

MATH4426
Robert Gross
Homework 1
Due September 11, 2020

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EDT on September 11. I will try to acknowledge receipt of each e-mail.

1. When area codes were originally invented, they consisted of 3 digits so that:

- The first digit could not be 0 or 1.
- The second digit had to be 0 or 1.
- The third digit could not be 0.

How many area codes were possible? *Note:* It turned out that this was not enough area codes, and so some of these restrictions were removed eventually.

2. A nursery school class has 3 girls and 3 boys.

- (a) How many ways can they be put in a single row of 6 chairs?
- (b) How many ways can they be put in a single row of chairs if the boys must sit together and the girls must sit together?
- (c) How many ways can they be put in a single row of chairs if no two students of the same gender can sit next to each other?

3. Suppose that Alice, Bob, Claire, Dennis, Estelle, Fred, Gloria, and Herbert gather for a party. Assume that the party has 4 women and 4 men.

- (a) How many ways can they be lined up in a single line for a photo?
- (b) How many ways can they be lined up for a photo if Alice and Bob insist on being next to each other?
- (c) How many ways can they be lined up for a photo so that no two people of the same gender are adjacent?
- (d) Suppose that these are 4 married couples. How many ways can they be lined up for a photo if spouses are adjacent to each other?

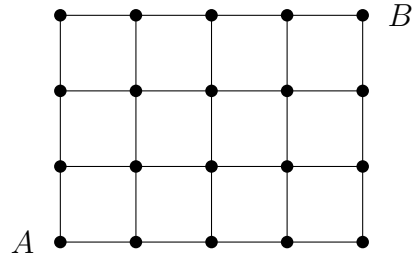
4. Suppose that a “3 digit number” means any whole number from 000 to 999.

- (a) How many 3 digit numbers are there with *at least* 2 equal digits? The equal digits need not be adjacent, so that 121 and 000 should be counted.
- (b) How many 3 digit numbers are there with *exactly* 2 equal digits? The equal digits need not be adjacent, so that 020 should be counted, but not 000.

5. Show that

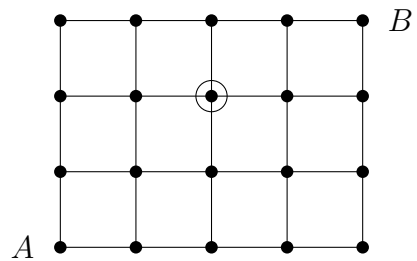
$$\sum_{k=0}^n \binom{n}{k} 2^k = 3^n.$$

6. Consider this grid:



Suppose that starting at the point A , you can go one step up or one step to the right, continuing until B is reached. How many different paths are there from A to B ? **HINT:** Notice that to go from A to B , your path must contain 4 steps to the right, and 3 steps upwards.

7. Consider this grid:



Continuing with the previous problem, how many paths are there from A to B that pass through the circled point?

8. Suppose that n , m , and r are non-negative integers.

(a) Prove that

$$\binom{n+m}{r} = \binom{n}{0} \binom{m}{r} + \binom{n}{1} \binom{m}{r-1} + \cdots + \binom{n}{r} \binom{m}{0}.$$

Hint: Consider a group of n men and m women. How many different ways are there to choose r of them?

(b) Show that

$$\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2.$$