarrow Can get complicated! Distribute each term of the first polynomial to each term of the second polynomial

Example: Perform the indicated operation and simplify.

i) $[(3z)^2]^3$

ii) $(3y^3 + 9y^2 - 11y) - (-4y^2 + 10y - 6)$

iii) $-6r(2r^2 + 5r - 7)$

iv) $(7z - 3)(2z + 5)$

v) $(8x + 3)(x^4 - 2x^2 + 6x)$

Definition [Business Applications]: When evaluating

i) Revenue = Price per item \times Number of items sold

ii) Cost = Fixed Costs + Variable Costs

Fixed Costs: Do not depend on how many items made: Buildings, Machinery

Variable Costs; Do depend on how many items made: Labor, Materials

iii) Profit = Revenue - Cost

Example: Find expressions for revenue, cost, and profit from selling x **thousand** items if item price is \$2.00, Fixed Costs are \$100,000, and Variable Costs are $1200x$.

Example: Hot Rocks Movies sells DVDs for \$8 each (wholesale) and can produce a maximum of 200,000 DVDs. The variable cost of producing x thousand DVDs is $3550x - 8x^2$ dollars, and the fixed costs for the manufacturing operation are \$215,000. If x thousand DVDs are manufactured and sold, find expressions for the revenue, cost, and profit.

Example: The number of knee implants (in millions) in year x is approximated by the polynomial $0.002722x^2 + 0.003x + 0.37$, where $x = 0$ corresponds to the year 2000. Assuming the polynomial approximation is accurate through 2021, use it to determine whether the following statement is true or false. There will be more than a million knee implants in 2014.

1.2 Problems: 5, 11, 12, 19, 21, 23, 27, 31, 34, 36, 37, 39, 49, 51, 53, 56

1.3 Factoring

Factoring : Write the number 18 as a product at least three ways: _____

i) The numbers in each product are called **factors**:

ii) The process of writing 18 as a product of its factors is called **factoring**.

*Anytime you are asked to factor a polynomial, first look for a **greatest common factor** (largest number that divides each term). You will factor out this GCF first.

Example: Factor out the greatest common factor.

i) $12r + 9k$

ii) $6m^4 - 9m^3 + 12m^2$

iii) $3(2x + 1)^3 + 4(2x + 1)^4$

Factoring Quadratics: AC Method for a quadratic of the form $ax^2 + bx + c$ follow the steps below:

i) Multiply a and c and consider the factors m, n of ac .

ii) Make the following chart:

iii) Work through combinations to find the factors that will work and substitute $(n+m)$ in for b in the original polynomial then factor by grouping as below:

*Note: When done correctly, the (polynomial) above will be the same in each factor.

****Difference of Squares:** Special case for a quadratic of the form $a^2 - b^2$

Use the formula: $a^2 - b^2 = (a + b)(a - b)$

*****Perfect Square:** Special case for a quadratic of the form $a^2 \pm 2ab + b^2$

Use the formula: $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$

Helpful factoring videos: Khan Academy and YouTube

Example: Factor Completely

i) $x^2 + 11x + 24$

ii) $y^2 + 3y - 10$

iii) $z^2 - 4z + 3$

iv) $3m^2 + 5m - 2$

v) $4y^2 - 11y + 6$

vi) $4x^2 + 12x + 9$

vii) $6p^2 - 7pq - 5q^2$

viii) $25y^2 - 80yz + 64z^2$

Example: Factor Completely

i) $a^2 - 81$

ii) $a^2 + 81$

*How do the previous two examples differ and why?

iii) $9p^2 - 4q^2$

iv) $128x^2 - 98$

v) $(x + 3)^2 - 64$

vi) $16x^4 - y^4$

1.3 Problems: 5, 6, 7, 11, 12, 17, 22, 27, 28, 35, 39, 47, 48, 73

1.4 Rational Expressions

Definition [Rational Expression]: an expression that can be written as the quotient of two polynomials. In this section, we will assume that all denominators are nonzero!

Rules for Rational Expressions

i) Cancellation Property: For all expressions P, Q, R , and S with $Q \neq 0$ and $S \neq 0$,

$$\frac{PS}{QS} = \frac{P}{Q}$$

ii) Multiplication Rule: For all expressions P, Q, R , and S with $Q \neq 0$ and $S \neq 0$,

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$$

iii) Division Rule: For all expressions P, Q, R , and S with $Q, R, S \neq 0$,

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR}$$

iv) Addition and Subtraction Rule: For all expressions P, Q , and R with $Q \neq 0$,

$$\frac{P}{Q} \pm \frac{R}{Q} = \frac{P \pm R}{Q}$$

Example: Simplify Completely

i) $\frac{12m}{-18}$

ii) $\frac{15a + 30a^2}{5a}$

iii) $\frac{4(w - 3)}{(w - 3)(w + 6)}$

iv) $\frac{2p^2 + 3p + 1}{p^2 + 3p + 2}$

Example: Perform the indicated operation and Simplify Completely

i) $\frac{3r^2}{5} \cdot \frac{20}{9r}$

ii) $\frac{5y}{16} \div \frac{3y^2}{10}$

iii) $\frac{x^2 + 5x + 6}{x + 3} \cdot \frac{2x^2 + 5x - 3}{x^2 + 3x + 2}$

iv) $\frac{2z - 8}{6} \div \frac{5z - 20}{3}$

v) $\frac{a^2 - 2a - 3}{a^2 + a} \div \frac{a + 4}{5a}$

vi) $\frac{7}{2p} + \frac{1}{p + 2} - \frac{9}{2p}$

vii) $\frac{\frac{2}{(x+h)^2} - \frac{2}{x^2}}{h}$

1.4 Problems: 3, 4, 5, 6, 9, 10, 12, 13, 15, 16, 21, 23, 31, 37, 38, 49, 50, 57, 60

1.5 Exponents and Radicals

Properties of Exponents and Radicals:

- i) $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ and $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- ii) $(a^m)(a^n) = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$
- iii) $(a^m)^n = a^{mn}$
- iv) $(ab)^n = a^n b^n$ and $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- v) $(a)^0 = 1$, for $a \neq 0$ Note: 0^0 is not defined
- vi) $a^{1/n} = \sqrt[n]{a}$ for real number a and positive integer n
- vii) $a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$, real number a , integer m , and positive integer n
*Note: If n is **even**, the n th root of a is the **positive** real number whose n th power is a . **In this course, we will only take even roots of positive numbers.**
If n is **odd**, the n th root of a is the real number whose n th power is a .

Example: Perform the indicated operation and Simplify Completely

i) $\frac{3^{14}}{3^{10}}$

ii) $\frac{(-5)^9}{(-5)^7}$

iii) $\frac{(xy)^{17}}{(xy)^{12}}$

iv) $(r^2 s^5)^6$

v) $\left(\frac{2}{z}\right)^5$

vi) $\left(\frac{3a^5}{(ab)^3}\right)^2$

vii) 4^{-3}

viii) -5^{-4}

ix) $\left(\frac{3}{x^3}\right)^{-4}$

Example: Perform the indicated operation and Simplify Completely

i) $36^{1/2}$

ii) $-64^{1/2}$

iii) $(-49)^{1/2}$

iv) $(-27)^{1/3}$

v) $16^{3/4}$

vi) $(-8)^{6/3}$

vii) $\left(\frac{27}{64}\right)^{-2/3}$

viii) $\frac{9^{14}}{9^{-16}}$

ix) $7^{-4} \cdot 7^6$

1.5 Problems: 5, 7, 8, 10, 15, 16, 17, 18, 19*, 20*, 23*, 25*, 53-60, 86, 95
(*without a calculator)

1.6 First-Degree Equations

Definition [Equation]: A statement that two mathematical expressions are equal.

First-Degree Equation: Equations that involve only constants and the first power of the variable. Ex: $5x - 3 = 13$, $8y = 4$; Not Ex: $2x^2 = 5x$, $\sqrt{x + 2} = 4$

Solutions: A number that can be substituted for the variable in an equation to produce a true statement. Ex: $2x + 1 = 19 \Rightarrow 2(9) + 1 = 19 \Rightarrow 18 + 1 = 19 \Rightarrow 19 = 19$
True, so 9 is a solution to this given equation.

Properties of Equality:

i) The same number may be added to or subtracted from both side of an equation:

If $a = b$, then $a + c = b + c$ and $a - c = b - c$

ii) Both sides may be multiplied or divided by the same nonzero number:

If $a = b$ and $c \neq 0$, then $ac = bc$ and $\frac{a}{c} = \frac{b}{c}$

Example: Solve for the appropriate variable (check your solution in the original equation):

i) $-2(y + 3) + 4y = 3(y + 1) - 6$

ii) $\frac{2x}{3} + \frac{1}{2} = \frac{x}{4} - \frac{9}{2}$

iii) $\frac{4}{3k + 6} - \frac{k}{3(k + 2)} = \frac{5}{3}$

iv) $\frac{8y}{y - 4} = \frac{32}{y - 4} - 3$

Example: Solve for x :

i) $2x - 7y = 3xk$

ii) $8(4 - x) + 6p = -5k - 11yx$

Example: The percentage y of workers in private industry who participate in a defined contribution retirement plan (such as a 401(k)) in year x is approximated by the equation $0.08(x - 1992) = 14y - 4.9$. Assuming this equation remains valid; determine when 40% of workers will participate in such a plan.

Example: A financial manager has \$14,000 to invest for her company. She plans to invest part of the money in tax free bonds at 6% interest and the remainder in taxable bonds at 9%. She wants to earn \$1,005 per year in interest from the investments. Find the amount she should invest at each.

1.6 Problems: 4, 6, 10, 12, 13, 18, 31, 32, 45, 51, 59, 60, 61, 62

1.7 Quadratic Equations

Definition [Quadratic Equation]: an equation that can be written in the form $ax^2 + bx + c = 0$, where a , b , and c are real numbers with $a \neq 0$.

Real Solution: a solution of an equation that is a real number.

Zero-Factor Property: If a and b are real numbers, with $ab = 0$, then $a = 0$ or $b = 0$ or both.

Square-Root Property: If $b > 0$, then the solutions of $x^2 = b$ are \sqrt{b} and $-\sqrt{b}$ ($\pm\sqrt{b}$)

Quadratic Formula: The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: Solve for the appropriate variable:

i) $y^2 + 3y = 10$

ii) $6x^2 = 3 - 7x$

iii) $4k^2 = 9k$

iv) $m(m + 7) = -10$

v) $x^2 = 21$

vi) $(2y - 3)^2 = 5$

Example: Solve for the appropriate variable:

i) $x^2 + 1 = 4x$ (use QF)

ii) $9x^2 - 30x + 25 = 0$ (use QF)

iii) $5 - \frac{4}{k} = \frac{1}{k^2}$

Example: The yearly amount y of food distribution by the St. Louis Area Foodbank is approximated by the equation $0.027x^2 - 0.037x + 7.16 = y$ where $x \geq 0$, where $x = 0$ corresponds to the year 1990 and y is measured in millions of pounds. Find the year in which 17,000,000 pounds of food were distributed.

Example: A landscape architect wants to make an exposed gravel border of uniform width around a small shed behind a company plant. The shed is 10 feet by 6 feet. He has enough gravel to cover 36 square feet. How wide should this border be?

Example: The length of a picture is 2 inches more than the width. It is mounted on a mat that extends 2 inches beyond the picture on all sides. What are the dimensions of the picture if the area of the mat is 99 square inches?

Example: If an object is thrown upward, dropped, or thrown downward and travels in a straight line subject only to gravity (with wind resistance ignored), the height h of the object above the ground (in feet) after t seconds is given by $h = -16t^2 + v_0t + h_o$ where h_o is the height of the object where $t = 0$ and v_0 is the initial velocity at time $t = 0$. The value v_0 is taken to be positive if the object moves upward and negative if it moves downward. Suppose that a golf ball is thrown downward from the top of a 625 foot high building with an initial velocity of 65 feet per second. How long does it take to reach the ground?

1.7 Problems: 9, 10, 16, 19, 21, 29, 30, 37, 53, 55, 61, 62, 63, 67, 69