

MATH4426  
Robert Gross  
Homework 3  
Due September 25, 2020

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EDT on September 25. Please name your file hw03-lastname-firstname.pdf. My solution file is hw03-gross-robert.pdf.

I will try to acknowledge receipt of each e-mail.

1. What is the probability that a random bridge hand contains no queens, kings, or aces?
2. Suppose that I give you a list of 15 problems, and I tell you that a random selection of 8 of them will comprise the first examination. Suppose that you decide to prepare answers for 10 of them, choosing randomly.
  - (a) What is the probability that you have prepared answers for all 8 of the problems on the exam?
  - (b) What is the probability that you have prepared answers for exactly 7 of the 8 problems?
3. Two cards are chosen randomly from a standard 52-card deck. What is the probability that they
  - (a) are both jacks?
  - (b) have the same denomination?
4. Two standard 6-sided dice have 3 sides painted red, 2 sides painted green, and 1 side painted purple. Roll the two dice. What is the probability that the dice land with the same color facing up?
5. Four red, 8 purple, and 5 green balls are arranged randomly in a line.
  - (a) What is the probability that the first 5 balls are all purple?
  - (b) What is the probability that none of the first 5 balls are purple?
  - (c) What is the probability that the last 3 balls are 3 different colors?
  - (d) What is the probability that all of the red balls are adjacent?
6. Suppose that
  - the probability that a new car needs no repairs in its first 10,000 miles of driving is 0.8,
  - the probability that a new car needs no repairs in its first 20,000 miles of driving is 0.4, and
  - the probability that a new car needs no repairs in its first 30,000 miles of driving is 0.1.Suppose that you have already driven your new car 10,000 miles and it has not needed any repairs.
  - (a) What is the probability that you can drive your car another 10,000 miles and it still will not need to be repaired?
  - (b) What is the probability that you can drive your car another 20,000 miles and it still will not need to be repaired?

7. You ask your roommate to water your houseplant while you go away for a long weekend. If your roommate does not water the plant, the probability that the plant will die is 0.8. If your roommate waters the plant, the probability that it will die is 0.05. You estimate that your roommate will remember to water the plant with probability 0.9.

- (a) What is the probability that the plant will be alive when you return from your long weekend?
- (b) Suppose that the plant is dead when you return from your long weekend. What is the probability that your roommate forgot to water it?

8. Five people,  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$ , are put in a line randomly (so that all  $5!$  permutations are equally likely).

- (a) What is the probability that  $A$  and  $B$  are adjacent?
- (b) What is the probability that there is exactly one person between  $A$  and  $B$ ?
- (c) What is the probability that there are exactly two people between  $A$  and  $B$ ?
- (d) What is the probability that there are exactly three people between  $A$  and  $B$ ?

In all of these questions, we do not care whether  $A$  is to the left or to the right of  $B$ .

9. Suppose that we choose 5 numbers from the set  $\{1, 2, \dots, 14\}$  and we put them in order. What is the probability that 9 is the middle number on the ordered list?