3.1 Functions and 3.2 Graphs of Functions

Properties of Functions: What is a function?

Equation that consists of inputs called _________, the set of outputs called _________,

and a rule that each input determines _____________ output.

Graphically, we can check to see if a graph is a function by applying the _____________.

Example: State whether each rule defines a function, stating the domain and range:

i) $\{(5,1),(3,2),(4,9),(7,6)\}$

Function: _____

Domain:

Range:

ii) $\{(2,4),(0,2),(2,5)\}$

Function: _____

Domain:

Range:

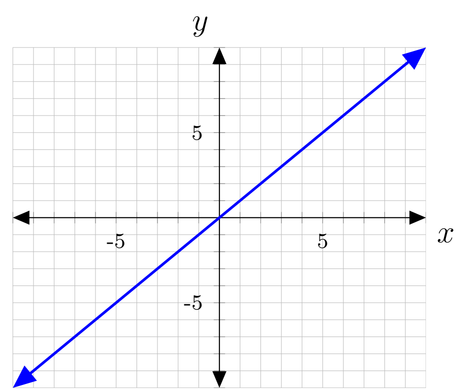
*Note: These questions can also be given as tables or drawings.

Example: Determine if the graph is a function, and state its domain and range:

i) Function: _____

Domain:

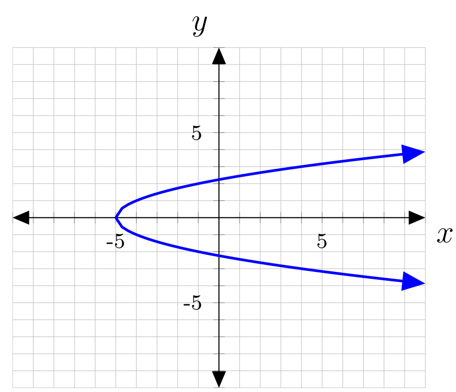
Range:



ii) Function: _____

Domain:

Range:



Function Notation:

$$f(x) = \sqrt{x^2 + 1}$$

Example: Decide whether the relation defines a function of x, and give the domain, range:

i) $y = 2x + 6$

Function: _____

Domain:

ii) $y = \frac{5}{x+2}$

Function: _____

Domain:

iii) $y = \sqrt{2x - 1}$

Function: _____

Domain:

iv) $y^6 = x$

Function: _____

Domain:

Example: Let $f(x) = -3x + 4$ and $g(x) = -x^2 + 4x + 1$. Find the following:

i) $f(0)$

ii) $g(-2)$

iii) $f(-x)$

iv) $f(x + 2)$

v) Difference Quotient: $\dfrac{g(x+h) - g(x)}{h}$, where $h \neq 0$

Definition [Piecewise Linear Function]:

Functions whose graphs consist of straight-line segments and whose rules are typically defined with different equations for different parts of the domain.

Example: If you were a single person in Connecticut in 2008 with a taxable income of x dollars, then your state income tax, $T(x)$, was determined by the rule below. Find the income tax paid by a single person with the given taxable income.

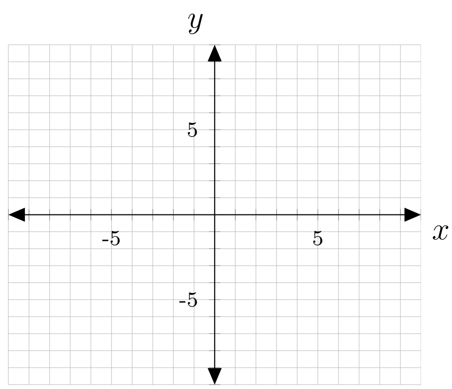
$$T(x) = \begin{cases} .03x & \text{if } 0 \leq x \leq 10,000 \\ 300 + .05(x - 10,000) & \text{if } x > 10,000 \end{cases}$$

i) \$48,750.

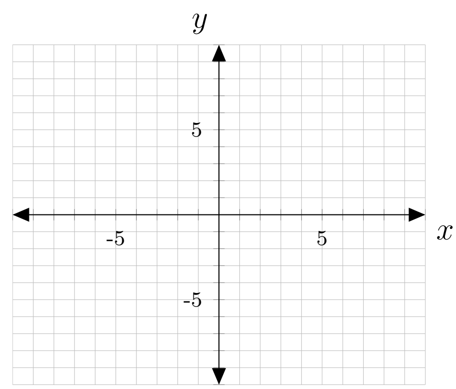
ii) \$7,345

Example: Graph each of the following functions:

i) $f(x) = \begin{cases} x + 1 & \text{if } x \leq 2 \\ -2x + 7 & \text{if } x > 2 \end{cases}$



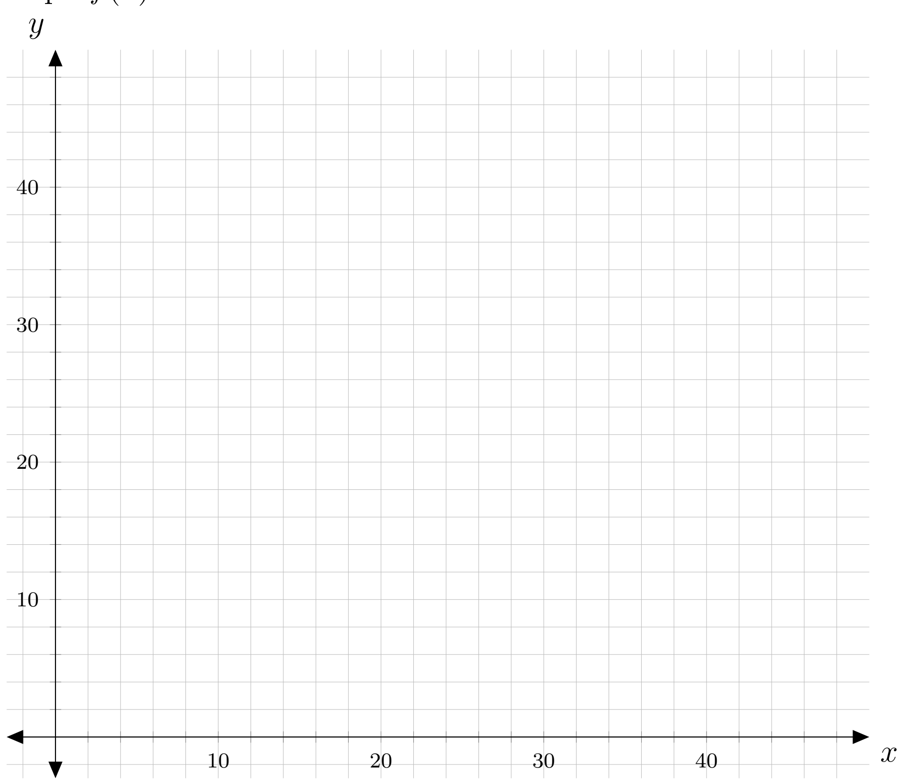
ii) $f(x) = \begin{cases} -3x - 2 & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$



Example: The price of Starbucks stock in 2008 can be modeled by the function where x is the number of business days past June 16, 2008:

$$f(x) = \begin{cases} 18.35 - .18x & \text{if } 0 \leq x \leq 22 \\ 12.49 + .10(x-3) & \text{if } 22 < x \leq 44 \end{cases}$$

i) Graph $f(x)$.



ii) What is the lowest stock price during the period given by $f(x)$?

3.1 Problems: 1, 3, 6, 7, 15-18, 23, 26, 33, 34, 35, 42-46, 47, 49, 51, 57, 58

3.2 Problems: 3-6, 27-32, 37, 40, 41, 45, 46

Note: When homework asks find the rule of the _______ function, just find the equation as usual and check to make sure the equation is a function. To find rule for piecewise functions, find rule between first two points, then the second and third points, and put this together as a piecewise function for each of the respective x values.

3.3 Applications of Linear Functions

Definition : Recall:

$R(x) =$ (Price per Item)(Number of Items) and $C(x) =$ (Fixed Costs)+(Variable Costs)

i) **Average Cost-** If $C(x)$ is the total costs to produce x items, then the average cost per item is given by Total Costs divided by Total Number of Items:

$$\overline{C(x)} = \frac{C(x)}{x}$$

ii) **Rate of Change-** For a linear function $y = mx + b$, the rate of change is given by the slope m.

iii) **Linear Depreciation-** The rate of change of a decreasing linear function (will be negative).

iv) **Marginal Cost-** The rate of change of the cost function.

v) **Marginal Revenue-** The rate of change of the revenue function.

vi) **Break-Even Point-** The number of units at which revenue equals cost (or equivalently, Profit = Zero).

vii) **Equilibrium Point-** The point where supply and demand are equal.

Example: A company manufactures a 42-inch plasma HDTV that sells to retailers for \$550. The cost of making x of these TVs for a month is given by the cost function $C(x) = 250x + 213,000$.

i) Find the function $R(x)$ that gives the revenue from selling x TVs.

ii) Find the profit function $P(x)$.

iii) What is the profit from selling 500 TVs?

iv) How many TVs would the company have to sell in a month in order to break even?

Example: The total cost of producing 10 calculators is \$100. The variable costs per calculator are \$4.

i) Find the rule of the linear cost function.

ii) What are the fixed costs?

iii) Find the average cost per calculator when 50 and 100 are produced.

iv) Why will the average cost per calculator always decrease as more calculators are produced?

Example: According to Kelley Blue Book, a Ford Mustang two-door convertible that is worth \$14,776 today will be worth \$10,600 in two years (if it is in excellent condition with average mileage).

i) Find the Depreciation Function for this car (assuming linear depreciation).

ii) What will the car be worth in 6 years (assuming it is in excellent condition)?

iii) At what rate is the car depreciating?

Example: The cost in dollars to produce x kilograms (kg) of chocolate candy is given by $C(x) = 3.5x + 800$.

i) Find the fixed costs for producing this product.

ii) What is the total cost for producing 12 kg?

iii) Find the marginal cost per kilogram.

iv) After 49 units have been produced, what is the cost of producing one more?

Example: Bill Fullington, an economist, has studied the supply and demand for aluminum siding and has determined that the price per unit (p) and the quantity demanded (q) are related by the linear equation $p = 60 - 3/4q$.

i) Find the demand at a price of \$40 per unit.

ii) Find the demand if the price rises to \$60 per unit.

iii) Find the price if the demand is 32 units.

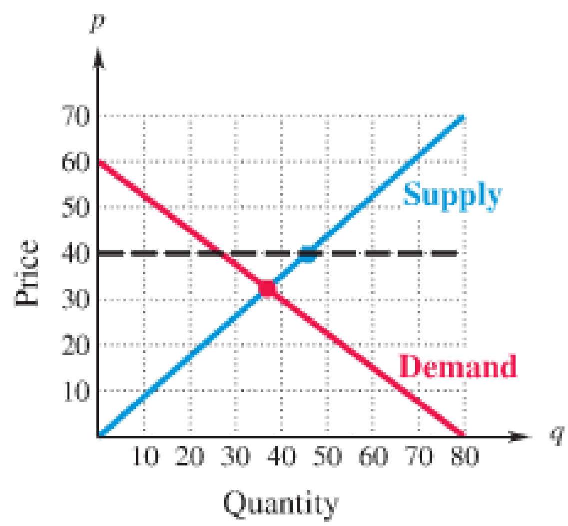
Example: Suppose the economist concludes that the supply (q) of siding is related to its price (p) by the equation $p = .85q$.

i) Find the supply if the price is \$51 per unit.

ii) Find the price per unit if the supply is 20 units.

Example: The supply and demand curves of the previous examples are shown.

i) Determine graphically whether there is a surplus or a shortage of supply at a price of \$40 per unit.



ii) Find the equilibrium quantity and price algebraically. Compare your answer to the equilibrium point shown on the graph.

3.3 Problems: 1, 4, 5, 7, 10, 13, 15, 18-20, 23, 24, 27, 28, 35, 36, 39, 40, 45-49, 52

3.4 Quadratic Functions and Applications

Properties of Quadratic Functions:

Quadratic Function in Standard Form:

$$f(x) = ax^2 + bx + c, \text{ with } a \neq 0 \text{ and } a,\, b,\, c \text{ constants}$$

Quadratic Function in Vertex Form:

$$f(x) = a(x-h)^2 + k, \text{ where } (h,k) \text{ is the vertex}$$

We can find the vertex of a quadratic function in standard form by:

i) Use Completing the Square on $f(x)$ (if you already know how to do this!)

ii) Use the equation for the vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

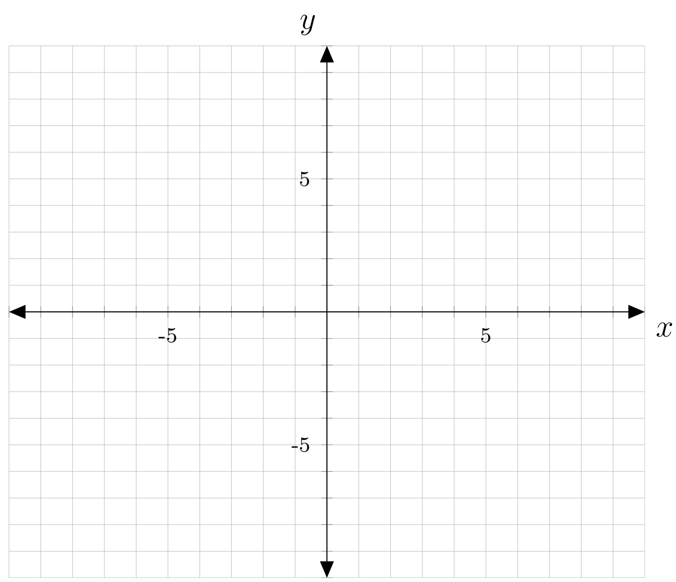
The graph of a quadratic function is called a _______________.

If $a > 0$, _______________.

If $a < 0$, _______________.

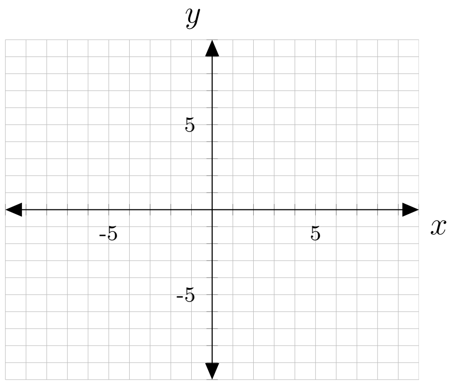
If $|a| > 1$, _______________.

If $|a| > 1$, _______________.

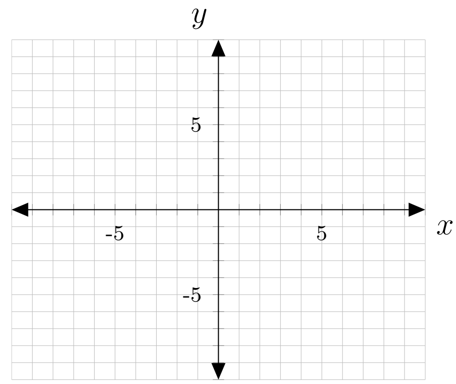


Example: Sketch each of the functions below and label the intercepts:

i) $f(x) = -1/2(x+1)^2 - 3$



ii) $f(x) = 4x^2 + 8x - 1$



Example: Find the rule of a quadratic function that has the given vertex and passes through the given point.

i) Vertex (-3,2); Point (2,1)

ii) Vertex (-2,0); Point (-3,1)

Example: Anne Kelly owns and operates Aunt Emma's Blueberry Pies. She has hired a consultant to analyze her business operations. The consultant tells her that her profits, $P(x)$, from the sale of x cases of pies are given by $P(x) = -2x^2 + 40x + 280$. How many cases of pies should she sell in order to maximize profits? What is the maximum profit possible?

Example: Suppose that the price of and demand for an item are related by $p = 150 - 6q^2$, where p is price (in dollars) and q is the number of items demanded (in hundreds). Also, suppose that the price and supply are related by $p = 10q^2 + 2q$, where p is price (in dollars) and q is the number of items supplied (in hundreds).

i) Graph the supply and demand functions on your graphing calculator with a viewing window [-10,10] x [-10,200].

ii) Find the equilibrium quantity and price algebraically. Compare this to the equilibrium point on the graph.

Example: The rental manager of a small apartment complex with 16 units has found from experience that each \$40 increase in monthly rent results in an empty apartment. All 16 apartments will be rented at a monthly rent of \$500.

i) Let x be the number of apartments that are not rented. Find an expression for the price per apartment.

ii) Find an expression for the number of apartments that are rented.

iii) Write an equation for the total monthly revenue (income) for the complex.

iv) How many \$40 increases will produce maximum monthly income for the complex? What is the maximum monthly income?

v) What amount should they charge for monthly rent?

Example: The annual revenues (in billions of dollars) as reported by Morningstar.com for Verizon Communications, Inc. are given in the following table for the years 2000 2007, with $x = 0$ representing the year 2000.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Revenue	64.7	67.2	67.6	67.8	71.3	75.1	88.1	93.5

Using the year 2000 as the minimum and the data from 2005, find the quadratic model $f(x) = a(x - h)^2 + k$ for the data.

3.4 Problems: 3, 4, 6, 7, 9-18, 21, 24, 25, 28, 33, 37, 39, 40, 42, 46, 47, 51, 54, 55, 58, 59, 61, 67

3.5 Polynomial Functions

Definition of Polynomial Function of Degree n:

A function whose rule is a polynomial whose highest degree is n.

Examples:	$f(x) = 3x - 2$	Polynomial function of degree 1
	$g(x) = 3x^2 + 4x - 6$	Polynomial function of degree 2
	$f(x) = x^4 + 5x^3 - 6x^2 + x - 3$	Polynomial function of degree 4

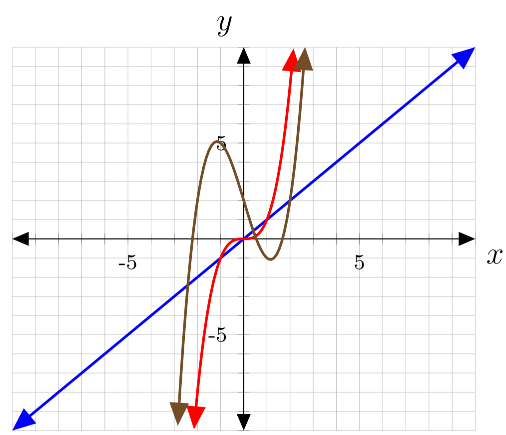
Graphing odd degree polynomials with leading coefficient greater than 0:

Behavior of the ends:

Behavior of the number of zeros:

Behavior of the number of peaks/valleys:

What changes if we make the leading coefficients negative?



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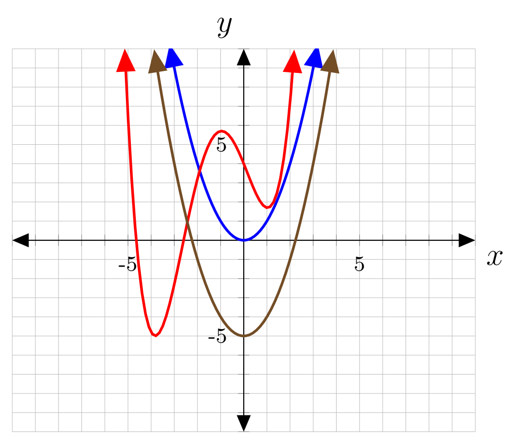
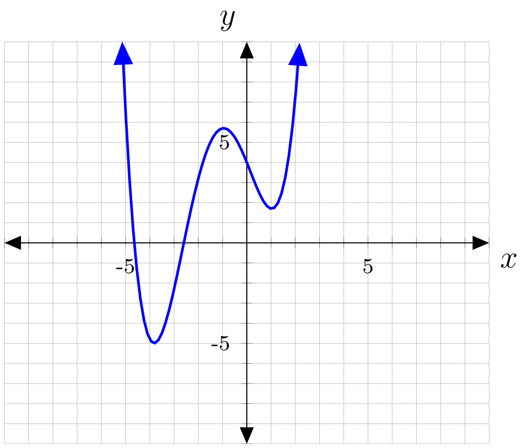
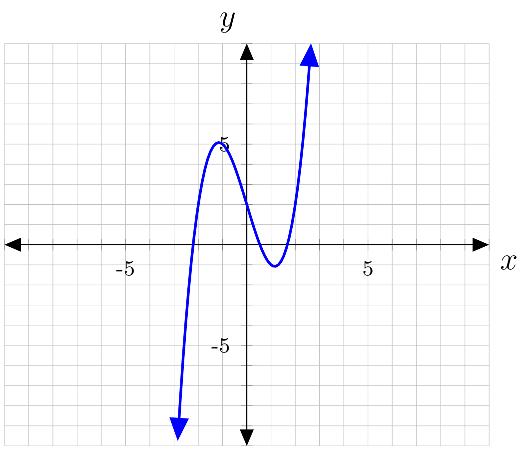


Example: State whether each graph could be the graph of a polynomial function. If it is, state whether it could be a polynomial function of degree 3, 4, or 5.

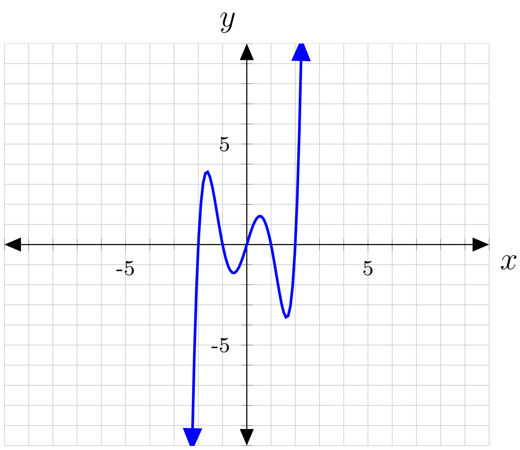
i)



ii)



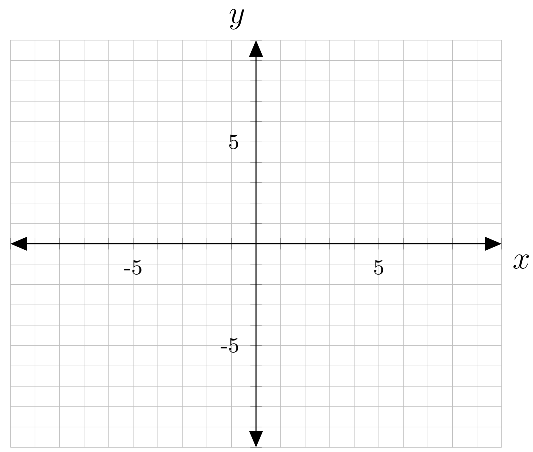
iii)



Example: Graph each of the functions and identify its degree: (without using a calculator):

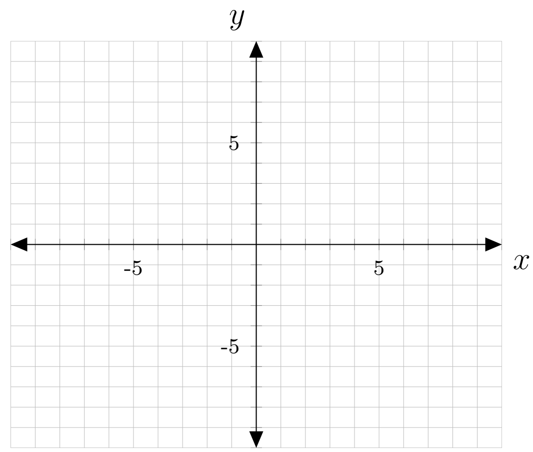
i) $f(x) = (2x+3)(x-1)(x+2)$

Degree: _____



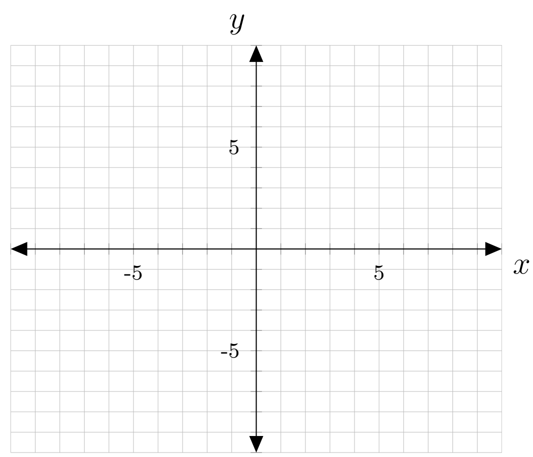
ii) $f(x) = x^2(x+4)(2x-5)$

Degree: _____



iii) $f(x) = x^3 + 4x^2 - 7x$

Degree: _____



3.5 Problems: 5, 6, 7, 9-15, 17, 18, 20, 21, 23, 25, 27

3.6 Rational Functions

Properties of Polynomial and Rational Inequalities:

A function whose rule is a quotient of two polynomials:

$$f(x) = \frac{P(x)}{Q(x)},\ Q(x) \neq 0 \text{ and undefined at } x \text{ such that } Q(x) = 0$$

If a number, c, makes the denominator zero, but numerator nonzero, than the line $x = c$ is a vertical asymptote.

If the graph of a function approaches a horizontal line very closely when x is very large (or small), we say this line is a horizontal asymptote.

Properties of Horizontal Asymptotes:

For $f(x) = \dfrac{P(x)}{Q(x)}$ where the leading term of $P(x)$ is ax^n and of $Q(x)$ is bx^m, we have,

i) If $n < m$: Then the x-axis ($y = 0$) is the horizontal asymptote

ii) If $n > m$: Then there is no horizontal asymptote

iii) If $n = m$: Then $y = \dfrac{a}{b}$ is the horizontal asymptote

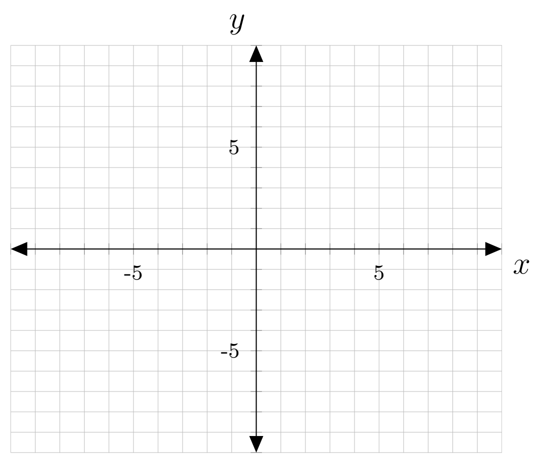
Example: Find the equations of the vertical and horizontal asymptotes of the given rational functions:

i) $f(x) = \dfrac{x-3}{x^2+x-2}$ ii) $f(x) = \dfrac{x^2+x-2}{x-3}$ iii) $f(x) = \dfrac{x-2}{x-3}$

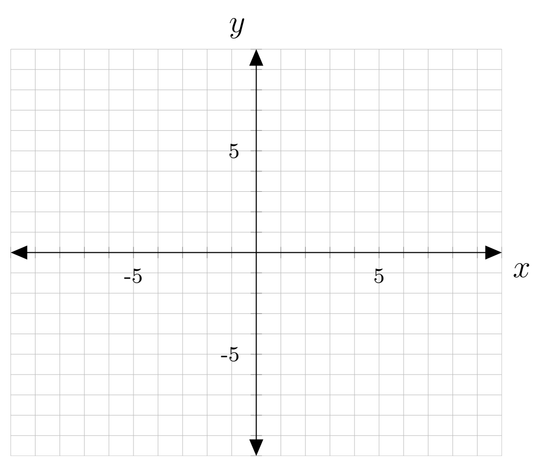
Example: Graph each function:

i) Find any vertical asymptotes, horizontal asymptotes, and x-intercepts.
ii) Label items from i) on the coordinate grid. Remember asymptotes are dotted lines.
iii) Plot a point in each interval of the x-axis created by x-ints and vertical asymptotes.
iv) Use the information and points from iii) to finish sketching the graph.

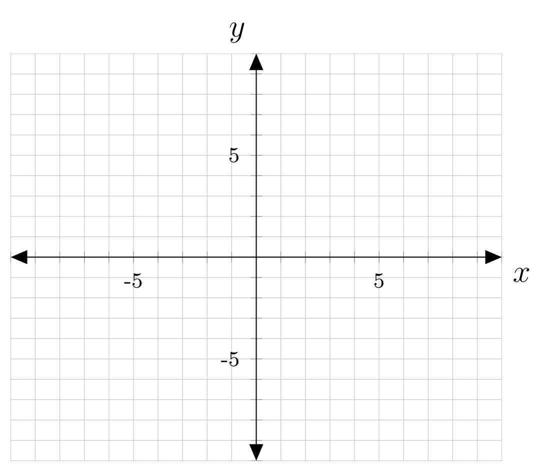
i) $f(x) = \dfrac{-4}{x+4}$



i) $f(x) = \dfrac{5+2x}{x-2}$



iii) $f(x) = \dfrac{3x^2}{x^2-4}$



Example: The product exchange function for the Fruits of the Earth Winery for red wine x and white wine y, in number of cases is $y = (150,000 - 75x)/(1200 + x)$. Graph the function on a graphing calculator and find the maximum quantity of each kind of wine that can be produced.

Example: A retailer buys 2500 specialty lightbulbs from a distributor each year. In addition to the cost of each bulb, there is a fee for each order, so she wants to order as few times as possible. However, since storage costs are higher when there are fewer orders (and hence more bulbs per order to store). Past experience shows that the total annual cost (for bulbs, ordering fees, and storage costs) is given by the rational function $C(x) = (.98x^2 + 1200x + 22,000)/x$, where x is the number of bulbs ordered each time. Graph this function on a graphing calculator. How many bulbs should be ordered each time in order to have the smallest possible cost?

3.6 Problems: 5, 8, 9, 11, 13, 14, 15, 21, 22, 23, 25, 27, 29