

3.2 Quadratic Functions

Properties of Quadratic Functions:

Quadratic Function in **General Form**:

$$f(x) = ax^2 + bx + c, \text{ with } a \neq 0 \text{ and } a, b, c \text{ constants}$$

Quadratic Function in **Vertex (or Standard) Form**:

$$f(x) = a(x - h)^2 + k, \text{ where } (h, k) \text{ is the vertex}$$

We can find the vertex of a quadratic function in standard form by:

- i) Use Completing the Square on $f(x)$ (if you already know how to do this!)
- ii) Use the equation for the vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

The graph of a quadratic function is called a _____.

If $a > 0$, _____.

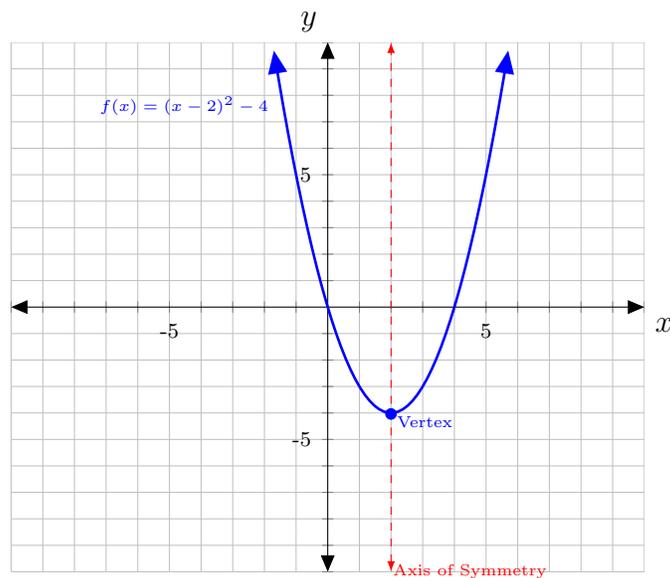
If $a < 0$, _____.

If $|a| > 1$, _____.

If $|a| < 1$, _____.

Domain: _____

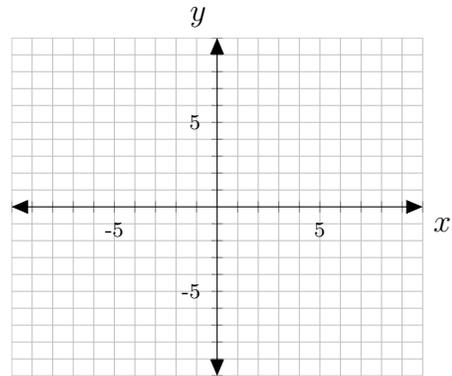
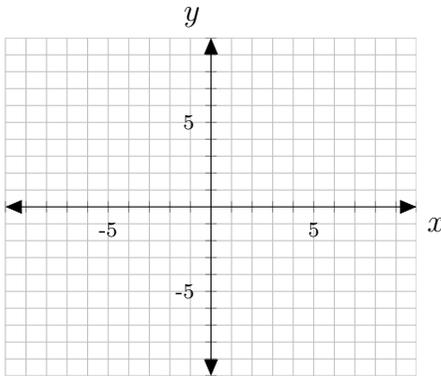
Range: $a > 0$, _____; $a < 0$, _____



Example: Sketch each function below, labelling the intercepts, and find domain and range:

i) $f(x) = -1/2(x + 1)^2 - 3$

ii) $f(x) = 4x^2 + 8x - 1$



Example: Find the rule of a quadratic function that has the given vertex and passes through the given point.

i) Vertex $(-3,2)$; Point $(2,1)$

ii) Vertex $(-2,0)$; Point $(-3,1)$

Example: If an object is thrown upward, dropped, or thrown downward and travels in a straight line subject only to gravity (with wind resistance ignored), the height h of the object above the ground (in feet) after t seconds is given by $h = -16t^2 + v_0t + h_o$ where h_o is the height of the object where $t = 0$ and v_0 is the initial velocity at time $t = 0$. The value v_0 is taken to be positive if the object moves upward and negative if it moves downward. Suppose that a golf ball is thrown downward from the top of a 625 foot high building with an initial velocity of 65 feet per second. How long does it take to reach the ground?

Example: A backyard farmer wants to enclose a rectangular space for a new garden within her fenced backyard. She has purchased 80 feet of wire fencing to enclose three sides, and will use a section of the backyard fence as the fourth side. Find a formula for the area enclosed. What dimensions should she make her garden to maximize the enclosed area?

Factoring Quadratics: AC Method for a quadratic of the form $ax^2 + bx + c$ follow the steps below:

- i) Multiply a and c and consider the factors m, n of ac .
- ii) Make the following chart:

- iii) Work through combinations to find the factors that will work and substitute $(n+m)$ in for b in the original polynomial then factor by grouping as below:

*Note: When done correctly, the (polynomial) above will be the same in each factor.

****Difference of Squares:** Special case for a quadratic of the form $a^2 - b^2$

Use the formula: $a^2 - b^2 = (a + b)(a - b)$

*****Perfect Square:** Special case for a quadratic of the form $a^2 \pm 2ab + b^2$

Use the formula: $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$

Helpful factoring videos: Khan Academy and YouTube

Example: Factor Completely

i) $x^2 + 11x + 24$

ii) $y^2 + 3y - 10$

iii) $z^2 - 4z + 3$

iv) $3m^2 + 5m - 2$

3.2 Problems: 15, 16, 21, 24, 45, 46, 53, 55, 59, 62, 85, 86, 91, 92

3.3 Power Functions and Polynomial Functions

Definition of Polynomial Function of Degree n :

A function whose rule is a polynomial whose highest degree is n .

Examples: $f(x) = 3x - 2$

Polynomial function of degree 1

$g(x) = x^4 + 5x^3 - 6x^2 + x - 3$

Polynomial function of degree 4

$h(x) = 3x^5$

Power function of degree 5

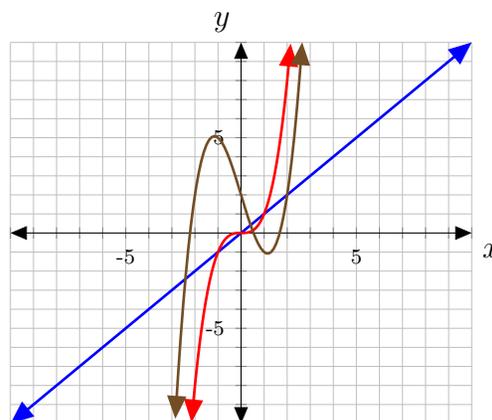
Graphing odd degree polynomials with leading coefficient greater than 0:

Behavior of the ends:

Behavior of the number of zeros:

Behavior of the number of turning points:

What changes if we make the leading coefficients negative?



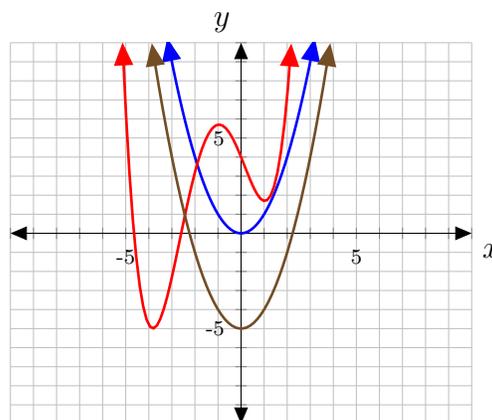
Graphing even degree polynomials with leading coefficient greater than 0:

Behavior of the ends:

Behavior of the number of zeros:

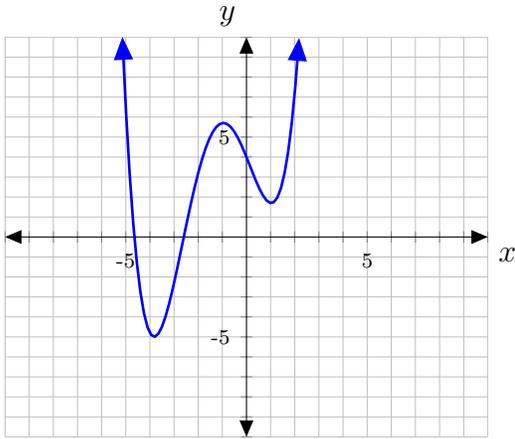
Behavior of the number of turning points:

What changes if we make the leading coefficients negative?

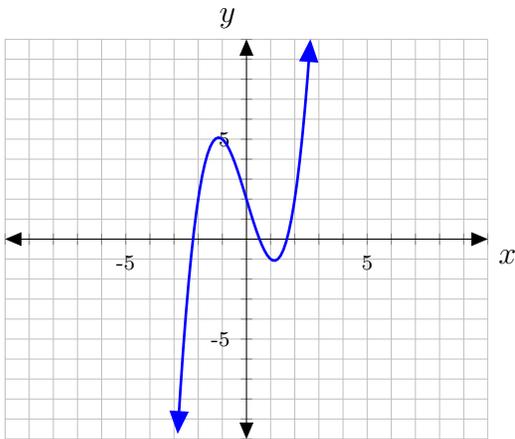


Example: State whether each graph could be the graph of a polynomial function. If it is, state whether it could be a polynomial function of degree 3, 4, or 5.

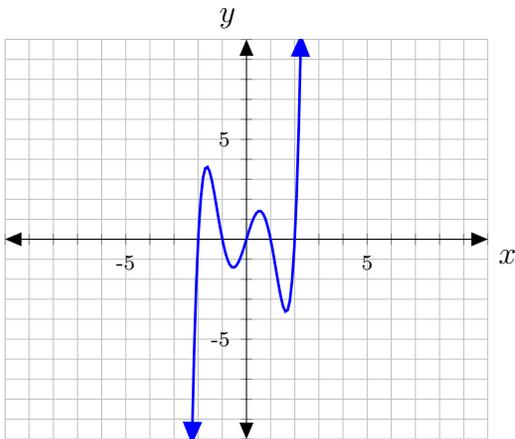
i)



ii)



iii)



Example: Use the following information about a polynomial to determine the function.
 y -int: $(0,9)$; x -ints: $(-3,0)$, $(3,0)$; Degree 2; End Behavior: as $x \rightarrow \pm\infty$, $f(x) \rightarrow -\infty$.

Example: A cube has an edge of 3 feet. The edge is increasing at the rate of 2 feet per minute. Express the volume of the cube as a function of m , the number of minutes elapsed.

3.3 Problems: 21, 23, 25, 30-32, 36, 61, 63, 66, 68-70

3.4 Graphs of Polynomial Functions

Properties of Polynomial Functions

Domain and Range: The domain of all polynomial functions is all real numbers. The range of odd polynomial functions is all real numbers (think line), while the range of even polynomial functions is restricted by its minimum or maximum (think parabola).

Identifying Zeros and Their Multiplicity: Set the function equal to zero and factor to find when the function is zero. We will define the number of times a given factor appears in the factored form of the equation of a polynomial as that zero's **multiplicity**.

Examples:

$$f(x) = (x + 3)(x + 2)(x - 1)$$

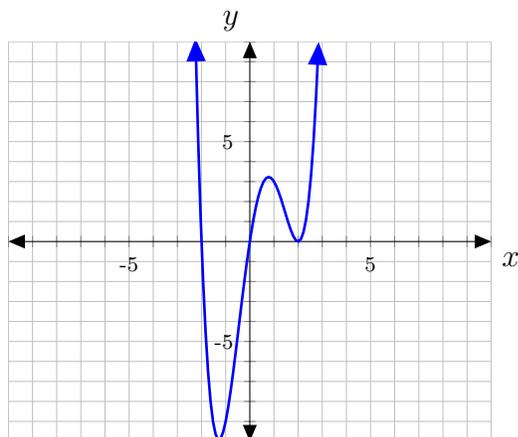
Each zero has multiplicity 1

$$g(x) = (x - 2)^2$$

The zero has multiplicity 2

Behavior Near Zeros: A polynomial function's behavior near its zeros is given by the multiplicity of that zero. The graph of a polynomial function will touch the x -axis at zeros with even multiplicity, and will cross the x -axis for zeros with odd multiplicity.

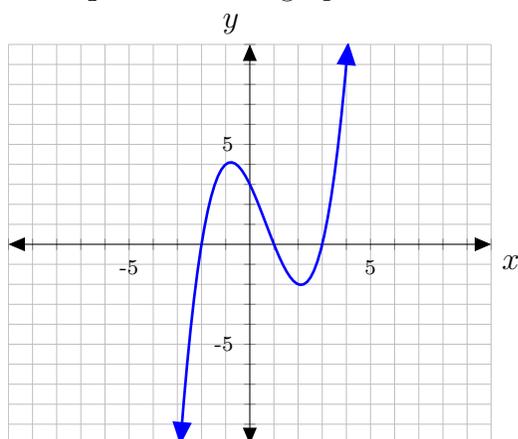
Example: State the multiplicity (assuming the lowest) of each zero for the following graph.



Example: Use the following information about a polynomial to determine the function.

Deg 5; y -int: $(0,9)$; roots of multiplicity 2 at $x = 3$, $x = 1$; root of multiplicity 1 at $x = -3$.

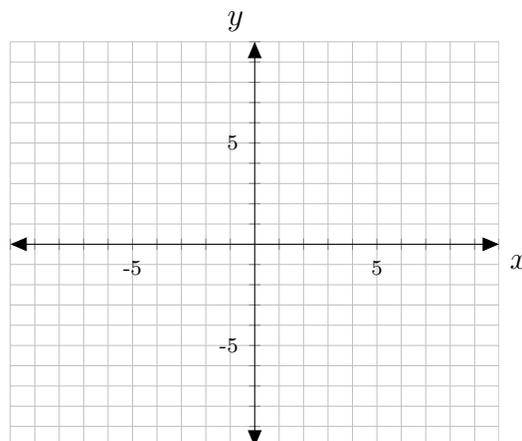
Example: Use the graph to write the formula of a polynomial function of least degree.



Example: Sketch the graph of the function and identify its degree:

i) $f(x) = x^2(x + 4)(2x - 5)$

Degree: _____



Theorem [Intermediate Value]: Let f be a polynomial function. If $f(a)$ and $f(b)$ have opposite signs, then there exists at least one value c between a and b such that $f(c) = 0$.

Example: Show that the polynomial $p(x) = x^3 + x^2 + x - 1$ has zeros in the interval $(0,1)$.

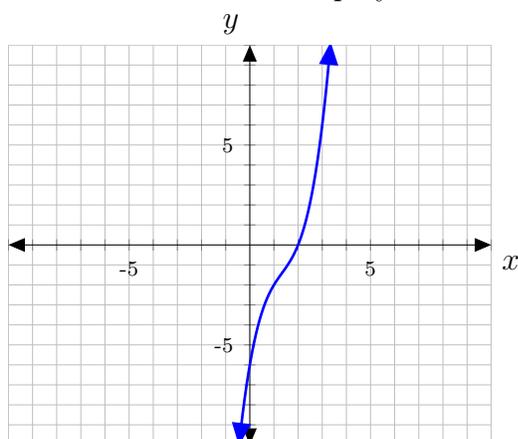
So, even if we do not know the zeros of a function, if we find that the output values change signs, there is at least one zero between the inputs that correspond to that change!

3.4 Problems: 25, 28, 33, 37, 48, 50, 57, 60, 78, 79

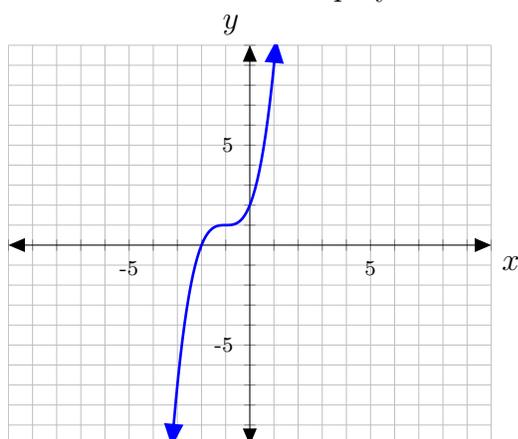
3.5 Dividing Polynomials

The book section covers long division and synthetic division, however we will focus on finding factors using graphs as below.

Example: Use the graph of the third-degree polynomial and the factor $x^2 - 2x + 3$ to write the factored form of the polynomial suggested by the graph. The leading coefficient is one.



Example: Use the graph of the third-degree polynomial and the factor $x^2 + x + 1$ to write the factored form of the polynomial suggested by the graph. The leading coefficient is one.



3.5 Problems: 44, 47

3.7 Rational Functions

Properties of Rational Functions:

A function whose rule is a quotient of two polynomials:

$$f(x) = \frac{P(x)}{Q(x)}, Q(x) \neq 0 \text{ and undefined at } x \text{ such that } Q(x) = 0$$

If a number, c , makes the denominator zero, but numerator nonzero, then the line $x = c$ is a **vertical asymptote**.

If a number, c , makes both the denominator and numerator zero, then $x = c$ is a **removable discontinuity**.

If the graph of a function approaches a horizontal line very closely when x is very large (or small), we say this line is a **horizontal asymptote**.

Properties of Horizontal Asymptotes:

For $f(x) = \frac{P(x)}{Q(x)}$ where the leading term of $P(x)$ is ax^n and of $Q(x)$ is bx^m , we have,

- i) If $n < m$: Then the x -axis ($y = 0$) is the horizontal asymptote
- ii) If $n > m$: Then there is no horizontal asymptote
- iii) If $n = m$: Then $y = \frac{a}{b}$ is the horizontal asymptote

Example: Find the equations of the vertical and horizontal asymptotes as well as any removable discontinuities of the given rational functions:

i) $f(x) = \frac{x - 3}{x^2 - 2x - 3}$

ii) $f(x) = \frac{x^2 + x - 2}{x - 3}$

iii) $f(x) = \frac{x - 2}{x - 3}$

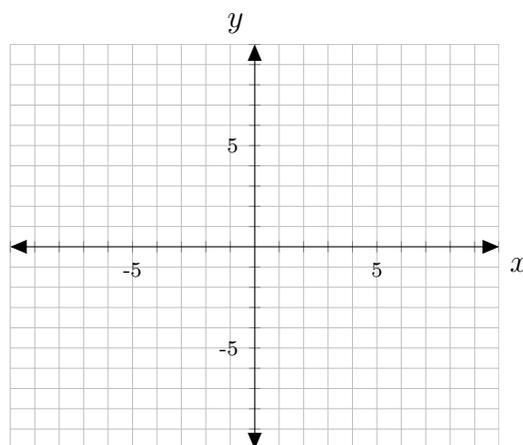
Example: Write an equation for a rational function with the given characteristics.

Vertical asymptotes: $x = -4$, $x = -1$; x -ints: $(1,0)$, $(5,0)$; y -int at $(0,7)$

Example: Graph each function:

- i) Find any vertical asymptotes, horizontal asymptotes, and x -intercepts.
- ii) Label items from i) on the coordinate grid. Remember asymptotes are dotted lines.
- iii) Plot a point in each interval of the x -axis created by x -ints and vertical asymptotes.
- iv) Use the information and points from iii) to finish sketching the graph.

$$f(x) = \frac{3x^2}{x^2 - 4}$$



Example: Use approaches (right arrow) symbols to describe the local and end behavior of the function above.

3.7 Problems: 10, 12, 18, 25-27, 51, 55, 60, 61, 75, 77, 79, 84-86