MATH4426
Robert Gross
Homework 4
Due October 2, 2020

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EDT on October 2. Please name your file hw04-lastname-firstname.pdf. My solution file is hw04-gross-robert.pdf.

I will try to acknowledge receipt of each e-mail.

1. Suppose that blue jeans are made at two factories, A and B. The jeans made at factory A are defective with probability 0.01, and the jeans made at factory B are defective with probability 0.005. Suppose that you buy 2 pairs of blue jeans at the same time. You assume that they came from the same factory, and that they are equally likely to have come from either factory. You look at the first pair, and it is defective. What is the probability that the second pair is also defective?

2. Suppose that 6 cards are selected randomly from a standard deck of cards. What is the probability that all 4 suits occur among those 6 cards?

3. A list of 14 students contains 3 first-year students, 4 sophomores, 4 juniors, and 3 seniors. Suppose that 4 students are chosen randomly from this list.
(a) What is the probability that there is one student from each class?
(b) What is the probability that the 4 students are 2 sophomores and 2 juniors?
(c) What is the probability that the 4 students do not include any sophomores or juniors?

4. A red die, a green die, and a purple die, all standard cubical dice, are rolled. We want the probability that the red die is less than the green die, which in turn is less than the purple die. With the obvious notation of R, P, and G for the numbers on the three dice, we want $P(R < G < P)$.
(a) What is the probability that the three dice land on 3 different numbers?
(b) Given that the three dice show 3 different numbers, what is the conditional probability that $R < G < P$?
(c) What is the unconditional probability that $R < G < P$?

5. Suppose that an urn contains 17 white and 9 black balls. A fair die is rolled, and that number of balls is removed from the urn.
(a) What is the probability that all of the balls removed from the urn are white?
(b) If all of the balls removed from the urn are white, what is the conditional probability that the die roll was a 3?

6. Prostate cancer is a common type of cancer in men. A test measuring PSA (Prostate Specific Antigen) is commonly employed but is unreliable. The probability that a noncancerous man will have an elevated PSA is approximately 0.135, and the probability that a cancerous man will have an elevated PSA is approximately 0.268. Suppose that based on family history and a physical examination, a physician is 70% certain that a man has prostate cancer, and measures his PSA.
(a) If the test indicates elevated PSA level, what is the probability that the man has cancer?

(*b*) If the test indicates normal PSA level, what is the probability that the man has cancer?

7. Suppose that E and F are any events.
(*a*) Show that $P(E|E \cup F) \geq P(E|F)$.
(*b*) Suppose that $EF = \emptyset$. Show that

$$P(E|E \cup F) = \frac{P(E)}{P(E) + P(F)}.$$