1.1 The Real Numbers

There is a good bit of information in Sec. 1.1 that we will not review.

Order of Operations: P E MD AS

1) **Parenthesis:** Within each set of parentheses or brackets, start with the innermost set and work outward. Also, work separately above and below any fraction bars.
2) **Exponents:** Find all powers and roots, working from left to right.
3) **Multiplication/Division:** Do any multiplications or divisions in the order in which they occur, left to right.
4) **Addition/Subtraction:** Do any additions or subtractions in the order in which they occur, left to right.

Example: Simplify the following:

i) $\dfrac{2(3-7)+4(8)}{4(-3)+(-8+5)(7-9)}$

ii) $\dfrac{x^2-3\sqrt{y+12}}{\sqrt{x^2+y}}$ where $x=6$, $y=13$

Notation of Inequalities/Intervals:

$<$ Less Than	$\leq$ Less Than or Equal To
$>$ Greater Than	$\geq$ Greater Than or Equal To
(a, b) # is NOT included	$[a, b]$ # IS included

Note: Infinity (∞) can never be actually "reached," thus it is always associated with parenthesis in interval notation.

Example: State the following solution as an interval and graph it on a number line.

i) $-5 < x < 1$

<————————————>

ii) $4 \leq y < 9$

<————————————>

iii) $z \geq -3$

<————————————>

iv) $x < 2$

<————————————>

1.1 Problems: 13, 14, 16, 22, 25, 26, 28, 45-50

1.2 Polynomials

Definition [Exponents]: Consider a^n: Here a is the base and n is the exponent.

This denotes the product $a \cdot a \cdot a....a$ (n times)

i) Consider 4^2 $\quad -4^2$ $\quad (-4)^2$

ii) Consider 4^3 $\quad -4^3$ $\quad (-4)^3$

iii) Consider how do -4^2 and $(-4)^2$ differ? How about -4^3 and $(-4)^3$?

Properties of Exponents:

i) $(a^m)(a^n) = a^{m+n}$

ii) $(a^m)^n = a^{mn}$

iii) $(ab)^n = a^n b^n$

iv) $(a)^0 = 1$, for $a \neq 0$ Note: 0^0 is not defined

Properties of Polynomials:

Examples of polynomials: $5x^2 + x + 1$, $-2x^2$ (monomial), 5

i) Here x is a **variable** and the constant before each variable is a **coefficient**

ii) The degree of a polynomial is the largest exponent of x with nonzero coefficient.

Adding and Subtracting Polynomials

i) Remember to ONLY add/sub **like** terms! $5x^2y^9z$, $5x^3y^5z^7$, $-2x^2y^9z$

ii) Be careful when subtracting: you will need to **distribute** the negative if there are parenthesis.

Multiplying Polynomials

i) Monomial $\times$ Polynomial $\Rightarrow$ Distribute the Monomial to each term in the polynomial

ii) Binomial $\times$ Binomial $\Rightarrow$ FOIL (First, Outside, Inside, Last)

iii) Polynomial $\times$ Polynomial $\Rightarrow$ Can get complicated! Distribute each term of the first polynomial to each term of the second polynomial

Example: Perform the indicated operation and simplify.

i) $[(3z)^2]^3$

ii) $(3y^3 + 9y^2 - 11y) - (-4y^2 + 10y - 6)$

iii) $-6r(2r^2 + 5r - 7)$

iv) $(7z - 3)(2z + 5)$

v) $(8x + 3)(x^4 - 2x^2 + 6x)$

Definition [Business Applications]: When evaluating

i) Revenue = Price per item $\times$ Number of items sold

ii) Cost = Fixed Costs + Variable Costs

Fixed Costs: Do not depend on how many items made: Buildings, Machinery

Variable Costs; Do depend on how many items made: Labor, Materials

iii) Profit = Revenue - Cost

Example: Find expressions for revenue, cost, and profit from selling x **thousand** items if item price is \$2.00, Fixed Costs are \$100,000, and Variable Costs are $1200x$.

Example: Hot Rocks Movies sells DVDs for \$8 each (wholesale) and can produce a maximum of 200,000 DVDs. The variable cost of producing x thousand DVDs is $3550x - 8x^2$ dollars, and the fixed costs for the manufacturing operation are \$215,000. If x thousand DVDs are manufactured and sold, find expressions for the revenue, cost, and profit.

Example: The number of knee implants (in millions) in year x is approximated by the polynomial $0.002722x^2 + 0.003x + 0.37$, where $x = 0$ corresponds to the year 2000. Assuming the polynomial approximation is accurate through 2021, use it to determine whether the following statement is true or false. There will be more than a million knee implants in 2014.

1.2 Problems: 5, 11, 12, 19, 21, 23, 27, 31, 34, 36, 37, 39, 49, 51, 53, 56

1.3 Factoring

Factoring : Write the number 18 as a product at least three ways: ______________

i) The numbers in each product are called **factors**:

ii) The process of writing 18 as a product of its factors is called **factoring**.

*Anytime you are asked to factor a polynomial, first look for a **greatest common factor** (largest number that divides each term). You will factor out this GCF first.

Example: Factor out the greatest common factor.

i) $12r + 9k$

ii) $6m^4 - 9m^3 + 12m^2$

iii) $3(2x + 1)^3 + 4(2x + 1)^4$

Factoring Quadratics: AC Method for a quadratic of the form $ax^2 + bx + c$ follow the steps below:

i) Multiply a and c and consider the factors m, n of ac.

ii) Make the following chart:

iii) Work through combinations to find the factors that will work and substitute $(n+m)$ in for b in the original polynomial then factor by grouping as below:

*Note: When done correctly, the (polynomial) above will be the same in each factor.

****Difference of Squares**: Special case for a quadratic of the form $a^2 - b^2$

Use the formula: $a^2 - b^2 = (a + b)(a - b)$

*****Perfect Square**: Special case for a quadratic of the form $a^2 \pm 2ab + b^2$

Use the formula: $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$

Helpful factoring videos: Khan Academy and YouTube

Example: Factor Completely

i) $x^2 + 11x + 24$

ii) $y^2 + 3y - 10$

iii) $z^2 - 4z + 3$

iv) $3m^2 + 5m - 2$

v) $4y^2 - 11y + 6$

vi) $4x^2 + 12x + 9$

vii) $6p^2 - 7pq - 5q^2$

viii) $25y^2 - 80yz + 64z^2$

Example: Factor Completely

i) $a^2 - 81$

ii) $a^2 + 81$

*How do the previous two examples differ and why?

iii) $9p^2 - 4q^2$

iv) $128x^2 - 98$

v) $(x+3)^2 - 64$

vi) $16x^4 - y^4$

1.3 Problems: 5, 6, 7, 11, 12, 17, 22, 27, 28, 35, 39, 47, 48, 73

1.4 Rational Expressions

Definition [Rational Expression]: an expression that can be written as the quotient of two polynomials. In this section, we will assume that all denominators are nonzero!

Rules for Rational Expressions

i) Cancellation Property: For all expressions P, Q, R, and S with $Q \neq 0$ and $S \neq 0$, $\frac{PS}{QS} = \frac{P}{Q}$

ii) Multiplication Rule: For all expressions P, Q, R, and S with $Q \neq 0$ and $S \neq 0$, $\frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$

iii) Division Rule: For all expressions P, Q, R, and S with $Q, R, S \neq 0$, $\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR}$

iv) Addition and Subtraction Rule: For all expressions P, Q, and R with $Q \neq 0$, $\frac{P}{Q} \pm \frac{R}{Q} = \frac{P \pm R}{Q}$

Example: Simplify Completely

i) $\dfrac{12m}{-18}$

ii) $\dfrac{15a + 30a^2}{5a}$

iii) $\dfrac{4(w-3)}{(w-3)(w+6)}$

iv) $\dfrac{2p^2 + 3p + 1}{p^2 + 3p + 2}$

Example: Perform the indicated operation and Simplify Completely

i) $\dfrac{3r^2}{5} \cdot \dfrac{20}{9r}$

ii) $\dfrac{5y}{16} \div \dfrac{3y^2}{10}$

iii) $\dfrac{x^2+5x+6}{x+3} \cdot \dfrac{2x^2+5x-3}{x^2+3x+2}$

iv) $\dfrac{2z-8}{6} \div \dfrac{5z-20}{3}$

v) $\dfrac{a^2-2a-3}{a^2+a} \div \dfrac{a+4}{5a}$

vi) $\dfrac{7}{2p} + \dfrac{1}{p+2} - \dfrac{9}{2p}$

vii) $\dfrac{\dfrac{2}{(x+h)^2} - \dfrac{2}{x^2}}{h}$

1.4 Problems: 3, 4, 5, 6, 9, 10, 12, 13, 15, 16, 21, 23, 31, 37, 38, 49, 50, 57, 60

1.5 Exponents and Radicals

Properties of Exponents and Radicals:

i) $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ and $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$

ii) $(a^m)(a^n) = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$

iii) $(a^m)^n = a^{mn}$

iv) $(ab)^n = a^n b^n$ and $(\frac{a}{b})^n = \frac{a^n}{b^n}$

v) $(a)^0 = 1$, for $a \neq 0$ Note: 0^0 is not defined

vi) $a^{1/n} = \sqrt[n]{a}$ for real number a and positive integer n

vii) $a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$, real number a, integer m, and positive integer n

*Note: If n is **even**, the nth root of a is the **positive** real number whose nth power is a. **In this course, we will only take even roots of positive numbers.**

If n is **odd**, the nth root of a is the real number whose nth power is a.

Example: Perform the indicated operation and Simplify Completely

i) $\frac{3^{14}}{3^{10}}$

ii) $\frac{(-5)^9}{(-5)^7}$

iii) $\frac{(xy)^{17}}{(xy)^{12}}$

iv) $(r^2 s^5)^6$

v) $\left(\frac{2}{z}\right)^5$

vi) $\left(\frac{3a^5}{(ab)^3}\right)^2$

vii) 4^{-3}

viii) -5^{-4}

ix) $\left(\frac{3}{x^3}\right)^{-4}$

Example: Perform the indicated operation and Simplify Completely

i) $36^{1/2}$

ii) $-64^{1/2}$

iii) $(-49)^{1/2}$

iv) $(-27)^{1/3}$

v) $16^{3/4}$

vi) $(-8)^{6/3}$

vii) $\left(\frac{27}{64}\right)^{-2/3}$

viii) $\frac{9^{14}}{9^{-16}}$

ix) $7^{-4} \cdot 7^{6}$

1.5 Problems: 5, 7, 8, 10, 15, 16, 17, 18, 19*, 20*, 23*, 25*, 53-60, 86, 95 (*without a calculator)

1.6 First-Degree Equations

Definition [Equation]: A statement that two mathematical expressions are equal.

First-Degree Equation: Equations that involve only constants and the first power of the variable. Ex: $5x - 3 = 13$, $8y = 4$; Not Ex: $2x^2 = 5x, \sqrt{x+2} = 4$

Solutions: A number that can be substituted for the variable in an equation to produce a true statement. Ex: $2x + 1 = 19 \Rightarrow 2(9) + 1 = 19 \Rightarrow 18 + 1 = 19 \Rightarrow 19 = 19$

True, so 9 is a solution to this given equation.

Properties of Equality:

i) The same number may be added to or subtracted from both side of an equation:

If $a = b$, then $a + c = b + c$ and $a - c = b - c$

ii) Both sides may be multiplied or divided by the same nonzero number:

If $a = b$ and $c \neq 0$, then $ac = bc$ and $\frac{a}{c} = \frac{b}{c}$

Example: Solve for the appropriate variable (check your solution in the original equation):

i) $-2(y+3) + 4y = 3(y+1) - 6$

ii) $\frac{2x}{3} + \frac{1}{2} = \frac{x}{4} - \frac{9}{2}$

iii) $\frac{4}{3k+6} - \frac{k}{3(k+2)} = \frac{5}{3}$

iv) $\frac{8y}{y-4} = \frac{32}{y-4} - 3$

Example: Solve for x:

i) $2x - 7y = 3xk$

ii) $8(4 - x) + 6p = -5k - 11yx$

Example: The percentage y of workers in private industry who participate in a defined contribution retirement plan (such as a 401(k)) in year x is approximated by the equation $0.08(x - 1992) = 14y - 4.9$. Assuming this equation remains valid; determine when 40% of workers will participate in such a plan.

Example: A financial manager has \$14,000 to invest for her company. She plans to invest part of the money in tax free bonds at 6% interest and the remainder in taxable bonds at 9%. She wants to earn \$1,005 per year in interest from the investments. Find the amount she should invest at each.

1.6 Problems: 4, 6, 10, 12, 13, 18, 31, 32, 45, 51, 59, 60, 61, 62

1.7 Quadratic Equations

Definition [Quadratic Equation]: an equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers with $a \neq 0$.

Real Solution: a solution of an equation that is a real number.

Zero-Factor Property: If a and b are real numbers, with $ab = 0$, then $a = 0$ or $b = 0$ or both.

Square-Root Property: If $b > 0$, then the solutions of $x^2 = b$ are $\sqrt{b}$ and $-\sqrt{b}$ $(\pm\sqrt{b})$

Quadratic Formula: The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example: Solve for the appropriate variable:

i) $y^2 + 3y = 10$

ii) $6x^2 = 3 - 7x$

iii) $4k^2 = 9k$

iv) $m(m + 7) = -10$

v) $x^2 = 21$

vi) $(2y - 3)^2 = 5$

Example: Solve for the appropriate variable:

i) $x^2 + 1 = 4x$ (use QF)

ii) $9x^2 - 30x + 25 = 0$ (use QF)

iii) $5 - \frac{4}{k} = \frac{1}{k^2}$

Example: The yearly amount y of food distribution by the St. Louis Area Foodbank is approximated by the equation $0.027x^2 - 0.037x + 7.16 = y$ where $x \geq 0$, where $x = 0$ corresponds to the year 1990 and y is measured in millions of pounds. Find the year in which 17,000,000 pounds of food were distributed.

Example: A landscape architect wants to make an exposed gravel border of uniform width around a small shed behind a company plant. The shed is 10 feet by 6 feet. He has enough gravel to cover 36 square feet. How wide should this border be?

Example: The length of a picture is 2 inches more than the width. It is mounted on a mat that extends 2 inches beyond the picture on all sides. What are the dimensions of the picture if the area of the mat is 99 square inches?

Example: If an object is thrown upward, dropped, or thrown downward and travels in a straight line subject only to gravity (with wind resistance ignored), the height h of the object above the ground (in feet) after t seconds is given by $h = -16t^2 + v_0 t + h_o$ where h_o is the height of the object where $t = 0$ and v_0 is the initial velocity at time $t = 0$. The value v_0 is taken to be positive if the object moves upward and negative if it moves downward. Suppose that a golf ball is thrown downward from the top of a 625 foot high building with an initial velocity of 65 feet per second. How long does it take to reach the ground?

1.7 Problems: 9, 10, 16, 19, 21, 29, 30, 37, 53, 55, 61, 62, 63, 67, 69