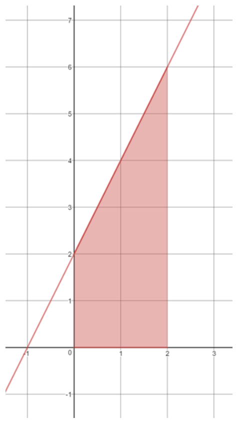
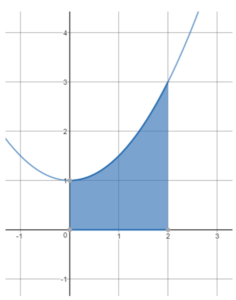
6.1: Area of Regions in the Plane

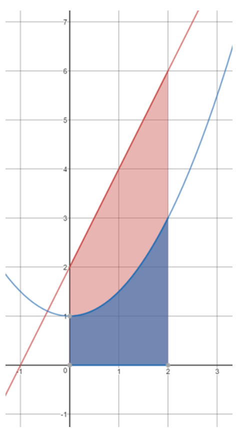
Example: Find the area under the graph of $f(x) = 2x + 2$ from $x = 0$ to $x = 2$.



Example: Find the area under the graph of $g(x) = .5x^2 + 1$ from $x = 0$ to $x = 2$.



Example: Find the area between the graphs of $f(x) = 2x + 2$ and $g(x) = .5x^2 + 1$ from $x = 0$ to $x = 2$.



Definition [Area Bounded By Two Curves]: If $f(x)$ and $g(x)$ are two continuous functions for which $f(x) \geq g(x)$ on $[a, b]$, then the area bounded by the two functions and the vertical lines $x = a$ and $x = b$ on $[a, b]$ is

$$\int_a^b [f(x) - g(x)]\, dx.$$

Example: Find the area bounded by the graphs of $f(x) = e^x$, $g(x) = x^4$, and the lines $x = 0$ and $x = 1$.

Example: Find the area bounded by the graphs of $f(x) = -3x^2 + 6x + 4$, $g(x) = -x + 3$, and the lines $x = 0$ and $x = 2$.

Example: Find the area completely enclosed by $f(x) = x$ and $g(x) = 2 - x^2$.

Example: Suppose a farm's revenue stream (in thousands of dollars) is given by $R(t) = t^2 + 99$, and the farm's annual cost (in thousands of dollars) is given by $C(t) = -.01t^3 + t + 90$, where t is the number of years since the owner acquired the farm. What is the farm's accumulated profit in it's first 10 years?

Example: The current profit for a company is \$10 million per year. The manager estimates that over the next 5 years, annual profit will increase at a continuous rate of between 2% and 5%. If the profit increases at a rate of 2%, the new annual profit will be given by the function $P_{2\%}(t) = 10e^{.02t}$. If the profit increases at a rate of 5%, the new annual profit will be given by the function $P_{5\%}(t) = 10e^{.05t}$. Both functions are in millions of dollars, and for both, t is the number of years from now. Approximate the total difference in accumulated profit under the two models.

6.1 Problems: 1, 2, 5, 7, 10, 11, 12, 15, 17, 18, 21, 22, 25, 27, 30, 31

6.2: Consumer and Producer Surplus

Definition [Supply/Demand Functions, Equilibrium]:

i) The **demand function**, $D(x) = p$, is the price, p, that consumers are willing to pay when x units are available in the market.

ii) The **supply function**, $S(x) = p$, is the price, p, that consumers are willing to pay when x units are available in the market.

iii) The **equilibrium point**, (x_e, p_e), is the intersection of the supply and demand graphs.

Definition [Consumer/Producer Surplus]:

i) **Consumer Surplus** is the difference between what consumers were willing to pay for a certain number of items and what they actually did pay.

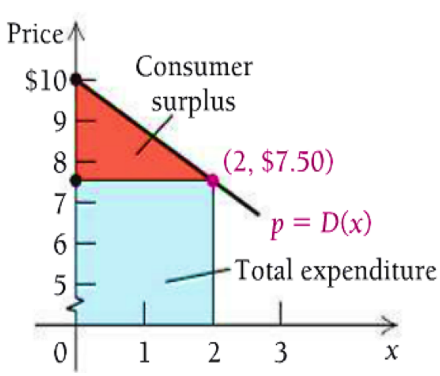
ii) **Producer Surplus** is the difference between the revenue from the sale of a certain number and what the producer was willing to sell it for.

Example:

Price Per Movie Ticket	Number of Movies I'll Go To Per Month
\$10	0
\$8.75	1
\$7.50	2

The table shows that the demand function for going to the movies is $D(x) = 10 - 1.25x$. The area under the demand curve, from 0 to a, i.e. $\int_0^a D(x)\, dx$, represents what going to a movies is worth to me.

If the ticket price is \$7.50, Ill go see two movies, and so I will spend \$15.00. However, each movie is worth \$8.75 to me, so I would have been willing to spend \$17.50 to see 2 movies. This means I have a consumer surplus of \$2.50.



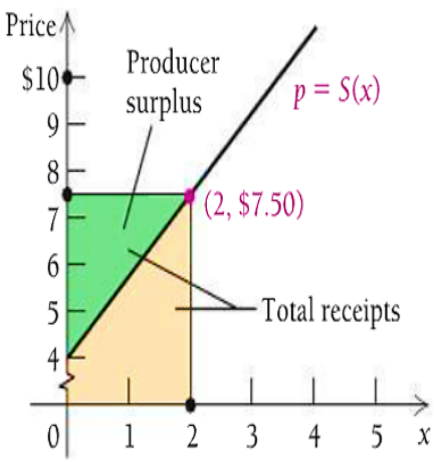
Definition [**Consumer Surplus Formula**]: If $D(x)$ is the demand function for a commodity, the **consumer surplus** at the equilibrium point (x_e, p_e) is given by

$$CS = \int_0^{x_e} D(x) - p_e \, dx.$$

Example Continued: Let's say the movie theater won't sell tickets for less than \$4, but will sell one ticket for \$5.75 or two tickets for \$7.50. Then we have the supply function $S(x) = 4 + 1.75x$. The area under the supply curve, from 0 to a, i.e. $\int_0^a S(x)\, dx$, represents how much revenue the theater obtains (per person) from showing a movies.

At a price of \$7.50, remember that I will go to two movies, and the theater will take in \$15 in revenue. However, the theater was willing to charge \$5.75 for one ticket, and so would have charged \$11.50 for two tickets.

This means there is a producer surplus of 15.00-11.50 = \$3.50.



Definition [**Producer Surplus Formula**]: If $S(x)$ is the supply function for a commodity, the **producer surplus** at the equilibrium point (x_e, p_e) is given by

$$PS = \int_0^{x_e} p_e - S(x) dx.$$

Example: The demand for a particular item is $D(x) = 1,450 - 3x^2$. Find the consumer surplus if the equilibrium price is $250.

Example: The supply for a particular item is $S(x) = x^2 + 5x + 20$. Find the producer's surplus if the equilibrium price is $434.

Example: The monthly demand for an item is $D(x) = -x^2 + 34.8x + 1928$, where the price is in dollars and x is in thousands of units. The supply is modeled by $S(x) = 1.4x^2 - 50x + 1480$. Find both the consumer's and producer's surplus.

6.2 Problems: 1, 2, 4, 5, 11, 16, 21, 22