

Useful Identities

Properties of Logarithms

Let x , y , and a be positive real numbers, $a \neq 1$, and r any real number. Then we have:

- i) $\log_a(1) = 0$ and $\ln(1) = 0$ since $a^0 = 1$
- ii) $\log_a(a) = 1$ and $\ln(e) = 1$ since $a^1 = a$
- iii) $\log_a(a^r) = r$ and $\ln(e^r) = r$ since $(a)^r = (a^r)$
- iv) $a^{\log_a(x)} = x$ and $e^{\ln(x)} = x$
- v) $\log_a(xy) = \log_a(x) + \log_a(y)$
- vi) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
- vii) $\log_a(x^r) = r \cdot \log_a(x)$

Summary of Trig Identities

i) Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

ii) Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

iii) Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

iv) Even-Odd Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

v) Sum/Difference Formulas

$$\cos(u - v) = \cos u \cos v + \sin u \sin v \quad \cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v \quad \sin(u + v) = \sin u \cos v + \cos u \sin v$$

vi) Double-Angle Formulas

$$\sin(2u) = 2 \sin u \cos u \quad \cos(2u) = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1$$

$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u} \quad \cos(2u) = \cos^2 u - \sin^2 u = 1 - 2 \sin^2 u$$