5.2 Compound Interest

Properties of Simple and Compound Interest:

Simple Interest: earn interest each year on your original investment

To find the future value, we have the following:

$$FV = P(1 + rt)$$

P = original investment, r = interest rate, t = number of years

Compound Interest: earn interest on both your original investment and previously earned interest

To find the future value (or compound amount), we have the following:

$$FV = P(1 + i)^n$$

P = original investment, i = interest rate per period, n = number of periods

*Note: i is not simply the interest rate!

$$i = \frac{\text{interest rate}}{\text{times per year compounded}} = \frac{r}{m}$$

Comparing compound and simple interest:

Example: If $7000 is deposited into an account that pays 4% interest compounded annually, how much money is in the account after 9 years? What if it is compounded quarterly?

Example: Find the amount of interest for both cases in the previous example.

Example: If a \$16,000 investment grows to \$50,000 in 18 years, what is the interest rate (assuming annual compounding)?

Example: Suppose that the inflation rate is 3.5% (which means that the overall level of prices is rising 3.5% a year). How many years will it take for the prices to double?

Example: Keisha Jones must pay a lump sum of \$6000 in 5 years. What amount deposited today at 6.2% compounded annually will amount to \$6000 in 5 years?

Continuous Compounding: informally, interest compounded as frequently as possible

Lets say we invest \$1 for one year at an annual interest rate of 100%, compounded n times per year. What happens?

$$FV = P(1+i)^n = 1\left(1+\frac{1}{n}\right)^n$$

Compounded	n	Future Value
Monthly	12	
Daily	365	
Every Minute	525,600	
Every Second	31,536,000	

Continuous Compounding:

$$FV = Pe^{rt}$$

P = original investment, r = interest rate, t = years compounded

Example: Suppose \$5000 is invested at an annual rate of 4% compounded quarterly for 5 years. Find the compound amount.

Example: Find the compounded amount if the interest in the previous example is compounded continuously. How much more interest is earned?

Example: Suppose that to settle a debt, you will be paid \$10,000 at the end of 10 years.

i) If you can get an interest rate of 4% compounded continuously, what is the present value of the \$10,000?

ii) If the person that owes you the money offers you \$8000 immediately to settle his/her debt, should you take the deal?

Nominal Rate: stated interest rate that is being paid (monthly, weekly, etc.)

Effective Rate (Annual Percentage Yield): the interest rate needed, compounded annually, to equal the nominal rate compounded (monthly, weekly, etc.)

*Wanting to find the interest rate compounded only annual to receive the same amount of money as compounded more often at a given interest rate

$$APY = r_E = \left(1 + \frac{r}{m}\right)^m - 1$$

r = interest rate, m = times per year compounded

Example: In October 2008, National City Bank in Cleveland offered its customers a 4-year $25,000 CD at 5.12% interest, compounded daily. Find the APY.

Example: Bank A is now lending money at 10% interest compounded annually. The rate at Bank B is 9.6% compounded monthly, and the rate at Bank C is 9.7% compounded quarterly. If you need to borrow money, at which bank will you pay the least interest?

5.2 Problems: 7, 10, 11, 16, 17, 21, 24, 29, 30, 31, 33, 35, 41, 44, 47, 52, 53, 55, 58, 59

5.3 Annuities, Future Value, and Sinking Funds

Properties of Annuities

i) **Annuity:** A sequence of equal payments made at equal periods of time.

ii) **Term:** The time from the beginning of the first payment period to the end of the last period.

iii) **Ordinary Annuity:** Annuities where the payments are made at the END of each period and the frequency of the payments is the SAME as the frequency of compounding the interest.

*Algebraic Note: $1 + x + x^2 + ... + x^{n-1} = \dfrac{x^n - 1}{x - 1}$

Example: $1500 is deposited at the end of each year for the next 6 years in an account paying 8% interest compounded annually. Find the future value of this annuity.

Future Value of an Ordinary Annuity

$$FV = P\left[\frac{(1+i)^n - 1}{i}\right]$$

P = payment at the end of each quarter, i = interest per period, n = number of periods

Example: Chris Webber is an athlete who feels that his playing career will last for 7 years. To prepare for his future, be deposits $10,000 at the end of each month for 7 years in an account paying 4% compounded monthly. How much will he have on deposit after 7 years?

> **Sinking Fund:** A fund set up by corporations to receive periodic payments. They form an Ordinary Annuity if the payments are equal and made at the end of regular periods.

Example: A business sets up a sinking fund so that it can pay off bonds it has issued when they mature. If it deposits $12,000 at the end of each quarter in an account that earns 5.2% interest, compounded quarterly, how much will be in the sinking fund after 10 years?

Example: Francisco Arce needs $8000 in 6 years so that he can go on an archaeological dig. He wants to deposit equal payments at the end of each quarter to save enough money. Find the amount of each payment if the bank pays 8% interest compounded quarterly.

Example: A firm borrows $6 million to build a factory. The bank requires it to set up a $200,000 sinking fund to replace the roof after 15 years. If the firms deposits earn 6% interest, compounded annually, find the payment it should make at the end of each year.

5.3 Problems: 3, 4, 5, 16, 17, 37, 41, 43, 48, 49